The Optimisation and Analysis of Multi-moded Feed Horn Structures at Terahertz Frequencies

D. McCarthy, N. Trappe, J. A. Murphy, M. Gradziel, C. O'Sullivan, and S. Doherty
Department of Experimental Physics, Maynooth University, Ireland

Abstract— Carrying out astronomical observations at far-infrared wavelengths is critical in enabling further progress in the fields of cosmology and astrophysics. Such observations will allow additional insight into the birth and evolution of the Universe. To allow progress in these areas, it is necessary to improve the sensitivity and resolution of the instrumentation that is used to carry out these observations.

At the high frequencies in question (terahertz), the instruments typically make use of horn antenna fed detector systems. To achieve the required performance, the horn designs must be highly optimised. Full electromagnetic solvers (CST, HFSS, COMSOL etc.) struggle to predict the performance of horn antennas at such high frequencies in a timely manner due to the large electrical size of the structures. It is therefore very challenging to perform the optimisation using such solvers particularly for multi-mode systems where each mode would have to be considered individually.

In this paper we outline an alternative technique for modelling multi-mode (partially coherent) horn antennas based on the mode-matching technique, which allows electrically large structures to be modelled in a highly efficient manner. This technique returns a set of scattering matrices which gives a full vector definition of the transmission and re ection characteristics of the resulting design at a given frequency. We demonstrate how this can be used to extract field patterns and other figures of merit that are important for evaluating the electromagnetic performance of a horn design.

An efficient genetic algorithm based optimisation technique (using mode-matching) is also presented. The optimisation process is based on a piecewise conical profile horn design and produces a geometry that is optimised with respect to some figure of merit that is of critical importance for the application in question. This allows the instrument to realise the high levels of optical performance that are required for astronomical applications.

1. INTRODUCTION

In order to further progress in the fields of cosmology and astrophysics, it is necessary to carry out astronomical observations at far-infrared wavelengths. Observations carried out at these wavelengths will allow insight into the birth and conditions necessary for the evolution of galaxies, stars, planetary systems and the evolution of matter. Such observations will allow further constraints to be placed on the theories and models that govern the origin and evolution of the Universe. In order to make progress in these areas, it is necessary to improve the sensitivity and resolution of the instruments that are being used.

One method that is used to increase the sensitivity is to make use of multi-moded horns. These horns provide additional channels of power relative to their single-modes counterparts, resulting in increased throughput to the incoherent detectors that they feed, which results in increased sensitivity. Since in incoherent systems these modes will couple independently to the detector, a higher level of control over the beam definition is possible, resulting in an increase in the packing factor when arrays of such detector pixels are used, which will further increase sensitivity. Multi-moded horns have already been used on the European Space Agency Planck satellite, allowing the mapping of the Cosmic Microwave Background radiation with hitherto unseen levels of sensitivity and angular resolution [1]. The horns were used on the HFI instrument [2, 3] for the 545 GHz and 857 GHz channels. These channels were used to remove foreground sources from the data, and were an excellent example of how such horns can be used to retain high levels of sensitivity, resolution and beam pattern control. In terms of application to far infrared missions, multi-mode horn antennas are being investigated as candidates to feed transition edge sensor based detector arrays, for example in the case of the SAFARI instrument on board the proposed SPICA satellite [4]. In this case, smooth-walled profiled horns are being examined as the feeds.

At the high frequencies in question for far infrared missions, the horn antennas must be carefully optimised in order to comply with the stringent optical performance requirements that are enforced.

A full electromagnetic analysis can be carried out using solvers such as CST, HFSS, COMSOL etc., however such structures tend to be electrically large at these frequencies, and so these solvers will struggle to predict their performance in a timely manner. The mode-matching technique offers an alternative method for modelling waveguide structures, and allows electrically large multi-moded structures like those in question to be modelled in a more efficient manner than is typically encountered when using full electromagnetic solvers.

2. ELECTROMAGNETIC MODELLING OF MULTI-MODED STRUCTURES — THE MODE-MATCHING TECHNIQUE

The mode-matching technique [5] efficiently implements a scattering matrix technique which gives a full vector definition of the transmission and reflection characteristics of a waveguide structure at a given frequency for each mode at the input port of the system. In this approach (where we assume a waveguide structure of cylindrical geometry), the structure is split into sections of varying radii such that when placed together, return the original structure. The mode-matching approach works by starting at one end of the horn, typically the throat or narrower end, considering the first two sections and then working through the entire structure considering all consecutive sections such that each junction between adjacent sections is considered.

In any given step of the process, two arbitrary adjacent sections are considered, as shown in Figure 1, where the field is nominally propagating either from a narrower waveguide to a larger waveguide (depicted on the left of Figure 1), or vice versa. The fields propagating in the sections either side of a given junction are represented using circular transverse electric (TE) and transverse magnetic (TM) modes. In a given section, the distribution of power across the modes that are supported in the section (based on the radius of the section and the frequency of the radiation being used as the source) is fixed, with the only change being the phase of the modes as they propagate through the section. At the junction between two adjacent sections, there is change in radius and it is this change that results in power scattering between the supported modes, in both the forward and backward propagating directions.

The mode-matching technique calculates this inter-modal scattering by matching the transverse fields across the junction in both directions of propagation and by applying the usual boundary conditions at the waveguide boundaries [5]. If $[A]$ and $[C]$ are vectors containing the mode coefficients for the input TE and TM modes to the sections either side of the junction, representing the forward and backwards propagating modes respectively, then they can be related to the vectors $[B]$ and $[D]$ (containing the mode coefficients of the output modes on the input and output sides of the junction respectively) by the scattering matrix S according to

$$\begin{bmatrix}[B]\\ [D]\end{bmatrix} = [S]\begin{bmatrix}[A]\\ [C]\end{bmatrix} = \begin{bmatrix}[S_{11}] & [S_{12}]\\ [S_{21}] & [S_{22}]\end{bmatrix}\begin{bmatrix}[A]\\ [C]\end{bmatrix}. \tag{1}$$

The sub-matrices of the S matrix, S_{mn}, are defined such that they give the amount of each waveguide mode arriving at port m, relative to the amount of the mode excited at port n. For example, assuming that the input of the horn (the throat) is port one, then the S_{21} matrix represents the amount of each mode arriving at the horn aperture relative to the amount by which the mode was exited at the throat, which we assume to be unity power. By calculating the scattering matrices for each junction, it is possible to cascade them in sequence [6] in order to find the set of S matrices that determine the behaviour of the bulk waveguide structure, allowing the transmitted and returned modal distributions obtained by illuminating the structure with a single frequency to be evaluated.

Typically for single or few-moded structures, it is only necessary to consider modes within the $n = 1$ azimuthal order. This results in a relatively quick calculation, with the modes adding in a partially incoherent manner. When dealing with multi-moded structures, the situation becomes more complex. in the case where the waveguide structure is of cylindrical geometry (which this

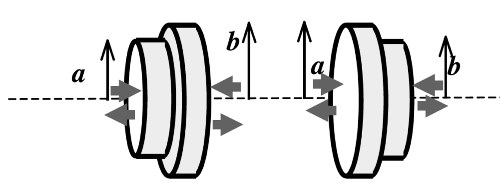

Figure 1: Two adjacent sections of waveguide, as considered in the mode-matching technique, with $a < b$ and $a > b$. Taken from [6].

paper deals with), the different azimuthal orders (i.e., modes with different values of n) are independent and so no intermodal scattering occurs between modes with different values for n. Such modes are said to be incoherent. This results due to certain symmetries that exist between modes in which n is different [3]. Of course, the overall field distribution of the structure consists of contributions from all of the modes present, and so to find the composite field, the fields due to the individual azimuthal orders must be added correctly in quadrature.

In order to achieve this, since all azimuthal orders are independent, the waveguide structures are considered one azimuthal order at the time, and so from this the modal composition of the field distribution of the waveguide structure will be known for a given value of n. These mode coefficients are then used to expand the appropriate basis set which gives rise to the field distribution that is required (Bessel functions for the aperture field and their Fourier Transformed equivalents for the farfield [6]. The field due to each individual azimuthal order is then calculated using a partially coherent summation as in the single-moded case, however the field patterns due to the different azimuthal orders are then added together incoherently to form the composite beam. This effectively means that there is no phase interaction between independent modes, and so they do not mix.

The mode-matching technique assumes a fixed number of modes throughout the entire calculation, typically the number of modes needed to model the largest section at the frequency in question, plus several additional modes to account for evanescent modes. When it comes to carrying out post-processing on the resulting scattering matrices, it is found that many modes contribute very little to the overall performance, if anything at all. Although these modes do not contribute to the overall performance of the horn, the algorithm will still calculate this null contribution, which is clearly time consuming. This is especially true of multi-moded horns were higher azimuthal orders are considered and will increase the computational time significantly. For this reason, it is desirable to make the simulation process more efficient. This can be easily accomplished by using the method of singular value decomposition (SVD) [7].

If $\mathbf{S}$ is an $m \times n$ matrix, in this context the S_{21} matrix of the horn, then the SVD of $\mathbf{S}$ is defined as

$$\mathbf{S} = \mathbf{U}.\mathbf{\Sigma} \cdot \mathbf{V}^{\dagger}, \tag{2}$$

where $\mathbf{U}$ is an $m \times m$ unitary matrix, $\mathbf{\Sigma}$ is an $m \times n$ diagonal matrix with non-negative real numbers on the diagonal. and $\mathbf{V}^{\dagger}$, the complex transpose of $\mathbf{V}$, is an $n \times n$ unitary matrix. The diagonal entries of $\mathbf{\Sigma}$, Σ_{ii}, are known as the singular values of $\mathbf{S}$, hereafter represented by σ_i, where $\sigma_i = \Sigma_{ii}$, and are arranged in order of decreasing value. It is usual that only the first β diagonal elements are non-zero. The columns of $\mathbf{V}$ form a basis set for the input modes of the system and the columns of $\mathbf{U}$ form a basis set for the output modes of the system.

Owing to the diagonal nature of σ, only input modes (columns) with a non-zero singular value will propagate to the output of the system, with the appropriate singular value giving the amplitude factor by which the mode is transmitted to the output. Given the new basis set for the output modes (the columns of $\mathbf{U}$) and the corresponding singular value, the field at the output of the system, for example at the output of a multi-moded horn, can be easily reconstructed. To do this, the fields corresponding to each of the individual "singular modes" are added in quadrature, even if the modes originate within the sane azimuthal order, as each singular mode is mathematically independent and so the modes are incoherent.

This new basis set represents all of the information contained within the original scattering matrix in a much more compact form, allowing the fields to be reconstructed in a much more timely manner. This is due to the fact that the number of non-zero singular values is typically significantly less than the number of modes considered in the calculation. For example, in a single-moded system which considers m modes throughout the scattering matrix calculation, one would traditionally calculate the field using an $m \times m$ matrix, but by using SVD this matrix is replaced by a $m \times 1$ matrix which significantly reduces the computational time [8]. This also applies to multi-moded systems, whereby in the traditional approach to field calculations one would need to consider n $m \times m$ matrices for a system that uses n azimuthal orders. Using the SVD approach, only the azimuthal orders contributing useful power would be considered. In the worst case scenario (computationally), all azimuthal would be considered, but in each case only the modes with non-zero singular values would be retained, which would mean that the size of the matrix representing each azimuthal order would be significantly reduced, resulting in a significant reduction in the computational time for the radiation patterns of multi-moded horn antennas.

3. OPTIMISATION OF MULTI-MODED STRUCTURES

The above approach to the calculation of field patterns for multi-moded horns can be applied to the optimisation of these structures, as often the figure of merit that is being optimised with respect to is derived from the analysis of several field patterns. For example, if a horn were being designed to realise maximum beam symmetry in the farfield, then each horn design that was tested (i.e., each iteration of the algorithm) would require the calculation of two orthogonal cuts of the farfield in order to calculate the symmetry of the field pattern. Considering the fact that most optimisation algorithms will require many iterations to converge to a solution, this is a time consuming process even in the single-moded case, unless SVD is used. The use of SVD can reduce the time required to converge to a solution to a matter of hours [8]. When few or multi-moded designs (for example Winston Cones [10]) are being considered and several scattering matrices are required when calculating field patterns, the advantages of using SVD as a part of this process become even more apparent, as multi-moded optimisations carried out without using this technique have been found to run out of memory prior to obtaining a solution.

It is necessary to combine this efficient approach to modelling multi-moded waveguide structures with an efficient optimisation algorithm. The algorithm must also be able to deal with multi-variable stochastic functions, as antenna optimisation problems tend to be of this nature, with several input variables defining the geometry of the horn and the figure of merit function altering unpredictably with these variables. The genetic algorithm [9] satisfies these requirements and provides an efficient, robust optimisation algorithm. It is based on the theory of evolution and iterates towards a global minimum.

By specifying the figure of merit in such a way that minimising it results in optimal horn performance, the ideal horn design can be found. The genetic algorithm also allows users to specify the allowed range for each of the optimisation variables. Since these correspond ultimately to the physical parameters of the horn either directly or indirectly (for example overall length, aperture radius, severity of any profiling being used etc.) this allows the overall minimum or maximum dimensions of the horn or portions thereof, to be specified. This is useful if for example the horn is being used in a confined space, such as a cryostat or in a focal plane array, as the size of the horn can be constrained. It is particularly useful to be able to specify the allowed ranges for the horn geometry at various points in multi-moded antennas, as the geometry (radii, step size between sections) determines the modal content of the horn and the relative amplitudes of the modes, which determines the performance of the horn antenna. This is critical in terms of determining horn performance, and so being able to specify limits for the horn geometry assists the optimisation process by bounding the problem and so reduces the execution time and ensures that the solution space being investigated is the desired one.

The method has already been successfully demonstrated for a single-mode, smooth-walled conical profile horn, operating at 100 GHz. The optimisation process returned a horn geometry which was optimised with respect to minimising the maximum cross-polar level. The optimiser achieved this by minimising the impedance mismatch between the horn and free space, with the resulting decrease in re ections giving rise to low levels of return loss, low sidelobe levels and a high degree of symmetry in the co-polar beam. As this horn was being designed for use in future CMB missions, these were the performance metrics that were of interest and so the horn met the performance requirements. The simulated results were further verified in CST. The horn was manufactured and measured using a vector network analyser based testbed, with the measurements agreeing very well with simulation, showing that the horn was able to largely meet the performance requirements, despite being of a simple design. A more complete description can be found in [11].

4. CONCLUSION

Observations in the far-infrared will require greater sensitivity in the future, with multi-moded detection systems allowing this due to the additional channels of power that are available in such systems. Pixels for these systems will likely make use of multi-moded feed horn antennas, and so the design and optimisation of such structures is a relevant problem. In this paper, an efficient method for simulating multi-moded waveguide structures based on the mode-matching technique was described. A modification to the traditional field pattern calculation method was also described, which brings significant time savings to the calculation, particularly when multi-moded structures are being considered, allowing simulations of these structures to be carried out in a relatively short amount of time using standard desktop computing power. This has large implications for the optimisation of such structures which typically involve thousands of iterations of this code, as demonstrated by its application to single-moded systems. A description of the optimisation technique that would be used was also given, illustrating how an efficient modelling technique can provide excellent results for multi-moded structures when coupled with an efficient optimisation algorithm.

ACKNOWLEDGMENT

The authors wish to acknowledge the financial support of the European Space Agency under the Irish Announcement of Opportunity scheme and the Irish Research Council EMBARK initiative.

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