Power allocation for distortion outage minimization in clustered wireless sensor networks

Chih-Hong Wang and Subhrakanti Dey, *Senior Member, IEEE*

***Abstract*— In this paper we study a clustered wireless sensor network observing a Gaussian random field. Within a cluster, multiple sensors amplify and forward their measurements using uncoded analog transmission to the clusterhead (CH). In turn the clusterheads transmit also amplify and forward their received signals to a Fusion Centre (FC) located at some distance using an orthogonal multiple access scheme such as frequency division multiple access (FDMA). The distortion of the signal reconstructed at the FC is required to be within a certain threshold D_{max}. Due to random fading suffered by the channels from the CH's to the FC, the distortion achieved at the FC can exceed D_{max} in which case a distortion outage occurs. We propose a novel optimal power allocation scheme at the CH transmitters that minimizes this distortion outage probability subject to an average total power constraints across the CH's. While this optimal scheme performs very well, it requires full (instantaneous) channel state information (CSI) at the receiver (FC) as well as the CH transmitters. We also study some sub-optimal power allocation methods based on the knowledge of the statistics of the fading channels between the CH's and the FC at the transmitters (and full CSI at the FC). Simulation studies show that the statistical power allocation methods perform poorly compared to the full CSI based algorithm, which points to the need for designing efficient power allocation algorithms based on quantized channel feedback from the FC to the CH's.**

***Index Terms*— wireless sensor networks, optimal power allocation, fading channels, distortion outage probability**

I. Introduction

WIRELESS sensor networks have many potential and useful applications that have already been implemented and those yet to emerge as newer technologies are made available. They can be used in environmental and wildlife habitat monitoring, in tracking targets for defense applications, in aged healthcare and many other areas of human life. They usually involve large numbers of sensor nodes that are distributed geographically to collect some data of interest. The sensor nodes send their data to some central processing unit such as a fusion centre where the data are combined to obtain an estimate of the physical phenomenon observed. Sensors are usually cheap, mass-manufactured, battery-operated devices that have limited energy and communication capabilities. Replacement of batteries are usually costly if not impossible or unnecessary. Hence how to efficiently manage the energy/power consumption of sensors is a problem that is particularly important for wireless sensor networks.

Many recent studies have been dedicated to cross-layer optimisation to deal with energy concerns in wireless sensor networks [1], [2]. [2] shows that cooperative MIMO and rate adaptation coupled with cross-layer optimisation can significantly improve the energy-delay trade off in wireless networks. Recent results in [3] demonstrating the asymptotic optimality of uncoded analog forwarding of measurements by multiple sensors as opposed to separate source channel coding have motivated a lot of researchers to investigate multi-sensor estimation problems and related energy/power efficiency issues within this uncoded transmission framework.

The authors are with the ARC Special Research Centre for Ultra Broadband Information Networks (CUBIN) within the Department of Electrical & Electronic Engineering at the University of Melbourne, Parkville, Victoria 3010 (e-mail: [chwang,sdey]@ee.unimelb.edu.au). CUBIN is an affiliated program of National ICT Australia (NICTA).

In [4] an optimal power allocation scheme is obtained for analog forwarding based transmission in an inhomogeneous Gaussian sensor network. [5] looks at estimation diversity and energy efficiency in distributed sensing. It shows that the estimation diversity gain increases as order of the number of sensors and derive optimal power allocation schemes for minimum distortion under power constraint and minimum power under distortion constraints. However optimal power allocation strategies found [4] and [5] are based on static channels and do not explicitly take into account fading channels, for which meeting a strict distortion constraint may not be always possible.

In this paper we study a wireless sensor network where sensors are organised into clusters. Each cluster has an elected clusterhead. Sensors within a cluster observe a Gaussian random field and send their observed (noisy) information to the clusterhead (CH) by uncoded analog transmission using distributed beamforming, and the cluster heads amplify-and-forward the combined signal to the fusion centre (FC) using orthogonal FDMA. The FC is required to compute the minimum mean square error (MMSE) estimate of the source within a certain distortion threshold. However, the channels between the CH's and the FC are subject to random fading which make meeting the distortion constraint with probability one impossible. The main purpose of this paper is therefore to design an optimal power allocation scheme at the CH's (based on full channel state information (CSI) at the FC and the CH's) to minimise the probability that the distortion exceeds the required maximum threshold under a long term sum power constraint. Simulation studies demonstrate the performance of this algorithm for varying average sum power constraints at the CH's, varying sensor powers and numbers of sensors within clusters etc. Since obtaining full CSI at the CH's can be costly, we also study some sub-optimal power allocation algorithms based on the statistics of the fading channels, by minimising some upper bounds of the outage probability as obtaining an explicit expression for the outage probability proves to be difficult. It is seen that these statistical power allocation schemes do not fare well compared to the performance of the full CSI base algorithm, thus mandating the need for power allocation algorithms based on finite rate channel feedback.

II. Sensor Network Model and Problem Formulation

A schematic diagram of the wireless sensor network studied in this paper is shown in figure 1. N clusters of sensors are distributed around a source $\theta[k]$ that is to be measured. Here $k = 0, 1, 2\dots$ denotes discrete time instants. We assume that $\theta[k]$ is an independent and identically distributed (i.i.d.) Gaussian (band-limited) random process of mean zero and variance σ_θ^2. Each cluster contains M_n sensors which observe the source and send their measurements to a pre-selected cluster head. The observed sample $x_m^n[k]$ of the mth sensor in the nth cluster at time k is given as

$$x_m^n[k] = \theta[k] + N_m^n[k] \tag{1}$$

where $N_m^n[k]$ is the measurement noise which is i.i.d., Gaussian distributed of zero mean and variance $(\sigma_m^n)^2$. We

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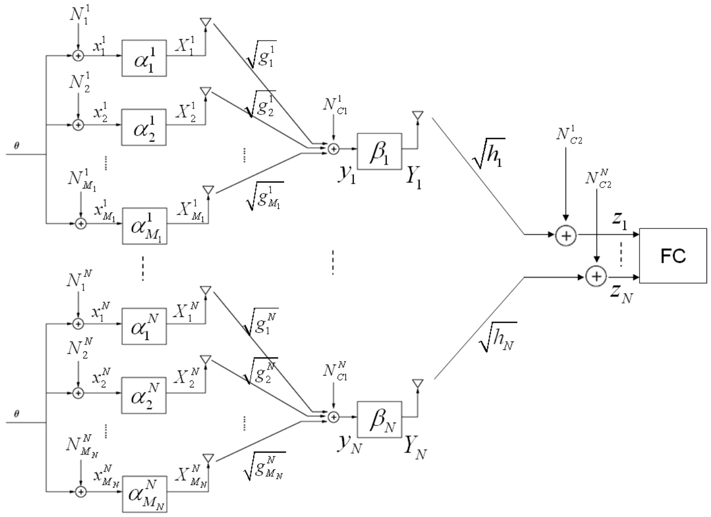

Fig. 1. Schematic diagram of wireless sensor network.

assume that $\theta[k]$ is independent of $N_m^n[k]$, $\forall k, n, m$. $(\sigma_m^n)^2$ is assumed to be proportional to the square of the distance from the source to the sensor (although the noise model at the sensors can be appropriately adjusted depending on the sensing model of the sensors). A more realistic model may include spatial correlation of the observed signal amongst sensors. For simplicity, this paper treats the observed signal as being spatially independent of each other. We also assume that CHs are selected by some chosen protocol, and that CHs are capable of transmitting with greater power than the sensors since the transmission distance between a CH and the fusion centre is larger in general.

Motivated by recent results showing asymptotic optimality of uncoded analog transmission from multiple sensors observing a Gaussian source [3], we assume that the sensors within a cluster simply amplify-and-forward (using equal power) their observations to the CH via a multi-access channel using distributed beamforming so that the received signals at the CH add up coherently. This is referred to as the first stage of transmission. Although distributed beamforming may not be easy to implement (see [6] for details of implementing distributed beamforming), specially in the case of large number of sensors within each cluster, it has been shown that even under the presence of random phase errors, the average loss in performance is not significant unless the variance of the phase errors is severely large. The signal received at the nth CH is given as

$$y_n[k] = \sum_{m=1}^{M_n} \left[\alpha_m^n \sqrt{g_m^n}(\theta[k] + N_m^n[k])\right] + N_{C1}^n[k] \quad (2)$$

where α_m^n is the power gain factor, $\sqrt{g_m^n}$ is the first stage channel gain and N_{C1}^n is the zero-mean AWGN (additive white Gaussian noise) channel noise of variance $(\sigma_{C1}^n)^2$. We also assume that the channels between sensors and CHs are static, where the channel gains are assumed to be proportional to the inverse of the square of the transmission distance. We also assume that the signal received at each CH is not interfered by any signals from other clusters (which can be achieved by a time-division access protocol where each cluster operates in a different time slot). For simplicity, we let the CHs also use the amplify and forward scheme to transmit $y_n[k]$ to the FC using an orthogonal multiple access protocol such as FDMA, referred to here as the second stage of transmission. We assume that full (instantaneous) CSI is available at both the CH transmitters and the receiver (FC) (which can be obtained by delayless and error-free feedback from the FC once FC has estimated the channels using pilot tones). We do not consider the effects of channel estimation errors or power consumptions due to channel estimation in this paper. The signal received at the FC from the nth CH is given as

$$z_n[k] = \beta_n \sqrt{h_n} y_n[k] + N_{C2}^n[k] \quad (3)$$

where β_n is the power gain factor, $\sqrt{h_n}$ is the second stage channel gain and N_{C2}^n is the zero mean AWGN channel noise of variance $(\sigma_{C2}^n)^2$.The received signal vector is given as $\mathbf{z} = \mathbf{s}\theta + \mathbf{v}$ where

$$\mathbf{z} = [z_1[k], \ldots, z_N[k]]^T$$
$$\mathbf{s} = \left[\beta_1\sqrt{h_1}\sum_{m=1}^{M_1}\alpha_m^1\sqrt{g_m^1}, \ldots, \beta_N\sqrt{h_N}\sum_{m=1}^{M_N}\alpha_m^N\sqrt{g_m^N}\right]^T$$
$$\mathbf{v} = \left[\beta_1\sqrt{h_1}\left(\sum_{m=1}^{M_1}\alpha_m^1\sqrt{g_m^1}N_m^1[k] + N_{C1}^1[k]\right) + N_{C2}^1[k], \ldots, \beta_N\sqrt{h_N}\left(\sum_{m=1}^{M_N}\alpha_m^N\sqrt{g_m^N}N_m^N[k] + N_{C1}^N[k]\right) + N_{C2}^N[k]\right]^T$$

where ^{T} denotes transposition.

In what follows, we suppress the time index k for simplicity. The FC uses the MMSE estimator to reconstruct the source θ, since we have the prior pdf (probability density function) of θ. The MMSE estimator is given as $\hat{\theta} = \frac{\mathbf{s}^T\mathbf{C}^{-1}\mathbf{z}}{\frac{1}{\sigma_\theta^2} + \mathbf{s}^T\mathbf{C}^{-1}\mathbf{s}}$ where $\mathbf{C}$ is a diagonal matrix with its nth diagonal element given as $C_{nn} = \beta_n^2 h_n \left(\sum_{m=1}^{M_n}(\alpha_m^n)^2 g_m^n (\sigma_m^n)^2 + (\sigma_{C1}^n)^2\right) + (\sigma_{C2}^n)^2$. The variance of $\hat{\theta}$ is given by $\text{var}(\hat{\theta}) = \left[\frac{1}{\sigma_\theta^2} + \mathbf{s}^T\mathbf{C}^{-1}\mathbf{s}\right]^{-1}$.

From figure 1 we can obtain $X_m^n = \alpha_m^n(\theta + N_m^n)$ and $Y_n = \beta_n\left(\sum_{m=1}^{M_n}\sqrt{g_m^n}X_m^n + N_{C1}^n\right)$. Define q_n as the total power of sensors in the nth cluster and P_n the power of the nth CH. We then obtain $q_n = \sum_{m=1}^{M_n}(\alpha_m^n)^2\left(\sigma_\theta^2 + (\sigma_m^n)^2\right)$ and $P_n = \beta_n^2\left(\sum_{m=1}^{M_n}(\alpha_m^n)^2\left(\sigma_\theta^2 + (\sigma_m^n)^2\right)g_m^n + (\sigma_{C1}^n)^2\right)$. Here we are interested in obtaining the optimal power allocation scheme that minimises the total power of sensors and CHs subject to a distortion constraint at the FC, i.e.,

$$\begin{aligned} \underset{P_n, q_n}{\text{minimise}} \quad & \sum_{n=1}^{N}(P_n + q_n) \\ \text{subject to} \quad & \text{var}[\hat{\theta}] \le D_{\max}. \end{aligned} \quad (4)$$

The optimisation problem (4) can be easily shown to be non-convex. In order to avoid this difficulty, we assume that the sensors within clusters have fairly limited functionality and have only a few adjustable transmission power levels (e.g. low, medium and high transmission power). With this assumption we drop the optimisation variable q_n and assume that it is fixed at a value within a finite set of a small number of elements. We investigate the effect of q_n later via simulations in Section IV. Furthermore we assume all sensors within a cluster transmit with equal power (q_n/M_n). Hence the expressions for sensor power gain, CH power and distortion become

$$\alpha_m^n = \sqrt{\frac{q_n}{M_n\left[\sigma_\theta^2 + (\sigma_m^n)^2\right]}},\ P_n = \beta_n^2 C_n \tag{5}$$

$$\mathrm{var}[\hat{\theta}] = \sigma_\theta^2 \left(1 + \sum_{n=1}^{N} \frac{\beta_n^2 h_n U_n}{\beta_n^2 h_n V_n + (\sigma_{C2}^n)^2}\right)^{-1} \tag{6}$$

where $C_n = (q_n/M_n)\sum_{m=1}^{M_n} g_m^n + (\sigma_{C1}^n)^2$, $U_n = (q_n/M_n)\left(\sum_{m=1}^{M_n}\sqrt{g_m^n/\left(1+(\gamma_m^n)^{-1}\right)}\right)^2$, $V_n = (q_n/M_n)\sum_{m=1}^{M_n}(g_m^n(\gamma_m^n)^{-1})/\left(1+(\gamma_m^n)^{-1}\right) + (\sigma_{C1}^n)2$ and $\gamma_m^n = \sigma_\theta^2/(\sigma_m^n)^2$.

We now solve this optimisation problem for static (modelling only distance based attenuation) and fading (modelling random channel variations in addition to distance based attenuations) channels in the second stage of transmission (CH's to the FC), and describe the corresponding problem formulations in the following two subsections respectively. Note that the fading channel gain, $\sqrt{h_n}$ is assumed to be i.i.d. Rayleigh-distributed, and hence the signal power gain, i.i.d. h_n is exponentially distributed (although the analysis can be extended to any other fading distribution). The fading channel power gain is modelled as

$$h_n = \zeta_n f_n \tag{7}$$

where ζ_n is the mean channel gain and f_n is i.i.d. exponentially distributed with unity mean (any non-unity mean value of f_n can be absorbed into ζ_n). The mean channel gain is assumed to be equal to the inverse of the transmission distance squared.

A. Static Channel

In this section, we assume that the channel gains are static and distance-based, and are given by ζ_n. The optimisation problem becomes

$$\begin{aligned} \min_{\beta_n^2} \quad & \sum_{n=1}^{N} \beta_n^2 C_n \\ \text{s.t.} \quad & \sigma_\theta^2\left(1 + \sum_{n=1}^{N}\frac{\beta_n^2 \zeta_n U_n}{\beta_n^2 \zeta_n V_n + (\sigma_{C2}^n)^2}\right)^{-1} \le D_{max} \\ & \beta_n^2 \ge 0,\ n = 1,\ldots,N. \end{aligned} \tag{8}$$

The solution a variation to this problem (for a best linear unbiased estimator (BLUE) instead of the MMSE estimator) can be found in [5], and we just state it below as it will be useful in later sections where we solve the problem for fading channels. The optimal power gain for problem (8) is given as

$$\beta_n^{2\,*} = \begin{cases} 0, & n > N_1 \\ \frac{G_n}{H_n}\left(\frac{1}{\sqrt{\eta_n^{-1}\rho_0}} - 1\right), & n \le N_1. \end{cases} \tag{9}$$

where $G_n = U_n/V_n$, $H_n = \zeta_n U_n/(\sigma_{C2}^n)^2$, $\eta_n = H_n/C_n$ and $\rho_0 = D(N_1)/C(N_1)$. $D(n) = \sum_{j=1}^{n} G_j - (\sigma_\theta^2/D_{max} - 1)$ and $C(n) = \sum_{j=1}^{n} G_j/\sqrt{\eta_j}$. N_1 is given by ordering $\eta_1 \ge \ldots \ge \eta_N$ and finding $g(N_1) > 0$ and $g(N_1+1) \le 0$, where $g(n) = 1 - D(n)/\left(\sqrt{\eta_n}C(n)\right)$, $n = 1,\ldots,N$.

Similarly, the solution for the dual problem given as

$$\begin{aligned} \min_{\beta_n^2} \quad & \sigma_\theta^2\left(1 + \sum_{n=1}^{N}\frac{\beta_n^2 \zeta_n U_n}{\beta_n^2 \zeta_n V_n + (\sigma_{C2}^n)^2}\right)^{-1} \\ \text{s.t.} \quad & \sum_{n=1}^{N}\beta_n^2 C_n \le P_{tot} \\ & \beta_n^2 \ge 0,\ n = 1,\ldots,N. \end{aligned} \tag{10}$$

can also be found in [5]. The optimal power allocation is given as

$$\beta_n^{2\,*} = \begin{cases} 0, & n > N_1 \\ v_n\left(\frac{1}{\sqrt{\xi_n^{-1}c_0}} - 1\right), & n \le N_1 \end{cases} \tag{11}$$

where $v_n = (\sigma_{C2}^n)^2/\zeta_n V_n$, $\xi_n = \zeta_n U_n/C_n(\sigma_{C2}^n)^2$ and $c_0 = A(N_1)/B(N_1)$. $A(n) = \sum_{j=1}^{n} v_j\sqrt{\xi_j}C_j$ and $B(n) = \sum_{j=1}^{n} v_j C_j + P_{tot}$. N_1 is given by ordering $\xi_1 \ge \ldots \ge \xi_N$ and finding $f(N_1) > 0$ and $f(N_1+1) \le 0$, where $f(n) = \sqrt{\xi_n}B(n)/A(n) - 1$, $n = 1,\ldots,N$.

B. Fading Channel

In this subsection we assume Rayleigh-faded channels between the CH's and the FC as given by (7) and define the probability of distortion outage. The probability of distortion outage is defined as the probability that the distortion exceeds some predefined threshold, D_{max}. We want to minimise this distortion outage probability subject to a long term power constraint, stated as

$$\begin{aligned} \min \quad & P_r\left(D(\mathbf{P}(\mathbf{h}),\mathbf{h}) > D_{max}\right) \\ \text{s.t.} \quad & E[\langle \mathbf{P}(\mathbf{h})\rangle] \le P_{av},\ \mathbf{P}(\mathbf{h}) \ge 0. \end{aligned} \tag{12}$$

where $\mathbf{P}(\mathbf{h}) \triangleq (P_1(\mathbf{h}),\ldots,P_N(\mathbf{h}))$ and $\mathbf{h} \triangleq (h_1,\ldots,h_N)$. Here $P_r(x)$ denotes probability of the event x, and $\langle \mathbf{x}\rangle$ denotes the arithmetic mean of the vector $\mathbf{x}$ of length M defined by $\langle \mathbf{x}\rangle \triangleq (1/M)\sum_{i=1}^{M} x_i$. $\mathbf{h}$ is a vector of length N of random variables that model the channel gains from the CH's to the FC. $D(\mathbf{P}(\mathbf{h}),\mathbf{h})$ is the distortion as a function of channel gains and CH transmission power, which is also a function of channel gains. Note that we assume instantaneous channel knowledge at the FC (receiver) and at the transmitters (CH's) (where the transmitter CSI can be accurately obtained via feedback channels which are error-free and have zero delay).

III. Solution and Optimal Power Allocation Schemes for Fading Channel

The problem given in (12) can be solved in the same way as in [7]. We first consider the following minimisation problem given as

$$\begin{aligned} \min \quad & \langle \mathbf{P}(\mathbf{h})\rangle \\ \text{s.t.} \quad & D(\mathbf{P}(\mathbf{h}),\mathbf{h}) \le D_{max} \\ & \mathbf{P}(\mathbf{h}) \succeq 0 \end{aligned} \tag{13}$$

where $\succeq$ denotes componentwise inequality.

We have the following lemma:

Lemma 3.1: Without loss of generality, assume $h_1 \ge h_2 \ge \ldots \ge h_N$. With the knowledge of $\mathbf{h}$, the solution for (13) has already been given in (9). Hence the nth optimal power is given as

$$P_n^*(\mathbf{h}) = \frac{C_n G_n}{\bar{H}_n}\left[\frac{\sqrt{\bar{\eta}_n}}{\bar{\rho}_0(\mathbf{h},N_1)} - 1\right]^+,\ \text{for } n = 1,\ldots,N \tag{14}$$

where N_1 is a unique integer in $\{1,\ldots,N\}$ required to evaluate $\bar{\rho}_0(\mathbf{h},N_1)$. $\bar{H}_n$, $\bar{\eta}_n$ and $\bar{\rho}_0(\mathbf{h},N_1)$ are defined similarly to the static channel case except that ζ_n is now replaced by h_n and the explicit dependence on $\mathbf{h}$ is shown. Note also that $[x]^+$ denotes $\max(x, 0)$.

One can also obtain the following Lemma which is necessary to find the final optimal solution to problem (12).

Lemma 3.2: The optimal power function, $\mathbf{P}^*(\mathbf{h}) \triangleq (P_1^*(\mathbf{h}), ..., P_N^*(\mathbf{h}))$, is a continuous function of $\mathbf{h}$. Furthermore, $\langle \mathbf{P}^*(\mathbf{h}) \rangle$ is a non-increasing function of h_n, for $n = 1, \ldots, N$.

Proof: The first statement can be proved in a similar way to the one given in [7] and is omitted. The second statement can be proved by differentiating $\langle \mathbf{P}^*(\mathbf{h}) \rangle$ with respect to h_n, which yields

$$\frac{\partial \langle \mathbf{P}^*(\mathbf{h}) \rangle}{\partial h_n} = -\frac{2G_n}{N h_n \sqrt{\bar{\eta}_n}} \left[\frac{1}{\bar{\rho}_0(\mathbf{h}, N_1)} - \frac{1}{\sqrt{\bar{\eta}_n}} \right]^+ \leq 0. \quad (15)$$

The gradient of $\langle \mathbf{P}^*(\mathbf{h}) \rangle$ with respect to h_n is non-positive for $n = 1, \ldots, N$, which proves the second statement. ∎

We define two regions, $\mathcal{R}(s)$ and $\overline{\mathcal{R}}(s)$ and the boundary surface $\mathcal{B}(s)$ for some non-negative s as in [7]:

$$\begin{aligned} \mathcal{R}(s) &= \{\mathbf{h} \in \mathbb{R}_+^N : \langle \mathbf{P}(\mathbf{h}) \rangle < s\} \\ \overline{\mathcal{R}}(s) &= \{\mathbf{h} \in \mathbb{R}_+^N : \langle \mathbf{P}(\mathbf{h}) \rangle \leq s\} \\ \mathcal{B}(s) &= \{\mathbf{h} \in \mathbb{R}_+^N : \langle \mathbf{P}(\mathbf{h}) \rangle = s\} \end{aligned} \quad (16)$$

We then define two average power sums as

$$\begin{aligned} P(s) &= \int_{\mathcal{R}(s)} \langle \mathbf{P}(\mathbf{h}) \rangle dF(\mathbf{h}) \\ \overline{P}(s) &= \int_{\overline{\mathcal{R}}(s)} \langle \mathbf{P}(\mathbf{h}) \rangle dF(\mathbf{h}) \end{aligned} \quad (17)$$

where $F(\mathbf{h})$ denotes the cdf (cumulative density function) of $\mathbf{h}$. Finally, the power sum threshold s^* and the weight w^* are given as

$$\begin{aligned} s^* &= \sup\{s : P(s) < P_{av}\} \\ w^* &= \frac{P_{av} - P(s^*)}{\overline{P}(s^*) - P(s^*)} \end{aligned} \quad (18)$$

With the above lemma and definitions we can now present the solution to (12). The proof follows using similar techniques as in [7] and is excluded due to space reasons.

Theorem 1: The solution to problem (12) is given as

$$\hat{\mathbf{P}}(\mathbf{h}) = \begin{cases} \mathbf{P}^*(\mathbf{h}), & \text{if } \mathbf{h} \in \mathcal{R}(s^*) \\ \mathbf{0}, & \text{if } \mathbf{h} \notin \overline{\mathcal{R}}(s^*) \end{cases} \quad (19)$$

while if $\mathbf{h} \in \mathcal{B}(s^*)$, $\hat{\mathbf{P}}(\mathbf{h}) = \mathbf{P}^*(\mathbf{h})$ with probability s^* and $\hat{\mathbf{P}}(\mathbf{h}) = 0$ with probability $1 - w^*$.

The optimal power allocation scheme states that if the channel condition is above some threshold then the CHs transmit with power allocation given by (9), or else none should transmit to save power.

Remark 1: Note that the solution given in (19) is of a general form, which can be applied to both continuous and discontinuous fading distributions. If the fading distribution is continuous (which is true for this problem), then the probability that $\mathbf{h} \in \mathcal{B}(s^*)$ is zero, hence discarding the need for randomisation at the boundary.

Remark 2: Note also that while the computations necessary to implement the above solutions are carried out at the FC (such as those of s^* (based on P_{av}) and $\bar{\rho}_0(\mathbf{h}, N_1)$) and the decision whether the CH's should transmit or not transmit can be broadcast by the FC, the optimal power allocation for individual CH can be easily implemented in a distributed fashion (in the case where the CH's transmit). The FC has to just broadcast the quantity $\bar{\rho}_0(\mathbf{h}, N_1)$ to all CH's and the CH's can then update their transmission power according to (14) which only involves local variables at the CH's (apart from $\bar{\rho}_0(\mathbf{h}, N_1)$).

IV. Simulation Results

Two sensor network topologies are simulated in MATLAB. Topology A has six clusters deployed equally spaced around the source and Topology B deploys six clusters on one side of the source only as shown in figure 2. Topology B models environments where it is difficult or impossible to deploy sensors in certain parts of the landscape, for example, when the source is located at the edge of a cliff. The sensors in each cluster are organised in four equally spaced concentric circles and the number of sensors in each circle are 6,12,18 and 24 from the smallest to the biggest circle respectively. All clusters have a radius of 40m. All sensors transmit with $q_n/M_n = 1mW$ in topology A. In topology B sensors transmit with $1.33mW$, $1mW$ and $0.67mW$ in the two clusters closest to the source, two clusters second closest and two clusters farthest away from the source respectively. The CHs are located at the center of each cluster for simplicity. CHs are 100m and 60m apart from the next closest CH in topology A and B respectively. The FC is located 500m away from the source in both topologies. The channel noise variances are set to $(\sigma_{C1}^n)^2 = 10^{-12}$ Watts and $(\sigma_{C2}^n)^2 = 10^{-10}$ Watts for $n = 1, \ldots, 6$ in the first and second stage of transmission respectively in both topologies. Source variance is set to $\sigma_\theta^2 = 1$ Watt.

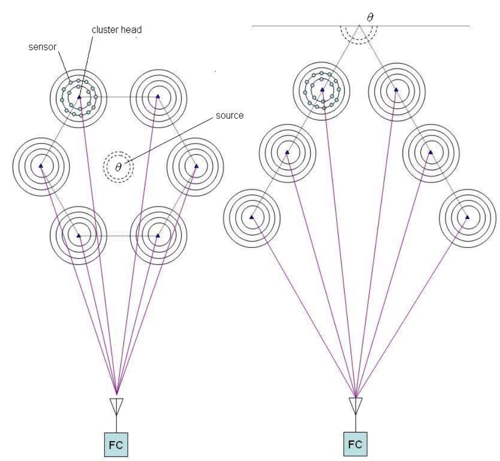

Fig. 2. Wireless sensor network topologies. Left: topology A. Right: topology B.

A. Static Channel

Figure 3 shows total power consumption, $\sum_{n=1}^N (P_n + q_n)$, versus total sensor power within clusters, $\sum q_n$, in topology A. In this simulation only the total sensor power of one of the six clusters is varied. As $\sum q_n$ increases, more power is allocated to the sensors, and hence signals received at the CH have a lower distortion. Therefore, CHs can transmit with less power to achieve the same distortion. However total power starts to increase after some point of $\sum q_n$ since allocating extra q_n cannot bring down the distortion anymore and this power is wasted. Asymptotic analysis shows that as q_n goes to infinity for all n distortion is given as

$$\lim_{q_n \to \infty} D = \sigma_\theta^2 \left(1 + \sum_{n=1}^{N} \frac{\left(\sum_{m=1}^{M_n} \sqrt{\frac{g_m^n}{1 + (\gamma_m^n)^{-1}}} \right)^2}{\sum_{m=1}^{M_n} \frac{g_m^n (\sigma_m^n)^2}{1 + (\sigma_m^n)^{-1}}} \right)^{-1}. \quad (20)$$

This is in fact the same expression as the minimum distortion achievable at the FC if $\beta_n^2 \to \infty$, and characterizes the feasible set of the distortion constraint for the optimisation problem

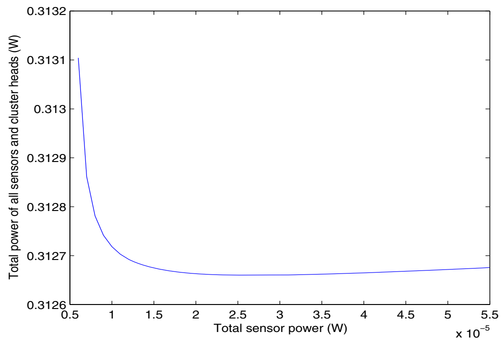

Fig. 3. Total cluster head power versus distortion (Topology A).

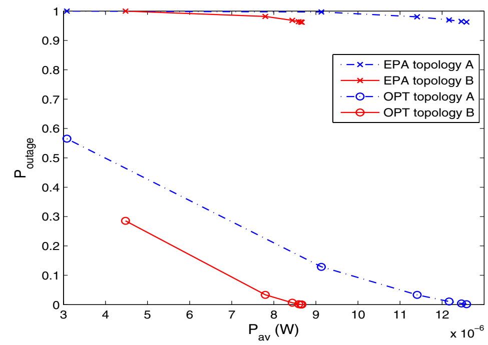

Fig. 5. P_{outage} against P_{av}.

in (8). It is also seen (but not included here due to space limitations) that as the number of sensors per cluster increases (while keeping q_n fixed), more observations are transmitted to the CH. This lowers the distortion at the CH and hence CHs need less power to meet the distortion requirement at the fusion center. Numerical analysis shows that distortion decreases like $1/M_n$ which conforms with the asymptotic analysis given in [3].

B. Fading Channel

In this section, the channels between the CH's and the FC are modelled as Rayleigh-faded channels. The following results are obtained over 1,000,000 realisations of exponentially-distributed channel power gains of mean equal to the inverse of the distance squared for each average power given. The distortion requirement is set to 0.0043, which is a hundred times the minimum achievable distortion.

Figure 4 shows P_{av} versus s^* (the sum power threshold that determines whether the CH's should transmit or not). This graph allows us to obtain the value of s^* that corresponds to a given P_{av}. This is used for calculating the probability of distortion outage.

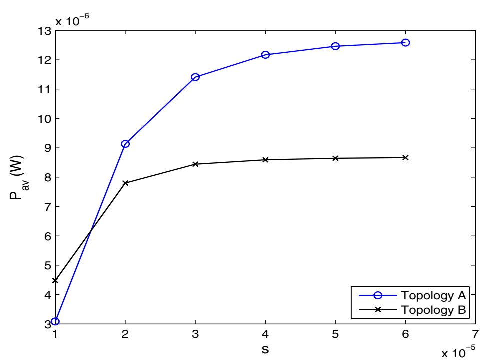

Fig. 4. P_{av} against s.

Figure 5 shows the distortion outage probability against average power for optimal power allocation (OPT) and equal power allocation (EPA). EPA allocates all the CHs with equal transmitting power, which equals P_{av}. As shown in this figure, optimal power allocation scheme performs significantly better than EPA scheme for both network topologies.

In the (simplified) problem formulation we assumed that the sensors can only transmit with a finite number of power levels and hence q_n is no longer a variable of optimisation. Here we investigate the effect of q_n on the outage performance via simulation. Figure 6 shows how the outage probability varies with q_n using optimal power allocation in topology A (essentially total power consumed by sensor transmissions in all clusters are kept at the same value q_n). D_{max} is set to a hundred times the minimal achievable distortion. As q_n increases, the outage probability obviously decreases. However,the effect of lowering the outage probability by increasing q_n quickly saturates when q_n reaches around -70dB; any q_n higher than this power level does not lower the outage probability significantly. This is because adjusting q_n only affects the first stage of transmission and the resulting distortion achieved at the CHs. The saturation level outage probability then depends on the channel conditions in the second stage of transmission (cluster heads to the fusion centre). One can similarly plot the outage probability versus P_{av} for various values of q_n. While increasing average transmit power for the clusterheads decreases the outage probability, increasing q_n beyond a certain level does not result in any significant reduction in the outage probability.

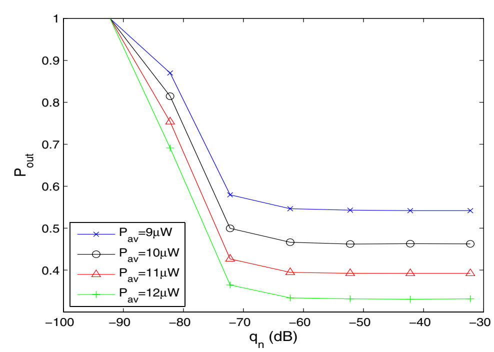

Fig. 6. P_{outage} against q_n with different long-term average power using optimal power allocation in topology A ($D_{max} = 0.0043$).

Power allocation based on statistics of the fading channels: As acquiring full instantaneous channel knowledge at the cluster head transmitters can be costly, here we look at some optimal power allocation methods based on statistical knowledge of the fading channels between the cluster-heads and the fusion centre. Since the fading statistics do not necessarily vary rapidly with time, this requires very little overhead communication between the cluster heads and the fusion centre. It is however difficult to obtain an explicit expression of the outage probability for $N > 1$ (an observation which was also made in [5]). Hence we choose to minimise an upper bound on the outage probability by minimising the expected distortion, which is motivated by Markov's inequality $P_r(D > D_{max}) = P_r(D \geq D_{max}) \leq \frac{E[D]}{D_{max}}$. To simplify the analysis even further, we obtain an approximation to the expected distortion by obtaining a lower bound on it, which is given by

$$
\begin{aligned}
\min E&\left[\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\theta^2}\sum_{i=1}^{N}\frac{U_i\beta_i^2 h_i}{V_i\beta_i^2 h_i+\delta_i}\right]^{-1}\\
&\geq \min\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\theta^2}\sum_{i=1}^{N}\frac{U_i\beta_i^2 E[h_i]}{V_i\beta_i^2 E[h_i]+\delta_i}\right)^{-1}\\
&\equiv \sigma_\theta^2(1+\max\sum_{i=1}^{N}\frac{U_i\beta_i^2 E[h_i]}{V_i\beta_i^2 h_i E[h_i]+\delta_i})\\
&\geq \sigma_\theta^2(1+\max E\left[\sum_{i=1}^{N}\frac{U_i\beta_i^2 h_i}{V_i\beta_i^2 h_i h_i+\delta_i}\right])
\end{aligned}
\tag{21}
$$

where $\delta_i = (\sigma_{C2}^i)^2$. The inequalities in the above analysis follow from Jensen's inequality due to convexity of the distortion function with respect to the channel gains. Although the distortion outage probability may not be strictly upper bounded by this lower bound on the expected distortion, it provides a heuristic for obtaining a statistical power allocation scheme.

We can now solve an optimisation problem by minimizing the lower bound of expected distortion given by the last line of (21) as

$$
\begin{aligned}
\max \quad & E\left[\sum_{i=1}^{N}\frac{U_i\beta_i^2 h_i}{V_i\beta_i^2 h_i+\delta_i}\right]\\
\text{s.t.} \quad & \sum_{i=1}^{N}\beta_i^2 C_i \leq P_{tot},\ \beta_i^2 \geq 0.
\end{aligned}
\tag{22}
$$

For Rayleigh-faded channels, the objective function can be expressed as

$$
\begin{aligned}
E\left[\sum_{i=1}^{N}\frac{U_i\beta_i^2 h_i}{V_i\beta_i^2 h_i+\delta_i}\right] &= \frac{U_i}{V_i}-\frac{U_i\delta_i}{V_i}E\left[\frac{1}{V_i\beta_i^2 h_i+\delta_i}\right]\\
&= K_{1i}-\frac{U_i\delta_i}{V_i}\int_0^\infty\frac{\lambda_i e^{-\lambda_i h_i}}{V_i\beta_i^2 h_i+\delta_i}dh_i\\
&= K_{1i}-\frac{K_{2i}}{\beta_i^2}e^{\frac{K_{3i}}{\beta_i^2}}E_1\left(\frac{K_{3i}}{\beta_i^2}\right)
\end{aligned}
$$

where $K_{1i} = U_i/V_i$, $K_{2i} = U_i\lambda_i\delta_i/V_i^2$, $K_{3i} = \lambda_i\delta_i/V_i$ and $E_1(z) = \int_z^\infty e^{-t}/t\ dt$. Hence the optimisation problem is given as

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{N}\left(\frac{K_{2i}}{\beta_i^2}e^{\frac{K_{3i}}{\beta_i^2}}E_1\left(\frac{K_{3i}}{\beta_i^2}\right)-K_{1i}\right)\\
\text{s.t.} \quad & \sum_{i=1}^{N}\beta_i^2 C_i \leq P_{tot},\ \beta_i^2 \geq 0.
\end{aligned}
\tag{23}
$$

It can be easily shown that this problem is a standard convex optimization problem and by solving the KKT conditions and letting $z_i = K_{3i}/{\beta_i}^2$, we get the following set of nonlinear equations

$$
\begin{cases}
\mu > 0, \quad \sum_{i=1}^{N}\frac{K_{3i}C_i}{z_i}-P_{tot}=0,\\
z_i^2\left[e^{z_i}E_1(z_i)(1+z_i)-1\right]=\frac{C_iK_{3i}^2}{K_{2i}}\mu, \quad 0\leq z_i<\infty\\
z_i^2\left[e^{z_i}E_1(z_i)(1+z_i)-1\right]\leq\frac{C_iK_{3i}^2}{K_{2i}}\mu, \quad z_i=\infty
\end{cases}
\tag{24}
$$

The optimal power values can be obtained by solving the above set of nonlinear equations numerically by using provably convergent fixed point iterative methods.

We can also look at minimising the lower bound on expected distortion given by the third line of (21), which is equivalent to problem (10). Figure 7 shows the outage probability achieved by problem (22) (heuristic method 1) and problem (10) (heuristic method 2) for Topology A. Clearly, the sub-optimal statistical power allocation methods based on minimizing the upper bounds on the outage probability do not fare well compared to the performance of the optimal power allocation method based on full CSI at the clusterhead transmitters. A similar observation was also made in [8] in the context of outage probability performance of beamforming in multiple antenna systems. This motivates the need for optimal power allocation for distortion outage minimization based on quantized or finite rate channel feedback from the fusion centre to the clusterheads, a topic that is currently under investigation.

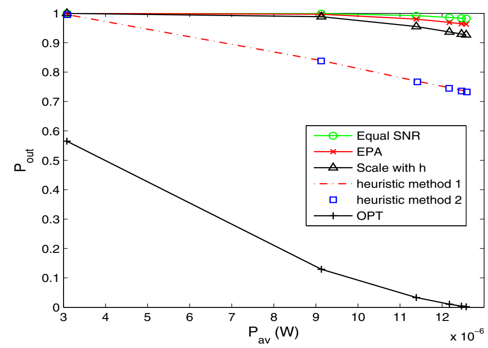

Fig. 7. Performance of heuristic methods that use knowledge of channel statistics (topology A).

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