2018 IEEE Conference on Decision and Control (CDC)
Miami Beach, FL, USA, Dec. 17-19, 2018

SNR–triggered Communication Rate for LQG Control over Wi-Fi

M. Pezzutto, F. Tramarin, S. Dey, L. Schenato

***Abstract*— In this work we address the problem of LQG control where the communication between the sensor and the controller/actuator is performed via Wi-Fi. We exploit one feature of Wi-Fi (standard IEEE 802.11) which gives the ability to switch among different data-rates in real-time: a lower transmission rate provides a lower packet loss probability at a price of a larger sampling period. As a matter of fact, it is not obvious how to select the optimal rate from a control perspective. Nevertheless, the packet error probability as a function of the perceived SNR can be obtained either analytically or empirically. Based on these curves, we determine the optimal rate and the optimal LQG controller for any fixed SNR. In a scenario with a time-varying SNR, we also propose a rate adaptation strategy which is triggered by the measured SNR. Numerical simulations and comparisons with current literature are included to show the benefits of our approach.**

***Index Terms*— Packet loss, LQG, Wi-Fi, Rate Adaptation**

I. INTRODUCTION

In the last years extensive improvements have been made in wireless networks in terms of data-rate, reliability, and coverage [1] [2], allowing control engineers to employ this technology on Networked Control Systems (NCSs). A NCS consists of a control system where at least one of the two links between the plant and the controller is implemented through a network. This solution ensures many advantages: the amount of wiring can be drastically decreased, resulting in lower costs and in more comfortable connections, less computational capabilities can be located to the plant, and more complex controllers can be implemented. These interesting properties come at the cost of the two main network-induced constraints, namely the stochastic time delays and the random packet losses, which deteriorate the performances of the estimation and of the control, and may result in the loss of stability, as it has been shown since the early works, e.g. [3]. Many attempts have been made to deal with these issues and many results have been obtained, see e.g. [4] [5].

In the past fifteen years, a large body of literature on Wireless Sensors Networks [6] has exploited mainly low data-rate protocols, e.g. ZigBee, WirelessHart, and ISA100.11a. More recently, the high date-rate standards of the IEEE 802.11 family, also refereed to as Wi-Fi, have attracted the interest of the NCS community. The high transmission rates of Wi-Fi (up to $54\ \mathrm{Mbit/s}$ with 802.11g and up to $150\ \mathrm{Mbit/s}$ with 802.11n) allow higher sampling rate, but high latency and heavy stochastic timing behaviour make Wi-Fi not suitable for control systems. A possible solution for these issues is done in [7] by applying TDMA-like methodology. An interesting feature provided by Wi-Fi is the possibility to switch among different transmission rates by the user on a packet basis. It is well known that higher rates allow to transmit a greater number of packets and hence to reduce the sampling period. At the same time, thanks to more robust modulations, lower rates are able to guarantee lower error probabilities. The objective of this work is to exploit this flexibility to devise an optimal (in the sense of LQG cost) rate selection algorithm which is triggered by the measured Signal-to-Noise Ratio (SNR).

A rate selection algorithm is recently proposed in [8], which, for each packet and for the current SNR condition, finds the number N of transmission attempts and the sequence of N transmission rates that minimize the residual packet loss probability. In industrial scenario, Minstrel algorithm [9] searches a trade-off between the throughput and the arrival probability within a maximum delay. Differently from our technique, these two algorithms consider a communication metric (i.e. the transmission probability or the throughput) that affects the control rather than the actual control performance. In [10], the sampling rates of multiple plants that share a network are selected according to an non-linear constrained optimization problem. In contrast to our approach, it does not consider the LQG cost but a metric which measures the degradation of the performances due to the discretization. In [11] the MAC parameters of the network (e.g. the backoff exponent, the maximum number of backoffs, the retry limit) are chosen to minimize the energy consumption subject to an admissible LQG cost.

The main contribution of our work is the simultaneous analysis of the optimal control and the rate selection problem via an LQG framework. The main difference with respect to the previous works is that the packet loss and the delay (i.e. the sampling period) are a function of the transmission rate and of the SNR level. We devise the optimal LQG regulator where the control input is updated at the highest rate possible, independently on the transmission rate. We consider two scenarios. First we assume the SNR to be constant and we find the joint optimal rate and LQG gains. In the second scenario, we consider the more realistic case of time-varying SNR. In this context, we propose to select the transmission rate in real-time as if the SNR will be indefinitely constant, i.e. the optimal rate obtained in the first case. Interestingly, the optimal regulator has the same gains both with constant and with time-varying SNR. Our approach of switching between different rates has the potential to provide the best possible performance at any SNR level.

M. Pezzutto, F. Tramarin, and L. Schenato are with the Department of Information Engineering, University of Padova, Via Gradenigo 6/b, 35131 Padova, Italy, S. Dey is with Inst for Telecomm Research, University of South Australia while being on leave of absence from the Dept. of Signals & Systems, Uppsala University, Uppsala, SE 75121 Sweden. This work is partially supported by University of Padova, project MAgIC SCHE_SID17_01.

II. Problem formulation

Preliminary, it is important to clarify what we mean by control rate and transmission rate and what is their relation to the sampling period.

Control rate R^{ctr}: it is the inverse of the sampling period to which the system is discretized. It corresponds to the rate at which the input and the estimate are updated.

Transmission rate R^{tx}: it is the inverse of the period between two consecutive packets sent form the sensor to the estimator. This definition does not coincide with the usual sense for which transmission rate indicates the bytes sent over the network per second, also called data-rate. Although they are not exactly interchangeable, we consider to statically associate a transmission rate to a data-rate, and we will possibly refer to the data-rate by the associated transmission rate.

We consider the set of periods T_i $i \in \{1, 2, \dots, L\}$, which represent both the sampling periods and the periods between two measurements. For sake of simplicity, we assume that $T_i = i{\cdot}T_1$. The smallest period is simply indicated by T, so the set of given periods is $\{T, 2T, \dots, L{\cdot}T\}$. From the relation $R_i = 1/T_i$, it is possible to derive the set of rates R_i $i \in \{1, 2, \dots, L\}$; it holds that $R_i = R_1/i$.

A. Communication

We consider to connect the sensor and the estimator through a Wi-Fi network. Our choice arises from the widespread adoption of Wi-Fi, but our analysis can be extended to any network protocol that allows the user to dynamically select the transmission rate in real-time. Our work does not employ a detailed protocol, but it relies on a simplified network model. In the following we introduce our main assumptions.

In our model, at each measurement instant, the sensor samples the output, it accesses the network and sends the measurement to the estimator at the chosen transmission rate. The packet arrives with a certain probability λ which depends on the transmission rate and on the SNR level:

$$\lambda = \psi(R^{\text{tx}}, \mathit{SNR}).$$

It is well known that, for a given SNR level, lower transmission rates achieve lower error probability thanks to the adoption of more robust modulations. We consider that the packets have fixed length; we do not consider any packet retransmissions. We assume that either the measurement arrives within the following measurement instant or it never arrives. Under this assumption, the packet loss probability $1-\lambda$ is the probability with which the packet is not arrived before the following measurement instant.

Under these assumptions, approximated curves of packet loss probability vs SNR are shown in Fig. 1. See for example [12] for IEEE 802.11g. They can be inferred analytically or empirically. Although the curves are quite typical, other curves are also possible: our methodology can be applied with any other curves, as long as the assumptions made above hold.

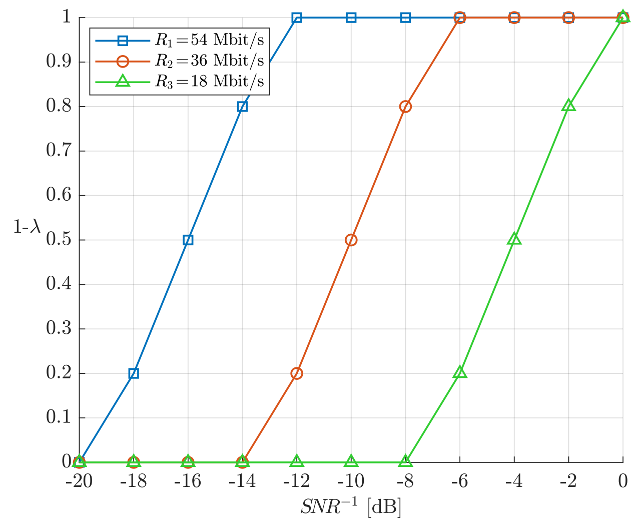

Fig. 1: Curves of loss probability vs SNR.

B. System dynamics

Consider the continuous-time linear system:

$$\begin{cases} \mathrm{d}x(t) = A_c x(t)\mathrm{d}t + B_c u(t)\mathrm{d}t + \mathrm{d}w(t) \\ y(t) = C_c x(t) \end{cases}$$

where $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^p$, $u(t) \in \mathbb{R}^m$, and $w(t)$ is a Wiener process such that $w(t+\tau) - w(t) \sim \mathcal{N}(0,\, Q_c\tau)$ with $Q_c \geq 0$.

Consider to control the system by a Networked Control System. The sensor packetizes and sends the output to the estimator through a wireless link, which is subject to random packet loss. The controller communicates with the actuator and with the estimator through reliable links, without delays and packet losses: for example, when they are co-located.

To deal with this set-up, given the finite set of sampling periods T_i $i \in \{1, 2, \dots, L\}$, we need to consider a set of discrete-time systems:

$$\begin{cases} x_{k+1} = A_i x_k + B_i u_k + w_k \\ y_k = C_i x_k + v_k \end{cases} \tag{1}$$

with $A_i = A(T_i)$, $B_i = B(T_i)$, $C_i = C_c$ and $w_k \sim \mathcal{N}(0,\, Q(T_i))$, $v_k \sim \mathcal{N}(0,\, R)$ with $R > 0$, where

$$A(\tau) = e^{A_c\tau} \qquad B(\tau) = \int_0^\tau e^{A_c t} B_c \mathrm{d}t$$

$$Q(\tau) = \int_0^\tau e^{A_c t} Q_c e^{A_c' t} \mathrm{d}t.$$

Consider w_k and v_k independent and identically distributed, and w_k independent of v_k. Assume that the initial state is a Gaussian random vector with mean x_0. Define the measurement model at the estimator as:

$$y_h^k = \gamma_h^k y_h = \gamma_h^k (C_i x_h + v_h)$$

where $\gamma_h^k \in \{0, 1\}$ indicates if the measurement y_h has been present at the estimator location at time $k > h$. In the following, to simplify the notation, if $i = 1$ we omit it, so: $A_1 = A$, $B_1 = B$, $C_1 = C$, and $Q_1 = Q$.

C. LQG cost

Consider the following quadratic cost measure:

$$J_M(u(t)) = \mathbb{E}\left[\frac{1}{M}\int_0^M \Big(x'(t)W_c x(t) + u'(t)U_c u(t)\Big)\mathrm{d}t \;\middle|\; u(t),\ t\in[0,M]\right]$$

with $W_c \geq 0$ and $U_c > 0$. At discrete time, with the sampling period T, and K such that $M = KT$, the cost can be rewritten as:

$$J_K(u_k) = c + \mathbb{E}\left[\frac{1}{K}\sum_{k=0}^{K-1} x_k' W x_k + 2x_k' N u_k + u_k' U u_k \;\middle|\; \{u_k\}_{k=0}^{K-1}\right]$$

with

$$\begin{aligned} W &= \frac{1}{T}\int_0^T A'(\tau)W_c A(\tau)\mathrm{d}\tau \\ N &= \frac{1}{T}\int_0^T A'(\tau)W_c B(\tau)\mathrm{d}\tau \\ U &= U_c + \frac{1}{T}\int_0^T B'(\tau)W_c B(\tau)\mathrm{d}\tau \\ c &= \frac{1}{T}\int_0^T \mathrm{trace}\left(Q(\tau)W_c\right)\mathrm{d}\tau. \end{aligned}$$

III. Optimal regulator for constant SNR

In this section, we derive the *Single-Rate Controller* (SRC). It has a fixed control rate, equal to the highest rate, i.e. $R^{\mathrm{ctr}} = R_1$, that can be associated to any transmission rate R^{tx}. With this definition, the sampling period of the system is $T = 1/R^{\mathrm{ctr}}$ and it is independent of R^{tx}. For now, consider R^{tx} fixed to a generic rate and the SNR constant: it follows that the arrival probability is constant.

We define the information set $\mathcal{I}_k$, that is the information available at the controller/estimator at time instant k:

$$\mathcal{I}_k = \left\{\{y_h^k\}_{h=0}^{k-1},\ \{\gamma_h^k\}_{h=0}^{k-1},\ \{u_h\}_{h=0}^{k-1}\right\}.$$

Note that at time instant k the measurement y_k is not available, since we assume a non-zero transmission time to send it over the network. We define the following variables:

$$\begin{aligned} \widehat{x}_{k|k-1}^t &:= \mathbb{E}[x_k \mid \mathcal{I}_t] \\ P_{k|k-1}^t &:= \mathbb{E}[(x_k - \widehat{x}_{k|k-1}^t)(x_k - \widehat{x}_{k|k-1}^t)' \mid \mathcal{I}_t]. \end{aligned}$$

Since the process noise and the measurement noise are Gaussian, $\widehat{x}_{k|k-1}^t$ is the optimal estimator [13].

With delayed packets, the optimal estimator can be found following [14]:

$$\begin{aligned} \widehat{x}_{k|k-1}^t &= A\widehat{x}_{k-1|k-2}^t + Bu_{k-1} \\ &\quad + \gamma_{k-1}^t K_{k-1}(y_{k-1}^t - C\widehat{x}_{k-1|k-2}^t) \\ K_{k-1} &= AP_{k-1|k-2}^t C'(CP_{k-1|k-2}^t C' + R)^{-1} \\ P_{k|k-1}^t &= AP_{k-1|k-2}^t A' + Q \\ &\quad -\gamma_{k-1}^t AP_{k-1|k-2}^t C'(CP_{k-1|k-2}^t C' + R)^{-1}CP_{k-1|k-2}^t A' \end{aligned}$$

The optimal estimator is time-varying and depends on the particular realization of the packet arrival process γ_{k-1}^t. The filter equations do not directly depend on R^{tx}, but it affects the values assumed by γ_{k-1}^t, that in cascade affect the error covariance $P_{k|k-1}^t$, the prediction gain K_{k-1}, and $\widehat{x}_{k|k-1}^t$.

In our framework we have assumed that a measurement arrives within the following measurements instant or it never arrives. With the SRC, the period between two following measurements contains $R^{\mathrm{ctr}}/R^{\mathrm{tx}}$ sampling periods. It follows that, if a measurement arrives before the following measurement instant, it can arrive with a delay up to $R^{\mathrm{ctr}}/R^{\mathrm{tx}}$ sampling periods. The optimal estimator works similar to [14] with bounded delay: when the packet arrives, the filter returns to the instant associated to the delivered measurement and updates the estimates from that instant to the current one.

Now we focus on the control input. We look for the causal feedback law that minimizes the LQG cost:

$$J_K^* = \min_{\{u_k\}_{k=0}^{K-1}} J_K(u_k), \quad \text{s.t.}\quad u_k = f_k(\mathcal{I}_k). \tag{2}$$

Proposition III.1 *Consider the discrete system (1) with sampling period T. Consider the finite-horizon LQG problem (2). Then, the cost can be rewritten as:*

$$J_K^* = c + \frac{1}{K}\Bigg(\hat{x}_0' S_0 \hat{x}_0 + \mathrm{trace}(S_0 P_0) + \sum_{k=0}^{K-1}\mathrm{trace}(S_{k+1}Q) + \mathrm{trace}\Big((A'S_{k+1}A + W - S_k)P_{k|k-1}^k\Big)\Bigg) \tag{3}$$

with

$$\begin{aligned} S_k &= A'S_{k+1}A + W \\ &\quad - (A'S_{k+1}B + N)(U + B'S_{k+1}B)^{-1}(B'S_{k+1}A + N') \end{aligned}$$

starting from $S_K = 0$. The optimal control is:

$$\begin{aligned} u_k &= L_k \hat{x}_{k|k-1}^k \\ L_k &= (U + B'S_{k+1}B)^{-1}(B'S_{k+1}A + N'). \end{aligned}$$

Proof: The theorem and its proof are an adaptation from [15], including the mixed term in the quadratic cost, excluding the penalty term on the final state, and using the one-step predictor instead of the filter. ■

The SRC is the combination of the optimal estimator and the optimal control given in this section. The control gain L_k is time-varying but it can be computed off-line and it does not depend on R^{tx}. On the other hand, the LQG cost depends on the adopted R^{tx}, due to the dependence on $P_{k|k-1}^k$. It follows that, if we consider two SRCs that adopt two different R^{tx}, they have the same control gain but different LQG costs.

IV. Optimal rate for constant SNR

In this section, under constant SNR, we compute the LQG cost specifically for each R^{tx}. We start from (3), that is the cost obtained by the SRC on the optimal control input. Note that, since the optimal control gain is independent of the choice of the transmission rate, to minimize the LQG cost first over only the input and then over only the transmission rate is equivalent to the joint minimization for the control and the rate. Since the error covariance depends on the arrival process, we consider the expected value of the infinite-horizon cost. If $\lim_{k\to\infty} S_k = S_\infty$ exists, then:

$$J^*_\infty(R^{\text{tx}}, SNR) := \lim_{K\to\infty} \sup \mathbb{E}_\gamma \left[J^*_K\right] = c + \text{trace}(S_\infty Q)$$
$$+ \lim_{K\to\infty} \sup \frac{1}{K} \sum_{k=0}^{K-1} \text{trace}\left((A' S_\infty A - S_\infty + W)\mathbb{E}_\gamma[P^k_{k|k-1}]\right).$$

where we emphasize the dependence on R^{tx} and on the SNR. Then, the optimal (i.e. minimum infinite-horizon LQG cost) transmission rate is:

$$R^* = \underset{R^{\text{tx}}}{\operatorname{argmin}}\, J^*_\infty(R^{\text{tx}}, SNR).$$

In order to evaluate $J^*_\infty(R^{\text{tx}}, SNR)$, we need to evaluate $\lim_{k\to\infty} \mathbb{E}_\gamma[P^k_{k|k-1}]$. As shown in [16], it can not be computed in closed-form, but it can be upper-bounded. In the following we use the operator:

$$g^{T_i}_\lambda(X) = A_i X A_i' + Q_i - \lambda A_i X C_i'(C_i X C_i' + R)^{-1} C_i X A_i'$$

with T_i the sampling period. As in [16], we define $\lambda_{i,c}$ as the $\arg\inf$ of the set of λ_i for which $g^{T_i}_{\lambda_i}(X) = X$ as a unique positive semidefinite solution.

First, we consider $R^{\text{tx}} = R^{\text{ctr}} = R_1$ and $\mathbf{P}[\gamma^k_{k-1} = 1] = \lambda_1$. The inequality

$$\mathbb{E}[P^k_{k|k-1}] \leq \overline{P}^k_{k|k-1}$$

where

$$\overline{P}^k_{k|k-1} = g^T_{\lambda_1}\left(\overline{P}^{k-1}_{k-1|k-2}\right), \quad \overline{P}^0_{0|-1} = \mathbb{E}\left[P^0_{0|-1}\right]$$

holds for every instant and so it is true also for $k \to \infty$. As stated in [16], if $\lambda_1 > \lambda_{1,c}$, the sequence of upper-bounds converges, i.e.:

$$\lim_{k\to\infty} \overline{P}^k_{k|k-1} = \overline{P}$$

and we can find the limit by solving $\overline{P} = g^T_{\lambda_1}(\overline{P})$. Then we can upper-bound the optimal LQG cost as:

$$\overline{J}^*_\infty(R_1) = c + \text{trace}(S_\infty Q) + \text{trace}\left((A' S_\infty A + W - S_\infty)\overline{P}\right)$$

We show the computation for the case of $R^{\text{tx}} = R_2$. In this scenario, the plant does not send the output at each sampling time, but it sends a packet every 2 sampling times. Without loss of generality, we assume that only the output at every even sampling times is sent, while the output at odd sampling times is never sent. Recalling that the measurement y_{2h} arrives in the time interval $[2h, 2h+2]$ or it never arrives, we can summarize:

$$\gamma^{2h+2+j}_{2h} = \gamma^{2h+2}_{2h} \quad \forall j \in \mathbb{N}$$
$$\gamma^{2h+2+j}_{2h+1} = 0 \quad \forall j \in \mathbb{N}.$$

The sent packets are successfully delivered within the next measurement instant with a probability λ_2. Within the period between two measurements, which is equal to 2 sampling periods, the packet arrives at the controller at a random time instant, that is assumed to be uniformly distributed. It follows that the output y_{2h} is available at the controller before the next control instant (i.e. $2h+1$) with a probability equal to $\lambda_2/2$. In conclusion, the arrival process can be modelled as:

$$\begin{cases} \mathbf{P}\left[\gamma^{2h+1}_{2h} = 1\right] = \frac{1}{2}\lambda_2 \\ \mathbf{P}\left[\gamma^{2h+2}_{2h} = 1\right] = \lambda_2 \end{cases}$$

With this hypothesis, we compute the cost. The error covariance assumes different steady-state values on the odd and on the even sampling instants. For this reason we can split the sum in two parts:

$$J^*_\infty(R_2, SNR) = c + \text{trace}(S_\infty Q)$$
$$+ \lim_{K\to\infty} \sup \frac{1}{K} \left(\sum_{\substack{k=0,\\ \text{even}}}^{K-1} \text{trace}\left((A' S_\infty A - S_\infty + W)\mathbb{E}_\gamma[P^k_{k|k-1}]\right) + \sum_{\substack{k=0,\\ \text{odd}}}^{K-1} \text{trace}\left((A' S_\infty A - S_\infty + W)\mathbb{E}_\gamma[P^k_{k|k-1}]\right) \right).$$

Proposition IV.1 *If $\lambda_2 > \lambda_{2,c}$, the optimal LQG cost can be upper-bounded by*

$$\overline{J}^*_\infty(R_2, SNR) = c + \text{trace}(S_\infty Q) + \frac{1}{2}\text{trace}\left((A' S_\infty A + W - S_\infty)\left(\overline{P}^{2h} + \overline{P}^{2h+1}\right)\right)$$

where

$$\overline{P}^{2h} = g^{2T}_{\lambda_2}(\overline{P}^{2h})$$
$$\overline{P}^{2h+1} = g^T_{\frac{\lambda_2}{2}}(\overline{P}^{2h}).$$

Proof: The proof is omitted for reason of space. It follows from $g^T_\lambda \circ g^T_0(X) = g^{2T}_\lambda(X)$ and the properties studied in [16]. ■

Following the same procedure, we can find the steady-state upper-bound of error covariance and the cost for every transmission rates. For example, if $R^{\text{tx}} = R_3$ it holds:

$$\overline{P}^{3h} = g^{3T}_{\lambda_3}(\overline{P}^{3h})$$
$$\overline{P}^{3h+1} = g^T_{\frac{\lambda_3}{3}}(\overline{P}^{3h})$$
$$\overline{P}^{3h+2} = g^{2T}_{\frac{2}{3}\lambda_3}(\overline{P}^{3h})$$

and

$$\overline{J}^*_\infty(R_3, SNR) = c + \text{trace}(S_\infty Q) + \frac{1}{3}\text{trace}\left((A' S_\infty A + W - S_\infty)\left(\overline{P}^{3h} + \overline{P}^{3h+1} + \overline{P}^{3h+2}\right)\right)$$

For the general case where $R^{\text{tx}} = R_i$, the cost is:

$$\overline{J}^*_\infty(R_i, SNR) = c + \text{trace}(S_\infty Q) + \frac{1}{i}\text{trace}\left((A' S_\infty A + W - S_\infty) \cdot \sum_{j=0}^{i-1} \overline{P}^{ih+j}\right).$$

V. Optimal solution for time-varying SNR

In the time-varying case we do not simultaneously optimize for the rate and control, since this requires the knowledge of statistics on the behaviour of the SNR. We decide to fix the rate policy based on the static scenario. At each step, the rate $R(k)$ to transmit y_k is selected to minimize the infinite-horizon LQG cost as if the SNR remains constant and equal to the value of time instant k, i.e. SNR_k:

$$R(k) = \underset{R^{\text{tx}}}{\operatorname{argmin}}\, J^*_\infty(R^{\text{tx}}, SNR_k) \tag{4}$$

Note that $R(k)$ determines also the period before the following sent packet. Without any modification, the SRC is able to switch among different rates and so it can adopt this rate selection algorithm. Indeed, the SRC requires to have an estimate at every sampling period but it is not sensitive to how the estimate is obtained. The time-varying estimator works with every transmission rate, because the missing of a packet due to the adoption of a lower transmission rate is treated as a packet lost. Interestingly, the SRC is still optimal also with time-varying SNR. The optimality follows from the Proposition III.1, that gives the optimal control input if the sampling period is constant, which is the case of the SRC. The optimality of the estimator is preserved for the given information set.

VI. Simulations

The system used for the test is a pendulum on a cart, for which we consider the state-space model linearized in the neighbourhood of the origin [17]:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -33.746 & -2.1107 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 111.33 & 39.327 & 0 \end{bmatrix} \qquad B_c = \begin{bmatrix} 0 \\ 5.3725 \\ 0 \\ -17.724 \end{bmatrix}$$

$$C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \qquad D_c = [0]$$

and we choose the following parameters:

$$\sqrt{Q_c} = \begin{bmatrix} 10^{-8} & 0 & 10^{-8} & 0 \end{bmatrix}' \quad Q_c = \sqrt{Q_c}\sqrt{Q_c}' \quad R = 10^{-8}$$

$$\sqrt{W_c} = \begin{bmatrix} 10 & 100 & 0 & 0 \end{bmatrix} \quad W_c = \sqrt{W_c}'\sqrt{W_c} \quad U_c = 1.$$

The sampling periods are $\{T, 2T, 3T\}$, where $T = 0.005$ s, and the inverses give the rates. We associate R_1, R_2, R_3 to the data-rates $54, 36, 18$ Mbit/s, respectively, and the loss probabilities from Fig. 1. The asymptotic costs plotted in Fig. 2 are computed in according to the formulas of the Sec. IV with constant SNR. Note that the cost is not evaluated in the entire range of SNR but only in the range for which the arrival probability allows to compute the steady-state upper-bound of the error covariance, i.e. $\lambda_i > \lambda_{i,c}$. According to our algorithm, the optimal rate for a given fixed SNR is the one which gives the lowest cost, i.e. the lowest curve. The cost obtained by choosing always the optimal rate is indicated by the dot line. As it is depicted in Fig. 3, we see that the optimal rate is not always the one for which the loss probability is the lowest. To illustrate the benefits of our adaptive scheme, we decide to compare the SRC to

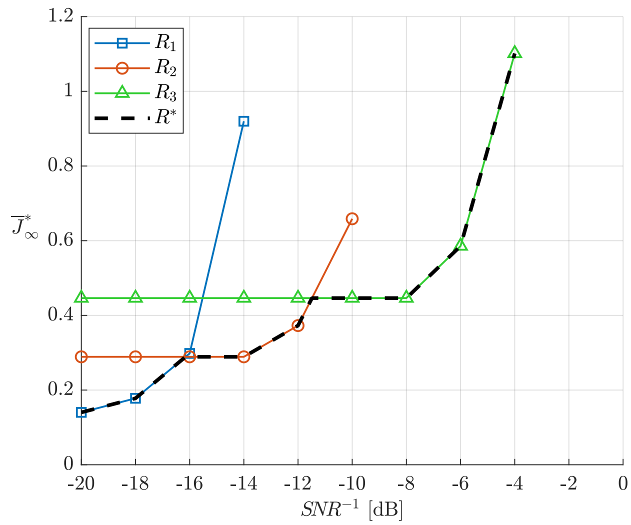

Fig. 2: LQG cost.

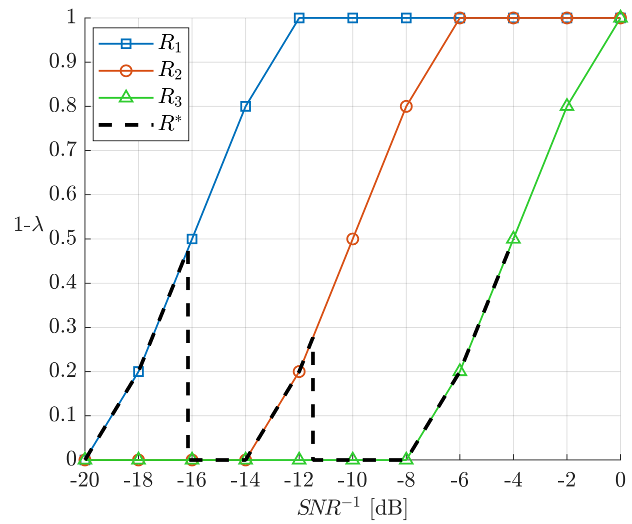

Fig. 3: Optimal loss probability.

the LQG controller over lossy network devised in [15]. The implementations of the two regulators are quite similar in term of complexity and both require approximately the same computational capabilities. In both, we add integral action for tracking of step reference. The testbed is the response to a square wave under a piece-wise constant SNR:

$$SNR(t) = \begin{cases} 20 \text{ dB} & t \in (0 \text{ s}, 25 \text{ s}) \\ 14 \text{ dB} & t \in (25 \text{ s}, 50 \text{ s}) \\ 6 \text{ dB} & t \in (50 \text{ s}, 75 \text{ s}) \end{cases}$$

By construction, the SRC is able to switch among the three rates without any modification; on the other hand, we consider two LQG controllers over lossy network with fixed rate, one with the highest rate (that gives the sampling period T), and one with the slowest rate (that gives the sampling period $3T$). The results are reported in Fig. 4. We see that, in ideal condition with high SNR, i.e. in $(0 \text{ s}, 25 \text{ s})$, the SRC and the LQG with the highest rate clearly

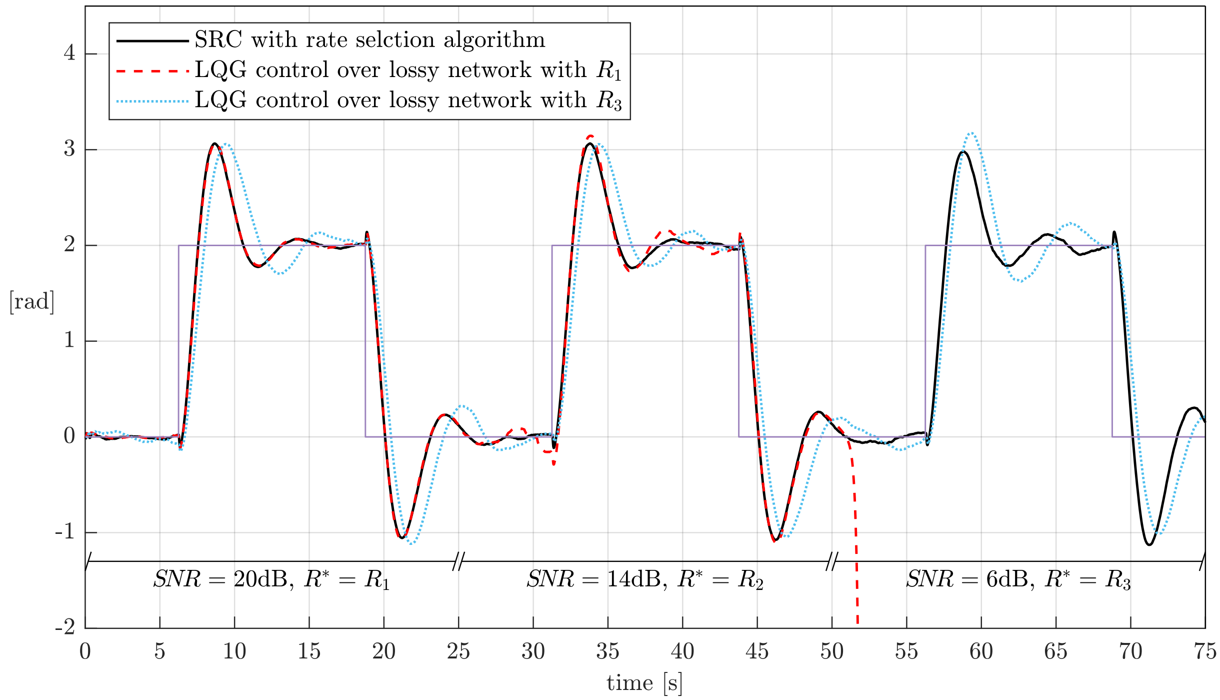

Fig. 4: Response to square wave among the three controllers.

outperform the LQG controller with the slowest rate. The latter achieves nearly the same overshoot but a settling time approximately 5 s longer than the other two. When the SNR decreases, the loss probability obtained with R_1 becomes relevant. In $(25\text{ s}, 50\text{ s})$, the controller with R_1 achieves worse performances. In $(50\text{ s}, 75\text{ s})$, it becomes unstable, while the SRC and the LQG with the smallest rate are still reliable, but with worse performances due to the higher packet loss. The SRC with adaptive rate achieves both the reliability of the controller with the most robust transmission rate and the performance (in terms of overshoot and settling time) of the controller with the smallest sampling time.

VII. Conclusion

In this paper, we have considered the problem of LQG control of a continuous system, where the sensor and the controller are connected through a wireless link based on Wi–Fi. We devised an LQG controller, named SRC, which is able to deal with time-varying SNR, and we provided an optimal choice of the transmission rate. We shown that our controller outperforms other LQG controllers over lossy network.

References

[1] J. Kim and I. Lee, "802.11 WLAN: history and new enabling mimo techniques for next generation standards," *IEEE Communications Magazine*, vol. 53, no. 3, pp. 134–140, 2015.

[2] F. Tramarin, S. Vitturi, M. Luvisotto, and A. Zanella, "On the use of IEEE 802.11 n for industrial communications," *IEEE Transactions on Industrial Informatics*, vol. 12, no. 5, pp. 1877–1886, 2016.

[3] J. Eker, A. Cervin, and A. Hörjel, "Distributed wireless control using bluetooth," *IFAC Proceedings Volumes*, vol. 34, no. 22, 2001.

[4] Y.-Q. Xia, Y.-L. Gao, L.-P. Yan, and M.-Y. Fu, "Recent progress in networked control systems—a survey," *International Journal of Automation and Computing*, vol. 12, no. 4, pp. 343–367, 2015.

[5] X.-M. Zhang, Q.-L. Han, and X. Yu, "Survey on recent advances in networked control systems," *IEEE Transactions on Industrial Informatics*, vol. 12, no. 5, pp. 1740–1752, 2016.

[6] J. Yick, B. Mukherjee, and D. Ghosal, "Wireless sensor network survey," *Computer networks*, vol. 52, no. 12, pp. 2292–2330, 2008.

[7] Y.-H. Wei, Q. Leng, S. Han, A. K. Mok, W. Zhang, and M. Tomizuka, "RT-wiFi: Real-time high-speed communication protocol for wireless cyber-physical control applications," in *Real-Time Systems Symposium (RTSS), 2013 IEEE 34th*. IEEE, 2013, pp. 140–149.

[8] F. Tramarin, S. Vitturi, and M. Luvisotto, "A dynamic rate selection algorithm for IEEE 802.11 industrial wireless LAN," *IEEE Transactions on Industrial Informatics*, vol. 13, no. 2, pp. 846–855, 2017.

[9] D. Xia, J. Hart, and Q. Fu, "Evaluation of the minstrel rate adaptation algorithm in IEEE 802.11 g WLANs," in *Communications (ICC), 2013 IEEE International Conference on*. IEEE, 2013, pp. 2223–2228.

[10] J. Bai, E. P. Eyisi, F. Qiu, Y. Xue, and X. D. Koutsoukos, "Optimal cross-layer design of sampling rate adaptation and network scheduling for wireless networked control systems," in *Proceedings of the 2012 IEEE/ACM Third International Conference on Cyber-Physical Systems*. IEEE Computer Society, 2012, pp. 107–116.

[11] P. Park, J. Araújo, and K. H. Johansson, "Wireless networked control system co-design," in *Networking, Sensing and Control (ICNSC), 2011 IEEE International Conference on*. IEEE, 2011, pp. 486–491.

[12] T. Karhima, A. Silvennoinen, M. Hall, and S.-G. Haggman, "IEEE 802.11 b/g WLAN tolerance to jamming," in *Military Communications Conference, 2004. MILCOM 2004. 2004 IEEE*, vol. 3. IEEE, 2004, pp. 1364–1370.

[13] B. D. Anderson and J. B. Moore, "Optimal filtering," *Englewood Cliffs*, vol. 21, pp. 22–95, 1979.

[14] L. Schenato, "Optimal estimation in networked control systems subject to random delay and packet drop," *IEEE transactions on automatic control*, vol. 53, no. 5, pp. 1311–1317, 2008.

[15] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry, "Foundations of control and estimation over lossy networks," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 163–187, 2007.

[16] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan, and S. S. Sastry, "Kalman filtering with intermittent observations," *IEEE transactions on Automatic Control*, vol. 49, no. 9, pp. 1453–1464, 2004.

[17] N. Muskinja and B. Tovornik, "Swinging up and stabilization of a real inverted pendulum," *IEEE transactions on industrial electronics*, vol. 53, no. 2, pp. 631–639, 2006.