Lifetime Optimization for Wireless Sensor Networks with Outage Probability Constraints [1]

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Abstract: **Due to the limited energy resources in a wireless sensor network (WSN), lifetime of a WSN is a key parameter. In this paper, we address a lifetime optimization problem of a wireless TDMA/CDMA sensor network for joint transmit power and rate allocations in a time-varying fast Rayleigh fading environment. The effect of fast fading is captured by including rate outage and link outage constraints on each link and a given time-slot. A resulting nonconvex problem is then reduced to an approximate convex optimization problem using an appropriate functional approximation and variable substitutions. This centralized problem is then solved by standard barrier-method based optimization algorithms. A partially distributed algorithm is also provided to illustrate how most of the computations can be done locally at each node in a decentralized manner. The novelty of the paper lies in considering fast fading channels via outage probability constraints for the first time in lifetime maximization problems and obtaining a better convex approximation than previously used approximations in the literature. Simulation results illustrate that our centralized algorithm results in optimal power and rate allocations that result in a substantially extended lifetime of the WSN compared to previously published algorithms. The convergence of the partially distributed algorithms to the optimal power and rate solutions is also illustrated.**

1. Introduction

Wireless sensor networks (WSN) have become a key technology for the 21st century due to its widespread applications in security, health, disaster response, defense, telecommunications, structural health monitoring etc. These networks usually consist of a collection of sensor nodes connected by wireless communication links. The sensor nodes usually have on-board battery, communication and computation capability. However, due to limited energy resources and a distinct lack of coordination (compared to cellular networks), the usefulness of these networks can become limited unless special care is taken to optimize energy consumption in communication and computation. Optimizing the lifetime of a WSN is thus an important problem. Due to the *ad hoc* nature of the WSN deployment, and the random nature of the wireless links, proper cross-layer optimization techniques are needed to maximize the lifetime of these WSN's. The physical layer requires the devices in the network to be energy-conserving [11] and transmit power control is needed, specially for interference limited networks. The MAC protocol in the network layer also needs to be designed efficiently including sleep coordination of sensors, transmission scheduling, as this has important consequences on ensuring longer battery life. Similarly, energy efficient routing schemes need to be designed to extend the lifetime of WSNs [3]. Furthermore, optimal congestion control in the transmission layer is needed. Finally, if the quality of a connection drops below a certain level, the application layer can be configured to lower the QoS requirements to save energy. In summary, maximizing the lifetime of a WSN is a complex cross-layer optimization problem that involves both continuous and discrete variables, delays, randomly time-varying parameters etc. There are several definitions of the lifetime of a sensor network, such as those adopted in [3], [5] and [12]. In this paper, we adopt the same definition as in [3], in that the lifetime is defined as the length of time before the first node runs out of energy.

In [4], a lifetime optimization problem for sensor networks is considered where the multiple access scheme is TDMA and the optimization problem is to allocate the length of time-slots according to the average power consumption. Because of only one link being activated each time, no interference is considered. In addition, the transmission rate is kept fixed over each link and the transmission power is proportional to some integral power of the distance. In [8], the authors study a TDMA/CDMA based sensor network system in the Gaussian multi-access channel and interference is taken into consideration. In our work, we adopt this interference limited TDMA/CDMA based sensor network model and formulate the lifetime optimization problem as a nonlinear non-convex optimization problem over link transmit powers and rates. The novelty lies in considering fast Rayleigh fading in the wireless links (due to mobility) in terms of rate and link outage probability constraints, as opposed to static fading considered in previous works. The transmit powers and the rates over the various links in individual time-slots are allocated as a function of the slow fading parameters and the statistics of the fast fading channels. This is important because in a fast fading environment, having to track the rapidly changing channel can result in costly overheads which can substantially reduce the lifetime of the network. We also obtain a more efficient convex approximation scheme compared to [8], as the high signal-to-noise ratio assumption used in [8] is not applicable to our situation. It is shown through simulation studies that our optimization algorithm achieves a substantially ex-

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tended lifetime than obtained by the one presented in [8]. A partially distributed algorithm is also presented and simulation studies confirm that the iterative optimization technique to update the link rates and transmit powers converges to the optimal values found by the centralized algorithm.

2. Network Model and Problem Formulation

We consider a sensor network consisting of several homogeneous sensor nodes, which is a synchronous time-slotted joint TDMA/CDMA system. Sensors transmit their data through a Gaussian multi-access channel that is interference limited. The signal-to-interference and noise ratio (SINR) at the receiver is defined as the ratio of the received power of the desired transmitter and interference from other simultaneously transmitting users plus additive white Gaussian noise (AWGN). The channels between any transmitter and any receiver are modelled as fast Rayleigh fading channels where the mean received power from a specific transmitter is governed by the distance between the transmitter and receiver. More details on this fading channel model will be provided later. Routing methods are not within the scope of this paper; the multi-hop route between the source and sink node is assumed to be fixed during the process of data relay. Similarly, we do not optimize over transmission scheduling in this paper either, as this leads to a complex mixed-integer non-linear optimization problem. In determining the optimal power and transmission rates, an arbitrary but fixed scheduling scheme is employed.

Before proceeding any further, we introduce the following notations. The directed graph $\mathcal{G}(V, L, H)$ represents the geometric configuration of the wireless sensor network. V is the set of all the $|V|$ nodes. L symbolizes the links and $H \in \mathbb{R}^{|L|\times|L|}$ denotes the propagation matrix. A valid link is established when the receiver can interpret the signal sent with the maximum power from the transmitter in an interference-free environment. The propagation channel between the transmitter of link k and the receiver of link l is given by $H_{lk} = G_{lk}F_{lk}$ where G_{lk} denotes the slow fading gain and F_{lk} denotes the fast fading gain. The component G_{lk} is given by $G_{lk} = \frac{c}{d_{lk}^m}$ where d_{lk} is the distance between the transmitter of link k and the receiver of link l and m is between 2 and 6, depending on the nature of the terrain and c is a constant. Note that G_{lk} can be extended to also include the random shadow fading component. However, fixing G_{lk} to be only distance-dependent does not result in any loss of generality, as in practice, G_{lk} (even if it includes shadow fading) can be accurately estimated (due to its time scale of change being much slower compared to the time scale of operation) and is a known constant for the purpose of the optimization problem considered in this paper. F_{lk} denotes the fast fading gain for the same transmission channel and in this paper is modelled as a unity mean exponentially distributed random process (denoting Rayleigh fading). We use three matrices T^+, T^-, and $T(= T^+ - T^-) \in \mathbb{Z}^{|V|\times|L|}$ to denote the network topology where

$$T_{vl}^+ = \begin{cases} 1 & \text{if } v \text{ is the transmitter of link } l \\ 0 & \text{otherwise} \end{cases}$$

$$T_{vl}^- = \begin{cases} 1 & \text{if } v \text{ is the receiver of link } l \\ 0 & \text{otherwise} \end{cases}$$

and $T_{vl} = T_{vl}^+ - T_{vl}^-$. Each frame is equally divided into $|N|$ time slots. $S \in \mathbb{Z}^{|N|\times|L|}$ denotes the arbitrary scheduling matrix, where $S_{nl} = 1$ if there exists a traffic load on link l in time-slot n and it is 0 otherwise. Accordingly, if $S_{nl} = 1$, the transmission power P_{nl} and the rate r_{nl} are positive; otherwise, $P_{nl} \equiv r_{nl} \equiv 0$. This implies that the total number of variables, as well as that of constraints of the optimization problem of interest, is equal to the sum of 1's in matrix S. Since the route from the source to the sink node is fixed during the transmission, we obtain the average rate in a frame on certain link l as $R_l = \sum_{l \in \text{route of } s_i} I(s_i)$ where $I(s_i)$ is the input Rate from source s_i. $E_v \in \mathbb{R}^{|V|}$ is the initial battery energy in the v-th node, and the average power of the background Gaussian noise over the system bandwidth is N_0.

In this paper, we consider a sensor network lifetime optimization problem, which is an extension of the problem considered in [8]. The novelty lies in considering the propagation matrix H to be randomly time-varying with a two-time scale (slow and fast) nature and knowledge is assumed only of the slow fading component G. Due to the fast variation of the component F, SINR and the hence the capacity on any given channel cannot be always guaranteed. This results in two kinds of possible outages in this system- link outage (when SINR falls below the required threshold for a particular receiver) and rate (or capacity) outage (when the sustainable rate over a channel falls below the actual rate over that channel). Noting that the instantaneous SINR at the receiver for link i in the time-slot n is given by $\frac{G_{ii}F_{ii}P_{ni}}{\sum_{k\neq i} G_{ik}F_{ik}P_{nk}+N_0}$, the Link Outage Probability (LOP) in time-slot n for link i (where each F_{ik} is independently identically exponentially distributed with unity mean) is given by [7] (γ_{th} is the required SINR threshold to meet the target bit error rate (BER))

$$\begin{aligned} O_{ni} &= Pr\left(\frac{G_{ii}F_{ii}P_{ni}}{\sum_{k\neq i} G_{ik}F_{ik}P_{nk} + N_0} < \gamma_{th}\right) \\ &= 1 - e^{\frac{-N_0\gamma_{th}}{G_{ii}P_i}} \prod_{k\neq i} \frac{1}{1 + \frac{\gamma_{th}G_{ik}P_{nk}}{G_{ii}P_{ni}}} \end{aligned} \quad (1)$$

This probability can be upper bounded as (see [7]) $O_{ni} \leq 1 - e^{-\frac{\gamma_{th}(\sum_{k\neq i} G_{ik}P_{nk}+N_0)}{G_{ii}P_{ni}}}$. It was shown in [7], [9] that this upper bound is very tight when the outage probability is less than 20%.

Note also that the Shannon capacity of a Gaussian multi-access channel with digitally modulated data is given by $\log(1 + K * SINR)$ where K is a constant depending on the modulation scheme. For example, $K = \frac{-1.5}{\log(5BER)}$ for M-QAM [6]. Now, one can define the Rate Outage Probability (ROP) on link l in time-slot

n as ROP_{nl}

$$\triangleq Pr\Big(\log\big(1+K\frac{G_{ll}F_{ll}P_{nl}}{\sum_{k\neq l}G_{lk}F_{lk}P_{nk}+N_0}\big)<r_{nl}\Big)$$
$$=1-e^{-\frac{N_0\frac{e^{r_{nl}}-1}{K}}{G_{ll}P_{nl}}}\prod_{k\neq l}\Big(\frac{1}{1+\frac{\frac{e^{r_{nl}}-1}{K}G_{lk}P_{nk}}{G_{ll}P_{nl}}}\Big)$$
$$\leq 1-e^{-\frac{\sum_{k\neq l}G_{lk}P_{nk}+N_0}{G_{ll}P_{nl}}\bullet\frac{e^{r_{nl}}-1}{K}}$$

In the optimization problem addressed in this paper, we require that the ROP and/or the LOP are less than their respective maximum outage probability targets O_{th}^{Rate} and O_{th}^{link}. Clearly, this can be guaranteed by ensuring the ROP and LOP upper bounds described above are less than the corresponding targets. With these outage probability constraints in mind, we consider the following optimization problem (with the ROP constraint only, the LOP constraint will be included later):

$$\max_{\mathbf{P},\mathbf{r}}\ \min_{v=1,\ldots,|V|} t_v \quad (2)$$
$$\text{s.t.}\quad \frac{1}{N}\sum_{n=1}^{N} r_{nl}\geq R_l \quad l=1,\ldots,|L|$$
$$\log\big(1+\beta_{\text{ROP}}\frac{G_{ll}P_{nl}}{\sum_{k\neq l}G_{lk}P_{nk}+N_0}\big)\geq r_{nl}\quad \text{if } S_{nl}=1$$
$$\frac{t_v}{N}\sum_{n=1}^{N}\sum_{l=1}^{|L|}(T_{vl}^{+}P_{nl})\leq E_v \quad v=1,\ldots,|V|$$
$$r_{nl},P_{nl}\geq 0$$

where $\beta_{\text{ROP}}=-K\log(1-O_{th}^{Rate})$. Here the objective function is the network lifetime defined as $t_{\text{net}}=\min_{v=1,\ldots,|V|}t_v$ (the first time instant when a node uses up all its energy). The first group of constraints guarantees the amount of traffic transmitted over one frame is equal to the requirement from different sources on each link. The second group of constraints relates to the rate outage probability being less than a certain threshold, whereas the third set describes the energy constraints and the last constraint simply ensures that all power and rate variables are nonnegative. The above optimization problem (2) is a variant of the one considered in [8] where only static fading was considered. Just like the one in [8], this is also a non-convex optimization problem. Similar to [8], we only include transmit powers as variables of optimization and ignore power consumption due to receiving and processing data etc. In [8], the authors use a convex approximation to their non-convex problem by simply assuming $\log(1+K*SINR)\approx\log(K*SINR)$ under a high $SINR$ assumption and then derive a convex optimization problem via suitable variable substitutions. For our problem stated in (2), this approximation does not work, as we show in the following example. Suppose $BER=10^{-4}$, $O_{th}^{rate}=1\%$, and $SINR=10dB$ in a M-QAM case, then

$$\beta_{\text{ROP}}\bullet\text{SINR}=\frac{-1.5}{\log(5\text{BER})}\big(-\log(1-O_{th}^{rate})\big)10^{\frac{10}{10}}$$
$$=0.0198\ll 1$$

Clearly, we need to come up with a more suitable convex approximation, which is discussed in the next section.

3. Convex Approximation and Solutions

Define $\alpha_{nl}=\frac{1}{\text{SINR}}=\frac{\sum_{k\neq l}G_{lk}P_{nk}+N_0}{G_{ll}P_{nl}}$ where $SINR$ here stands for the signal-to-interference and noise ratio for the l-th link and n-th time-slot. The rate outage probability constraint (for the l-th link and n-th time-slot can now be expressed as $\log(1+\frac{\beta_{\text{ROP}}}{\alpha_{nl}})\geq r_{nl}$ or $\frac{\alpha_{nl}}{\beta_{\text{ROP}}+\alpha_{nl}}\leq e^{-r_{nl}}$. If SINR is sufficiently large, then the above constraint is given by $\frac{\alpha_{nl}}{\beta_{\text{ROP}}+\alpha_{nl}}<\frac{\alpha_{nl}}{\beta_{\text{ROP}}}\leq e^{-r_{nl}}$. The right hand side inequality is actually the approximation $\log(1+\frac{\beta_{\text{ROP}}}{\alpha_{nl}})\approx\log(\frac{\beta_{\text{ROP}}}{\alpha_{nl}})$ which is similar to the one used in [8]. Our objective is to find a better approximation $f(\alpha_{nl})$, which satisfies $\frac{\alpha_{nl}}{\beta_{\text{ROP}}+\alpha_{nl}}\leq f(\alpha_{nl})\leq\frac{\alpha_{nl}}{\beta_{\text{ROP}}}$ We can simply use a tangent approximation $f(\alpha_{nl})=a_{nl}\alpha_{nl}+b_{nl}$ by appropriately choosing a tangent point α_{nl}. This can be done by scheduling initial transmissions for various links where the transmitters use some known or average powers. Once the $SINR$ for the desired signal is measured, α_{nl} can be estimated as $\hat{\alpha}_{nl}=\frac{\sum_{k\neq l}G_{lk}\bar{P}_{nk}+N_0}{G_{ll}\bar{P}_{nl}}$ where $\bar{P}_{nl}$, $\bar{P}_{nk}$ are the transmit powers for the various transmitters for the n-th time slot. Another way to estimate the tangent point is to divide the flow requirement by the total number of effective time-slots to obtain $\hat{r}_{nl}=\frac{R_l N}{\sum_{i=1}^{N}S_{il}}\quad\forall n$ s.t. $S_{nl}=1$ α_{nl} is then estimated by $\hat{\alpha}_{nl}=\frac{\beta_{\text{ROP}}}{e^{\hat{r}_{nl}}-1}$. a_{nl},b_{nl} for the tangent approximation are then obtained by $a_{nl}=\frac{\beta_{\text{ROP}}}{(\hat{\alpha}_{nl}+\beta_{\text{ROP}})^2}\geq 0$, $b_{nl}=\frac{\hat{\alpha}_{nl}^2}{(\hat{\alpha}_{nl}+\beta_{\text{ROP}})^2}\geq 0$ Note that one can iteratively update the tangent approximation during the various intermediate points in the optimization procedure. However, the effect of this iterative approximations on the convergence of the optimization procedure is not well understood and therefore we stick to a fixed initial tangent approximation only. It is shown through simulation studies that our method achieves substantial improvement in the network lifetime compared to the convex approximation used in [8]. As in [8], we do the following variable transformations to form a convex approximation problem: we substitute P_{nl} with $e^{Q_{nl}}$ and replace the network lifetime by its inverse q. This results in the following reformulation of the optimization problem (2):

$$\min_{\mathbf{Q},\,\mathbf{r}} q \quad (3)$$
$$\text{s.t.}\quad \frac{1}{N}\sum_{n=1}^{N} r_{nl}\geq R_l \quad l=1,\ldots,|L|$$
$$\log\big(a_{nl}\sum_{k\neq l}\frac{G_{lk}}{G_{ll}}e^{r_{nl}+Q_{nk}-Q_{nl}}+a_{nl}\frac{N_0}{G_{ll}}e^{r_{nl}-Q_{nl}}$$
$$+b_{nl}e^{r_{nl}}\big)\leq 0\quad \text{if } S_{nl}=1$$
$$\sum_{n=1}^{N}\sum_{l=1}^{|L|}(T_{vl}^{+}e^{Q_{nl}})-E_v qN\leq 0\quad v=1,\ldots,|V|$$
$$r_{nl}\geq 0\quad \text{if } S_{nl}=1$$

It can be shown [2] that (3) is a convex optimization problem in $\mathbf{Q},\ \mathbf{r}$. If one wishes to include the LOP constraints as well, one can similarly obtain the following convex optimization problem:

$$\min_{\mathbf{Q},\,\mathbf{r}} q \tag{4}$$

$$\text{s.t.}\quad \frac{1}{N}\sum_{n=1}^{N} r_{nl} \geq R_l \quad l=1,\dots,|L|$$

$$\log\big(a_{nl}\sum_{k\neq l}\frac{G_{lk}}{G_{ll}}e^{r_{nl}+Q_{nk}-Q_{nl}} + a_{nl}\frac{N_0}{G_{ll}}e^{r_{nl}-Q_{nl}} + b_{nl}e^{r_{nl}}\big) \leq 0 \quad \text{if } S_{nl}=1$$

$$\sum_{n=1}^{N}\sum_{l=1}^{|L|}(T_{vl}^{+}e^{Q_{nl}}) - E_v q N \leq 0 \quad v=1,\dots,|V|$$

$$\log\big(\beta_{\text{LOP}}\sum_{k\neq l}\frac{G_{lk}}{G_{ll}}e^{Q_{nk}-Q_{nl}} + \beta_{\text{LOP}}\frac{N_0}{G_{ll}}e^{-Q_{nl}}\big) \leq 0 \quad \text{if } S_{nl}=1,\ \beta_{\text{LOP}} = \frac{\gamma_{th}}{-\log(1-O_{th}^{link})}$$

$$r_{nl} \geq 0$$

Note that in these optimization problems, the maximum ROP and LOP targets are taken to be the same for all links and time-slots to maintain simplicity but the current formulation can accommodate unequal rate and link outage targets also.

There are a number of optimization algorithms to solve the convex optimization problems (3) and (4), if one is only interested in a *centralized* solution. We use the well-known Barrier Method [2]. The drawback of this method is that the initial point has to be within the feasible region. Therefore, before the actual optimization starts, we use some initial computations to find a feasible point. In order to make the algorithm converge fast, we use the BFGS Hessian Matrix Approximation [1] which is one of the Quasi-Newton methods. It is well known that due to the energy, communication and computational limitations of ad hoc wireless sensor networks, distributed algorithms are preferred to centralized ones. In the next section, we provide a partially distributed algorithm to solve the above optimization problems. Simulations results for all (centralized and distributed) algorithms are presented in Section 5..

4. A Partially Distributed Algorithm

Given that the above nonlinear optimization problems are convex, we can use the standard primal-dual method to form a Lagrangian and then use the sub-gradient method to derive the optimal solutions via iterative optimization techniques. We assume that Slater's condition holds such that there is no duality gap. We define the following optimization problem:

$$\max_{\lambda,\mu,\mathbf{v},\mathbf{w}}\ \inf_{\mathbf{Q},\mathbf{r},q} L(q,\mathbf{Q},\mathbf{r},\lambda,\mu,\mathbf{v},\mathbf{w})$$

where $L(q,\mathbf{Q},\mathbf{r},\lambda,\mu,\mathbf{v},\mathbf{w})$

$$= q^2 + \sum_{l=1}^{|L|}\lambda_l\big(\frac{1}{N}\sum_{n=1}^{N} r_{nl} - R_l\big) + \sum_{v=1}^{|V|}\mu_v\big(\sum_{n=1}^{N}\sum_{l=1}^{|L|}T_{vl}^{+}e^{Q_{nl}} - E_v q N\big)$$
$$+ \sum_{n,l:\,S_{nl}=1}\nu_{nl}\big(\log(a_{nl}\frac{N_0}{G_{ll}}e^{-Q_{nl}} + a_{nl}\sum_{k\neq l}\frac{G_{lk}}{G_{ll}}e^{Q_{nk}-Q_{nl}} + b_{nl}) + r_{nl}\big)$$
$$+ \sum_{n,l:\,S_{nl}=1}\omega_{nl}\big(\log(\beta_{\text{LOP}}) + \log(\frac{N_0}{G_{ll}}e^{-Q_{nl}} + \sum_{k\neq l}\frac{G_{lk}}{G_{ll}}e^{Q_{nk}-Q_{nl}})\big)$$

where $\lambda,\mu,\mathbf{v},\mathbf{w}$ are the various Lagrangian multipliers. Note that we substitute the objective function q with q^2, in order to increase the speed of convergence with constant stepsizes. κ's denote the stepsizes of the following iterative update equations for the variables of optimization (lifetime, rate, power) and the Lagrange multipliers. The superscript t denotes the iteration index. Below, we summarize the update equations for our partially distributed algorithm. Due to lack of space, the derivations are not shown here.

Updates of Lifetime, Rate, and Power:
Inverse of lifetime q :

$$q^{(t+1)} = q^{(t)} - \kappa_q\big(2q^{(t)} - N\sum_{v=1}^{|V|}\mu_v^{(t)}E_v\big) \tag{5}$$

It is clear that the update of the inverse lifetime - q - needs information from all nodes in the network and can be implemented by a central coordinator that has access to information to and from all nodes or by sequential processing through the various sensor nodes concerned. All other computations (rate, power etc.) can be distributed among the individual nodes as will be seen below.
Rate variables $\mathbf{r}$ *if* $S_{nl}=1$:

$$r_{nl}^{(t+1)} = r_{nl}^{(t)} - \kappa_r\big(\frac{\lambda_l^{(t)}}{N} + \nu_{nl}^{(t)}\big) \tag{6}$$

The 'old' r_{nl}, λ_l and ν_{nl} can be stored in the transmitter of link l and the 'new' flow $r_{nl}^{(t+1)}$ information can be included in the next transmission frame.
Power variables $\mathbf{Q}$ *or* $\mathbf{P}$ *if* $S_{nl}=1$: Suppose v is the transmitting node for the link l. Then,

$$Q_{nl}^{(t+1)} = Q_{nl}^{(t)} - \kappa_Q\Big[\mu_v^{(t)}P_{nl}^{(t)} - \frac{a_{nl}\nu_{nl}^{(t)}}{a_{nl}+b_{nl}\text{SINR}_{nl}^{(t)}} - \omega_{nl}^{(t)} + P_{nl}^{(t)}\sum_{i\neq l}G_{il}\Phi_{ni}^{(t)}\Big] \tag{7}$$

$$\Phi_{ni} = \frac{\nu_{ni}a_{ni}\text{SINR}_{ni}}{G_{ni}P_{ni}(a_{ni}+b_{ni}\text{SINR}_{ni})} + \frac{\omega_{ni}\text{SINR}_{ni}}{G_{ii}P_{ni}}$$

$$P_{nl}^{(t+1)} = e^{Q_{nl}^{(t+1)}} \tag{8}$$

Note that each receiver of link i can calculate Φ_{ni} locally by estimating the SINR in the n-th time-slot and then this can be communicated to the transmitter of link l, where the node collects this information as interference. The rest of the terms within the square brackets of the r.h.s. of (7) can be calculated locally at the transmitter of link l.

Updates of Lagrangian Parameters:

Note that all the Lagrangian parameters for each time slot can be computed locally at the various nodes.

$$\lambda_l^{(t+1)} = \lambda_l^{(t)} + \kappa_\lambda(\frac{1}{N}\sum_{n=1}^{N} r_{nl}^{(t+1)} - R_l) \quad (9)$$

$$\mu_v^{(t+1)} = \left[\mu_v^{(t)} + \kappa_\mu(\sum_{n=1}^{N}\sum_{l=1}^{|L|} T_{vl}^{+} P_{nl}^{(t+1)} - E_v q^{(t+1)} N)\right]^+ \quad (10)$$

$$\nu_{nl}^{(t+1)} = \left[\nu_{nl}^{(t)} + \kappa_\nu\big(\log(\frac{a_{nl}}{\mathrm{SINR}_{nl}^{(t+1)}} + b_{nl}) + r_{nl}^{(t+1)}\big)\right]^+ \quad (11)$$

$$\omega_{nl}^{(t+1)} = \left[\omega_{nl}^{(t)} + \kappa_\omega\big(\log(\frac{\beta_{\mathrm{LOP}}}{\mathrm{SINR}_{nl}^{(t+1)}})\big)\right]^+ \quad (12)$$

In summary, we have designed a partially distributed algorithm where in each iteration the central coordinator calculates q by formula (5) and sends the new value to all the nodes in the network. The receiver of each link sends the updated value of Φ to the transmitters which use these values to compute the new rates and powers via (6) and (8), respectively. The Lagrangian parameters are locally updated through (9), (10), (11), (12) at each link and associated nodes. Finally, the value of μ on each node is sent back to the central coordinator, and the next iteration of this process starts. This continues until the variables of optimization converge within a prescribed degree of accuracy.

5. Simulation Results

Linear Topology:

We consider a linear sensor networks topology consisting of 10 nodes and 9 links and the distance between each pair of nodes is 100m as shown in Figure 1. Source node ① sends the data stream to the destination - node ⑩ - through all the intermediate nodes one by one. The scheduling method used is so-called Spatially Periodic Time Sharing 3 (SPTS-3) with 12 time-slots. Transmitted data cascade through all the 9 links in a 3-time-slot routine as follows: links (I), (IV), (VII) are activated in the first time-slot, links (II), (V), (VIII) are activated in the second time-slot and links (III), (VI), and (IX) are activated last. The system then repeats the schedule again until the 12-time-slot frame is finished. SPTS-4 is similar with the difference being a routine has 4 time-slots and a cascade is comprised of 4 stairs. The operation frequency was chosen to be 2.4GHz, the total bandwidth 1MHz, and the power of background noise 1×10^{-15} Watts. The slow fading component of the propagation gain is inversely proportional to the distance with the loss factor $m = 4$. The BER is equal to 10^{-3} and the initial energy on each node is 1000 Joules.

Operation Frequency	2.4 (GHz)
Bandwidth B	1 (MHz)
Scattering Exponent m	4
Noise Power per Bandwidth N_0	10^{-15} (W)
Battery Energy on Each Node E_v	1000 (J)
Data Rate from Each Ending Node s_i	38.4 (Kbps)
SINR Threshold γ_{th}	6 (dB)

Table 1: Simulation Parameters for Clustered Sensor Network

Figures in 2 compare the network lifetime obtained by solving (3) and (3), respectively, using various tangent approximation methods by equally distributing rates, by averaging powers, and the approximation used by Madan *et. al.* in [8]. The maximum ROP target is 20%, and the required rate is varied between 40kbps to 120kbps. It is clearly seen that the tangent approximation methods always perform better than that of [8]. As the transmission rate increases, the network lifetime decreases accordingly and the performance gaps among the various approximations diminish. It is also seen that the tangent approximation by equally distributing rates performs better than that by averaging power. It can be explained by the symmetry property of SPTS scheduling; the optimal solution for the optimization problem (3) always appears at the point where the data rate is equally distributed among the activated time-slots. However, if we take Link Outage Probability (LOP) constraints into consideration, this feature is not so pronounced. While including link outage probabilities in the optimization, $\gamma_{th} = 0$dB and $O_{th}^{Link} = 15\%$ are used. It is seen that there is little difference between the cases with LOP constraints and without LOP constraints when the required rate is high (120kbps for SPTS-3, 100 and 120kbps for SPTS-4). However, when the rate is low, the LOP constraints cap the network lifetime by raising the transmission power in order to make the communication robust for fast Rayleigh Fading, which explains why the lifetime does not increase by much when the required rate decreases in the right hand side graph of Figure 2.

A Clustered Sensor Network:

In this example, the sensor network is composed of 13 nodes; 8 sensors - ⑤ $\sim$ ⑫ are distributed in the field; routers ① $\sim$ ④ fuse all the data collected from the sensors nearby and send them to the central base station or sink ⓪. Figure 3 shows the geometric configuration of the system and the scheduling method. Table 1 lists the parameters used in the simulations.

The pyramid in Figure (4) shows that the network lifetime increases with the relaxation of maximum ROP and LOP constraints. This graph reveals that these two kinds of constraints are similar in terms of SINR requirement on links as far as the network lifetime is concerned. The only difference between them is that for ROP constraints it seems we have to optimally distribute rate over multiple time-slots.

Performance of the partially distributed algorithm:

For simplicity, we choose a linear topology for this simulation; the scheduling method is SPTS-3 with 6 time-slots. The distributed algorithm iteratively finds the optimal values for the transmitting powers and the rates allocated to each activated time-slot on every link. Apart from the data rate (120kbps in this part), the simulation parameters are the same as those in the Linear Topology simulations described before. We include the LOP constraints in the optimization problem.

Figure 5 show how the powers and rates converge to their optimal values on the 3 activated links (i.e. 1, 4, 7) in the first time slot. The optimal rates seem to be equal to each other as in the previous simulations.

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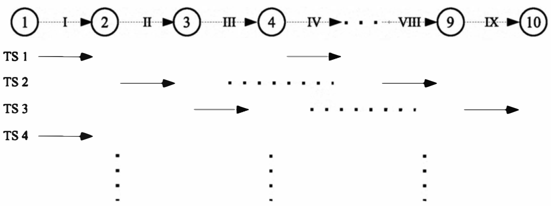
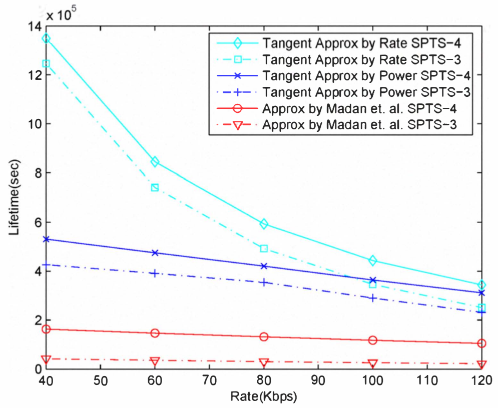
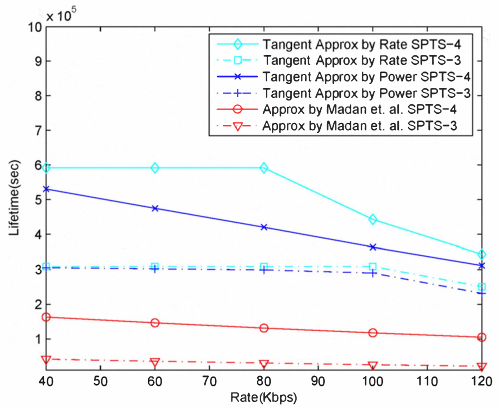


Figure 1: Linear Topology with the SPTS-3 Scheduling Method

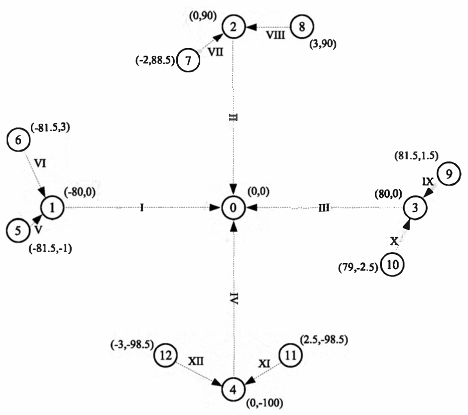


(a) ROP constraints only

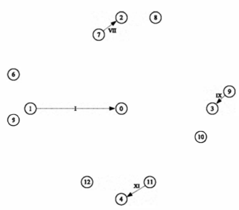


(b) ROP and LOP constraints

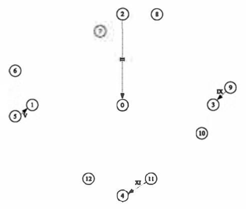
Figure 2: Network Lifetime with and without LOP constraints for Linear Topology with the SPTS-3 and SPTS-4 Scheduling Method



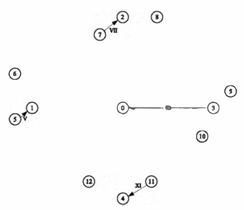
(a) Geometric Configuration



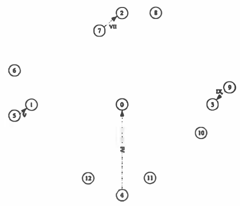
(b) Time Slot 1



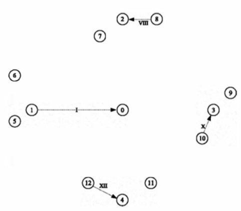
(c) Time Slot 2



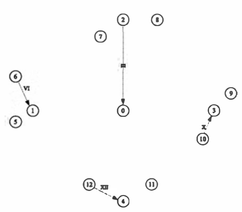
(d) Time Slot 3



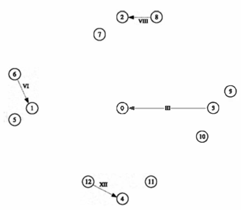
(e) Time Slot 4



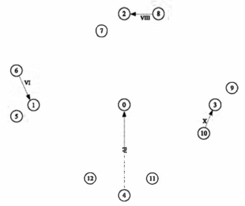
(f) Time Slot 5



(g) Time Slot 6



(h) Time Slot 7



(i) Time Slot 8

Figure 3: Scheduling Method and configuration for clustered network

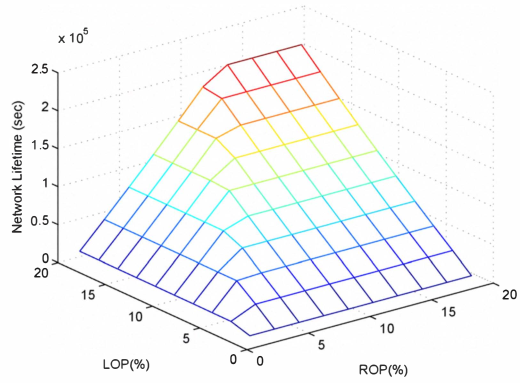
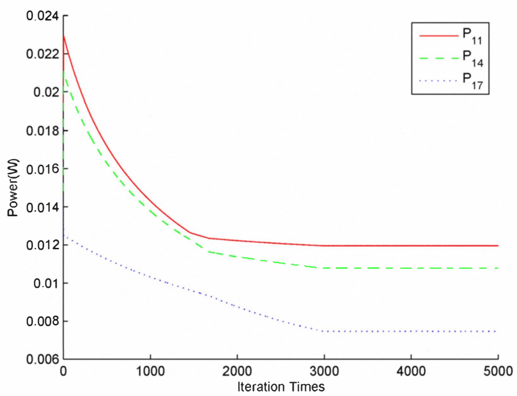
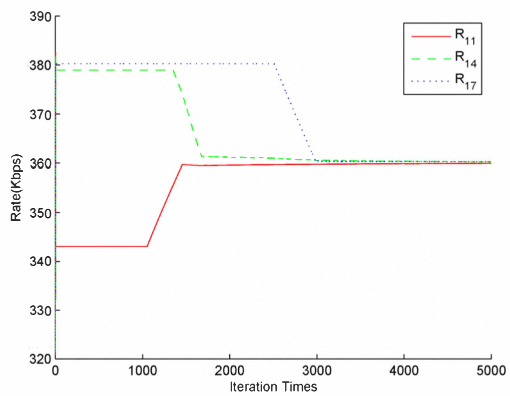


Figure 4: Lifetime v.s. ROP and LOP



(a) Convergence of transmit powers



(b) Convergence of allocated rates

Figure 5: Convergence of Allocated Transmit Powers and Rates