On Diversity Orders of Distortion Outage for Coherent Multi-Access Channels

Chih-Hong Wang, Alex S. Leong, and Subhrakanti Dey

***Abstract*— In this paper we investigate the distortion outage performance of distributed estimation schemes in wireless sensor networks, where a distortion outage is defined as the event that the estimation error or distortion exceeds a pre-determined threshold. The sensors transmit their observation signals using analog amplify and forward through coherent multi-access channels to the fusion center, which reconstructs a minimum mean squared error (MMSE) estimate of the physical quantity observed. We consider three power allocation schemes - 1) equal power allocation (EPA), 2) short-term optimal power allocation (ST-OPA), and 3) long-term optimal power allocation (LT-OPA). We study their diversity orders of distortion outage in terms of increasing numbers of sensors, and show that under Rayleigh fading EPA and ST-OPA achieve the same diversity order of $N \log N$, where N is the number of sensors. On the other hand, in LT-OPA, we find that for $N > 1$ the outage probability can be driven to zero with a finite amount of total power.**

I. Introduction

Wireless sensor networks have recently attracted research interests and practical implementations in many areas of human life due to the numerous applications WSNs (wireless sensor networks) can achieve such as in environmental monitoring, tracking in defense technology, monitoring chemical levels in factories, and health monitoring, just to name a few.

One important issue in WSNs is the utilization of battery energy, since replacing batteries is considered expensive. Many works in the literature have considered energy-efficient protocols, power allocation schemes and cross-layer optimization to optimize the use of energy in WSNs under various different network assumptions and protocols. Recently it was shown that in a sensor network with a Gaussian source, it is exactly optimal [1] to transmit using uncoded analog forwarding of measurements by multiple sensors as opposed to separate source channel coding. Many works have since studied power-allocation problems (such as minimizing the transmit power subject to a distortion constraint) in multi-sensor estimation under the framework of analog-forwarding transmission, e.g. [2]–[4]. When time varying fading channels are considered, distortion becomes a random variable and it is not always possible to satisfy the distortion constraint. In such cases a *distortion outage* occurs [2]. This leads to the notion of *distortion outage probability*, which is defined as the probability that the distortion exceeds a given threshold D_{max}. The estimation diversity achieved by wireless sensor networks was first studied in [2] for equal power allocation in orthogonal multi-access channels with Rayleigh fading. They showed that such a network can achieve an estimation diversity on the order of the number of sensors in the network.

In this paper we will look at a WSN where multiple sensors take noisy measurements of a single i.i.d. Gaussian source and transmit, using amplify-and-forward, their noisy measurements to the fusion center (FC). We assume that the sensors transmit coherently to the FC so that the signals add up in phase at the FC [1], [3]. Under this setting we consider three power allocation schemes - equal power allocation, short-term optimal power allocation (minimizing distortion) and long-term optimal power allocation (minimizing distortion outage probability) - and give theoretical analysis on the diversity order of distortion outage using these power allocation schemes. We show that the diversity order achieved by the equal power allocation and the short-term power allocation is $N \log N$, where N is the number of sensors. In the long-term optimal power allocation we show that we can drive the outage probability to zero using finite total power for $N > 1$.

This paper is organized as follows. In Section II we give the network model. We define and state the three different power allocations in Section III, and perform theoretical analysis to find their diversity orders of distortion outage in Section IV. Simulation results are given in Section V.

In this paper, symbols in bold indicate that they are column vectors, e.g., $\mathbf{x} = [x_1, \ldots, x_N]^T$, where ^{T} denotes vector transposition. Given a random variable X, its p.d.f. (probability density function) and c.d.f. (cumulative distribution function) are denoted as $f_X(x)$ and $F_X(x)$ respectively, while $E[X]$ denotes its expectation.

II. Network model

A schematic diagram of the wireless sensor network model is shown in Fig. 1. We assume that there are N sensors in the network and the sensors observe a single point Gaussian source, denoted by $\theta[k]$, which has zero mean and variance σ_θ^2, and is i.i.d. (independent and identically distributed) in time (k denotes the discrete time index). The measurements of the ith sensor at time k are given as

$$x_i[k] = \theta[k] + w_i[k]$$

where w_i is Gaussian with zero mean and variance σ_i^2 and denotes the sensor measurement noise. The sensors amplify and forward their signals to the fusion center (FC) via a coherent MAC channel [1], [3] with a gain of $\beta_i[k]$. The transmitted signal is given as

$$y_i[k] = \beta_i[k] x_i[k].$$

The authors are with the Department of Electrical & Electronic Engineering, University of Melbourne, Parkville, Victoria 3010 (e-mail: chwang@ee.unimelb.edu.au, [asleong,sdey]@unimelb.edu.au).

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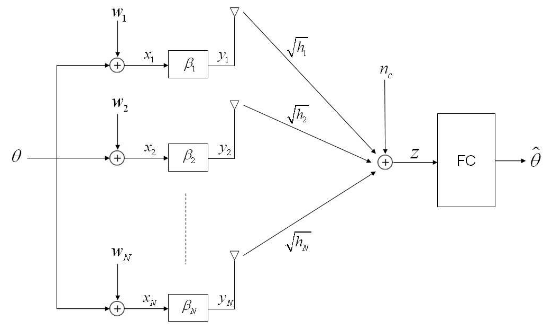

Fig. 1. Schematic diagram of the wireless sensor network using coherent MAC scheme.

We assume that the instantaneous channel gains, denoted as $\sqrt{h_i[k]}$, are time-varying random quantities that are i.i.d. over time. The channel noise is i.i.d. AWGN denoted as $n_c[k]$, with zero mean and variance σ_c^2. We assume that full CSI (channel state information) is available at the receiver, i.e., the FC is aware of all the values of $h_i[k]$, $\forall i,k$, while each sensor i has knowledge of their own channel $h_i[k]$. Hence the signal received by the FC is given by

$$z[k] = \sum_{i=1}^{N} \sqrt{h_i[k]}\beta_i[k]\theta[k] + \sum_{i=1}^{N} \sqrt{h_i[k]}\beta_i[k]w_i[k] + n_c[k].$$

We define the transmission power of the ith sensor as $P_i[k] \triangleq E\left[y_i^2[k]\right]$, and obtain $P_i[k] = C_i\beta_i^2[k]$, where $C_i = \sigma_\theta^2 + \sigma_i^2$.

It is well known that the optimal estimator for θ is the linear MMSE (minimum mean square error) estimator [5], given as $\hat{\theta} = \frac{E[\theta z]}{E[z^2]}z$. The mean squared error or *distortion* D_k of this estimator, is given as

$$\left(\frac{1}{\sigma_\theta^2} + \left(\sum_{i=1}^{N}\sqrt{\frac{h_i[k]P_i[k]}{C_i}}\right)^2 \left(\sum_{i=1}^{N}\frac{h_i[k]P_i[k]\sigma_i^2}{C_i} + \sigma_c^2\right)^{-1}\right)^{-1} \tag{1}$$

Note that (1) gives the expression of the *instantaneous* distortion, i.e., it is a function of the channel realizations h_i, $\forall i,k$. Due to the randomness of the fading channels, the instantaneous distortion at the FC changes randomly over time. Such estimation networks usually impose a distortion threshold at the FC to guarantee acceptable estimation, and if the instantaneous distortion D_k exceeds the distortion threshold $D_{\max}$, a *distortion outage* event occurs. We define the *distortion outage probability*, or simply *outage probability*, as the probability that the distortion exceeds the maximum distortion threshold, expressed as $P_{outage} \triangleq \Pr(D_k > D_{max})$.

Remark: Due to the i.i.d. (in time) nature of the model, we will drop the time index k from the rest of the paper.

III. Full-CSI power control schemes

In the following subsections we introduce three different power control schemes for our proposed wireless sensor network model. The diversity order of distortion outage achieved by these three schemes will be studied in Section IV.

Remark: In this paper we assume that the power allocations are limited by a total power $\mathcal{P}_{tot}$ that is fixed as the number of sensors N varies. Analysis can also be carried out for the case where the total power $\mathcal{P}_{tot}$ scales linearly with the number of sensors N, but are omitted to avoid repetition.

A. Equal power allocation

A very simple power allocation scheme is to have all the sensors transmit with the same power. Given a fixed total power constraint $\mathcal{P}_{tot}$, the individual sensor power is then given as $P_i = \mathcal{P}_{tot}/N$, $\forall i$.

B. Short-term optimal power allocation

Since the transmitters have CSI, we can formulate a power control scheme that minimizes the distortion while satisfying a total power constraint in every transmission. We will call this power allocation the short-term optimal power allocation (ST-OPA). ST-OPA can be obtained by solving the following optimization problem

$$\begin{array}{ll} \min & D(\mathbf{P}(\mathbf{h}),\mathbf{h}) \\ \text{s.t.} & \sum_{i=1}^{N} P_i(\mathbf{h}) \le \mathcal{P}_{tot},\ P_i(\mathbf{h}) \ge 0 \quad \forall i. \end{array} \tag{2}$$

Problem (2) has been solved in [3]. The short-term optimal power allocation of the ith sensor is given by

$$P_i^*(\mathbf{h}) = \mathcal{P}_{tot}c_i(h_i)\left(\sum_{j=1}^{N} c_j(h_j)\right)^{-1} \quad \forall i \tag{3}$$

where $c_i(h_i) = C_ih_i/\left(C_i + \mathcal{P}_{tot}h_i\sigma_i^2/\sigma_c^2\right)^2$.

C. Long-term optimal power allocation

We now consider imposing a long-term total power constraint to the wireless sensor network, where the total power usage is averaged over time. Since the problem now deals with an extra dimension in time, an appropriate performance measure is the distortion outage probability introduced in Section II. We are interested in finding the optimal power allocation that minimizes the outage probability subject to a long-term total power constraint. We call this power allocation scheme the long-term optimal power allocation (LT-OPA). The problem is given as

$$\begin{array}{ll} \min & \Pr\left(D\left(\mathbf{P}(\mathbf{h}),\mathbf{h}\right) > D_{max}\right) \\ \text{s.t.} & E\left[\sum_{i=1}^{N} P_i(\mathbf{h})\right] \le \mathcal{P}_{tot},\ P_i(\mathbf{h}) \ge 0 \quad \forall i. \end{array} \tag{4}$$

Before we give the solution to problem (4), we first need the following definitions and notations, similar to [6]. We define the power allocation $\mathbf{P}^*(\mathbf{h}) = [P_1^*(\mathbf{h}), \ldots, P_N^*(\mathbf{h})]^T$, where

$$P_i^*(\mathbf{h}) = P_{tot}(\mathbf{h})c_i(h_i)\left(\sum_{j=1}^{N} c_j(h_j)\right)^{-1}, i = 1,\ldots,N \tag{5}$$

with $c_i(h_i) = C_i h_i / \left(C_i + P_{tot}(\mathbf{h}) h_i \sigma_i^2/\sigma_c^2\right)^2$, and $P_{tot}(\mathbf{h})$ being the solution of

$$\frac{1}{D_{max}} - \frac{1}{\sigma_\theta^2} = \sum_{i=1}^{N} \frac{h_i}{\left(\frac{\sigma_c^2 C_i}{P_{tot}(\mathbf{h})} + \sigma_i^2 h_i\right)} \quad (6)$$

Define the regions $\mathcal{R}_T(t) = \left\{\mathbf{h} : \sum_{i=1}^{N} P_i^*(\mathbf{h}) < t\right\}$, $\overline{\mathcal{R}}_T(t) = \left\{\mathbf{h} : \sum_{i=1}^{N} P_i^*(\mathbf{h}) \le t\right\}$ and $\mathcal{B}_T(t) = \left\{\mathbf{h} : \sum_{i=1}^{N} P_i^*(\mathbf{h}) = t\right\}$. We also define two power sum quantities as $P_T(t) = \int_{\mathcal{R}_T(t)} \sum_{i=1}^{N} P_i^*(\mathbf{h}) dF(\mathbf{h})$ and $\overline{P}_T(t) = \int_{\overline{\mathcal{R}}_T(t)} \sum_{i=1}^{N} P_i^*(\mathbf{h}) dF(\mathbf{h})$ where $F(\mathbf{h})$ denotes the joint c.d.f. of $\mathbf{h}$. Finally, the power sum threshold t^* and the weight u^* are given as $t^* = \sup\{t : P_T(t) < \mathcal{P}_{tot}\}$ and $u^* = \frac{\mathcal{P}_{tot} - P_T(t^*)}{\overline{P}_T(t^*) - P_T(t^*)}$. With the above definitions we now present the solution to problem (4).

Theorem 1: The solution of problem (4) is given as

$$\hat{\mathbf{P}}(\mathbf{h}) = \begin{cases} \mathbf{P}^*(\mathbf{h}), & \text{if } \mathbf{h} \in \mathcal{R}_T(t^*) \\ \mathbf{0}, & \text{if } \mathbf{h} \notin \overline{R}_T(t^*) \end{cases} \quad (7)$$

while if $\mathbf{h} \in \mathcal{B}_T(t^*)$, $\hat{\mathbf{P}}(\mathbf{h}) = \mathbf{P}^*(\mathbf{h})$ with probability u^* and $\hat{\mathbf{P}}(\mathbf{h}) = 0$ with probability $1 - u^*$.

The proof follows using similar techniques as in [6] together with results from [3], and is hence excluded.

The long-term optimal power allocation scheme that minimizes the outage probability subject to a long-term total power constraint says that if the vector of channel gains falls inside the region defined by $\mathcal{R}_T(t^*)$, where t^* is a quantity that is associated with $\mathcal{P}_{tot}$, then the sensors should transmit with powers given by (5) and achieve a distortion of exactly D_{max}. Otherwise, none should transmit to save power, and this is where outage occurs.

IV. Diversity orders of distortion outage

We are interested in seeing how the outage probability decays as the number of sensors N increases. In this section we will obtain asymptotic expressions of $\log P_{outage}$ (log here denotes the natural logarithm), for the different power allocation schemes given in Section III. Such expressions characterize the *diversity order of distortion outage* introduced in [2], who showed that the outage probability decays exponentially with the number of sensors N for i.i.d. orthogonal MAC. For analytical tractability, in the following analysis, we will only consider a homogeneous wireless sensor network where all the measurement noise and fading distributions are i.i.d. As a consequence, we will denote $\sigma_i^2 = \sigma^2$ and $C_i = C = \sigma_\theta^2 + \sigma^2$, $\forall i$.

Notation: For two functions $f(\cdot)$ and $g(\cdot)$, we will use the standard asymptotic notation (see for example [7]) and say that $f \sim g$ as $t \to t_0$, if $\frac{f(t)}{g(t)} \to 1$ as $t \to t_0$.

A. Equal power allocation

Substituting $P_i = \mathcal{P}_{tot}/N$ into (1), after some algebraic manipulation we obtain

$$\frac{D}{\sigma_\theta^2} = \frac{\frac{\sum_{i=1}^N h_i}{N} + \frac{\sigma_c^2 C}{\sigma^2 \mathcal{P}_{tot}}}{\frac{\sum_{i=1}^N h_i}{N} + \frac{\sigma_c^2 C}{\sigma^2 \mathcal{P}_{tot}} + \frac{\sigma_\theta^2 N}{\sigma^2}\left(\frac{\sum_{i=1}^N \sqrt{h_i}}{N}\right)^2}. \quad (8)$$

Inspecting the RHS of (8), we note that $\frac{1}{N}\sum_{i=1}^N h_i$ and $\frac{1}{N}\sum_{i=1}^N \sqrt{h_i}$ converge to $E[h]$ and $E[\sqrt{h}]$ respectively by the strong law of large numbers as N gets large. However we find that $\text{var}\left(\frac{1}{N}\sum_{i=1}^N h_i\right) = \frac{1}{N}\text{var}[h]$ and $\text{var}\left(\frac{\sigma_\theta^2 N}{\sigma^2}\left(\frac{\sum_{i=1}^N \sqrt{h_i}}{N}\right)^2\right) \approx \frac{4\sigma_\theta^4 N}{\sigma^4}(E[\sqrt{h}])^2 \text{var}[\sqrt{h}]$ (obtained using the Delta method [8]). We see that the variance of $\frac{1}{N}\sum_{i=1}^N h_i$ decreases like $1/N$, whereas the approximate variance of $\frac{\sigma_\theta^2 N}{\sigma^2}\left(\frac{\sum_{i=1}^N \sqrt{h_i}}{N}\right)^2$ increases with N. We therefore choose to replace $\frac{1}{N}\sum_{i=1}^N h_i$ by its mean $E[h]$, and retain $\frac{\sigma_\theta^2 N}{\sigma^2}\left(\frac{1}{N}\sum_{i=1}^N \sqrt{h_i}\right)^2$ for large N. This gives us the result:

$$D - \sigma_\theta^2 \eta \left(\eta + \frac{\sigma_\theta^2 N}{\sigma^2}\left(\frac{\sum_{i=1}^N \sqrt{h_i}}{N}\right)^2\right)^{-1} \overset{a.s.}{\to} 0 \quad (9)$$

where $\eta = E[h] + \frac{\sigma_c^2 C}{\sigma^2 \mathcal{P}_{tot}}$.

The asymptotic distortion outage probability for large N can therefore be found as

$$\begin{aligned} P_{outage} &= \Pr(D > D_{max}) \\ &\sim \Pr\left(\frac{1}{N}\sum_{i=1}^N \sqrt{h_i} < \sqrt{\frac{\eta\sigma^2(\sigma_\theta^2 - D_{max})}{D_{max}\sigma_\theta^2 N}}\right) \end{aligned} \quad (10)$$

By inspecting (10) we see that the asymptotic outage probability is expressed in terms of the empirical mean of i.i.d. random variables $\sqrt{h_i}$ being less than a threshold that is a function of N. This resembles a more general form of the typical large deviation problem where the threshold is a constant. In Theorem 2 we will provide a generalized version of Cramer's Theorem which can be applied to (10). Before we give the theorem we need the following definitions. The moment-generating function of the random variable X is defined as $M_X(t) \triangleq E\left[e^{tX}\right]$. The cumulant-generating function of the random variable X is defined as $\Lambda_X(t) \triangleq \log M_X(t)$. The rate function of the random variable X is defined as $I_X(c) = \sup_t\{ct - \Lambda_X(t)\}$.

Theorem 2: Let $X_1, X_2, \ldots$ be i.i.d. random variables with mean $\mu_X > 0$, and suppose that their moment generating function $M_X(t) = E\left[e^{tX}\right]$ is finite in some neighborhood of the origin $t = 0$. Let $\tilde{Y}_{n,i}$ be the exponential change of distribution of $Y_i = -X_i + \mu_X$ defined as

$$dF_{\tilde{Y}_n}(y) = \frac{e^{\tau_n y}}{M_Y(\tau_n)} dF_Y(y) \quad (11)$$

Suppose that $\Pr\left(\frac{1}{n}\sum_{i=1}^n \tilde{Y}_{n,i} > E\left[\tilde{Y}_{n,i}\right]\right)$ is bounded away from zero as $n \to \infty$. Let $a_n = \frac{a}{n^p}$, $p \ge 0$ and $\Pr(X < a_n) >$

$0, \forall n$. Then $I_X(a_n) > 0$ for sufficiently large n, and

$$\log \Pr\left(\frac{1}{n}\sum_{i=1}^{n} X_i \le a_n\right) \sim -nI_X(a_n) \quad \text{as } n \to \infty. \quad (12)$$

Proof: Due to length restrictions, the proof is omitted, but can be found in [9]. ■

In order to apply Theorem 2 to (10), we need to verify the assumption that $\Pr\left(\frac{1}{n}\sum_{i=1}^{n}\tilde{Y}_{n,i} > E\left[\tilde{Y}_{n,i}\right]\right)$ is bounded away from zero as $n \to \infty$. The following lemma verifies this condition in the case of Rayleigh fading.

Lemma 4.1: Let $Y_i = -\sqrt{h_i} + E\left[\sqrt{h_i}\right]$, where $\sqrt{h_i}$ is Rayleigh distributed with parameter κ (i.e. $f_{\sqrt{h}}(x) = \frac{x}{\kappa^2}e^{-x^2/2\kappa^2}$). Denote $\tilde{Y}_{n,i}$ as the exponential change of distribution of Y_i as defined in (11). Then

$$\Pr\left(\frac{1}{n}\sum_{i=1}^{n}\tilde{Y}_{n,i} > E\left[\tilde{Y}_{n,i}\right]\right) \to 0.5 \quad \text{as } n \to \infty \quad (13)$$

Proof: See [9]. ■

Applying Theorem 2 to (10) we have

$$\log P_{outage} \sim -NI^{-}_{\sqrt{h}}\left(\frac{a}{\sqrt{N}}\right) \quad \text{as } N \to \infty \quad (14)$$

where

$$I^{-}_{\sqrt{h}}\left(\frac{a}{\sqrt{N}}\right) = \sup_{\theta<0}\left(\frac{a}{\sqrt{N}}\theta - \log M_{\sqrt{h}}(\theta)\right). \quad (15)$$

Lemma 4.2:

$$I^{-}_{\sqrt{h}}\left(\frac{a}{\sqrt{N}}\right) \sim -2-\log\left(\frac{a^2}{2\kappa^2}\right)+\log N \quad \text{for large } N. \quad (16)$$

Proof: See [9]. ■

Substituting (16) into (14) gives

$$\log P_{outage} \sim -N\left(-2 - \log\left(\frac{a^2}{2\kappa^2}\right) + \log N\right) \quad (17)$$

$$\sim -N\log N \quad \text{for large } N \quad (18)$$

which shows that the diversity order of distortion outage in i.i.d. coherent MAC with Rayleigh fading using EPA is $N\log N$ for large N. In [2], the authors obtained a diversity order of N for i.i.d. orthogonal MAC with Rayleigh fading using EPA. We thus see that the coherent MAC achieves a higher diversity order over the orthogonal MAC case by a factor of $\log N$ for i.i.d. Rayleigh-faded channels.

B. Short-term optimal power allocation

We first give the expression of distortion using ST-OPA. Substituting (3) into (1) gives

$$D = \left(\frac{1}{\sigma_\theta^2} + \frac{\left(\sum_{i=1}^{N}\sqrt{h_iP_i^*}\right)^2}{\sigma^2\sum_{i=1}^{N}h_iP_i^* + \sigma_C^2 C}\right)^{-1} = \frac{\sigma_\theta^2\sigma^2}{\sigma^2 + \sigma_\theta^2\sum_{i=1}^{N}Z_i}$$

where $Z_i = h_i/(h_i+\rho)$ with $\rho = C\sigma_c^2/\mathcal{P}_{tot}\sigma^2$, and the second equality follows after some algebraic manipulation. The distortion outage probability can therefore be written as

$$P_{outage} = \Pr(D > D_{max}) = \Pr\left(\frac{1}{N}\sum_{i=1}^{N}Z_i < g_N\right) \quad (19)$$

where $g_N = g/N$ and $g = \sigma^2\left(1/D_{max} - 1/\sigma_\theta^2\right)$.

Denote Z as the random variable distributed according to the common distribution of Z_i. We now apply Theorem 2 to (19). We have the following lemma needed for verifying one of the assumptions in Theorem 2 (similar to Lemma 4.1).

Lemma 4.3: Let $Y_i = -Z_i + E[Z_i]$, where $Z_i = h_i/(h_i+\rho)$, with h_i being exponentially distributed with mean $1/\lambda$. Denote $\tilde{Y}_{n,i}$ as the exponential change of distribution of Y_i as defined in (11). Then

$$\Pr\left(\frac{1}{n}\sum_{i=1}^{n}\tilde{Y}_{n,i} > E\left[\tilde{Y}_{n,i}\right]\right) \to 0.5 \quad \text{as } n \to \infty \quad (20)$$

Applying Theorem 2 to (19) we have

$$\log P_{outage} \sim -NI_Z^{-}(g_N) \quad \text{as } N \to \infty \quad (21)$$

where $I_Z^{-}(g_N) = \sup_{\theta<0}(g_N\theta - \log M_Z(\theta))$.

Lemma 4.4:

$$I_Z^{-}(g_N) \sim -1 - \log(\lambda\rho g) + \log N \quad \text{for large N.} \quad (22)$$

Proof: See [9]. ■

Substituting (22) into (21) gives

$$\log P_{outage} \sim -N(-1 - \log(\lambda\rho g) + \log N) \quad (23)$$

$$\sim -N\log N \quad \text{for large } N. \quad (24)$$

Hence the diversity order of distortion outage for i.i.d. coherent MAC with Rayleigh fading using ST-OPA is $N\log N$, which interestingly achieves the same diversity order of distortion outage as EPA.

C. Long-term optimal power allocation

In this section we show that it is possible to use LT-OPA in coherent MAC to achieve zero distortion outage with a finite amount of power, if the number of sensors $N > 1$. To do this, we will analyze the power required to achieve zero outage.

For $N = 1$, the sum power expression in (6) can be re-arranged and expressed as $P_{tot}(h) = \frac{K_1}{h}$ where $K_1 = \frac{\gamma_{th}\sigma_c^2 C}{(1-\sigma^2\gamma_{th})}$. The region $\mathcal{R}_T(t)$ can be easily found directly from the definition as $\mathcal{R}_T(t) = \{h : P_{tot}(h) < t\} = \left\{h : h > \frac{K_1}{t}\right\}$. The average power sum, $P_T(t)$, becomes

$$P_T(t) = \int_{\mathcal{R}_T(t)} P_{tot}(h)dF(h) = \int_{\frac{K_1}{t}}^{\infty}\frac{K_1}{h}\lambda e^{-\lambda h}dh \quad (25)$$

$$= \lambda K_1 E_1\left(\frac{\lambda K_1}{t}\right) \quad (26)$$

where $u = \lambda h$ and $E_1(x) = \int_x^\infty \frac{e^{-t}}{t}dt$ is the exponential integral. To find the maximum total power that achieves zero-outage, we simply let $t \to \infty$. However, as $t \to \infty$, $P_T(t) \to \infty$, implying that we need an infinite amount of power to achieve zero outage for $N = 1$.

For $N > 1$ it is difficult to obtain closed form expressions of the maximum power required to achieve zero-distortion outage. Instead, we show that it is possible to achieve zero-outage with finite power for $N > 1$. Suppose we have a sub-optimal power allocation scheme as follows. For every transmission, we select the sensor with the best channel gain

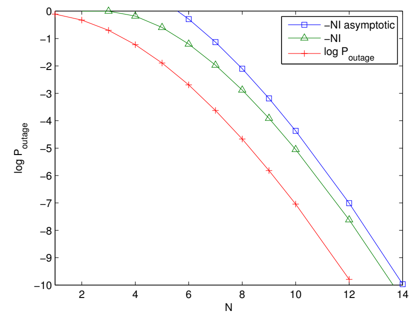

Fig. 2. EPA with $\mathcal{P}_{tot} = 10mW$. Simulation parameters: $\sigma = 0.0014$, $a = 0.003$, $\sigma_\theta^2 = 1$, $\sigma_i^2 = 10^{-3}$, $\sigma_c^2 = 10^{-8}$, $D_{max} = 0.1$.

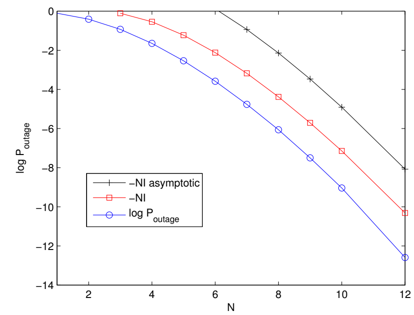

Fig. 3. ST-OPA with $\mathcal{P}_{tot} = 10mW$. Simulation parameters: $\lambda = 250,000$, $g = 0.09$, $\sigma_\theta^2 = 1$, $\sigma_i^2 = 10^{-3}$, $\sigma_c^2 = 10^{-8}$, $D_{max} = 0.1$.

and use only that sensor to transmit with just enough power to meet the distortion constraint. Denote the power as $\tilde{P}(h_{max})$ where $h_{max} = \max(h_1, \ldots, h_N)$. $\tilde{P}(h)$ can be obtained from the distortion constraint and it is given as $\tilde{P}(h_{max}) = \frac{\gamma_{th}\sigma_c^2 C}{(1-\sigma^2\gamma_{th})h_{max}}$. We can see that this power allocation scheme is simply a channel inversion scheme. The c.d.f. and p.d.f. of choosing the maximum channel gain out of a set of i.i.d. exponentially distributed random variables $\{h_1, \ldots, h_N\}$ with mean $1/\lambda$ is given respectively as $F_{h_{max}}(t) = \left(1 - \lambda e^{-\lambda t}\right)^N$ and $f_{h_{\max}}(t) = N\lambda\left(1 - \lambda e^{-\lambda t}\right)^{N-1} e^{-\lambda t}$. The transmission power averaged over all possible values of the channel realizations and time is then given as

$$E\Big[\tilde{P}(h_{max})\Big] = \int_0^\infty \frac{\gamma_{th}\sigma_c^2 C}{(1-\sigma^2\gamma_{th})h} \cdot N\lambda\left(1 - \lambda e^{-\lambda h}\right)^{N-1} e^{-\lambda h} dh.$$

The integral above is well-known to be finite for $N > 1$ [10]. Since this suboptimal power allocation scheme can achieve zero-outage with finite power, the optimal power allocation scheme will also achieve zero-outage with finite power.

V. Simulation Results

The following results, if not computed directly from the equations, are obtained via Monte Carlo simulation over 1,000,000 channel realizations. We first present the diversity order of distortion outage for EPA. We simulated the case where $\mathcal{P}_{tot} = 10mW$ and plotted the results in Fig. 2. We compare between plots of $\log P_{outage}$ obtained via Monte Carlo simulation, the values of $-NI_{\sqrt{h}}(a/\sqrt{N})$ obtained by solving (15) numerically, and plots of the asymptotic expression (17). Note that the asymptotic results $I_{\sqrt{h}}(a/\sqrt{N})$ and (17) only give us the slope of the outage probability when plotted on a log scale; these two lines may not necessarily converge to $\log P_{outage}$ but their gradients should coincide for large N, as can be seen in Fig. 2. Fig. 3 is an analogous plot for ST-OPA, again using $\mathcal{P}_{tot} = 10$mW. Again we see that the asymptotic expression (23) gives similar gradients to the Monte Carlo simulation as N increases.

In Fig. 4 we compare the outage performance as a function of N for the three different power allocation schemes considered in this paper, using $\mathcal{P}_{tot} = 1,600\mu$W. From this figure we can see that the gradients of EPA and ST-OPA are similar

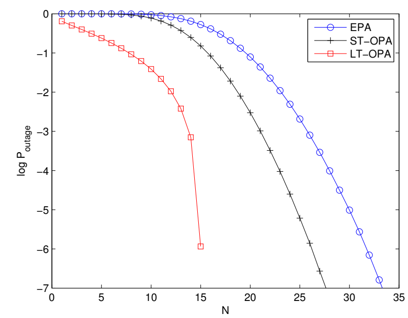

Fig. 4. P_{outage} versus N. Simulation parameters: $\sigma_\theta^2 = 1$, $\sigma^2 = 10^{-3}$, $\sigma_c^2 = 10^{-8}$, $D_{max} = 0.1$, $\lambda = 250,000$ and $\mathcal{P}_{tot} = 1,600\mu$W.

for large N, while the outage probability curve for LT-OPA approaches to a vertical asymptote due to the possibility of obtaining zero outage.

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