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Finite-Order hydrodynamic Approximation by Moment-Matching (FOAMM) toolbox for wave energy applications

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***Abstract*—Cummins' equation is commonly used to describe the motion of Wave Energy Converters (WECs), where the radiation force is characterised by a convolution operation. The computational effort associated with the solution of the convolution term, often represents a drawback for e.g. optimisation or exhaustive-search studies. To overcome this disadvantage, and given that the convolution operator intrinsically defines a dynamical system, the convolution term is commonly approximated using suitable finite-order parametric models. To this end, the Centre for Ocean Energy Research has recently presented a moment-matching based identification method for the radiation force subsystem and the complete force-to-motion WEC dynamics (i.e. wave excitation force to device velocity). Motivated by the theory and the obtained results, already reported by the authors, the FOAMM MATLAB application has been developed, which systematically implements the moment-matching based identification strategy from raw frequency-domain data, provided by hydrodynamic solvers, in a user-friendly fashion. The aim of this paper is to describe the theoretical background behind the identification strategy, and the structure, organisation and characteristics of the developed application. Additionally, the relevant modes of operation, along with the different options of the toolbox are explained, and, at the end, a step-by-step example of how to use the FOAMM application is provided, along with recommendations from the authors.**

***Index Terms*—Wave Energy, Radiation forces, Parametric form, Moment-matching, Frequency-domain identification**

I. INTRODUCTION

THE motion of a Wave Energy Converter (WEC) can be (linearly) expressed, in the time-domain, using the well-known Cummins' equation [1], which is a Volterra integro-differential equation of the convolution class. It includes a convolution term, accounting for the radiation forces, which represents the memory effect of the fluid. From a control/estimation (C/E) theory point of view, the convolution mapping complicates the application of well-established C/E strategies in the field, since modern C/E techniques are based on the availability of a state-space representation of the system [2]. Furthermore, from a simulation point of view, the convolution operator represents a drawback due to the associated computational effort, required for its solution.

Motivated by these drawbacks, the convolution term is commonly approximated using a parametric model, based on the frequency-domain hydrodynamic coefficients obtained from Boundary Element Method (BEM) codes, such as NEMOH [3] or WAMIT [4]. Several strategies can be found in the literature, reviewed in, for example, [5]–[7], attempting to approximate the radiation convolution term in terms of a linear time-invariant state-space representation.

Recently, the Centre for Ocean Energy Research (COER) presented an identification strategy, able to compute a parametric model of both, the radiation convolution term, and the complete force-to-motion WEC dynamics (wave excitation force to device velocity) [8]. The identification strategy is based on recent advances in model order reduction by moment-matching, which has been developed over several studies (see, for example, [9]–[12]). The approach presented in [8] identifies a parametric model (in state-space) for the WEC dynamics, which *exactly* matches the frequency response of the target system, at a set of user-selected frequencies. In fact, as shown in [8], this moment-based strategy inherently preserves the relevant physical properties in the identified model, such as internal stability.

Motivated by moment-matching theory and the results reported in the COER studies [8], [13], [14], a MATLAB toolbox has been developed, to disseminate this moment-based identification strategy for wave energy applications. The aim of the present paper is to introduce Finite-Order hydrodynamic Approximation by Moment-Matching (FOAMM) toolbox, developed at COER.

This study briefly describes the theoretical background behind the moment-based strategy, and details the structure, organisation and characteristics of the toolbox. In addition, the different operation modes and options, offered by the FOAMM application, are introduced, emphasising their impact on the obtained parametric model. Moreover, the required input variables are described, which are directly obtained from BEM solvers. Finally, a full step-by-step example is provided,

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along with authors' recommendations, to illustrate how to fully exploit its potential.

The remainder of this paper is organised as follows. In Section II, the equation of motion of a floating body is described in time- and frequency-domain. A brief summary of the theory behind FOAMM is given in Section III. The different modes and options, the required input variables, the obtained output variables, and how to install the toolbox is presented in Section IV. The step-by-step application example is given in Section V, and the authors' recommendations for the correct use of the software are given in Section VI. Finally, in Section VII, some conclusions are encompassed and future work is discussed.

II. WEC EQUATIONS OF MOTION

Given that FOAMM processes single-input single-output models, a WEC with a single Degree of Freedom (DoF) is considered in this study. Note that, for systems with multiple DoF, each relevant frequency-response coupling mapping needs to be identified separately, as shown in [15] and [16], [17].

A. Time-domain formulation

The motion of a 1-DoF WEC can be expressed, in the time-domain, according to Newton's second law, obtaining the following linear formulation:

$$m\ddot{x}(t) = \mathcal{F}_r(t) + \mathcal{F}_h(t) + \mathcal{F}_e(t) + \mathrm{u}_{\mathrm{PTO}}(t), \tag{1}$$

where m is the mass of the buoy, $\ddot{x}(t)$ the acceleration of the body, $\mathcal{F}_e(t)$ the wave excitation force, $\mathcal{F}_r(t)$ the radiation force, $\mathcal{F}_h(t)$ the hydrostatic restoring force, and $\mathrm{u}_{\mathrm{PTO}}(t)$ represents a control input, supplied by the means of a Power Take-Off (PTO) system. The linearised hydrostatic force is given by $\mathcal{F}_h(t) = -s_h x(t)$, where s_h denotes the hydrostatic stiffness. In this study, similarly to the analysis developed in [18], it is assumed that the PTO input is given by $\mathrm{u}_{\mathrm{PTO}}(t) = -m_{\mathrm{u}}\ddot{x}(t) - b_{\mathrm{u}}\dot{x}(t) - s_{\mathrm{u}}x(t)$, where m_{u}, b_{u} and s_{u} denote the mass, damping and stiffness of the PTO system, respectively. From linear potential theory, $\mathcal{F}_r(t)$ can be modelled using Cummins' equation [1] as,

$$\mathcal{F}_r(t) = -\mu_\infty\ddot{x}(t) - \int_0^{+\infty} k(\tau)\dot{x}(t-\tau)d\tau, \tag{2}$$

where $\mu_\infty = \lim_{\omega\to+\infty} A(\omega) > 0$ is the radiation added-mass $A(\omega)$ at infinite frequency, and $k(t)$ is the radiation impulse response. Therefore, Eq. (1) can be rewritten as:

$$(m+\mu_\infty)\ddot{x}(t) + k(t) * \dot{x}(t) + s_h x(t) = \mathcal{F}_e(t) + \mathrm{u}_{\mathrm{PTO}}(t), \tag{3}$$

where the symbol $*$ denotes the convolution integral of (2). Note that the internal stability of (3), for the WEC case, has been analysed and guaranteed for any physically meaningful values of the parameters and the convolution kernel $k(t)$ [19].

B. Frequency-domain formulation

Standard BEM solvers provide the frequency-domain response characteristic of the analysed device. Therefore, by applying the Fourier transform to (3), and considering velocity as the measured output[1], the following representation holds:

$$\hat{\dot{x}}(j\omega) = \mathcal{F}_e(j\omega)H(j\omega), \tag{4}$$

where $H(j\omega)$ represents the force-to-velocity frequency response, and is a function of a specific set of frequency-dependent parameters, namely

$$H(j\omega) = \frac{1}{b_{\mathrm{u}} + B(\omega) + j\omega\left[A(\omega) + m + m_{\mathrm{u}}\right] + \dfrac{s_h + s_{\mathrm{u}}}{j\omega}}, \tag{5}$$

where the coefficients $B(\omega)$ and $A(\omega)$ represent the radiation damping and added mass of the device, respectively, and can be efficiently obtained using BEM solvers [19].

It should be noted that BEM solvers compute the parameters $B(\omega)$ and $A(\omega)$ for a finite subset of user-defined frequency samples. However, if needed, the obtained data can be improved by using different reconstruction procedures, as shown in [20].

C. Ogilvie's relations: mapping between time and frequency

A direct relationship between the time-domain (3) and frequency-domain (4) is established by Francis Ogilvie in [21], as a function of the hydrodynamic coefficients $B(\omega)$ and $A(\omega)$, and the radiation kernel $k(t)$ as:

$$\begin{aligned} B(\omega) &= \int_0^{+\infty} k(t)\cos(\omega t)dt, \\ A(\omega) &= \mu_\infty - \frac{1}{\omega}\int_0^{+\infty} k(t)\sin(\omega t)dt. \end{aligned} \tag{6}$$

The impulse response $k(t)$ can then be written as a mapping involving the radiation damping coefficient as

$$k(t) = \frac{2}{\pi}\int_0^{+\infty} B(\omega)\cos(\omega t)d\omega. \tag{7}$$

Thus, the frequency-domain version of $k(t)$ is given by

$$\hat{k}(j\omega) = B(\omega) + j\omega\left[A(\omega) - \mu_\infty\right] \equiv K(j\omega). \tag{8}$$

Such a radiation kernel frequency response, $K(j\omega)$, has a set of particular properties which, as shown in [5] and [16], can be used to enforce a specific structure of the parametric model, used to identify the frequency-domain data, in an attempt to improve the quality of the computed representation.

III. MOMENT-BASED WEC FORMULATION

This section provides a brief summary of the theory behind FOAMM. For an extensive discussion on the specific underlying mathematical principles, the interested reader is referred to [8]. The development of model order reduction by moment-matching theory, is based on a state-space representation of the target system. Therefore, (3) needs to be re-written in a

[1]The force-to-position frequency response can be computed from (5) as $P(j\omega) = (j\omega)^{-1}H(j\omega)$.

more suitable structure, for which the following state-space representation is proposed:

$$\begin{aligned}\dot{\varphi}(t) &= A_\varphi \varphi(t) + B_\varphi \mathfrak{u}(t),\\ y_\varphi(t) &= C_\varphi \varphi(t),\end{aligned} \tag{9}$$

where $\varphi(t) = [x(t),\, \dot{x}(t)]^\intercal \in \mathbb{R}^2$ is the state-vector of the continuous-time model and $y_\varphi(t) = \dot{x}(t)$ is the output of the system (assuming velocity as the measurable output of the device). Finally, the function $\mathfrak{u}(t) \in \mathbb{R}$ (which is assumed to be the input of system (9)) is defined as $\mathfrak{u}(t) = \mathcal{F}_e(t) - k(t) * \dot{x}(t)$.

Therefore, under this assumption, the matrices in (9) are given by

$$\begin{aligned} A_\varphi &= \begin{bmatrix} 0 & 1 \\ -\dfrac{s_h + s_\mathfrak{u}}{m + \mu_\infty + m_\mathfrak{u}} & -\dfrac{b_\mathfrak{u}}{m + \mu_\infty + m_\mathfrak{u}} \end{bmatrix},\\ B_\varphi &= \begin{bmatrix} 0 \\ \dfrac{1}{m + \mu_\infty + m_\mathfrak{u}} \end{bmatrix},\ C_\varphi = \begin{bmatrix} 0 & 1 \end{bmatrix}. \end{aligned} \tag{10}$$

Then, the input $\mathcal{F}_e(t)$ is expressed as a *signal generator* (i.e. an exogenous dynamical system), written in implicit form as

$$\begin{aligned}\dot{\xi}_e(t) &= S\xi_e(t),\\ \mathcal{F}_e(t) &= L_e \xi_e(t),\end{aligned} \tag{11}$$

where $\xi_e(t) \in \mathbb{R}^\nu$, $S \in \mathbb{R}^{\nu\times\nu}$, $L_e \in \mathbb{R}^{1\times\nu}$ and the triple $(L_e, S, \xi_e(0))$ is assumed to be minimal[2]. Since the eigenvalues of S are simple and pure imaginary complex, S can be written in a real block-diagonal form as

$$S = \bigoplus_{p=1}^{\beta} S_p, \quad S_p = \begin{bmatrix} 0 & \omega_p \\ -\omega_p & 0 \end{bmatrix}, \tag{12}$$

where the symbol $\bigoplus$ denotes the direct sum of β matrices, i.e. $\bigoplus S_p = \text{diag}\{S_1, \ldots, S_\beta\}$, and $\nu = 2\beta$, with $\beta > 0$ (integer) the number of interpolation frequencies. Note that each $\omega_p > 0$ (integer) represents a desired interpolation point for the moment-matching-based model reduction process, i.e. a frequency where the transfer function of the parametric model matches the transfer function of the target system.

Following the theory developed in [8], a family of parametric models for the force-to-velocity response of the target WEC, achieving moment-matching at the set of frequencies $\mathscr{F} = \{\omega_1, \ldots, \omega_\beta\}$ can be described as

$$\tilde{\mathcal{H}}_\mathscr{F} : \begin{cases} \dot{\Theta}_\varphi(t) = (S - G_\varphi L_e)\, \Theta_\varphi(t) + G_\varphi \mathcal{F}_e(t), \\ \theta_\varphi(t) = \bar{V}\, \Theta_\varphi(t), \end{cases} \tag{13}$$

where $\bar{V}$ represents the so-called *moment-domain equivalent* of the velocity of the device $\dot{x}$ (see [8]), and can be readily computed using the frequency-domain data provided by BEM solvers as

$$\begin{aligned} \bar{V} &= L_e \Phi_\varphi^\mathscr{R},\\ \Phi_\varphi^\mathscr{R} &= \left[\left(\mathbb{I}_\nu + \Phi_\varphi \mathscr{R}^\intercal\right)^{-1} \Phi_\varphi \right]^\intercal,\\ \Phi_\varphi &= (\mathbb{I}_\nu \otimes C_\varphi) \left(S \,\hat{\oplus}\, A_\varphi\right)^{-1} (\mathbb{I}_\nu \otimes -B_\varphi), \end{aligned} \tag{14}$$

[2]The minimality of the triple $(L_e, S, \xi_e(0))$ implies the observability of (L_e, S) and the excitability of $(S, \xi_e(0))$.

where the symbol $\hat{\oplus}$ denotes the *Kronecker sum* (see [8]) and $\mathscr{R} \in \mathbb{R}^{\nu\times\nu}$ is a block-diagonal matrix defined by

$$\mathscr{R} = \bigoplus_{p=1}^{\beta} \begin{bmatrix} r_{\omega_p} & -m_{\omega_p} \\ m_{\omega_p} & r_{\omega_p} \end{bmatrix}, \tag{15}$$

where its entries depend on $A(\omega)$ and $B(\omega)$ of the device at each specific frequency induced by the eigenvalues of S, as

$$r_{\omega_p} = B(\omega_p), \text{ and } m_{\omega_p} = -\omega_p \left[A(\omega_p) - \mu_\infty\right]. \tag{16}$$

It should be noted that the model of (13) has dimension $\nu = 2\beta$, as the final finite order parametric model.

The additional degree of freedom provided by G_φ can be exploited to arbitrarily assign the eigenvalues of the reduced order model of (13). In this particular case, the set of desired eigenvalues is chosen within an optimisation formulation, which minimises the euclidean distance between the device frequency response $H(j\omega)$, shown in (5), constructed with data obtained with BEM solvers, and the frequency response of the parametric family (13) $\tilde{H}(j\omega)$, obtained from the transfer function

$$\tilde{H}_\mathscr{F}(s) = \bar{V} \left[s\mathbb{I}_\nu - (S - G_\varphi L_e)\right]^{-1} G_\varphi. \tag{17}$$

Note that the frequency-dependent device parameters $A(\omega)$ and $B(\omega)$ are calculated using BEM solvers, at a finite number of user-defined frequencies, with $\omega_i \in [\omega_l, \omega_u]$, and a frequency step of $\Delta\omega$, where ω_l and ω_u represent the lower and upper bound of the range, respectively. As further discussed in Section V, the definition of such a frequency range depends explicitly on the application under analysis. Defining the complex-valued vectors H_ω and $\tilde{H}_\omega$ as,

$$H_\omega = \begin{bmatrix} H(j\omega_l) \\ H(j(\omega_l + \Delta\omega)) \\ \vdots \\ H(j(\omega_u)) \end{bmatrix},\ \tilde{H}_\omega = \begin{bmatrix} \tilde{H}_\mathscr{F}(j\omega_l) \\ \tilde{H}_\mathscr{F}(j(\omega_l + \Delta\omega)) \\ \vdots \\ \tilde{H}_\mathscr{F}(j(\omega_u)) \end{bmatrix}, \tag{18}$$

the proposed optimisation procedure, to assign the eigenvalues of the parametric model (13) $\Sigma_\varphi^{\text{opt}}$, can be formulated as,

$$\Sigma_\varphi^{\text{opt}} = \arg\min_{\Sigma_\varphi} \|H_\omega - \tilde{H}_\omega\|_2^2, \tag{19}$$

where the elements of the set $\Sigma_\varphi^{\text{opt}}$ are chosen to have a negative real part, so that the system (13) is internally stable.

The method detailed above is described to obtain a parametric model of the force-to-velocity dynamics of the WEC. However, since the radiation convolution term in (2) defines a linear time-invariant system itself, an analogous process can be defined to obtain a parametric model for such a subsystem, as demonstrated in [8].

IV. Toolbox description

As mentioned in Section I, FOAMM is a MATLAB application that implements the moment-matching based frequency-domain identification algorithm described in Section III. The identification of both, the radiation force impulse response, and the force-to-motion dynamics of a WEC, can be performed using this tool. All the modes of operation and options that the software comprises are detailed in the following subsections.

D. Platform requirements

Since the application is created using a MATLAB compiler, FOAMM runs on a plain MATLAB distribution, i.e. no other toolboxes/applications are required. At the time of writing, two versions of FOAMM are available, one for the Windows operating system (OS), and another one for Linux OSs.

The compatibility with the Windows version has been tested for Windows 7, 8 and 10, using different MATLAB versions (years 2012, 2015, 2017 and 2018). Regarding the Linux version, the compatibility has been tested for Ubuntu 18 and CentOS (6.10 and 7.4).

E. Installation

The files can be downloaded from http://www.eeng.nuim.ie/coer/downloads/. Since FOAMM is an executable, having the correct Matlab runtime version, installed on the computer, is required[3], which is provided in the folder "Matlab Runtime". After installing the MATLAB runtime[4], the application effectively runs in a stand-alone fashion on a plain MATLAB.

It should be noted that the first run of the application is considerably slower than the subsequent ones, which is indeed a known issue, when using the MATLAB compiler. Additionally, for the Linux version, administrative access is required for both the installation of the MATLAB Runtime, and the use of the application.

F. Files

In this subsection, the files compressed in "FOAMM.rar" are listed and explained:

MatlabRuntime This file, located inside the "Matlab Runtime" folder, is the executable, required to install the correct version of the MATLAB runtime, as explained in Section IV-E.

Main.m This is the main file and the user interface to the application. The file loads the frequency-domain data contained in **"Data.mat"**, used to perform the identification, and allows to change between the different modes and options of the application. **"Main.m"** set-ups the required variables, and explicitly calls the executable file **"FOAMM"**. It should be noted that this file provides the only way to select between the different modes of the application.

FOAMM This is the executable file of the application, and is explicitly called by **"Main.m"**. For the Windows version, the extension of the file is ".exe".

Data.mat This file contains the frequency-domain data to be identified (computed with any BEM code) and it should be provided by the user, following the specific input format detailed in Section IV-I. In "FOAMM.rar" an example **"Data.mat"** file is provided, with the hydrodynamic coefficients of a cylinder with 5m radius and 10m draft.

[3]Note that installing the MATLAB runtime will not change any other functionality of the computer.

[4]Internet connection is needed to install the provided Matlab Runtime.

G. Identification methods

In order to select the matching frequencies, FOAMM offers the following 3 identification methods:

Manual method The user selects the desired set of frequencies to achieve moment-matching. The order will be twice the number of frequencies selected.

Automatic method The user selects a final number of interpolation points (β) and, additionally, a subset of frequencies to interpolate (with size denoted by α). This method optimises the value of the $\beta - \alpha$ interpolation frequencies. It should be noted that, if the user does not pre-select a set of frequencies i.e. $\alpha = 0$, all the β interpolation points are selected automatically.

Optimised-automatic method This method is essentially the automatic method for the selection of the matching frequencies, but it also selects the number of interpolation points β automatically. Starting from α, this method keeps adding (and optimising) interpolation frequencies, until the approximated model satisfies both an absolute and a relative thresholds specified by the user in the "Options" structure (see Section IV-H).

H. Application options

Every option of the application can (only) be changed using the structure **"Options"** from the "Main.m" file. The different variables stored in the structure can be accessed (and tuned) as follows:

Options.Mode (integer)

- default **0** Compute an approximated model of the radiation impulse response of the device.
- **1** Compute an approximated model of the force-to-velocity dynamics of the device.

Options.Method (integer) - see Section IV-G for a description of each method.

- default **0** Manual method.
- **1** Automatic method.
- **2** Optimised-automatic method.

Options.FreqRangeChoice (string or float)

- default **'G'** Select the frequency range from a plot.
- **'C'** When asked, enter a vector with the lower and upper bounds of the frequency range in the command window, as $[\omega_l, \omega_u]$.
- **VEC** Directly enter a vector with the lower and upper bounds of the frequency range as **VEC**=$[\omega_l, \omega_u]$.

Options.FreqChoice (string or float)

- default **'G'** Select the set of desired interpolation frequencies from a plot.
- **'C'** When asked, enter the desired set of interpolation frequencies in the command window as a vector, $[\omega_1, \dots, \omega_\beta]$.
- **VEC** Directly enter a vector with the set of interpolation frequencies as **VEC**=$[\omega_1, \dots, \omega_\beta]$.

Options.FreqNumChoice (string or integer) (if **Options.Method** = 1)

- default **'C'** When asked, enter the number of frequencies to interpolate $\beta > 0$ in the command window.

INT Directly enter the desired number of interpolation frequencies $\beta > 0$.

Options.Optim (optimisation-related options)

integer **Options.Optim.InitCond** Number of initial conditions considered on the optimisation. Default = 50.

float **Options.Optim.Tol** Tolerance on the final value of the optimisation. Default = $1e^{-5}$.

integer **Options.Optim.maxEval** Maximum number of evaluations considered for the optimisation. Default = $1e^{3}$.

integer **Options.Optim.maxIter** Maximum number of iterations considered for the optimisation. Default = 200.

float **Options.Optim.StepTol** Step tolerance value for the optimisation. Default = $1e^{-6}$.

float **Options.Optim.ThresRel** (if **Options.Method** = 2) Relative error threshold. Default = 0.03.

float **Options.Optim.ThresAbs** (if **Options.Method** = 2) Absolute error threshold. Default = 0.1.

If any of the labels inside the structure "Options" is changed, the application will not recognize the variables, and the default values will be used. Additionally, if the selected value is wrong, the application will ask the user to correct it.

I. Input variables

The frequency-domain data, used to perform the moment-matching based identification, is loaded from the user supplied file **"Data.mat"**. This file with ".mat" extension must contain all the information regarding the frequency-domain data, considered for the identification process. In the following, a detailed description of the required format is provide, so that the application can load the data correctly.

†**A** (vector, float, $n_\omega \times 1$) Radiation added mass ($A(\omega)$).
†**B** (vector, float, $n_\omega \times 1$) Radiation damping ($B(\omega)$).
†**w** (vector, float, $n_\omega \times 1$) Frequency vector (ω).
Mu (scalar, float, if **Options.Mode** = 0) Radiation infinite added mass (μ_∞). If this value is not supplied, the application will automatically calculate its value using Ogilvie's relations. However, the authors recommend users to provide it, in order to reduce inaccuracies.
Mass (scalar, float, if **Options.Mode** = 1) Mass of the structure under analysis ($m + m_{\text{u}}$).
K (scalar, float, if **Options.Mode** = 1) Sum of stiffness terms (usually the sum of s_h and s_{u}).
D (scalar, float, if **Options.Mode** = 1) Sum of (additional) damping terms (such as, for example, b_{u}).

If any of the variables denoted with † is named differently, the application will halt.

J. Output variables

This subsection gives a list with the variables obtained by running FOAMM. Explicitly:

A_ss (matrix, float, $\nu \times \nu$) Dynamic matrix of the final model.
B_ss (matrix, float, $\nu \times 1$) Input matrix of the final model.
C_ss (matrix, float, $1 \times \nu$) Output matrix of the final model.
MAPE (scalar, float) Mean Absolute Percentage Error (MAPE) of the approximation, which is defined as

$$\textbf{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\mathrm{f}(i) - \tilde{\mathrm{f}}(i)}{\mathrm{f}(i)}\right|, \tag{20}$$

where n is the number of frequencies contained in the considered frequency-range, and f and $\tilde{f}$ are the frequency responses of the target and the final model, respectively.
Frequencies (vector, float, $1 \times \beta$) Value of the chosen frequencies.
FreqRange (vector, float, 1×2) Value of the minimum and maximum frequencies of the chosen frequency range.
Mu (scalar, float, if **Options.Mode** = 0 and **Mu** not provided) Automatically calculated radiation infinite added mass.

V. Application Example

In this section a step-by-step example is shown, of how to use FOAMM to identify a finite order parametric model of the radiation impulse response and complete force-to-motion dynamics. The hydrodynamic parameters used, correspond to a heaving cylinder with a 5m radius, representing the example case, provided in the **"data.mat"** file. In the following subsections, the three identification methods are explained, and the options are varied, between the subsections, in order to exemplify all the different operating modes and options.

K. Manual method

In the following, an example of the **"Main.m"** is shown, for the operation of the toolbox in the manual mode. For the sake of clarity, the majority of the comments and spaces of the original **"Main.m"** file are omitted:

```
%% Load hydrodynamic parameters ----------
clear all; clc
load('data.mat')
K = Sh;

%% Options structure ----------
Options.Mode = 0;
Options.Method = 0;
Options.FreqRangeChoice = 'G';
Options.FreqChoice = 'G';
Options.FreqNumChoice = [];
Options.Optim.InitCond = 50;
Options.Optim.Tol = 1E-6;
Options.Optim.maxEval = 100;
Options.Optim.StepTol = 1E-6;
Options.Optim.ThresRel = 0.1;
Options.Optim.ThresAbs = 0.03;

%% Run application ----------
save('temp_file.mat')

system('FOAMM');

load('temp_file.mat')
delete('temp_file.mat')
%% ----------
```

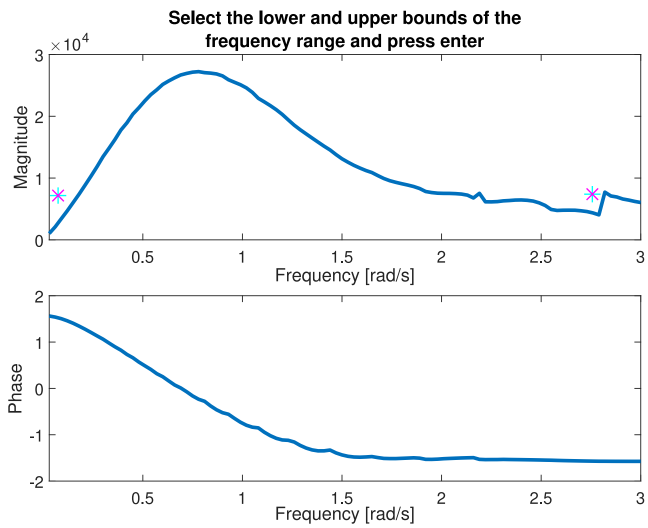

Fig. 1. Graphical interface to select the frequency range for the approximation.

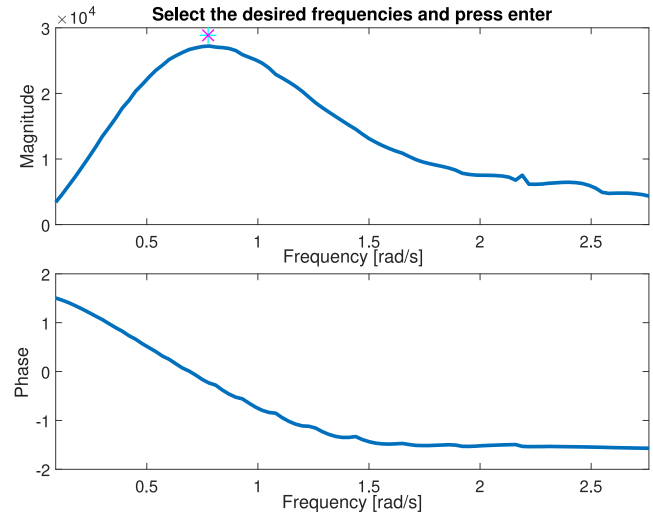

Fig. 2. Graphical interface to select the desired set of interpolation frequencies for the approximation.

As shown in the code, the file **"Main.m"** comprises three different parts. The commands in the first lines (from line 1 to 5) clear the variables of the MATLAB workspace, and load the hydrodynamic parameters saved in **"data.mat"**. Since no PTO is considered, the only stiffness term in this example is the hydrostatic stiffness and, therefore, the variable **K** is defined by s_h (**K=Sh**). For the same reason, no damping term (**D**) needs to be defined. In the middle section (from line 6 to 18) of the code in **"Main.m"**, the different working options are defined. Finally, at the end (from line 19 to 26), all the variables are saved in a temporary file (**"temp_file.mat"**), to be loaded by FOAMM and subsequently compute the identification. After the identification process is finished, the updated temporary file, which contains all the results obtained by the application, is loaded and deleted.

For this example, the parametric model of the radiation impulse response is identified (**Options.Mode=0**), using the manual method (**Options.Method=0**). Both, the frequency range and the frequencies are chosen, using the graphical interface (**Options.FreqRangeChoice='G'** and **Options.FreqChoice='G'**, respectively). Therefore, first, a graph, as the one shown in Fig. 1, will appear, asking the user to specify the frequency range.

As explained in [8], the frequency range will depend on the frequency distribution of the input signal. By way of example, lets assume that the model will be used in waves, characterised by the same JONSWAP spectrum [22] considered in [8] (peak period of 10s, significant wave height of 2m and peak enhancement factor of 3.3). For this case, choosing a frequency range from around 0.1 to 2.75 rad/s will cover the whole input frequency spectrum. Therefore, for any possible operational point, the identified model will behave as the original system.

Once the frequency range is correctly introduced, another graphical interface appears, asking for the interpolation frequencies, showing the radiation impulse response just for the previously chosen frequency range (see Fig. 2).

While FOAMM is optimising the value of the eigenvalues (as explained in Section III), a wait bar, as shown in Fig. 3, is displayed. The wait bar shows the progress of the optimisation, along with a cancel button, which allows the user to stop the optimisation process. In case of stopping the optimisation, FOAMM will return the best model obtained before stopping the process. In the particular case of using the optimised-automatic method and cancelling the optimisation, the predefined thresholds would not be taken into account, and the order (ν), giving the best fitting accuracy, will be returned as the resulting model.

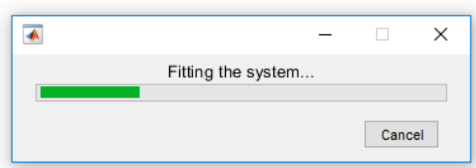

Fig. 3. Waiting bar showing the progress of the optimisation.

After the optimisation is finished, the resulting model, along with the target system, and the interpolation frequencies are displayed, as shown in Fig. 4. For this case, since a unique frequency was chosen, the error of the obtained model is **MAPE**$\approx$0.22.

L. Automatic method

In this subsection, apart from showing how to use the automatic mode, which is selected by defining **Options.Method=1**, the whole force-to-motion WEC dynamics are identified, instead of the radiation impulse response identified in Section V-K. Thus, in order to approximate the force-to-motion frequency response, the option **Options.Mode** needs to be set to 1. Additionally, the following changes are made with respect to the piece of code shown in Section V-K:

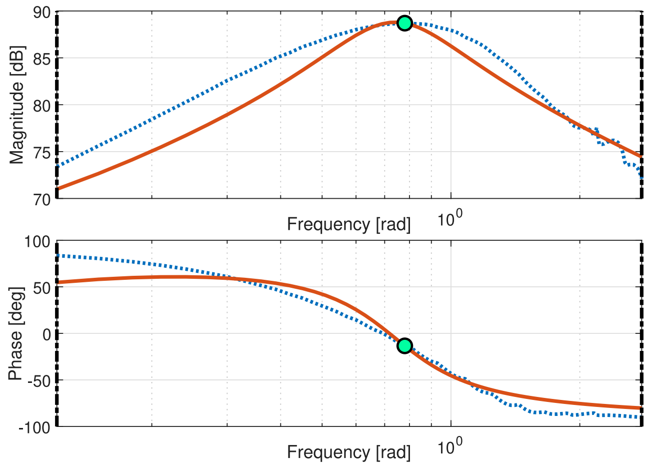

Fig. 4. Frequency response of the obtained parametric model of the radiation impulse response (solid-red), along with the target frequency response (dashed-blue) and the interpolation frequency (green-dot) using the manual method. Obtained error **MAPE**≈0.22.

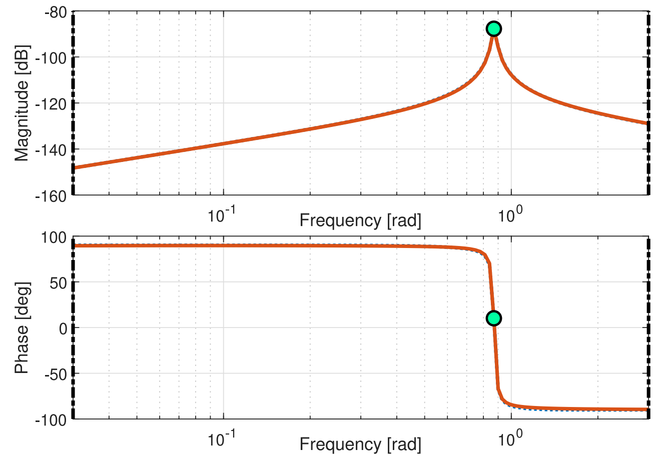

Fig. 5. Frequency response of the obtained parametric model of the WEC force-to-motion dynamics (solid-red), along with the target frequency response (dashed-blue) and the interpolation frequencies (green-dots) using the automatic method. Obtained error **MAPE**≈0.02.

```
%% Options structure ———————————
Options.FreqRangeChoice = 'C';
Options.FreqChoice = [];
Options.FreqNumChoice = 1;
```

In this case, the following message will appear in the command window of MATLAB, asking for the frequency range: *"Introduce a vector containing the lower and upper bounds of the desired frequency range (min=0.03, max=3):"*[5]. Then, the frequency range must be defined as a size 2 vector, with its values being inside the frequencies defined by the vector **w** from the file **"data.mat"**. If, as for this case example, enter is pressed without specifying anything (or writing an empty vector as [ ]), the whole vector of frequencies will be taken into account for the identification. Additionally, since no frequencies have been defined in **Options.FreqChoice**, the value of all the frequencies will be optimised. Since **Options.FreqNumChoice=1** ($\beta = 1$), a parametric model with a single frequency is returned as result, depicted in Fig. 5, and with a fitting error of **MAPE**≈0.02.

M. Optimised-automatic method

In order to run the optimised-automatic method, **Options.Method=2** needs to be selected. Additionally, the following changes are made with respect to the piece of code shown in Section V-K:

```
%% Options structure ———————————
Options.FreqRangeChoice = [0.1 2.75];
Options.FreqChoice = 0.78;
```

Since the frequency range and the interpolation frequencies are already defined for this case, FOAMM will directly proceed to the optimisation. During the optimisation, for this method, apart from the waiting bar, a graph will be displayed showing the **MAPE** obtained for the different model orders that were tested, as the one shown in Fig. 6.

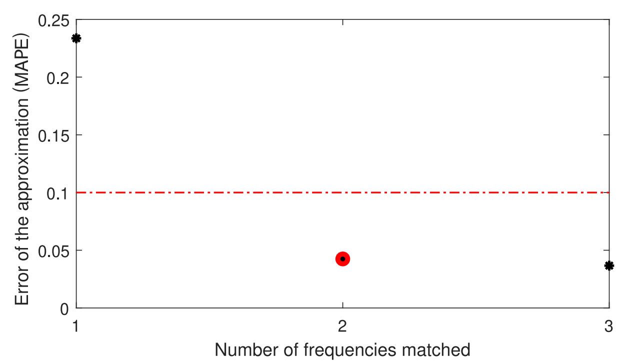

Fig. 6. Graph displayed while the optimised-automatic method is running.

This method will keep adding interpolation frequencies until the predefined absolute and relative thresholds are satisfied, or the optimisation is stopped using the cancel button of the waiting bar. It should be noted that the number of frequencies considered by the optimisation will begin from the frequencies specified in **Options.FreqChoice**. Since a unique frequency is predefined for this example, it starts from 1[6].

The dash-dotted red line of Fig. 6 represents the defined absolute threshold, which is set to 0.1 (as the default value). For this application example, the optimisation considers up to three interpolation frequencies ($\beta = 3$) but, since the **MAPE** improvement with respect to $\beta = 2$ is less than the defined relative threshold (which is 0.03, as the default value), the optimisation stops and chooses $\beta = 2$ as the optimal order. Finally, as for the other two methods, the resulting frequency response of the parametric model is displayed, shown in Fig. 7, which obtains MAPE≈0.04.

[5]Where the shown minimum and maximum values correspond to the minimum and maximum values of the input frequency vector **w**.

[6]This would also be the case if no frequency is predefined, with the only difference that the value of such interpolation frequency will also be subject to the optimisation.

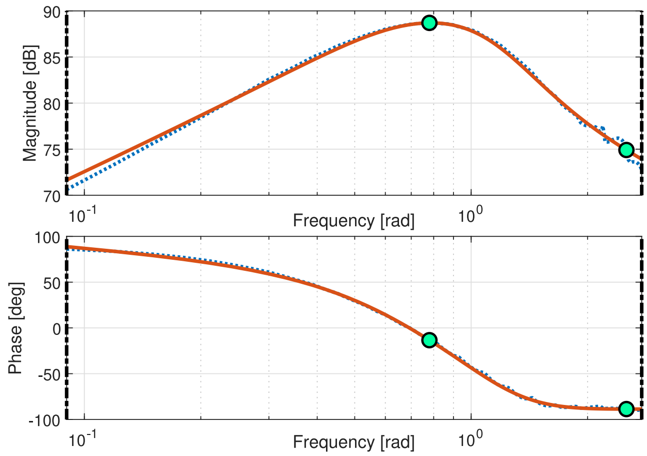

Fig. 7. Frequency response of the obtained parametric model of the radiation impulse response (solid-red), along with the target frequency response (dashed-blue) and the interpolation frequencies (green-dots) using the optimised-automatic method. Obtained error **MAPE**≈0.04.

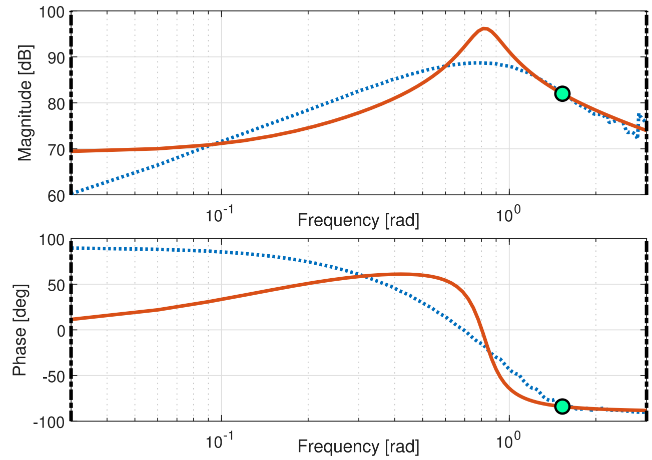

Fig. 8. Example of a suboptimal parametric model when using the automatic method (**Options.Method=1**) without specifying the most important frequency, and considering few initial conditions (for this example, **Options.Optim.InitCond=1**). Obtained error **MAPE**≈0.38.

VI. Authors' recommendations

This section provides some recommendations for the use of FOAMM, with which, in authors' opinion, the software runs optimally.

Use the manual method. Even though the application comes with different automated method, the authors' highly recommend a sensible (manual) choice of the interpolation frequencies, based on the system dynamics to be identify. In fact, this option, to perform a suitable selection of points to interpolate in the frequency-domain, is one of the most attractive characteristics of moment-matching. For example, the user might want to select, as one sensible choice, the resonant frequency of the structure to be identified, as shown in Fig. 2.

Specify the important frequencies. When not using the manual method, it is convenient to specify the most important frequencies to help the optimisation process, and improve the accuracy of the obtained model. Thus, instead of giving the algorithm the freedom to look for the best frequency, a sensible choice would be to pre-select the resonant frequency and, if $\beta = 1$ is not enough to accurately represent the system (and there is no other important frequency), let the algorithm check for the best value of the subsequent frequencies. Contrary, if the resonant frequency is not specified, and the optimisation variables are not correctly selected, FOAMM might not find the combination in which the resonant frequency is contained, obtaining a suboptimal parametric model. As an example, Fig. 8 shows a parametric model of the radiation impulse response of order 2 (as shown before in Fig. 4), where the resonant frequency is not pre-selected and the optimisation variables are not correctly selected, i.e. a single initial condition is chosen for the optimisation[7]. In this case, the obtained interpolation frequency is $\omega = 1.5$rad/s, which lead to an approximation error of **MAPE**≈0.38. However, if the resonant frequency is pre-selected, even though the optimisation variables are still not correctly selected, the obtained error is **MAPE**≈0.24.

[7]Even though is not realistic to optimise using a unique initial condition, this option is chosen to magnify the effect of this possible error.

Increase the number of initial conditions. Since the initial conditions for the optimisation are chosen randomly using a normal distribution over a pre-selected set, it is more likely to find an accurate model if a high number of initial conditions is considered. Therefore, when trying to identify large order models, or when optimising the value of the interpolation frequencies, using more initial conditions increases the chances of FOAMM to find the optimal model. For example, when using the optimised-automatic method, it may happen that, for some β, the obtained error is higher than for a given smaller β value. This can be fixed by increasing the number of initial conditions in **Options.Optim.InitCond**. As an example for this case, Fig. 9 shows the results obtained by the optimised-automatic method for the same case shown in Fig. 6, but considering only a unique initial condition (**Options.Optim.InitCond=1**). It is shown how, apart from the obtained MAPE which is higher than in Fig. 6, when considering 3 frequencies the obtained MAPE is higher than the one obtained for $\beta = 2$, which is due to the low number of initial conditions considered (a single one) in the example.

Use force-to-motion models. As can be appreciated from Fig. 5, force-to-motion frequency responses can be often approximated with a low order system, given the lower complexity behind its dynamical response when compared to the radiation force subsystem. Additionally, as detailed in [8], two (2) more states need to be added to the radiation impulse response approximated model, to obtain the complete WEC state-space representation (i.e. the user needs to embed the parametric model of the radiation force in Cummins' equation). Therefore,

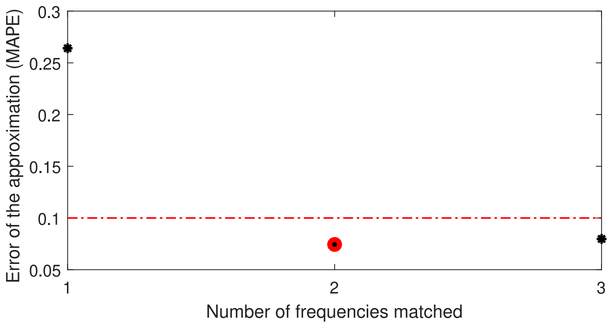

Fig. 9. Example of a possible error when using the optimised-automatic method (**Options.Method=2**) with few initial conditions (for this example, **Options.Optim.InitCond=1**).

to characterise the input-output response of the WEC, authors' recommend to directly parameterised the force-to-motion response of the target device.

VII. Conclusions and future work

This study describes how to use the FOAMM toolbox to identify a parametric model of the radiation impulse response subsystem, and the complete force-to-motion WEC dynamics. FOAMM is a MATLAB application based on a recently developed moment-matching-based strategy, reported in [8]. The paper explicitly shows the downloading procedure of FOAMM, and provides a thorough description on how to install the application, and the different options that it comprises, along with a step-by-step example with a cylindrical heaving point absorber WEC.

FOAMM runs in a plain MATLAB distribution, and can be installed in either Windows or Linux-based operating systems. The key advantage of this identification toolbox is that it allows the user to select a set of interpolation frequencies, where the approximated model exactly matches the behaviour of the target system, subsequently increasing the accuracy of the parametric model in dynamically relevant frequencies (such as, the resonant frequency of the device or the frequency associated with the peak period of the wave excitation force).

As future work, authors aim to include the already developed moment-matching-based passivity-enforcement method which, as shown in [13], allows for the preservation of the passivity property of radiation forces. Additionally, authors plan to include an extension of the software to compute parametric models for multiple DoF devices and WEC arrays, following the theory developed in [14].

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