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Throughput Scaling in Cognitive Multiple Access Networks with Power and Interference Constraints

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Abstract*—This paper focuses on the secondary network throughput scaling in cognitive radio networks when secondary users' transmission powers are optimally allocated. Throughput scaling laws are obtained for two different cognitive radio networks under two different communication scenarios. In the first network type called *power-interference limited* networks, secondary users' transmission powers are limited by both average total power constraint and the constraint on the average interference that they cause to primary users. In the second network type called *interference limited* networks, secondary users' transmission powers are only limited by average interference constraint. For both network types, an asymmetric communication scenario, in which the channels between secondary users and the secondary base station experience Rayleigh fading and those between secondary users and the primary base station experience Rician fading, and a symmetric communication scenario, in which both types of channels experience Rayleigh fading, are considered. It is shown that the secondary network throughput scales like $\log\log\left(\frac{K+1}{e^K}N\right)$ and $\log\left(\frac{K+1}{e^K}N\right)$ for power-interference limited and interference limited networks, respectively, under the asymmetric communication scenario, where N is the number of secondary users and $K > 0$ is the Rician factor. For the symmetric communication scenario, these scaling laws are given by $\log\log(N)$ and $\log(N)$ for power-interference limited and interference limited networks, respectively.

I. Introduction

Cognitive radio technology has been proposed as a possible solution to the spectrum scarcity problem by allowing unlicensed (secondary) users to access the spectrum reserved for licensed (primary) users [1]. The rationale is that secondary users can use this spectrum as long as they do not cause harmful degradation to the primary transmission. Various paradigms for such cognitive radio networks have been proposed such as overlay, underlay and interweave [2]. In the underlay model (which is the paradigm of interest in the current paper), also known as the spectrum sharing scenario, secondary users share the spectrum with primary users regardless of primary's ON/OFF status as long as the interference caused by the secondary transmitter to the primary receiver is kept low to guarantee a primary's Quality of Service (QoS). Various types of interference constraints have been considered in the literature including average or peak interference constraints, primary capacity loss constraint and primary outage probability constraint [3]. A large volume of papers considering optimal power allocation for a single secondary user under average or peak interference constraints and/or average or peak secondary transmission power constraints have also emerged, e.g., see [4] and [5]. Recently, optimal power allocation for ergodic sum capacity maximization in a multiple access and broadcast secondary network under various combinations of transmission power and interference constraints have been considered in [6]. It is shown in [6] that the optimal power allocation result for a multiple access secondary network with average transmission and average interference power constraints is to schedule the secondary user with the best joint power and interference channel states (see optimum power allocation policy in Section II). In essence, this reflects the opportunistic scheduling type of results derived for primary networks in [7] (for multiple access channels) and [8] (for broadcast channels).

This automatically triggers the need for multiuser diversity or sum rate scaling analysis in a cognitive radio multiple access or broadcast network with increasing numbers of secondary users under various forms of transmission power and interference constraints. An analysis for throughput scaling in MIMO broadcast primary networks can be found in [9]. Multiuser diversity has been investigated for cognitive radio networks in a number of papers. In [10], the authors investigate secondary network throughput scaling under peak transmission power and interference constraints with their ratio going to infinity, whereas in [11], throughput for a cognitive network with optimal pairing of N secondary users and M available spectrum bands is shown to scale as $M \log\log N$ under centralized scheduling. In [12], the authors study the gains obtained in a secondary user's capacity via selection diversity based on the best secondary channel (multiuser diversity) as well as the weakest interfering channel (multi-spectrum diversity). Finally, in [13], a related notion of multiuser interference diversity is analyzed for cognitive networks over multiple access, broadcast and parallel-access channels.

In our paper, we analyze the classical multiuser diversity or secondary sum throughput scaling for a cognitive multiple access network under average total transmission power and average interference constraints with a single primary user (note that the extension to multiple primary users is immediate), as described in Fig. 1. We consider the availability of full channel state information at the secondary base station of all secondary transmitter to secondary base station channels as well as the secondary transmitter to primary base station channels. We investigate a centralized scenario where the secondary base station schedules the optimum secondary user according to the results of [6]. We consider two possible networks: (1) the power-interference limited networks where both average total power and average interference constraints are active, and

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(2) the interference limited networks where the average transmission power is unlimited and only the average interference constraint is active. We also consider a more general fading scenario than those considered in [11] for cognitive networks or [9] for primary networks in that we allow secondary's own channels and secondary to primary interfering channels to have different (asymmetric) distributions instead of having symmetric distributions such as the exponential distribution (Rayleigh fading). In particular, we consider independent and identically distributed (*i.i.d.*) Rayleigh fading for all secondary transmitter to secondary base station channels and i.i.d. Rician fading channels (with a Rician factor of K) for all secondary transmitter to primary receiver channels (see also [14] for related capacity results for a single secondary user). We show that the sum throughput scaling rate for N secondary users is given by $\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$ and $\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$ for power-interference limited and interference limited networks, respectively. The symmetric fading scenario can be derived as a special case when $K=0$, in which case the scaling rates are given by $\log\log(N)$ and $\log(N)$ for power-interference limited and interference limited networks, respectively. These results are summarized in Table I.

The rest of the paper is organized as follows. Section II describes the system model and network configuration along with the modeling assumptions. Section III derives and presents the scaling laws for the power-interference limited networks, and Section IV presents the scaling laws for the interference limited networks. Section V presents some numerical results to illustrate the derived scaling laws followed by some concluding remarks in Section VI.

II. System model

We consider a cognitive radio network where N secondary users (SU) share a frequency band with one primary user (PU) as shown in Fig. 1. The extensions of the system model to communication scenarios with multiple PUs are possible but they are not considered in this paper for the sake of the clarity of final results. SUs form a multiple access channel to the secondary base station (SBS), and interfere with the signal reception at the primary base station (PBS). For $i \in \{1,2,\cdots,N\}$, let h_i be the ith SU's channel power gain to the SBS, and let the corresponding interference channel gain to the PBS be given by g_i. All channels are assumed to be ergodic block-fading channels with continuous power gain distributions. We assume that all h_i's and all g_i's are i.i.d. across the SUs, and the random vectors $\boldsymbol{h} = (h_1, h_2, \cdots, h_N)^\top$ and $\boldsymbol{g} = (g_1, g_2, \cdots, g_N)^\top$ are also independent. We assume the availability of full channel state information (CSI), i.e., availability of the random gain vectors $\boldsymbol{h}$ and $\boldsymbol{g}$ at the SBS.

We define a power allocation policy $\boldsymbol{P}(\boldsymbol{h},\boldsymbol{g}) = (P_1(\boldsymbol{h},\boldsymbol{g}), P_2(\boldsymbol{h},\boldsymbol{g}), \cdots, P_N(\boldsymbol{h},\boldsymbol{g}))^\top$ as a mapping from $\mathbb{R}_+^N \times \mathbb{R}_+^N$ to $\mathbb{R}_+^N$, where $P_i(\boldsymbol{h},\boldsymbol{g})$ represents the transmission power allocated to the ith SU at the joint fading state $(\boldsymbol{h},\boldsymbol{g})$. Let $\mathcal{P}$ be the space of all power allocation policies. We are interested in the solution of the following function optimization problem [15]:

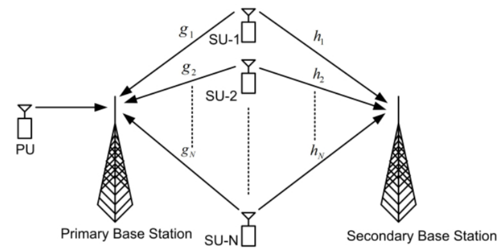

Fig. 1. N SUs forming a multiple access channel to the secondary base station and interfering with signal reception at the primary base station

$$\begin{array}{ll} \underset{\boldsymbol{P}\in\mathcal{P}}{\text{maximize}} & \mathsf{E}_{\boldsymbol{h},\boldsymbol{g}}\left[\log\left(1+\boldsymbol{h}^\top\boldsymbol{P}(\boldsymbol{h},\boldsymbol{g})\right)\right] \\ \text{subject to} & \mathsf{E}_{\boldsymbol{h},\boldsymbol{g}}\left[\boldsymbol{1}^\top\boldsymbol{P}(\boldsymbol{h},\boldsymbol{g})\right] \leq P_{\text{av}} \\ & \mathsf{E}_{\boldsymbol{h},\boldsymbol{g}}\left[\boldsymbol{g}^\top\boldsymbol{P}(\boldsymbol{h},\boldsymbol{g})\right] \leq Q_{\text{av}} \end{array}, \quad (1)$$

where all expectations are taken over random vectors $\boldsymbol{h}$ and $\boldsymbol{g}$, and $\boldsymbol{1}$ (in boldface) is the vector of ones. Our objective function for a given power allocation policy $\boldsymbol{P}$ represents the multiple access sum rate capacity under $\boldsymbol{P}$, which can be achieved by using complex Gaussian codebooks and successive signal decoding at the SBS [7]. For the sake of simplicity, we will assume that all channel power gains have unit mean.

The optimal power allocation policy $\boldsymbol{P}^\star$ for the function optimization problem in (1) is given by the following water-filling scheme:

$$\begin{array}{ll} P_i^\star(\boldsymbol{h},\boldsymbol{g}) = \left(\frac{1}{\lambda+\mu g_i} - \frac{1}{h_i}\right)^+ & \text{if } \frac{h_i}{\lambda+\mu g_i} \geq \frac{h_j}{\lambda+\mu g_j}, \forall j \neq i \\ P_i^\star(\boldsymbol{h},\boldsymbol{g}) = 0 & \text{otherwise} \end{array},$$

where $(x)^+ = \max(x,0)$, $\lambda \geq 0$ is the Lagrange multiplier associated with the average total transmission power constraint in (1), and $\mu \geq 0$ is the Lagrange multiplier associated with the average interference constraint in (1)[1]. We skip the proof as it is similar to the proof of lemma 3.2 in [6]. We define X_i and $X_N^\star$ as $X_i = \frac{h_i}{\lambda+\mu g_i}$ and $X_N^\star = \max_{1\leq i\leq N} X_i$. Then, under $\boldsymbol{P}^\star$, we schedule the ith SU for transmission with power $\left(\frac{1}{\lambda+\mu g_i} - \frac{1}{h_i}\right)^+$ if and only if ith SU has the best joint power and interference channel state, i.e., $X_i = X_N^\star$. Therefore, the sum-rate achieved by $\boldsymbol{P}^\star$ is given by $R_N = \mathsf{E}\left[\log(X_N^\star)\, 1_{\{X_N^\star \geq 1\}}\right]$. The purpose of this paper is to identify how the optimum sum rate R_N scales as the number of SUs becomes large. To this end, we will study the asymptotic distribution of $X_N^\star$, and characterize the asymptotic scaling behavior of $X_N^\star$ for two different network types, *power-interference limited* and *interference limited* networks, under two different communication scenarios, *asymmetric* and *symmetric* communication scenarios.

[1]Although Lagrange multipliers are functions of N, it can be shown that assuming fixed Lagrange multipliers does not affect the throughput scaling results.

TABLE I
THROUGHPUT SCALING IN COGNITIVE RADIO NETWORKS.

	Asymmetric Communication Scenario	Symmetric Communication Scenario
Power-interference Limited Networks	$\lim_{N\to\infty} \frac{R_N}{\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)} = 1$[a]	$\lim_{N\to\infty} \frac{R_N^{\mathrm{s}}}{\log\log(N)} = 1$
Interference Limited Networks	$\lim_{N\to\infty} \frac{R_N}{\log\left(\frac{K+1}{\mathrm{e}^K}N\right)} = 1$[a]	$\lim_{N\to\infty} \frac{R_N^{\mathrm{s}}}{\log(N)} = 1$

[a] K is the Rician factor.

III. THROUGHPUT SCALING: POWER-INTERFERENCE LIMITED NETWORKS

In this part of the paper, we will study the throughput scaling for cognitive radio networks when SUs' transmission powers are limited by both average total power and average interference constraints given in (1). To study the scaling behavior of R_N for these networks, we will first consider an asymmetric communication scenario in which the channel envelope gains between SUs and the SBS are Rayleigh distributed, and those between SUs and the PBS are Rician distributed with Rician factor $K \geq 0$. This implies that h_i's are exponentially distributed, and g_i's have non-central chi-square distribution with two degrees of freedom [16]. For $K = 0$, we recover the symmetric communication scenario where all channel envelope gains are Rayleigh distributed. Therefore, the scaling results provided in this work will be more general than those presented in [9]. In the next section, we will extend the analysis in this part to the special case (for both symmetric and asymmetric scenarios) where the network is interference limited; that is, each SU's transmission power is limited by the average interference constraint Q_{av} at the PBS but not average transmission power constraint P_{av}.

After some computation, it can be shown that the cumulative distribution function (CDF) of X_i, which we denote by $F(x)$, is given $F(x) = 1 - \frac{K+1}{\mu x+K+1}\mathrm{e}^{-x\left(\lambda+\frac{\mu K}{\mu x+K+1}\right)}$. $F(x)$ satisfies the Von Mises conditions [17], i.e., $\lim_{x\to\infty}\frac{d}{dx}\left[\frac{1-F(x)}{f(x)}\right] = 0$, where $f(x)$ denotes the probability density function (PDF) corresponding to $F(x)$. Therefore, we can find sequences of real numbers $\{a_N\}_{N=1}^{\infty}$ and $\{b_N\}_{N=1}^{\infty}$ such that $\frac{X_N^\star - b_N}{a_N}$ converges in distribution to a Gumbel distributed random variable. That is, if $F_N^\star$ is the CDF of $X_N^\star$, then $\lim_{N\to\infty} F_N^\star(a_N x + b_N) = \exp\left(-\mathrm{e}^{-x}\right)$ for all $x \in \mathbb{R}$. Furthermore, normalizing constants a_N and b_N can be chosen to satisfy $F(b_N) = 1 - 1/N$ and $a_N = \frac{1}{Nf(b_N)}$, and they can be further expanded as

$$b_N = \frac{1}{\lambda}\log\left(\frac{K+1}{\mathrm{e}^K}N\right) - \frac{1}{\lambda}\log\left(\frac{\mu}{\lambda}\log\left(\frac{K+1}{\mathrm{e}^K}N\right)\right) + O\left(\frac{\log\log(N)}{\log(N)}\right),$$

and

$$a_N = \frac{1}{\lambda}\left(1 - O\left(\frac{1}{\log(N)}\right)\right).$$

The following lemma, Lemma 1, characterizes the asymptotic behavior of $F_N^\star(a_N x + b_N)$. This lemma will be very helpful to estimate the tail probabilities lying under $F_N^\star(x)$. We will only focus on its applications by skipping the proof due to space limitations. The proof directly follows by using standard arguments in mathematical analysis. In Lemma 1, we allow x to vary with N but do not show this relation explicitly.

Lemma 1: For $x = O\left(\log(N)\right)$,

$$F_N^\star(a_N x + b_N) = \mathrm{e}^{-c_N(x) - O\left(\frac{c_N^2(x)}{N}\right)}, \qquad (2)$$

where $c_N(x)$ is given by (3).

As an application of Lemma 1, we can put $x = \log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$ and $x = -\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$ to obtain $\mathsf{Pr}(A) = 1 - O\left(\frac{1}{\log(N)}\right)$ where A is defined in (5). This implies that $\frac{X_N^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N\right)}$ converges in probability to $\frac{1}{\lambda}$, and $\frac{\log(X_N^\star)}{\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)}$ converges in probability to 1. These results intuitively suggest that the secondary network throughput scales like $\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$ as N tends to infinity. Since the convergence in probability does not always imply convergence in mean [18], we need some more work to establish the exact asymptotic behavior of the secondary network throughput in Theorem 1.

Theorem 1: $\lim_{N\to\infty}\frac{R_N}{\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)} = 1.$

Proof: It is easy to show $\liminf_{N\to\infty}\frac{R_N}{\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)} \geq 1$: we lower bound R_N for N large enough as

$$\begin{aligned} R_N &\geq \mathsf{E}\left[\log\left(X_N^\star\right)1_A\right] \\ &= \log\log\left(\frac{K+1}{\mathrm{e}^K}N\right) + O(1). \end{aligned} \qquad (4)$$

where A is defined in (5). The desired result follows by dividing R_N by $\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$, and then taking the $\liminf$ as N tends to infinity. To prove the other direction $\limsup_{N\to\infty}\frac{R_N}{\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)} \leq 1$, we will prove a stronger result given by the next lemma.

Lemma 2: $\lim_{N\to\infty}\mathsf{E}\left[\frac{X_N^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N\right)}\right] = \frac{1}{\lambda}.$

Proof: $\liminf_{N\to\infty}\mathsf{E}\left[\frac{X_N^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N\right)}\right] \geq \frac{1}{\lambda}$ follows from Fatou's lemma [18] and the convergence of $\frac{X_N^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N\right)}$ in

$$c_N(x) = \mathrm{e}^{-x}\left(\frac{\exp\left(O\left(\frac{x}{\log(N)}\right) - O\left(\frac{\log\log(N)}{\log(N)}\right)\right)\exp\left(\frac{K(K+1)}{\frac{\mu}{\lambda}x - O\left(\frac{x}{\log(N)}\right) + \frac{\mu}{\lambda}\log\left(\frac{K+1}{\mathrm{e}^K}N\right) - O(\log\log(N))}\right)}{1 + O\left(\frac{x}{\log(N)}\right) - O\left(\frac{\log\log(N)}{\log(N)}\right)}\right) \quad (3)$$

$$A = \left\{\frac{1}{\lambda}\log\left(\frac{K+1}{\mathrm{e}^K}N\right) - O\left(\log\log(N)\right) \leq X_N^\star \leq \frac{1}{\lambda}\log\left(\frac{K+1}{\mathrm{e}^K}N\right) - O\left(1\right)\right\} \quad (5)$$

probability to $\frac{1}{\lambda}$. To prove the other direction, we put $x = k\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$ in Lemma 1, where $k \geq 1$ is an integer. Then,

$$a_N x + b_N = \frac{k+1}{\lambda}\log\left(\frac{K+1}{\mathrm{e}^K}N\right) - O\left(\log\log(N)\right)$$

and

$$c_N(x) \quad = \quad \frac{\mathrm{e}^{O(k)}}{N^k O\left(k\right)} O\left(1\right).$$

Thus, for N large enough, we can write

$$\mathsf{Pr}\left\{X_N^\star \leq \frac{k+1}{\lambda}\log\left(\frac{K+1}{\mathrm{e}^K}N\right) - O\left(\log\log(N)\right)\right\} = 1 - O\left(\frac{\mathrm{e}^{O(k)}}{N^k O\left(k\right)}\right).$$

This implies that $\mathsf{Pr}\left\{X_N^\star > \frac{k+1}{\lambda}\log\left(\frac{K+1}{\mathrm{e}^K}N\right)\right\} \leq O\left(\frac{\mathrm{e}^{O(k)}}{N^k O(k)}\right)$. Take now a subsequence N_j such that

$$\limsup_{N\to\infty} \mathsf{E}\left[\frac{X_N^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N\right)}\right] = \lim_{N_j\to\infty} \mathsf{E}\left[\frac{X_{N_j}^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}\right] \quad (6)$$

and

$$\lim_{N_j\to\infty}\frac{X_{N_j}^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)} = \frac{1}{\lambda} \text{ almost surely.} \quad (7)$$

We can upper bound $\mathsf{E}\left[\frac{X_{N_j}^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}\right]$ as in (8). For N_j large enough, the summation $\sum_{k=2}^\infty \frac{k+1}{\lambda}O\left(\frac{\mathrm{e}^{O(k)}}{N_j^{k-2}O(k)}\right)$ is always finite and a decreasing function of N_j. Thus, we have

$$\mathsf{E}\left[\frac{X_{N_j}^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}\right] \leq \mathsf{E}\left[\frac{X_{N_j}^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}1_{\left\{\frac{X_{N_j}^\star}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}\leq\frac{2}{\lambda}\right\}}\right] + O\left(\frac{1}{N_j}\right).$$

We finish the proof by taking the limits, and using the dominated convergence theorem. ■

For N large enough, we have by Jensen inequality

$$\begin{aligned} R_N &\leq \mathsf{E}\left[\log\left(1 + X_N^\star\right)\right] \\ &\leq \log\left(1 + \mathsf{E}\left[X_N^\star\right]\right) \\ &\leq \log\log\left(\frac{K+1}{\mathrm{e}^K}N\right) + O\left(1\right). \end{aligned} \quad (9)$$

Thus, $\limsup_{N\to\infty}\frac{R_N}{\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)} \leq 1$, which completes the proof. ■

For the symmetric communication scenario in which $K = 0$, we have the following theorem characterizing the secondary network throughput scaling.

Theorem 2: Let R_N^{s} be the throughput of the secondary network in the symmetric communication scenario. Then,

$$\lim_{N\to\infty}\frac{R_N^{\mathrm{s}}}{\log\log(N)} = 1.$$

These two theorems completely characterize the throughput scaling for a power-interference limited cognitive radio secondary networks. In the next section, we turn our attention to the cognitive radio networks in which SUs' transmissions are only limited by the interference that they cause to PUs.

IV. Throughput Scaling: Interference Limited Networks

In this case, the average transmission power constraint is not active, and thus $\lambda = 0$ and $X_i = \frac{h_i}{\mu g_i}$. With a slight abuse of notation, we will still represent the CDF of X_i by F, which can be given by $F(x) = 1 - \frac{(K+1)\mathrm{e}^{-K}}{\mu x + K + 1}\mathrm{e}^{\frac{K(K+1)}{\mu x + K + 1}}$. $F(x)$ satisfies $\lim_{x\to\infty}\frac{xf(x)}{1-F(x)} = 1$, where $f(x)$ denotes the PDF corresponding to $F(x)$. Therefore, we can find a sequence of real numbers $\{a_N\}_{N=1}^\infty$ such that $\frac{X_N^\star}{a_N}$ converges in distribution to a Frechet distributed random variable. That is, for all $x > 0$, we have $\lim_{N\to\infty}F_N^\star(a_N x) = \exp\left(-\frac{1}{x}\right)$. The normalizing constants a_N can be chosen as $F(a_N) = 1 - 1/N$, which implies the asymptotic expansion

$$a_N = \frac{K+1}{\mu\mathrm{e}^K}\mathrm{e}^{\frac{K}{N}\mathrm{e}^K}N - \frac{K+1}{\mu} - O\left(\frac{1}{N}\right).$$

The following lemma characterizes the asymptotic behavior of $F_N^\star(a_N x)$. Again, we will only focus on the applications of Lemma 3, and omit its proof due to space limitations. In this lemma, we allow x to vary with N but do not show this relation explicitly.

Lemma 3: For $x = O\left(\log(N)\right)$,

$$F_N^\star(a_N x) = \mathrm{e}^{-c_N(x) - O\left(\frac{c_N^2(x)}{N}\right)}, \quad (10)$$

where

$$c_N(x) = \frac{\exp\left(\frac{K}{\exp\left(-K + \frac{K}{N}\mathrm{e}^K\right)Nx - O(x) + O(1)}\right)}{\exp\left(\frac{K}{N}\mathrm{e}^K\right)x - O\left(\frac{x}{N}\right) + O\left(\frac{1}{N}\right)}. \quad (11)$$

$$\begin{aligned}\mathsf{E}\left[\frac{X^{\star}_{N_j}}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}\right] &= \mathsf{E}\left[\frac{X^{\star}_{N_j}}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}1_{\left\{\frac{X^{\star}_{N_j}}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}\leq\frac{2}{\lambda}\right\}}\right]+\sum_{k=2}^{\infty}\mathsf{E}\left[\frac{X^{\star}_{N_j}}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}1_{\left\{\frac{k}{\lambda}<\frac{X^{\star}_{N_j}}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}\leq\frac{k+1}{\lambda}\right\}}\right]\\ &\leq \mathsf{E}\left[\frac{X^{\star}_{N_j}}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}1_{\left\{\frac{X^{\star}_{N_j}}{\log\left(\frac{K+1}{\mathrm{e}^K}N_j\right)}\leq\frac{2}{\lambda}\right\}}\right]+\frac{1}{N_j}\sum_{k=2}^{\infty}\frac{k+1}{\lambda}O\left(\frac{\mathrm{e}^{O(k)}}{N_j^{k-2}O(k)}\right) \qquad (8)\end{aligned}$$

For example, by putting $x = \frac{\log(N)}{\exp\left(\frac{K}{N}\mathrm{e}^K\right)}$ and $x = \frac{1}{\exp\left(\frac{K}{N}\mathrm{e}^K\right)\log(N)}$, we can obtain $\mathsf{Pr}(A) = 1 - O\left(\frac{1}{\log(N)}\right)$ where A is defined in (12). This implies that $\frac{\log(X^{\star}_N)}{\log(\frac{K+1}{\mathrm{e}^K}N)}$ converges in probability to 1. Similar to the power-interference limited networks, this result intuitively suggests that the secondary network throughput scales like $\log(\frac{K+1}{\mathrm{e}^K}N)$. This assertion is proved rigorously in the next theorem. With a slight abuse of notation, R_N will again represent the secondary network throughput when there are N SUs in the system.

Theorem 3: $\lim_{N\to\infty}\frac{R_N}{\log\left(\frac{K+1}{\mathrm{e}^K}N\right)} = 1.$

Proof: It is easy to prove $\liminf_{N\to\infty}\frac{R_N}{\log(\frac{K+1}{\mathrm{e}^K}N)} \geq 1$: we take the expectation of $\log(X^{\star}_N)1_{\{X^{\star}_N\geq 1\}}$ over event A which is defined in (12). To prove $\limsup_{N\to\infty}\frac{R_N}{\log(\frac{K+1}{\mathrm{e}^K}N)} \leq 1$, we define the following events for $k \geq 2$:

$$\begin{aligned} B &= \left\{1 \leq X^{\star}_N \leq \frac{K+1}{\mu\mathrm{e}^K}N\log^2(N)\right\}\\ B_k &= \left\{\frac{K+1}{\mu\mathrm{e}^K}N\log^k(N) < X^{\star}_N \leq \frac{K+1}{\mu\mathrm{e}^K}N\log^{k+1}(N)\right\}.\end{aligned}$$

By using Lemma 3, we can estimate probabilities $\mathsf{Pr}(B_k)$ as $\mathsf{Pr}(B_k) \leq O\left(\frac{1}{\log^k(N)}\right)$, and then upper bound R_N as in (13) by using these estimated probabilities and $R_N = \mathsf{E}[\log(X^{\star}_N)1_B] + \sum_{k=2}^{\infty}\mathsf{E}[\log(X^{\star}_N)1_{B_k}]$. For N large enough, $\sum_{k=2}^{\infty}O\left(\frac{O(k)}{\log^{k-1}(N)}\right)$ and $\sum_{k=2}^{\infty}O\left(\frac{1}{\log^{k-1}(N)}\right)$ are always finite and decreasing functions of N. Thus, we have

$$R_N \leq \log\left(\frac{K+1}{\mathrm{e}^K}N\right) + O(\log\log(N)).$$

We finish the proof by dividing R_N by $\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$, and then taking the $\limsup$ as N tends to infinity. ■

We now turn our attention to the symmetric communication scenario in which both secondary-to-secondary and secondary-to-primary channels experience Rayleigh fading. The throughput scaling for these networks can be found by simply putting $K = 0$ in Theorem 3, which is formally stated in the next theorem.

Theorem 4: Let R^{s}_N be the throughput of the secondary network in the symmetric communication scenario. Then,

$$\lim_{N\to\infty}\frac{R^{\mathrm{s}}_N}{\log(N)} = 1.$$

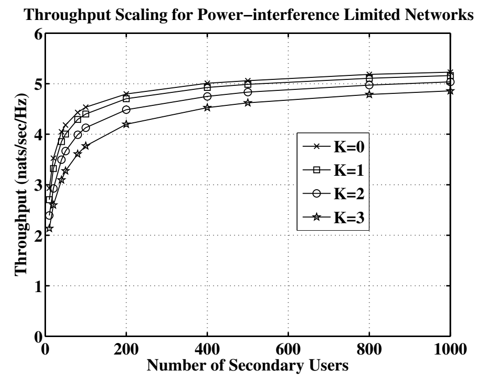

Fig. 2. Secondary network throughput scaling for power-interference limited networks with P_{ave} and Q_{ave} set to 16dB and 0dB, respectively.

These two theorems completely characterizes the throughput scaling for interference limited networks. Finally, one can verify our results in Theorems 2 and 4 directly by following the same steps that we used to derive Theorems 1 and 3 in the asymmetric communication scenario.

V. Simulation Results

In this section, we present our Monte-Carlo simulation results for the secondary network throughput scaling in which we set P_{ave} to 16dB and Q_{ave} to 0dB (by assuming unit power background noise), and allocate SUs' transmission powers optimally as in (2).

In Fig. 2, we show the secondary network throughput scaling for power-interference limited networks for different Rician factors. An increase in the number of SUs leads to a corresponding increase in the secondary network throughput due to multiuser diversity gains as predicted by Theorem 1. In addition, the scaling behavior of the throughput as shown in Fig. 2 is qualitatively similar to the $\log\log(N)$-type of behavior in Theorem 1, up to a scaling factor. We expect to see an exact throughput scaling of $\log\log\left(\frac{K+1}{\mathrm{e}^K}N\right)$ with more SUs, however we cannot simulate networks with large numbers of SUs due to computational constraints. Therefore, our asymptotic results can be considered as the lower bound, which becomes asymptotically tight for large numbers of SUs, on the secondary network throughput. As the Rician factor increases, SUs' transmission powers become more limited due to more severe interference caused to the PBS, and as a result, we start to observe a decrease in the secondary user network throughput. The maximum throughput is achieved when there

$$A = \left\{ \frac{1}{\log(N)} \left(\frac{(K+1)N}{\mu e^K} - \frac{K+1}{\mu \exp\left(\frac{K}{N} e^K\right)} - O\left(\frac{1}{N}\right) \right) < X_N^\star \leq \log(N) \left(\frac{(K+1)N}{\mu e^K} - \frac{(K+1)}{\mu \exp\left(\frac{K}{N} e^K\right)} - O\left(\frac{1}{N}\right) \right) \right\} \quad (12)$$

$$R_N \leq \log\left(\frac{K+1}{\mu e^K} N \log^2(N)\right) + \frac{\log\log(N)}{\log(N)} \sum_{k=2}^{\infty} O\left(\frac{O(k)}{\log^{k-1}(N)}\right) + \sum_{k=2}^{\infty} O\left(\frac{1}{\log^{k-1}(N)}\right) \quad (13)$$

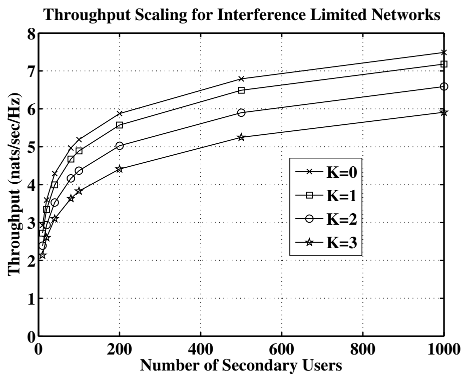

Fig. 3. Secondary network throughput scaling for interference limited networks with Q_{ave} set to 0dB.

is no line of sight between SUs and the PBS, i.e., symmetric communication scenario.

We show the secondary network throughput scaling for interference limited networks in Fig. 3. Since the same explanations given for the power-interference limited networks continue to hold for interference limited networks, we do not repeat them here, again.

VI. Conclusions

In this paper, we have focused on the secondary network throughput scaling in cognitive radio networks when secondary users' transmission powers are optimally allocated. We analyzed two different network types: power-interference limited networks and interference limited networks. In the power-interference limited networks, secondary users' transmissions are limited by both an average total power constraint and a constraint on the average interference that they cause to the primary users. In the interference limited networks, secondary users' transmissions are only limited by an average interference constraint. For both networks, we studied asymmetric and symmetric communication scenarios. In the symmetric scenario, all channels experience Rayleigh fading, whereas the channels between secondary users and the primary base station experience Rician fading in the asymmetric scenario. In the asymmetric case, we showed that as the number of secondary users N grows to infinity, the secondary network throughput scales like $\log\log\left(\frac{K+1}{e^K}N\right)$ and $\log\left(\frac{K+1}{e^K}N\right)$ for power-interference and interference limited networks, respectively, where $K \geq 0$ is the Rician factor. In the symmetric case, on the other hand, these scaling laws are given by $\log\log(N)$ and $\log(N)$ for power-interference limited and interference limited networks, respectively.

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