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On Estimating Maximum Sum Rate of MIMO Systems with Successive Zero-Forcing Dirty Paper Coding and Per-antenna Power Constraint

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***Abstract*—In this paper, we study the sum rate maximization for a multiple-input multiple-output (MIMO) system with successive zero-forcing dirty-paper coding (SZFDPC) and per-antenna power constraint (PAPC). Although SZFDPC is a low-complexity alternative to the optimal dirty paper coding, efficient algorithms to compute its sum rate are still open problems especially under practical PAPC. The existing solution to the considered problem is computationally inefficient due to employing high-complexity interior-point method. In this study, we propose two novel low-complexity approaches to this important problem. More specifically, the first algorithm achieves the optimal solution by transforming the original problem in the broadcast channel into an equivalent problem in the multiple access channel, then the resulting problem is solved by alternating optimization together with successive convex approximation. We also derive a suboptimal solution based on machine learning to which simple linear regressions are applicable. The approaches are analyzed and validated extensively to demonstrate their superiors over the existing approach.**

***Index Terms*—MIMO, alternating optimization, machine learning, successive zero-forcing dirty-paper coding, per-antenna power constraint, regression, successive convex approximation.**

I. Introduction

The preliminary studies on multiple-input multiple-output (MIMO) capacity showed that single-user MIMO capacity can be achieved by Gaussian input signaling [1], [2]. In [3], Weingarten *et al.* further proved that the entire capacity region of a Gaussian MIMO broadcast channel (BC) is achievable through optimal dirty paper coding method (DPC). In reality, the optimal precoding method is not only high-complexity but also difficult to implement. As a result, there are growing interests in suboptimal precoding techniques such as zero-forcing (ZF) and successive zero-forcing dirty paper coding (SZFDPC) [4]–[9].

Traditionally, the majority of the research on MIMO capacity assumes a sum power constraint (SPC) for which efficient algorithms can be derived [1], [2], [10]–[12]. Nevertheless, solutions to SPC problems may result in nonlinear distortions if the allocated power is beyond the power limit of one or several power amplifiers. Towards this end, per-antenna power constraint (PAPC) is more realistic and of particular interest [13]–[17].

To the best of authors' knowledge, only Tran *et al.* characterized the achievable rate region of SZFDPC under PAPC by customized interior-point method [8], [9]. However, this second-order-based method is not attractive to large-scale MIMO systems due to high complexity. In this study, we propose two approaches to obtain sum rates of MIMO systems under PAPC and SZFDPC. The first approach achieves the optimal solution by alternating optimization (AO) while the second one exploits machine learning (ML) to arrive at a suboptimal solution. In particular, our contributions include the following:

- A novel AO-based algorithm is proposed to obtain the optimal solution. Specifically, the original maximization in the BC is transformed into an equivalent minimax problem in the multiple access channel (MAC), then an efficient iterative algorithm is derived based on successive convex approximation (SCA).
- In case the optimal approach is feasible but inefficient, an ML-based approach, which trades off the complexity and the optimal solution, is a good alternative. In fact, our ML approach relies on linear regressions and thus is appealing to applications such as massive MIMO.
- We report for the first time the comparison of SZFDPC and others precoding methods under PAPC. Moreover, our ML-based approach can be applicable to similar capacity-related problems.

The remainder of the paper is organized as follows. A description of the system model is in Section II. The methods of computing the sum rate for SZFDPC under PAPC are described in Section III followed by the numerical results in Section IV. Finally, we conclude the paper in Section V.

Notation: Standard notations are used in this paper. Bold lower and upper case letters represent vectors and matrices, respectively. $\mathbf{I}$ defines an identity matrix, of which the size can be easily inferred from the context; $\mathbb{C}^{M\times N}$ denotes the space of $M \times N$ complex matrices; $\mathbf{H}^{\dagger}$ and $\mathbf{H}^{T}$ are Hermitian and normal transpose of $\mathbf{H}$, respectively; $|\mathbf{H}|$ is the determinant of $\mathbf{H}$; $\mathrm{null}(\mathbf{H})$ stands for a basis of the null space of $\mathbf{H}$; $|\mathbf{x}|$ denotes the absolute value of $\mathbf{x}$; $\mathrm{diag}(\mathbf{H})$, where $\mathbf{H}$ is a square matrix, returns the vector of diagonal elements of $\mathbf{H}$ and $\mathrm{diag}(\mathbf{x})$ denotes a diagonal matrix with the diagonal being $\mathbf{x}$. Furthermore, we denote the Euclidean norm by $||\cdot||$ and $[x]_+ = \max(x, 0)$.

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II. System Model

Consider a MIMO BC consisting of a base station (BS) and K users. The BS and each user k are equipped with N and M antennas, respectively. The channel matrix for user k is denoted by $\mathbf{H}_k \in \mathbb{C}^{M\times N}$. Normally, a user suffers interference from all other users in the system. For user k in the SZFDPC scheme, the interference caused by users $j<k$ is canceled by DPC, while that caused by users $j>k$ is nulled out by zero-forcing technique. In this way, a MIMO BC can be decomposed into parallel interference-free channels. We refer the interested reader to [8] and references therein for a more detailed description of the SZFDPC scheme.

The sum rate of SZFDPC can be characterized through solving the sum rate (SRMax) problem under PAPC which is formulated as

$$\underset{\{\mathbf{S}_k\succeq\mathbf{0}\}}{\text{maximize}} \quad \textstyle\sum_{k=1}^{K}\log|\mathbf{I}+\mathbf{H}_k\mathbf{S}_k\mathbf{H}_k^{\dagger}| \tag{1a}$$

$$\text{subject to} \quad \mathbf{H}_j\mathbf{S}_k\mathbf{H}_j^{\dagger}=0,\ \forall j<k \tag{1b}$$

$$\textstyle\sum_{k=1}^{K}[\mathbf{S}_k]_{i,i}\le P_i,\ i=1,2,\ldots,N \tag{1c}$$

where $\mathbf{S}_k\succeq\mathbf{0}$ is the input covariance matrix for user k and P_i is the power constraint of the amplifier associated with the ith transmit antenna. The constraint in (1b) is imposed to suppress the interference from users $j<k$ as mentioned above.

Due to the use of zero-forcing method, SZFDPC is a suboptimal transmission strategy compared to DPC. However, SZFDPC does not cancel multiuser inference only by zero-forcing technique since DPC is still invoked for this purpose. Thus, SZFDPC can achieve a performance close to that of DPC, which was reported in various previous studies [8], [9], [18]. We note that for SZFDPC (i.e. (1)) to be feasible, it should hold that $N>(K-1)M$ which is assumed in this paper. This condition in fact imposes a constraint on the maximum number of users that can be supported simultaneously. When the number of demanding users increases, a user scheduling algorithm is required and this problem was studied in [18] where several efficient user selection methods were proposed for SZFDPC. We also remark that the interference cancelling process is performed sequentially after each user, and thus user ordering in SZFDPC is important. Optimal user ordering requires solving a combinatorial optimization problem but efficient user order algorithms were also proposed in [18]. In this paper we simply assume the natural user ordering for SZFDPC so that we can focus on the precoder design.

In order to simplify the formulation in (1), let $\breve{\mathbf{H}}_k=[\mathbf{H}_1^{\dagger},\mathbf{H}_2^{\dagger},\ldots\mathbf{H}_{k-1}^{\dagger}]^{\dagger}$, $\breve{\mathbf{V}}_k=\text{null}(\breve{\mathbf{H}}_k)$, and $\dot{\mathbf{H}}_k=\mathbf{H}_k\breve{\mathbf{V}}_k$. Intuitively, $\dot{\mathbf{H}}_k$ is called the effective channel of user k. The optimal $\mathbf{S}_k$ in (1) is then given by $\mathbf{S}_k=\breve{\mathbf{V}}_k\dot{\mathbf{S}}_k\breve{\mathbf{V}}_k^{\dagger}$, where $\dot{\mathbf{S}}_k$ is the optimal solution to the following problem

$$\begin{array}{ll}\underset{\{\dot{\mathbf{S}}_k\succeq\mathbf{0}\}}{\text{maximize}} & \sum_{k=1}^{K}\log|\mathbf{I}+\dot{\mathbf{H}}_k\dot{\mathbf{S}}_k\dot{\mathbf{H}}_k^{\dagger}| \\ \text{subject to} & \sum_{k=1}^{K}[\breve{\mathbf{V}}_k\dot{\mathbf{S}}_k\breve{\mathbf{V}}_k^{\dagger}]_{i,i}\le P_i,\ \forall i.\end{array} \tag{2}$$

III. Proposed Algorithms

A. Alternating Optimization

Inspired by the work in [7], we extend the AO approach to our considered problem. More specifically, by extending Theorem 2 of [9], we can show that (2) can be equivalently transformed into the following minimax problem in the dual MAC

$$\begin{array}{ll}\underset{\mathbf{Q}\succeq\mathbf{0}}{\min}\ \underset{\{\bar{\mathbf{S}}_k\succeq\mathbf{0}\}}{\max} & \sum_{k=1}^{K}\log\frac{|\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}\breve{\mathbf{V}}_k+\dot{\mathbf{H}}_k^{\dagger}\bar{\mathbf{S}}_k\dot{\mathbf{H}}_k|}{|\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}\breve{\mathbf{V}}_k|} \\ \text{subject to} & \sum_{k=1}^{K}\text{tr}(\bar{\mathbf{S}}_k)=P \\ & \text{tr}(\mathbf{QP})=P,\mathbf{Q}:\text{diagonal}\end{array} \tag{3}$$

where $P=\text{tr}(\mathbf{P})$, $\mathbf{P}=\text{diag}([P_1,P_2,\ldots,P_N]^T)$. The relationship between optimal solutions of (2) and (3) is given by

$$\dot{\mathbf{S}}_k=(\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}\breve{\mathbf{V}}_k)^{-1/2}\mathbf{U}\mathbf{V}^{\dagger}\bar{\mathbf{S}}_k\mathbf{V}\mathbf{U}^{\dagger}(\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}\breve{\mathbf{V}}_k)^{-1/2} \tag{4}$$

where $\mathbf{U},\mathbf{V}$ are obtained from the singular value decomposition of $(\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}\breve{\mathbf{V}}_k)^{-1/2}\dot{\mathbf{H}}_k^{\dagger}$ [19]. In light of AO algorithm in [7], [15], [16], we first fix $\mathbf{Q}$ and consider the following problem

$$\begin{array}{ll}\text{maximize} & \sum_{k=1}^{K}\log|\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}^n\breve{\mathbf{V}}_k+\dot{\mathbf{H}}_k^{\dagger}\bar{\mathbf{S}}_k\dot{\mathbf{H}}_k| \\ \text{subject to} & \sum_{k=1}^{K}\text{tr}(\bar{\mathbf{S}}_k)=P;\{\bar{\mathbf{S}}_k\succeq\mathbf{0}\}.\end{array} \tag{5}$$

Problem (5) is the one of finding the capacity of parallel interference-free MIMO channels under a sum power constraint, which adopts the classical water-filling solution.

We now consider the problem of finding $\mathbf{Q}$ for given $\{\bar{\mathbf{S}}_k^n\}$. Recall that SCA-based methods are not only applicable to nonconvex problems but also convex problems [7], [17], we can therefore take advantages of this approach to solve the considered minimization problem. To this end, we apply the following logdet inequality [7, Eq. (7)]:

$$\log|\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}\breve{\mathbf{V}}_k+\dot{\mathbf{H}}_k^{\dagger}\bar{\mathbf{S}}_k^n\dot{\mathbf{H}}_k|\le\log|\mathbf{\Phi}_k^n|+\text{tr}\Big(\breve{\mathbf{V}}_k\mathbf{\Phi}_k^{-n}\breve{\mathbf{V}}_k^{\dagger}\big(\mathbf{Q}-\mathbf{Q}^n\big)\Big) \tag{6}$$

where $\mathbf{\Phi}_k^n\triangleq\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}^n\breve{\mathbf{V}}_k+\dot{\mathbf{H}}_k^{\dagger}\bar{\mathbf{S}}_k^n\dot{\mathbf{H}}_k$, and $\mathbf{\Phi}_k^{-n}$ stands for $\big(\mathbf{\Phi}_k^n\big)^{-1}$. As a result of this inequality, $\mathbf{Q}^{n+1}$ is the solution to the following problem

$$\begin{array}{ll}\min & \sum_{k=1}^{K}\text{tr}\big(\breve{\mathbf{V}}_k\mathbf{\Phi}_k^{-n}\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}\big)-\log|\breve{\mathbf{V}}_k^{\dagger}\mathbf{Q}\breve{\mathbf{V}}_k| \\ \text{s.t.} & \text{tr}(\mathbf{QP})=P,\mathbf{Q}:\text{diagonal};\mathbf{Q}\succeq\mathbf{0}.\end{array} \tag{7}$$

Note that SCA-based methods generally result in a local minimum for nonconvex problems. In our case, the original problem is convex and thus the local minimum is also the global minimum. Since $\mathbf{Q}$ is diagonal, (7) indeed reduces to

$$\begin{array}{ll}\min & \boldsymbol{\alpha}^T\mathbf{q}-\sum_{k=1}^{K}\log|\breve{\mathbf{V}}_k^{\dagger}\,\text{diag}(\mathbf{q})\breve{\mathbf{V}}_k| \\ \text{s.t.} & \mathbf{p}^T\mathbf{q}=P\end{array} \tag{8}$$

where $\boldsymbol{\alpha}=\sum_{k=1}^{K}\big(\text{diag}(\breve{\mathbf{V}}_k\mathbf{\Phi}_k^{-n}\breve{\mathbf{V}}_k^{\dagger})\big)$, $\mathbf{p}=\text{diag}(\mathbf{P})$.

It's worth noting that the feasible set of (8) is in fact a simplex. As shown shortly, projection onto a simplex can be done efficiently by closed-form expressions and this motivates

Algorithm 1: The Proposed GP Algorithm for Solving (8).

Input: $\mathbf{p}$, $\epsilon > 0$
1 Initialization: $\tau = 1 + \epsilon$, $m = 0$, $\mathbf{q}_0 = \mathbf{1}_N^T$.
2 **while** $\tau > \epsilon$ **do**
3 Calculate the gradient $\tilde{\mathbf{g}}_m = \nabla f(\mathbf{q}_m) = \boldsymbol{\alpha} - \sum_{k=1}^{K} \operatorname{diag}(\check{\mathbf{V}}_k^\dagger (\check{\mathbf{V}}_k \operatorname{diag}(\mathbf{q}_m) \check{\mathbf{V}}_k^\dagger)^{-1} \check{\mathbf{V}}_k)$.
4 Choose an appropriate scalar $s_m > 0$ and create $\tilde{\mathbf{q}}_m = \mathbf{q}_m - s_m \tilde{\mathbf{g}}_m$.
5 Project $\tilde{\mathbf{q}}_m$ onto $\mathcal{Q}_q = \{\mathbf{p}^T\mathbf{q} = P, \mathbf{q} \geq \mathbf{0}\}$ to obtain $\bar{\mathbf{q}}_m$.
6 Choose appropriate stepsize β_m and set $\mathbf{q}_{m+1} = \mathbf{q}_m + \beta_m(\bar{\mathbf{q}}_m - \mathbf{q}_m)$ using the Armijo rule [20].
7 $\tau = |\nabla f(\mathbf{q}_m)^T(\mathbf{q}_{m+1} - \mathbf{q}_m)|$.
8 $m := m + 1$.
9 **end**
Output: $\mathbf{q}^m$ as the optimal solution to (8).

Algorithm 2: Proposed Algorithm for SRMax Problem with SZFDPC and PAPC based on AO.

Input: $\mathbf{Q} := \mathbf{Q}^0$ diagonal matrix of positive elements, $\epsilon > 0$
Initialization: Set $n := 0$ and $\tau = 1 + \epsilon$, $\check{\mathbf{H}}_1 = \mathbf{H}_1$, and $\check{\mathbf{V}}_1 = \mathbf{I}$. For each $k \geq 2$, create $\check{\mathbf{H}}_k = [\mathbf{H}_1^\dagger, \mathbf{H}_2^\dagger, \dots \mathbf{H}_{k-1}^\dagger]^\dagger$, $\check{\mathbf{V}}_k = \operatorname{null}(\check{\mathbf{H}}_k)$, and $\dot{\mathbf{H}}_k = \mathbf{H}_k \check{\mathbf{V}}_k$.
while $\tau > \epsilon$ **do**
Apply the water-filling algorithm to obtain the optimal solution $\{\bar{\mathbf{S}}_k^n\}$ of (5).
If $n \geq 1$, let $\tau = |f^{\text{SZF-DPC}}(\mathbf{Q}^n, \{\bar{\mathbf{S}}_k^n\}) - f^{\text{SZF-DPC}}(\mathbf{Q}^{n-1}, \{\bar{\mathbf{S}}_k^{n-1}\})|$ where $f^{\text{SZF-DPC}}(.)$ denotes the objective in (3).
For each k, set $\boldsymbol{\Phi}_k^n = (\check{\mathbf{V}}_k^\dagger \mathbf{Q}^n \check{\mathbf{V}}_k + \dot{\mathbf{H}}_k^\dagger \bar{\mathbf{S}}_k^n \dot{\mathbf{H}}_k)$.
Find $\mathbf{Q}^{n+1}$ using Algorithm 1.
$n := n + 1$.
end
Output: $\{\bar{\mathbf{S}}_k^n\}_{k=1}^K$ and compute optimal $\{\dot{\mathbf{S}}_k^n\}_{k=1}^K$ using the BC-MAC transformation.

us to solve (8) by a gradient projection (GP) method, which is outlined in Algorithm 1.

In Algorithm 1 the subscript m denotes the iteration index and $\tilde{\mathbf{g}}_m$ is the gradient of the objective at iteration m computed as in line 3. Projection of $\tilde{\mathbf{q}}_m$ onto the feasible set of (8) is equivalent to solving the following problem

$$\begin{array}{ll} \text{minimize} & \frac{1}{2}||\mathbf{q} - \tilde{\mathbf{q}}_m||^2 \\ \text{subject to} & \mathbf{p}^T\mathbf{q} = P; \mathbf{q} \geq 0. \end{array} \tag{9}$$

This optimization problem can be solved efficiently by a water-filling-like algorithm. Specifically, the partial Lagrangian function of (9) is written as

$$\mathcal{L}(\mathbf{q}, \boldsymbol{\gamma}) = \frac{1}{2}||\mathbf{q} - \tilde{\mathbf{q}}_m||^2 + \gamma(\mathbf{p}^T\mathbf{q} - P). \tag{10}$$

For a given γ, it is easy to see that the optimal solution to $\max_{\mathbf{q} \geq 0} \mathcal{L}(\mathbf{q}, \boldsymbol{\gamma})$ is given by

$$\mathbf{q}^* = [\tilde{\mathbf{q}}_m - \gamma\mathbf{p}]_+. \tag{11}$$

The optimal γ such that $\mathbf{p}^T\mathbf{q}^* = P$ can be simply found by bisection method. Note that the feasible set for the case of the equal PAPC, i.e., $P_i = P/N$, for all $i = 1, 2, \dots, N$, becomes a canonical simplex for which more efficient projection algorithms are available [21]. The overall algorithm to solve the SRMax problem of SZFDPC under PAPC is summarized in Algorithm 2. Note that at convergence, the algorithm converges to the global optimum of the considered problem [7]. Since the convergence proof of Algorithm 2 is similar to those in [7, Appendix B], we refer interested readers to [7, Appendix B] for the details.

B. A Feature Design-based Approach

Following similar arguments to those in [7, Subsection III-B], the interior-point-based approach to solve the considered problem has the per-iteration complexity up to $\mathcal{O}(K^3N^6)$ while that of Algorithm 2 is $\mathcal{O}(KN^3)$ flops. On the one hand, Algorithm 2 dominates the existing approach and reduces the complexity significantly, but on the other hand, it still experiences high complexity in case of massive MIMO settings where $\frac{N}{K} \geq 10$. In such cases, we can employ the following ML approach to obtain a suboptimal solution since this approach can adapt quickly to any changes in the systems while retaining the satisfactory performance. Regarding ML-based methods, it is also worth mentioning that deep learning has been applied recently to the relevant problems [22], [23]. However, the performance of the deep learning-based methods depends heavily on the choice of the number of hidden layers as well as the number of neurons in each layer. More importantly, the tuning of the hyperparameters is difficult. Instead, we will show shortly that we can find an appropriate estimator using simple linear regression methods which are not only tractable but also easy to implement and analyze.

Assuming that we execute Algorithm 2 to generate optimal sum rates $\mathbf{y}$ based on $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_s] \in \mathbb{C}^{p \times s}$ inputs where s is the number of samples. Note that $\mathbf{x}_i$ contains p features of the power constraints and channel coefficients. In other words, we stack the power constraints and the channel coefficients of all users into a vector, i.e., $p = N + KMN$. If we simply apply arbitrary ML algorithms, the errors will be extremely prohibitive due to the fact that the considered problem is nonlinear in nature with respect to either the power constraint or the channel matrix (c.f. Fig. 3). On the other hand, nonlinear ML algorithms are much more difficult to investigate since there are no available solutions to this type of optimization. Even the optimal solution mentioned above already contains many nonlinear terms. In the following, we propose a novel two-step preprocessing method to transform the inputs into another feature space to which linear regression algorithms are applicable. Herein, we will refer to this approach as feature design (FD)-based approach.

Step 1: Select a set of features $\check{\mathbf{x}}$ by customizing the principle component analysis (PCA)-based algorithm in [24]:
- Choose the number of eigenvectors whose eigenvalues are larger than 1 .
- Select the features based on l largest contribution

Step 2: Transform $\check{\mathbf{x}}$ into higher feature space by $\boldsymbol{\phi}(\check{\mathbf{x}}) = [1, \log_b(|\check{\mathbf{x}}|)]^T$.

Note that instead of choosing a number of largest eigenvalues of the covariance matrix randomly [24], we empirically choose d eigenvalues which are larger than 1. As a result, we can form a new matrix $\tilde{\mathbf{U}} = [\tilde{\mathbf{u}}_1, \tilde{\mathbf{u}}_2, \ldots, \tilde{\mathbf{u}}_d]$ corresponding to those eigenvalues. To select the most dominant features, we first calculate the contribution measure

$$\vartheta_i = \sum_{j=1}^{d} |\tilde{u}_{i,j}| \tag{12}$$

where $i = 1, 2, \ldots, p$. Then we select the desired features with respect to l largest contribution ϑ_i. Again, we avoid random selection of l whose appropriate value is not easy to justify in practice. Instead, we propose to choose l based on the matrix size and the number of users:

$$l = N + Kr \tag{13}$$

where N and K are the number of transmit antennas and the number of users, respectively; $r = \min(M, N)$ where M is the number of receive antennas. Note that $l < p$ from (13) and we can therefore obtain a new matrix with reduced dimension $\check{\mathbf{X}} = [\check{\mathbf{x}}_1, \check{\mathbf{x}}_2, \ldots, \check{\mathbf{x}}_s] \in \mathbb{C}^{l \times s}$.

In fact there are no criteria to choose a function to transform the inputs into another space where efficient algorithms can be derived. In our approach, we rely on the characteristics of the problem to propose a transform function. Specifically, recall that the considered sum rate is a logdet function, thus we can transform these features into a new feature space where linear model are possible using the following

$$\boldsymbol{\phi}(\check{\mathbf{x}}) = \begin{bmatrix} 1 \\ \log_b(|\check{\mathbf{x}}|) \end{bmatrix} \tag{14}$$

where b is the base of the logarithm. Under this assumption, an output i.e., an estimate sum rate is given by

$$y_i \approx \boldsymbol{\phi}(\check{\mathbf{x}}_i)^T \hat{\mathbf{w}}. \tag{15}$$

As a result of this formulation, we can apply any linear regression algorithms such as ordinary least square (OLS), ridge regression or principal component regression (PCR) [25, Chapter 6] to find an appropriate estimator $\hat{\mathbf{w}}$. In the numerical results, we will show the effectiveness of our proposed approach in comparison with other algorithms which do not take the feature design into account.

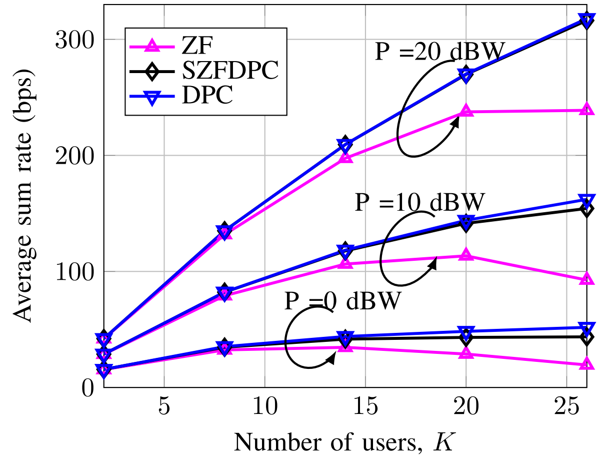

Fig. 1. Average sum rate versus the number of users for MIMO systems under PAPC, $N = 64$ transmit antennas.

IV. Numerical Results

In the following, we numerically evaluate the performance of the proposed algorithms under different MIMO settings. For all iterative algorithms taken into consideration, we set an error tolerance of $\epsilon = 10^{-6}$ as the stopping criterion. The number of receive antennas is fixed to $M = 2$, the power constraint for each transmit antenna is equal to P/N for all simulations and the total transmit power is simply set to 0 dBW, if not mentioned otherwise. We executed the MATLAB codes on a 64-bit desktop that supports 8 Gbyte RAM and Intel CORE i7.

In the first experiment, we compare the average sum rate of different precoding methods i.e., SZFDPC, ZF [7], and DPC [16] with PAPC over a large number of channel realizations. Under large-scale MIMO settings ($N/K \geq 10$), three methods obtain nearly identical values since the channel correlation are close to zero. However, under normal MIMO settings, there is still a big gap between the capacity for ZF and those of SZFDPC and DPC. In general, SZFDPC always achieves a near-capacity rate.

Since Algorithm 2 and the benchmark scheme all generate the optimal solution to the corresponding problem, we mainly compare their complexity. In particular, we average the runtime of Algorithm 2 as well as that of the interior-point method proposed in [8] over 100 channel realizations. Fig. 2 plots the average runtime as a function of the number of transmit antennas N for finding the maximum sum rate of SZFDPC under PAPC. Notice that the runtime takes account of both the number of iterations to converge and the per-iteration complexity mentioned in Subsection III-B. Thus Fig. 2 shows relative performance of the proposed algorithm and the solution in [8] under the same computer configurations. We observe that Algorithm 2 performs water-filling and GP to find $\bar{\mathbf{S}}, \mathbf{Q}$, which results in lower computation time compared with [8] which uses the barrier interior-point method. In general, the barrier method and other second-order optimization methods are known to achieve a superlinear rate but its per-iteration cost increases quickly with the problem size. This is actually consistent with what is shown in Fig. 2 for the barrier-method [8].

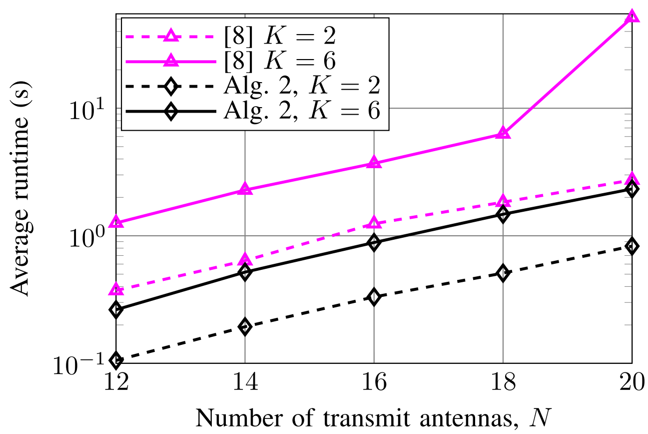

Fig. 2. Runtime versus the number of transmit antennas for solving the sum rate maximization problem with SZFDPC scheme and PAPC. Two methods are compared: Algorithm 2 and the interior-point method in [8].

As can be seen from Fig. 2, our AO-based algorithm, though low-complexity, may take a few seconds or more to execute when we increase the problem size. Thus, a method of lower complexity which can strike a balance between the optimal sum rate and complexity is of interest. In the following, we will investigate the performance of our ML-based approach to such scenarios. The PAPC ratio is chosen randomly, whereas the signal-to-noise ratio (SNR) is chosen from the set SNR $= \{0, 10, 20, 30, 40\}$ dB. For each MIMO setting we generated 240 samples using Algorithm 2. Also, we simply use natural logarithm to transform the feature space in (14).

In Fig. 3, we compare the cumulative distribution functions (CDFs) of the optimal and estimate sum rates of a MIMO system with SZFDPC and different PAPC settings using linear and nonlinear regression methods. More specifically, we utilize support vector regression (SVR) with radial basis function (RBF) kernel [26] for nonlinear regression. Here, we train on 216 samples and test on 24 samples. As can be seen from the figure, conventional OLS and SVR fail to fit the data due to nonlinear nature of the problem. However, the results of the simple OLS with the feature design are very close to optimal solutions. The performance has also proved the feasibility of our approach.

In the last experiment, we consider the effectiveness of our feature-design-based approach in terms of average relative root mean square error (aRRMSE) [27] over large samples with varying number of transmit and receive antennas. In particular, we obtain the aRRMSE by executing 10-fold cross-validation using three simple linear ML algorithms: OLS, Ridge and PCR. According to [27], a learning model is considered good and excellent when $10\% <$ aRRMSE $< 20\%$ and aRRMSE $< 10\%$, respectively. Interestingly, the ML-based methods show sufficiently low error rates, especially when $\frac{N}{K} \geq 10$. From our observations, the training matrices are invertible and the

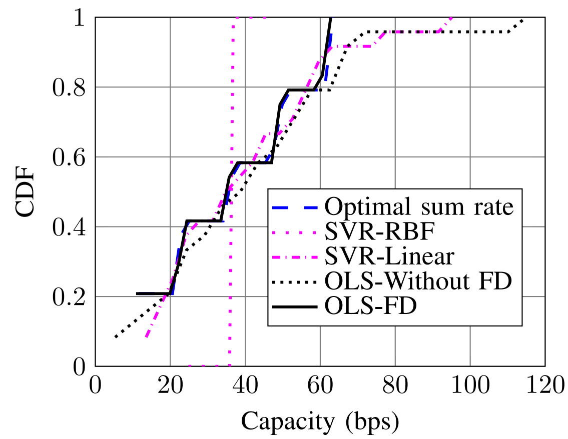

Fig. 3. Cumulative distribution functions of the optimal and estimate sum rates of a MIMO system with SZFDPC and random PAPC settings using linear and nonlinear regressions, $N = 32$ transmit antennas, $M = 2$ receive antennas and $K = 2$ users.

eigenvalues are larger than 1, thus the performance of OLS and PCR is the same and has minor difference in comparison with that of ridge regression. Unsurprisingly, these observations coincide with the properties of these regression methods.

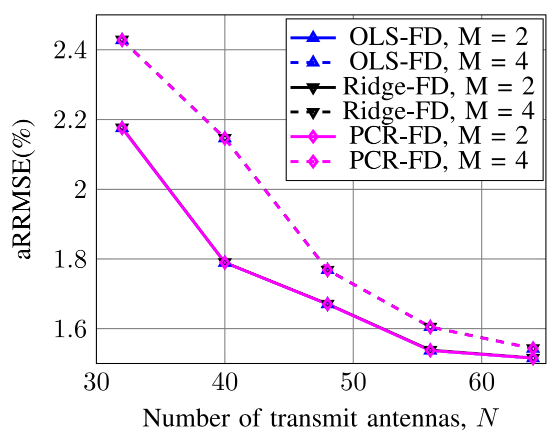

Fig. 4. aRRMSE of OLS, Ridge regression and PCR with feature design and $K = 2$ users.

V. Conclusions

We have proposed two low-complexity methods to compute sum rates of MIMO systems under PAPC and SZFDPC. The experiments using the optimal approach have stated that SZFDPC can obtain near-capacity rates whereas the ZF scheme still operates far from the optimal capacity boundary for a specified number of users. The suboptimal ML-based method is more advantageous in case of large-scale MIMO settings. Extensive numerical results have demonstrated the superiority of the proposed approaches over the existing interior-point method. More importantly, our ML-based approach can be applicable to a class of similar problems.

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