Optimal Power Policy and Throughput Analysis in Cognitive Broadcast Networks under Primary's Outage Constraint

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***Abstract*—This paper focuses on a spectrum-sharing based cognitive radio fading broadcast channel (BC) with a single-antenna secondary base station (SBS) and M secondary receivers (SRs) concurrently utilizing the same spectrum band with one delay-sensitive primary user (PU). The quality-of-service requirement in the primary network is given by the primary user's outage probability constraint (POC). We address the optimal power allocation problem for the ergodic sum capacity (ESC) maximization in the secondary BC network subject to a POC and an average transmit power constraint at SBS. Optimality conditions reveal that in each fading block SBS will choose only one SR with the highest channel power gain and allocate the block to that user. Furthermore, if PU's power strategy is assumed to be ON-OFF with constant power when ON, the secondary network throughput scaling for large M in Rayleigh fading is also investigated. It is shown that the secondary sum throughput in Rayleigh fading BC scales like $O(\log(\log M))$. Numerical results support the theoretical results derived in the paper.**

I. Introduction

Spectrum scarcity is quickly becoming one of the main concerns in wireless communications technology as most of the exclusively allocated spectrum is underutilized by licensed/primary users (PUs) [1]. This inspired the concept of cognitive radio (CR) technology, originally proposed in [2]. The rationale is that unlicensed/secondary users (SUs) are allowed to use the same spectrum with PUs as long as the quality-of-service (QoS) of the primary transmission is protected. This paper focuses on the underlay paradigm where SUs can share the spectrum regardless of the ON/OFF status of the primary network, provided that the QoS of the primary link is still guaranteed. To protect the service quality in primary transmission, several types of constraints have been proposed in literature including peak/average interference power constraint (PIPC/PAPC), primary capacity loss constraint, and primary outage probability constraint (POC) (see [3] and references therein). In this paper, we focus on a single-input single-output (SISO) fading cognitive broadcast channel which co-exists with a delay-sensitive primary link under an average transmit power constraint. The service quality in PU's link is protected by POC. Similar results for peak SU transmit power constraint have been derived but not included in this submission due to space constraints.

For a SISO block-fading non-cognitive BC, information theoretic capacity was investigated in, e.g. [4], [5]. In [4], the authors showed that the base station allocates a given fading block to the user with the strongest reception so as to maximize the total throughput, showing that dynamic time-division-multiple-access (D-TDMA) is the optimal scheme. In [6], the authors investigated the optimal power control for ESC maximization in the SISO fading cognitive MAC (C-MAC) and cognitive BC (C-BC) under both average/peak transmit power constraint and PIPC/PAPC, proving that D-TDMA is the optimal scheme for achieving the ESC in C-BC, reflecting the opportunistic scheduling type of results.

Opportunistic user selection strategy motivates researchers to analyze how the sum capacity scales as the number of users M increases. The analysis on throughput scaling in non-cognitive multiple-input multiple-output (MIMO) BC is provided in [7]. In an underlay cognitive radio network, there are a number of papers studying multiuser diversity (MUD) for secondary sum rate (See [8] and [9] and references therein). In [8], the MUD is examined for three types of cognitive networks, including C-MAC, C-BC, and cognitive parallel access channel (C-PAC), under peak transmit power and peak interference power constraints. Recently, [9] considers the MUD gain due to the optimal power control in C-MAC under average transmit and average interference power constraints with various types of fading channels. Different from [8] and [9], this paper study the MUD gain under optimal power control due to the effect of a *'probabilistic'* constraint as a QoS metric in the PU's link, i.e. POC.

In this paper, we first study the ESC maximization problem in a SISO C-BC under a POC. Description of the system model is provided in Section II. Under the assumption of full channel side information at SBS and PU's power strategy being known to SBS, we derive the optimal power control for the problem with average transmit power constraint in Section III, by using a rigorous probabilistic power allocation technique [10] [11] [12]. The optimal solutions show that D-TDMA is the optimal strategy when continuous fading channel state is considered. In Section IV, we present the analysis on SU sum throughput scaling according to the optimal power strategy under Rayleigh fading scenario, showing that the

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secondary throughput grows as $\log(\log M)$. The extended results on peak SU power constraint when PU adopts ON-OFF policy. Numerical results supporting the theoretical findings are presented in Section VI prior to concluding remarks in Section VII.

List of notations : $E[.]$ denotes expectation. $Pr\{.\}$ represents probability. The cumulative density function (CDF) of a random variable Z is given by $F_Z(z)$ whereas $F_Z(z|Y)$ expresses CDF of Z given Y. Let X_w be a Bernoulli w random variable such that $X_w = 1$ with probability w and $X_w = 0$ with probability $1-w$. $\frac{\partial y}{\partial x^*}$ denotes the partial derivative of y with respect to x, evaluated at $x = x^*$. $\mathbf{p}^T$ represents transpose of vector $\mathbf{p}$. $\mathcal{S}^c$ represents the compliment of the set $\mathcal{S}$.

II. System model

We consider a cognitive broadcast channel with one SBS and M secondary receivers (SRs), sharing the same spectrum as a primary transmitter-receiver pair (PT-PR). All terminals involved are equipped with a single antenna. The delay-sensitive primary network needs to meet a primary outage probability constraint (POC) with a service rate r_p^0 and an outage probability threshold ϵ_p. All channels involved in this cognitive radio network are assumed to be independent block fading additive white Gaussian noise (BF-AWGN) channels [10] and have continuous CDF. Let the channel gain from SBS to the i-th SR, PT to PR, PT to i-th SR, and SBS to PT are denoted by h_i, g, α_i, and β, respectively. Let $\boldsymbol{\chi}$ represent the combined channel state vector, i.e. $\boldsymbol{\chi} = [g, \beta, h_1, \ldots, h_M, \alpha_1, \ldots, \alpha_M]$. With SBS's transmit power policy given by $\mathbf{P}(\boldsymbol{\chi}) = [P_1(\boldsymbol{\chi}), \ldots, P_M(\boldsymbol{\chi})]$, the instantaneous rate expression can be written as follows

$$\begin{aligned} r_p(\boldsymbol{\chi}, \mathbf{P}(\boldsymbol{\chi})) = &\quad \log\left(1 + \frac{gP_p(g)}{\left(\beta \sum_{i=1}^{M} P_i(\boldsymbol{\chi})\right) + N_0}\right) \\ r_s(\boldsymbol{\chi}, \mathbf{P}(\boldsymbol{\chi})) = &\quad \log\left(1 + \sum_{i=1}^{M} \frac{h_i}{\alpha_i P_p(g) + N_0} P_i(\boldsymbol{\chi})\right) \end{aligned} \tag{1}$$

where $r_p(\cdot)$ and $r_s(\cdot)$ denote the PU's rate and the SBS's sum rate, respectively. Note that we drop the constant $\frac{1}{2}$ in the instantaneous rate expressions above and use natural logarithm for simplicity. In this problem, we aim to maximize downlink ESC from the base station to the SUs subject to a POC and a SBS long-term power budget. Generally, PU has its own power control strategy only based on the direct gain g between PT and PR regardless the interference from the secondary network. We assume that SBS has perfect CSI on $\boldsymbol{\chi}$ and PU's power policy, so that SBS also knows PU's power for every realization $\boldsymbol{\chi}$. In this work, PT is assumed to use an ON - OFF power strategy with constant power P_c : $P_p(g) = P_c$ when $g \geq g_T = \frac{(e^{r_p^0}-1)N_0}{P_c}$ and $P_p(g) = 0$ otherwise. Obviously, it implies that PU turns off when P_c is not enough to support the target rate r_p^0. So, when the secondary network does not exist, PU faces an outage with probability $Pr\left\{r_p < r_p^0\right\} = \epsilon_p^0$. This work thus focuses on the case that $\epsilon_p^0 \leq \epsilon_p$, such that the additional outage caused by SBS is already included in PU's maximum outage probability.

III. Problem formulation and the optimal power policy

The ESC maximization problem from SBS to SUs with a POC and a long-term transmit power constraint (LTPC), (**P1**), is defined as follows.

$$\begin{aligned} \max_{\mathbf{P}(\boldsymbol{\chi}) \succeq 0} \quad & C_{s,LT} = E\left[r_s(\boldsymbol{\chi}, \mathbf{P}(\boldsymbol{\chi}))\right] \\ \text{s.t.} \quad & \textbf{(a)}\ Pr\left\{r_p(\boldsymbol{\chi}, \mathbf{P}(\boldsymbol{\chi})) < r_p^0\right\} \ \leq\ \epsilon_p, \\ & \textbf{(b)}\ E\left[\sum_{i=1}^{M} P_i(\boldsymbol{\chi})\right] \ \leq\ P_{av}. \end{aligned} \tag{2}$$

By applying the same technique in [10] [11] [12], we can prove that the optimal power control for (**P1**) is randomized between two deterministic schemes, i.e. $\mathbf{p}_1(\boldsymbol{\chi}) = E\left[\mathbf{P}(\boldsymbol{\chi}) \mid r_p(\boldsymbol{\chi}, \mathbf{p}(\boldsymbol{\chi})) \geq r_p^0\right]$ and $\mathbf{p}_2(\boldsymbol{\chi}) = E\left[\mathbf{P}(\boldsymbol{\chi}) \mid r_p(\boldsymbol{\chi}, \mathbf{p}(\boldsymbol{\chi})) < r_p^0\right]$, with the probability indicated by the weighting function $w(\boldsymbol{\chi})$ which can be expressed as $w(\boldsymbol{\chi}) = Pr\left\{r_p(\boldsymbol{\chi}, \mathbf{P}(\boldsymbol{\chi})) \geq r_p^0 \mid \boldsymbol{\chi}\right\}$.

Lemma 3.1: The optimal solution to Problems (**P1**) can be expressed as $\mathbf{P}^*(\boldsymbol{\chi}) = w(\boldsymbol{\chi})\mathbf{p}_1(\boldsymbol{\chi}) + (1 - w(\boldsymbol{\chi}))\mathbf{p}_2(\boldsymbol{\chi})$, where $E\left[w(\chi)\right] \geq 1 - \epsilon_p$, $E\left[\sum_{i=1}^{M} P_i^*(\boldsymbol{\chi})\right]$ and $r_p(\boldsymbol{\chi}, \mathbf{p}_1(\boldsymbol{\chi})) \geq r_p^0$ for all $\boldsymbol{\chi}$. ■

Proof of Lemma 3.1 is provided in Appendix A. For convenience, we further define $\mathscr{P}_p = \mathscr{P}_p(g) = \left(\frac{gP_p(g)}{e^{r_p^0}-1} - N_0\right)^+$. Then, reformulating (**P1**) by Lemma 3.1, we obtain

$$\begin{aligned} \max_{\mathbf{p}_k(\boldsymbol{\chi}) \succeq 0,\ 0 \leq w(\boldsymbol{\chi}) \leq 1} \quad & E\left[w(\boldsymbol{\chi}) r_s(\boldsymbol{\chi}, \mathbf{p}_1(\boldsymbol{\chi}))\right. \\ & \left. + (1 - w(\boldsymbol{\chi})) r_s(\boldsymbol{\chi}, \mathbf{p}_2(\boldsymbol{\chi}))\right] \\ \text{s.t.} \quad & \textbf{(a)}\ E\left[\mathbf{1}^T(w(\boldsymbol{\chi})\mathbf{p}_1(\boldsymbol{\chi}) + (1 - w(\boldsymbol{\chi}))\mathbf{p}_2(\boldsymbol{\chi}))\right] \leq P_{av}, \\ & \textbf{(b)}\ E\left[w(\boldsymbol{\chi})\right] \geq 1 - \epsilon_p, \\ & \textbf{(c)}\ w(\boldsymbol{\chi})\left[\mathscr{P}_p - \beta \mathbf{1}^T \mathbf{p}_1(\boldsymbol{\chi})\right] \geq 0. \end{aligned} \tag{3}$$

Alluding to [11] and [12], the objective function can be proved to be concave while the other constraints are linear. Hence, we can solve the problem by using the necessary and sufficient Karush-Kuhn-Tucker (KKT) optimality conditions. For convenience, define $z_i = \frac{h_i}{\alpha_i P_p(g) + N_0}$ for all i, $p_{WF,i}^*(\boldsymbol{\chi}) = (\frac{1}{\Lambda^*} - \frac{1}{z_i})^+$ and $p_{RP}^*(\boldsymbol{\chi}) = \frac{\mathscr{P}_p}{\beta}$. Apply KKT conditions and the fact that channel state is continuous, the optimal power policy can be summarized in Theorem 1

***Theorem** 1:* The optimal power control for (**P1**) is $\mathbf{P}^* = X_{w^*}(\boldsymbol{\chi})\mathbf{p}_1^*(\boldsymbol{\chi}) + (1 - X_{w^*}(\boldsymbol{\chi}))\mathbf{p}_2^*(\boldsymbol{\chi})$, where λ^* and S^* are the solutions to $E\left[\sum_{i=1}^{M} P_i^*(\boldsymbol{\chi})\right] = P_{av}$ and $E\left[w^*(\boldsymbol{\chi})\right] \geq 1 - \epsilon_p$

and $\mathbf{p}_1^*(\boldsymbol{\chi})$, $\mathbf{p}_2^*(\boldsymbol{\chi})$ and $w^*(\boldsymbol{\chi})$ are defined as follows

$$p_{1i}^*(\boldsymbol{\chi}) = \begin{cases} p_{WF,i^*}^*(\boldsymbol{\chi}) & ,(w^*(\boldsymbol{\chi})=1), \\ & p_{WF,i^*}^*(\boldsymbol{\chi}) \leq p_{RP}^*(\boldsymbol{\chi}), \\ & i^* = \arg\max_{m\in\mathcal{I}} z_m \\ p_{RP}^*(\boldsymbol{\chi}) & ,(w^*(\boldsymbol{\chi})=1), \\ & p_{WF,i^*}^*(\boldsymbol{\chi}) > p_{RP}^*(\boldsymbol{\chi}), \\ & i^* = \arg\max_{m\in\mathcal{I}} z_m \\ 0 & , \textit{otherwise} \end{cases} \quad (4)$$

$$p_{2i}^*(\boldsymbol{\chi}) = \begin{cases} p_{WF,i^*}^*(\boldsymbol{\chi}) & ,(w^*(\boldsymbol{\chi})=0), \\ & i^* = \arg\max_{m\in\mathcal{I}} z_m \\ 0 & , \textit{otherwise} \end{cases} \quad (5)$$

$$w^*(\boldsymbol{\chi}) = \begin{cases} 1, & \mathscr{B}_{1,\boldsymbol{\chi}}^B > \mathscr{B}_{2,\boldsymbol{\chi}}^B \\ 0, & \mathscr{B}_{1,\boldsymbol{\chi}}^B < \mathscr{B}_{2,\boldsymbol{\chi}}^B \end{cases} \quad (6)$$

where $\mathscr{B}_{1,\boldsymbol{\chi}}^B = r_s(\boldsymbol{\chi},\mathbf{p}_1^*(\boldsymbol{\chi})) - \Lambda^*\mathbf{1}^T\mathbf{p}_1^*(\boldsymbol{\chi}) + S^*$ and $\mathscr{B}_{2,\boldsymbol{\chi}}^B = r_s(\boldsymbol{\chi},\mathbf{p}_2^*(\boldsymbol{\chi})) - \Lambda^*\mathbf{1}^T\mathbf{p}_2^*(\boldsymbol{\chi})$. ■

IV. Throughput Scaling in BC with Optimal Power Control

By the assumption that all the channel power gains are exponentially distributed and PU's power control is ON-OFF, we can analyze the throughput scaling according to the derived optimal power control policy. From the optimal solution in LTPC, we can divide $\boldsymbol{\chi}$ into four possible cases as summarized in Table I.

TABLE I
Four possible cases for the fading channel state $\boldsymbol{\chi}$ with LTPC

Case	Properties	Power control	Outage at PU
1	$g < g_T$	p_{WF,i^*}^*	Yes (PU turns OFF)
2	$g \geq g_T$, $p_{WF,i^*}^* \leq p_{RP}^*$	p_{WF,i^*}^*	No
3	$g \geq g_T$, $p_{WF,i^*}^* > p_{RP}^*$, $\mathscr{B}_{1,\boldsymbol{\chi}}^B < \mathscr{B}_{2,\boldsymbol{\chi}}^B$	p_{WF,i^*}^*	Yes
4	$g \geq g_T$, $p_{WF,i^*}^* > p_{RP}^*$, $\mathscr{B}_{1,\boldsymbol{\chi}}^B \geq \mathscr{B}_{2,\boldsymbol{\chi}}^B$	p_{RP}^*	No

If $S^*=0$, SBS can transmit with p_{WF,i^*}^* in LTPC case without making POC active, as if the PU never existed. It was shown in [7] that the SBS sum throughput in this case scales like $O(\log(\log M))$. Hence, this work will focus only on the case when $S^* > 0$. Let $\theta = \frac{\mathscr{P}_p(g)}{\beta}$. The CDF of θ given $g \geq g_T$, $F_\theta(\theta \,|\, \mathcal{S}_1^c)$, can be expressed as $1 - \frac{1}{1+c_o\theta}$ where $c_o = \frac{\exp(r_p^0)-1}{P_c} = \frac{g_T}{N_0}$. Also, we can find the cumulative density function (CDF) of z_i when $g < g_T$ and $g \geq g_T$.

With $z_{\max} = \max z_i$ and θ, we can re-characterize $\mathcal{S}_1$ to $\mathcal{S}_4$ corresponding to Table. I

$$\begin{aligned} \mathcal{S}_1 &= \{g < g_T\} \\ \mathcal{S}_2 &= \{g \geq g_T, z_{\max} \leq \Lambda^*\} \\ &\quad \bigcup \left\{ g \geq g_T, z_{\max} \geq \Lambda^*, \tfrac{1}{\Lambda^*} - \tfrac{1}{z_{\max}} \leq \theta \leq \infty \right\} \\ \mathcal{S}_3 &= \left\{ g \geq g_T, z_{\max} \geq \tfrac{\Lambda^*}{k_o}, 0 \leq \theta \leq \tfrac{k_o}{\Lambda^*} - \tfrac{1}{z_{\max}} \right\} \\ \mathcal{S}_4 &= \left\{ g \geq g_T, z_{\max} \geq \Lambda^*, \tfrac{k_o}{\Lambda^*} - \tfrac{1}{z_{\max}} \leq \theta \leq \tfrac{1}{\Lambda^*} - \tfrac{1}{z_{\max}} \right\} \end{aligned} \quad (7)$$

where k_o is the solution to $\log(k_o) - k_o + S^* + 1 = 0$.

Therefore, the sum throughput $C_s^* = E\left[r_s(\boldsymbol{\chi}, \mathbf{P}^*(\boldsymbol{\chi}))\right]$ can be computed as follows

$$\begin{aligned} C_s^* &= \sum_{k=1}^{3} Pr(\mathcal{S}_k) E\left[\log(\tfrac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}} \,|\, \mathcal{S}_k\right] \\ &\quad + Pr(\mathcal{S}_4) E\left[\log(1+\theta z_{\max}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}} \,|\, \mathcal{S}_4\right] \end{aligned} \quad (8)$$

Obviously, C_s^* is upper-bounded by $\sum_{k=1}^{4} Pr(\mathcal{S}_k) E\left[\log(\frac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}} \,|\, \mathcal{S}_k\right]$ which is $E\left[\log(\frac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}}\right]$. In Appendix B1, we will prove that $\lim_{M\to\infty} \Lambda_M^* > 0$.

When $\epsilon_p > \epsilon_p^0$, we can find the lower-bound of C_s^* as follows

$$\begin{aligned} C_s^* &\geq E\left[\log(\tfrac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}}\right] + Pr\left\{\mathcal{S}_4\right\} \log(k_o) \\ &\geq E\left[\log(\tfrac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}}\right] + \log(\tfrac{\Lambda^*}{c_o} \cdot \tfrac{\gamma_\epsilon}{1-\gamma_\epsilon}) \end{aligned} \quad (9)$$

where $\gamma_\epsilon = \frac{\epsilon_p - \epsilon_p^0}{1-\epsilon_p^0} > 0$. Note that the second inequality is from the fact that $k_o \geq \frac{\Lambda^*}{c_o} \cdot \frac{\gamma_\epsilon}{1-\gamma_\epsilon}$, which the coresponding proof can be found in Appendix B2.

When $\epsilon_p = \epsilon_p^0$, we can show that if $\Lambda^* \geq c_o$, $E\left[C_s^* \,|\, \mathcal{S}_1^c\right] \geq E\left[\log(\frac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}} \,|\, \mathcal{S}_1^c\right] + \frac{\frac{c_o}{\Lambda^*}}{1-\frac{c_o}{\Lambda^*}} \log \frac{c_o}{\Lambda^*}$. If $\Lambda^* \leq c_o$, $E\left[C_s^* \,|\, \mathcal{S}_1^c\right] \geq E\left[\log(\frac{z_{\max}}{c_o}) \mathit{1}_{\{z_{\max}\geq c_o\}} \,|\, \mathcal{S}_1^c\right] + \frac{1}{1-\frac{c_o}{\Lambda^*}} \log \frac{c_o}{\Lambda^*}$. The proof of these results can be found in Appendix B3.

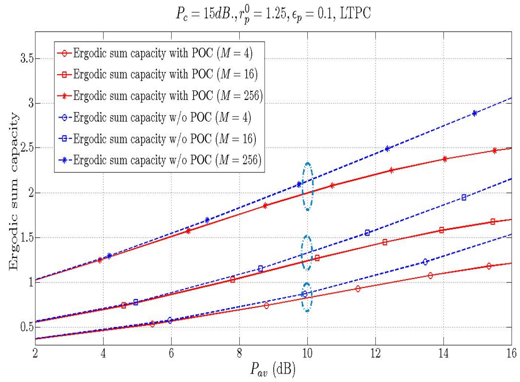
Further, we show that both $F_Z(z \,|\, \mathcal{S}_1)$ and $F_Z(z \,|\, \mathcal{S}_1^c)$ belong to the domain of attraction of the Gumbel distribution [13] as it satisfies the Von Mises conditions. The proofs are provided in Appendix B4. Thus, $\lim_{M\to\infty} \frac{E\left[\log(\frac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}} \,|\, g<g_T\right]}{\log(\log M)} = 1$ and $\lim_{M\to\infty} \frac{E\left[\log(\frac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max}\geq\Lambda^*\}} \,|\, g\geq g_T\right]}{\log(\log M)} = 1$. Finally, it is not hard to show that $\lim_{M\to\infty} \frac{C_s^*}{\log(\log M)} \geq 1$ and $\lim_{M\to\infty} \frac{C_s^*}{\log(\log M)} \leq 1$ leading to the result that C_s^* scales like $\log(\log M)$.

V. Extended Results in Throughput Scaling

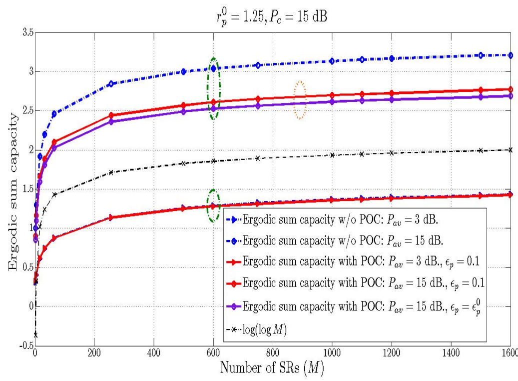
If all channel power gains are exponentially distributed with ON-OFF power policy at PU, the MUD analysis under peak SU power constraints can be investigated in similar manner. The result suggests that SU throughput also scales like $O(\log(\log M))$.

VI. Numerical Results

In this section, some numerical results on the performance of the proposed power policies are illustrated for the capacity maximization problem with POC and an average transmit



(a)



(b)

Fig. 1. Numerical results when $r_p^0 = 1.25$, $\epsilon_p = 0.1$, and $P_c = 15$ dB. with LTPC

power constraint at the SBS. We consider a symmetric network where all channel gains involved are assumed to be Rayleigh fading and their channel power gains are thus exponentially distributed, assumed to have a unit mean without loss of generality. Noises at PR and all SRs are presumed to be AWGN with unit variance, i.e. $N_0 = 1$. $P_c = 15$ dB when PU is ON with target rate is $r_p^0 = 1.25$ nats/sec and the outage probability threshold $\epsilon_p = 0.1$. The simulation results are based on a Monte-Carlo method averaged over 10^5 channel realizations.

Fig.1(a) shows that with an increment in average power budget (P_{av}), the downlink capacity is duly increases for both with POC and without POC. However, the effect of POC becomes more dominant as P_{av} becomes higher, making the rate of increase in downlink throughput significantly drop compared to that of without POC. The implication is that when POC becomes active, SBS must be aware of the QoS of primary user, thereby being forced to transmit by the highest possible power that still guarantee PU's service rate, i.e. p^*_{RP}, in some channel realizations. Nevertheless, the SBS is allowed to transmit when PU link encounters an outage in two scenarios. In the first scenario, PU faces an outage by itself when PU is OFF. In the second scenario, the benefit of PU being outage is greater than that of satisfying PU's target rate, i.e. SBS decides the strategy based on $w^*(\boldsymbol{\chi})$ in Theorem 1. Fig.1(a) shows the benefit of multiuser diversity. As the number of SRs M increases, the SBS downlink throughput is enhanced as SBS statistically has higher opportunity to obtain direct channel gains $z_{\max}$. In Fig. 1(b), it illustrates how the number of SRs, M, affects the growth of SBS downlink capacity when the proposed optimal power is applied under the fixed power budget P_{av}. It shows that the growth rate is the same as $\log(\log M)$, no matter whether $\epsilon_p > \epsilon_p^0$ (when the secondary network can cause additional outage to primary user) or $\epsilon_p = \epsilon_p^0$ (when the secondary network is not allowed to cause more outage to PU). The results in Fig. 1(b) thus confirm our theoretical findings in Section V.

VII. Conclusion

In this paper, we have investigated the optimal power allocation strategy to maximize a SU downlink ergodic sum capacity subject to a long-term transmit power constraint at the SBS and an outage probability constraint at PU using a probabilistic power allocation technique. We have also analyzed how the secondary sum-throughput scales as the number of secondary receivers increases when the optimal power policy is applied in Rayleigh fading channel and an ON-OFF power strategy for PU is assumed. Our results show that the ergodic sum throughput scales like $O(\log(\log M))$ as $M \to \infty$.

Appendix

A. Proof of Lemma 3.1

For an arbitrary feasible probabilistic power scheme $\mathbf{P}(\boldsymbol{\chi})$ with conditional PDF $f_{\mathbf{P}|\boldsymbol{\chi}}(\mathbf{p}(\boldsymbol{\chi}) \mid \boldsymbol{\chi})$,another feasible scheme $\mathbf{P}'_s(\boldsymbol{\chi})$, which is randomised between two deterministic power schemes with time-sharing factors $w(\boldsymbol{\chi})$, can achieve higher SU average rate. The feasibility of $\mathbf{P}(\boldsymbol{\chi})$ implies that $E[\sum_{i=1}^{M} P_i(\boldsymbol{\chi})] \leq P_{av}$ and $Pr\left\{r_p((\boldsymbol{\chi}), \mathbf{P}(\boldsymbol{\chi})) < r_p^0\right\} \leq \epsilon_p$.

Since $\mathbf{p}_1$ is feasible, we know that $r_p(\boldsymbol{\chi}, \mathbf{p}_1(\boldsymbol{\chi})) \geq E[r_p(\boldsymbol{\chi}, \mathbf{P}_s(\boldsymbol{\chi})) \mid r_p(\boldsymbol{\chi}, \mathbf{p}(\boldsymbol{\chi})) \geq r_p^0, \boldsymbol{\chi}]$. Therefore, all of possible $\mathbf{p}(\boldsymbol{\chi})$ such that $r_p(\boldsymbol{\chi}, \mathbf{p}(\boldsymbol{\chi})) \geq r_p^0$ lie in the halfspace defined by $\mathbf{p}^T(\boldsymbol{\chi})\boldsymbol{\beta} \leq \left(\frac{gP_p(g)}{e^{r_p^0}-1} - N_0\right)^+$. Thus, $\mathbf{p}_1(\boldsymbol{\chi})$ must also be in that halfspace, i.e. $\mathbf{p}_1^T(\boldsymbol{\chi})\boldsymbol{\beta} \leq \left(\frac{gP_p(g)}{e^{r_p^0}-1} - N_0\right)^+$.

Construct the new probabilistic scheme $\mathbf{P}'$ such that $\mathbf{P}' = \mathbf{p}_1(\boldsymbol{\chi})$ with probability $w(\boldsymbol{\chi})$ and $\mathbf{P}' = \mathbf{p}_2(\boldsymbol{\chi})$ with probability $1 - w(\boldsymbol{\chi})$, where $w(\boldsymbol{\chi}) = \quad Pr\left\{r_p(\boldsymbol{\chi}, \mathbf{P}(\boldsymbol{\chi})) \geq r_p^0 \mid \boldsymbol{\chi}\right\}$.

For the PU's outage probability based on the policy $\mathbf{P}'$, we can show that $Pr\left\{r_p(\boldsymbol{\chi}, \mathbf{P}'(\boldsymbol{\chi})) \geq r_p^0 \mid \boldsymbol{\chi}\right\} \geq w(\boldsymbol{\chi})$, so $E\left[w(\boldsymbol{\chi})\right] \geq 1 - \epsilon_p$.

Average power of the new power control is expressed by $E\left[\sum_{i=1}^{M} w(\boldsymbol{\chi})\mathbf{p}_1(\boldsymbol{\chi}) + (1 - w(\boldsymbol{\chi}))\mathbf{p}_2(\boldsymbol{\chi})\right] = E\left[\sum_{i=1}^{M} \mathbf{P}(\boldsymbol{\chi})\right] \preceq P_{av}$, so the new power control $\mathbf{P}'(\boldsymbol{\chi})$ satisfies the power constraint.

Finally, we can show that average SU rate by $\mathbf{P}'(\boldsymbol{\chi})$,

$$\begin{aligned}&E\left[r_s(\boldsymbol{\chi},\mathbf{P}'(\boldsymbol{\chi}))\right]\\&=E\left[w(\boldsymbol{\chi})r_s(\boldsymbol{\chi},\mathbf{p}_1(\boldsymbol{\chi}))+(1-w(\boldsymbol{\chi}))r_s(\boldsymbol{\chi},\mathbf{p}_2(\boldsymbol{\chi}))\right]\\&\geq E\left[r_s(\boldsymbol{\chi},\mathbf{P}(\boldsymbol{\chi}))\right]\end{aligned} \tag{10}$$

by the aid of Jensen's inequality for concave function.

B. Throughput scaling for LTPC

1) Λ^**:*

First,we will investigate the bound of Λ^*. As the average power constraint is always met, we have

$$\begin{aligned}P_{av}&=\sum_{k=1}^{3}Pr(\mathcal{S}_k)E\left[\left(\tfrac{1}{\Lambda^*}-\tfrac{1}{z_{\max}}\right)^+\mid\mathcal{S}_k\right]\\&\quad+Pr(\mathcal{S}_4)E\left[\left(\tfrac{1}{\Lambda^*+U^*(\chi)\beta}-\tfrac{1}{z_{\max}}\right)^+\mid\mathcal{S}_4\right]\\&\leq\tfrac{1}{\Lambda^*}\sum_{k=1}^{4}Pr(\mathcal{S}_k)\quad=\tfrac{1}{\Lambda^*}\end{aligned} \tag{11}$$

Suppose that $\lim_{M\to\infty}\Lambda_M^*=0$. For $\epsilon>0$, there exists M_o such that, for $M\geq M_o$, $\Lambda_M^*\leq\epsilon$. Therefore, $(\frac{1}{\Lambda_M^*}-\frac{1}{z_{\max}})^+\geq(\frac{1}{\epsilon}-\frac{1}{z_{\max}})^+$. For M is large enough, $\frac{1}{z_{\max}}$ converges to 0 in probability, implying that,$(\frac{1}{\epsilon}-\frac{1}{z_{\max}})^+$ converges to $\frac{1}{\epsilon}$ in probability. Finally, it means that $(\frac{1}{\Lambda_M^*}-\frac{1}{z_{\max}})^+\geq\frac{1}{\epsilon}$ with high probability. So, with an arbitrary small ϵ, SU will violate the power constraint with high probability if $\lim_{M\to\infty}\Lambda_M^*=0$. Consequently, $\lim_{M\to\infty}\Lambda_M^*>0$.

Now, we are sure that Λ_M^* will not converge to zero. Next, we will show that when M is large enough, Λ_M^* is lower-bounded by $\frac{\epsilon_p^0}{P_{av}}$.

$$\begin{aligned}P_{av}&=\sum_{k=1}^{3}Pr(\mathcal{S}_k)E\left[\left(\tfrac{1}{\Lambda_M^*}-\tfrac{1}{z_{\max}}\right)^+\mid\mathcal{S}_k\right]\\&\quad+Pr(\mathcal{S}_4)E\left[\left(\tfrac{1}{\Lambda_M^*+U^*(\chi)\beta}-\tfrac{1}{z_{\max}}\right)^+\mid\mathcal{S}_4\right]\\&>Pr(\mathcal{S}_1)E\left[\left(\tfrac{1}{\Lambda_M^*}-\tfrac{1}{z_{\max}}\right)^+\mid\mathcal{S}_1\right]\\&=\epsilon_p^0E\left[\left(\tfrac{1}{\Lambda_M^*}-\tfrac{1}{z_{\max}}\right)^+\mid\mathcal{S}_1\right]\end{aligned} \tag{12}$$

Since

$$\begin{aligned}&\lim_{M\to\infty}E\left[\left(\tfrac{1}{\Lambda_M^*}-\tfrac{1}{z_{\max}}\right)^+\mid\mathcal{S}_1\right]\\&=\lim_{M\to\infty}\Big[\tfrac{1}{\Lambda_M^*}(1-F_Z^M(\Lambda_M^*\mid\mathcal{S}_1))\\&\qquad-\int_{z_{\max}\Lambda_M^*}^{\infty}\tfrac{1}{z_{\max}}dF_Z^M(z_{\max}\mid\mathcal{S}_1)\Big]\\&=\tfrac{1}{\Lambda_M^*}\end{aligned} \tag{13}$$

It is because, as M is large enough, $F_Z^M(\Lambda_M^*\mid\mathcal{S}_1)$ and $\int_{z_{\max}=\Lambda_M^*}^{\infty}\frac{1}{z_{\max}}dF_Z^M(z_{\max}\mid\mathcal{S}_1)$ approach zero. Finally, we have $\lim_{M\to\infty}\Lambda_M^*>\frac{\epsilon_p^0}{P_{av}}$.

2) Lower bound on k_o *when* $\epsilon_p>\epsilon_p^0$*:*

Since $Pr\{\mathcal{S}_1\}=\epsilon_p^0$, $Pr\{\mathcal{S}_3\}=\epsilon_p-\epsilon_p^0$ and we have

$$\begin{aligned}&\epsilon_p-\epsilon_p^0=\\&\int_{z_{\max}=\frac{\Lambda^*}{k_o}}^{\infty}\int_{\theta=0}^{\frac{k_o}{\Lambda^*}-\frac{1}{z_{\max}}}dF_\Theta(.)\cdot dF_Z^M(.)\cdot\int_{g=g_T}^{\infty}dF_G(g)\\&\therefore\gamma_\epsilon=\frac{(\epsilon_p-\epsilon_p^0)}{\int_{g=g_T}^{\infty}dF_G(g)}=(\epsilon_p-\epsilon_p^0)/(1-\epsilon_p^0)\\&=\int_{z_{\max}=\frac{\Lambda^*}{k_o}}^{\infty}\left(\int_{\theta=0}^{\frac{k_o}{\Lambda^*}-\frac{1}{z_{\max}}}dF_\Theta(\theta\mid\mathcal{S}_1^c)\right)dF_Z^M(z_{\max}\mid\mathcal{S}_1^c)\\&=\int_{z_{\max}=\frac{\Lambda^*}{k_o}}^{\infty}\left(1-\frac{1}{1+c_o\left(\frac{k_o}{\Lambda^*}-\frac{1}{z_{\max}}\right)}\right)dF_Z^M(z_{\max}\mid\mathcal{S}_1^c)\\&\leq\left(1-\frac{1}{1+\frac{k_oc_o}{\Lambda^*}}\right)\end{aligned} \tag{14}$$

We finally have $k_o\geq\frac{\Lambda^*}{c_o}\frac{\gamma_\epsilon}{1-\gamma_\epsilon}$.

3) Lower bound on $E\left[C_s^*\mid\mathcal{S}_1^c\right]$ *when* $\epsilon_p=\epsilon_p^0$*:*

$$\begin{aligned}&E\left[C_s^*\mid\mathcal{S}_1^c\right]\\&=\int_{z_{\max}=\Lambda^*}^{\infty}\left(\int_{\theta=0}^{\infty}\log(1+z_{\max}p^*)\cdot dF_\Theta(\theta\mid\mathcal{S}_1^c)\right)\\&\qquad\cdot dF_Z^M(z_{\max}\mid\mathcal{S}_1^c)\\&=\int_{z_{\max}=\Lambda^*}^{\infty}\int_{\theta=\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}}^{\infty}\log(\tfrac{z_{\max}}{\Lambda^*})dF_\Theta(.)dF_Z^M(.)\\&\quad+\int_{z_{\max}=\Lambda^*}^{\infty}\int_{\theta=0}^{\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}}\log(1+z_{\max}\theta)dF_\Theta(.)dF_Z^M(.)\\&=\int_{z_{\max}=\Lambda^*}^{\infty}\log(\tfrac{z_{\max}}{\Lambda^*})\left(1-F_\Theta(\tfrac{1}{\Lambda^*}-\tfrac{1}{z_{\max}}\mid\mathcal{S}_1^c)\right)dF_Z^M(.)\\&\quad-\int_{z_{\max}=\Lambda^*}^{\infty}\log(\tfrac{z_{\max}}{\Lambda^*})\left(1-F_\Theta(\tfrac{1}{\Lambda^*}-\tfrac{1}{z_{\max}}\mid\mathcal{S}_1^c)\right)dF_Z^M(.)\\&\quad+\int_{z_{\max}=\Lambda^*}^{\infty}\frac{z_{\max}}{z_{\max}-c_o}\log\Big(\frac{\frac{z_{\max}}{\Lambda^*}}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}\Big)dF_Z^M(.)\\&=\int_{z_{\max}=\Lambda^*}^{\infty}\frac{z_{\max}}{z_{\max}-c_o}\log\Big(\frac{\frac{z_{\max}}{\Lambda^*}}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}\Big)dF_Z^M(z_{\max}\mid\mathcal{S}_1^c)\\&=\int_{z_{\max}=\Lambda^*}^{\infty}\frac{z_{\max}}{z_{\max}-c_o}\log(\tfrac{z_{\max}}{c_o})dF_Z^M(z_{\max}\mid\mathcal{S}_1^c)\\&\quad+\int_{z_{\max}=\Lambda^*}^{\infty}\frac{z_{\max}}{z_{\max}-c_o}\log\Big(\frac{c_o}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}\Big)dF_Z^M(z_{\max}\mid\mathcal{S}_1^c)\end{aligned} \tag{15}$$

The lower bound for $E\left[C_s^*\mid\mathcal{S}_1^c\right]$ is split in to two cases in (16) for $c_o\leq\Lambda^*$ and (17) for $c_o\geq\Lambda^*$

$$\begin{aligned}&E\left[C_s^*\mid\mathcal{S}_1^c\right]\\&=\int_{z_{\max}=\Lambda^*}^{\infty}\frac{z_{\max}}{z_{\max}-c_o}\log(\tfrac{z_{\max}}{c_o})dF_Z^M(z_{\max}\mid g\geq g_T)\\&\quad+\int_{z_{\max}=\Lambda^*}^{\infty}\frac{z_{\max}}{z_{\max}-c_o}\log\Big(\frac{c_o}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}\Big)dF_Z^M(z_{\max}\mid g\geq g_T)\end{aligned} \tag{16}$$

$$
\begin{aligned}
&> \int_{z_{\max}=\Lambda^*}^{\infty} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \int_{z_{\max}=\Lambda^*}^{\infty} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{c_o}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&= \int_{z_{\max}=\Lambda^*}^{\infty} \log(\tfrac{z_{\max}}{\Lambda^*}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad - \int_{z_{\max}=\Lambda^*}^{\infty} \log(\tfrac{c_o}{\Lambda^*}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \int_{z_{\max}=\Lambda^*}^{\infty} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{c_o}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\geq \int_{z_{\max}=\Lambda^*}^{\infty} \log(\tfrac{z_{\max}}{\Lambda^*}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad - \log(\tfrac{c_o}{\Lambda^*}) \left[1 - F_Z^M(z_{\max} = \Lambda^* \mid \mathcal{S}_1^c)\right] \\
&\quad + \tfrac{1}{1-\frac{c_o}{\Lambda^*}} \left[1 - F_Z^M(z_{\max} = \Lambda^* \mid \mathcal{S}_1^c)\right] \\
&= \int_{z_{\max}=\Lambda^*}^{\infty} \log(\tfrac{z_{\max}}{\Lambda^*}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \left[1 - F_Z^M(z_{\max} = \Lambda^* \mid \mathcal{S}_1^c)\right] \tfrac{\frac{c_o}{\Lambda^*}}{1-\frac{c_o}{\Lambda^*}} \log \tfrac{c_o}{\Lambda^*} \\
&> \int_{z_{\max}=\Lambda^*}^{\infty} \log(\tfrac{z_{\max}}{\Lambda^*}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) + \tfrac{\frac{c_o}{\Lambda^*}}{1-\frac{c_o}{\Lambda^*}} \log \tfrac{c_o}{\Lambda^*} \\
&= E\left[\log(\tfrac{z_{\max}}{\Lambda^*}) \mathit{1}_{\{z_{\max} \geq \Lambda^*\}} \mid \mathcal{S}_1^c\right] + \tfrac{\frac{c_o}{\Lambda^*}}{1-\frac{c_o}{\Lambda^*}} \log \tfrac{c_o}{\Lambda^*}
\end{aligned}
$$

For (16), the first inequality is from $\frac{z_{\max}}{z_{\max}-c_o} \log(\frac{z_{\max}}{c_o}) > \log(\frac{z_{\max}}{c_o})$. The second inequality is from the lower bound of $Q(z_{\max}) = \frac{z_{\max}}{z_{\max}-c_o} \log(\frac{c_o}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)})$ which is $\frac{1}{1-\frac{c_o}{\Lambda^*}} \log \frac{c_o}{\Lambda^*}$. Note that $Q(z_{\max})$ is increasing function and always negative but bounded over the range $\Lambda^* \leq z_{\max} \leq \infty$. Thus, $\frac{1}{1-\frac{c_o}{\Lambda^*}} \log \frac{c_o}{\Lambda^*} \leq Q(z_{\max}) \leq \log(\frac{\frac{c_o}{\Lambda^*}}{1+\frac{c_o}{\Lambda^*}}) < 0$. The last inequality is from $\frac{\frac{c_o}{\Lambda^*}}{1-\frac{c_o}{\Lambda^*}} \log \frac{c_o}{\Lambda^*} \leq 0$ and increasing in $z_{\max}$, while $\left[1 - F_Z^M(z_{\max} = \Lambda^* \mid g \geq g_T)\right] < 1$. Further, as $c_o \leq \Lambda^* \leq \frac{1}{P_{av}}$, the minimum of $\frac{\frac{c_o}{\Lambda^*}}{1-\frac{c_o}{\Lambda^*}} \log \frac{c_o}{\Lambda^*}$ is $-\frac{1}{c_o}$.

$$
\begin{aligned}
& E\left[C_s^* \mid \mathcal{S}_1^c\right] \\
&= \int_{z_{\max}=\Lambda^*}^{\infty} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \int_{z_{\max}=\Lambda^*}^{\infty} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{c_o}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&= \int_{z_{\max}=c_o}^{\infty} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \int_{z_{\max}=\Lambda^*}^{c_o} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \int_{z_{\max}=\Lambda^*}^{\infty} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{c_o}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&> \int_{z_{\max}=c_o}^{\infty} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \int_{z_{\max}=\Lambda^*}^{c_o} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \int_{z_{\max}=\Lambda^*}^{\infty} \tfrac{z_{\max}}{z_{\max}-c_o} \log(\tfrac{c_o}{1+c_o\left(\frac{1}{\Lambda^*}-\frac{1}{z_{\max}}\right)}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\geq \int_{z_{\max}=c_o}^{\infty} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \tfrac{\log(\frac{\Lambda^*}{c_o})}{1-\frac{c_o}{\Lambda^*}} \int_{z_{\max}=\Lambda^*}^{c_o} dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&\quad + \tfrac{1}{1-\frac{c_o}{\Lambda^*}} \log \tfrac{c_o}{\Lambda^*} \int_{z_{\max}=\Lambda^*}^{\infty} dF_Z^M(z_{\max} \mid \mathcal{S}_1^c) \\
&= \int_{z_{\max}=c_o}^{\infty} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(.) + \left[1 - F_Z^M(c_o \mid \mathcal{S}_1^c)\right] \tfrac{\log(\frac{c_o}{\Lambda^*})}{1-\frac{c_o}{\Lambda^*}} \\
&> \int_{z_{\max}=c_o}^{\infty} \log(\tfrac{z_{\max}}{c_o}) dF_Z^M(.) + \tfrac{\log(\frac{c_o}{\Lambda^*})}{1-\frac{c_o}{\Lambda^*}} \\
&= E\left[\log(\tfrac{z_{\max}}{c_o}) \mathit{1}_{\{z_{\max} \geq c_o\}} \mid \mathcal{S}_1^c\right] + \tfrac{\log(\frac{c_o}{\Lambda^*})}{1-\frac{c_o}{\Lambda^*}}
\end{aligned} \tag{17}
$$

For (17), the first inequality is from $\frac{z_{\max}}{z_{\max}-c_o} \log(\frac{z_{\max}}{c_o}) > \log(\frac{z_{\max}}{c_o})$. The second inequality is due to the minimum of $\frac{z_{\max}}{z_{\max}-c_o} \log(\frac{z_{\max}}{c_o})$ over $\Lambda^* \leq z \leq c_o$ is $\frac{\Lambda^*}{\Lambda^*-c_o} \log(\frac{\Lambda^*}{c_o})$ and the lower bound of $Q(z_{\max})$ which is $\frac{1}{1-\frac{c_o}{\Lambda^*}} \log \frac{c_o}{\Lambda^*}$. The last inequality is because of $0 < [1 - F_Z^M(c_o \mid g \geq g_T)] < 1$, while $\frac{\log(\frac{c_o}{\Lambda^*})}{1-\frac{c_o}{\Lambda^*}} \leq 0$. Also note that, as $0 < \Lambda^* \leq c_o$, $\frac{\log(\frac{c_o}{\Lambda^*})}{1-\frac{c_o}{\Lambda^*}}$ will not go to $-\infty$.

4) $\lim_{M \to \infty} \frac{E\left[\log(z_{\max}) \mathit{1}_{\{z_{\max} \geq \Lambda^*\}} \mid \mathcal{S}_1^c\right]}{\log(\log M)} = 1$*:*

We know that $F_Z(z \mid \mathcal{S}_1^c) = 1 - \frac{\exp(-N_0 z)}{1+P_c z}$ while $F_Z(z \mid \mathcal{S}_1) = 1 - \exp(-N_0 z)$. Hence, we will prove $\lim_{M \to \infty} \frac{E\left[\log(z_{\max}) \mathit{1}_{\{z_{\max} \geq \Lambda^*\}} \mid \mathcal{S}_1^c\right]}{\log(\log M)} = 1$ only, whereas the proof of $\lim_{M \to \infty} \frac{E\left[\log(z_{\max}) \mathit{1}_{\{z_{\max} \geq \Lambda^*\}} \mid \mathcal{S}_1\right]}{\log(\log M)} = 1$ is obviously similar.

The normalizing constants a_M and b_M for Type-I convergence can be determined by solving for $F(b_M) = 1 - \frac{1}{M}$ and $a_M = \psi(b_M)$, where ψ is the reciprocal hazard function, i.e. $\psi(z) = (1 - F_Z(z))/f_Z(z)$.

Afterwards, we can apply recursive method to find b_M and a_M as follows.

$$
\begin{aligned}
b_M &= \tfrac{1}{N_0}\left[\log M - \log(1 + P_c b_M)\right] \\
&= \tfrac{1}{N_0} \log M - \tfrac{1}{N_0} \log(1 + \tfrac{P_c}{N_0}\left[\log M - \log(1 + P_c b_M)\right]) \\
&= \tfrac{1}{N_0} \log M - \tfrac{1}{N_0} \log(\tfrac{P_c}{N_0} \log M) \\
&\quad - \tfrac{1}{N_0} \log\left[1 + \tfrac{1}{\frac{P_c}{N_0} \log M} - \tfrac{\log(1+P_c b_M)}{\frac{P_c}{N_0} \log M}\right] \\
&= \tfrac{1}{N_0} \log M - \tfrac{1}{N_0} \log(\tfrac{P_c}{N_0} \log M) \\
&\quad + \tfrac{1}{N_0} \log\left[\tfrac{\frac{P_c}{N_0} \log M}{\frac{P_c}{N_0} \log M + 1 - \log(1+P_c b_M)}\right] \\
&= \tfrac{1}{N_0} \log M - \tfrac{1}{N_0} \log\left(\tfrac{P_c}{N_0} \log M\right) + \mathcal{O}\left(\tfrac{\log\log M}{\log M}\right)
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
a_M &= \tfrac{1-F_X(b_M)}{f_X(b_M)} \\
&= \left(\tfrac{\exp(-N_0 b_M)}{1+P_c b_M}\right) / \left(\tfrac{\exp(-N_0 b_M)}{1+P_c b_M}\left[\tfrac{P_c}{1+P_c b_M} + N_0\right]\right) \\
&= \tfrac{P_c b_M + 1}{N_0 P_c b_M + N_0 + P_c} \\
&= \tfrac{1}{N_0}\left(1 - \tfrac{P_c}{N_0 + P_c + N_0 P_c b_M}\right) \\
&= \tfrac{1}{N_0}\left(1 - \tfrac{P_c}{\xi}\right) \\
&= \tfrac{1}{N_0}\left(1 - \mathcal{O}\left(\tfrac{1}{\log M}\right)\right)
\end{aligned} \tag{19}
$$

where $\xi = N_0 + P_c + N_0 P_c \left[\frac{1}{N_0} \log M - \frac{1}{N_0} \log\left(\frac{P_c}{N_0} \log M\right)\right.$

$+\mathcal{O}\left(\frac{\log\log M}{\log M}\right)\Big]$.

So, $b_M = \frac{1}{N_0}\log M - \frac{1}{N_0}\log\left(\frac{P_c}{N_0}\log M\right) + \mathcal{O}\left(\frac{\log(\log M)}{\log M}\right)$ and $a_M = \frac{1}{N_0}\left(1 - \mathcal{O}\left(\frac{1}{\log M}\right)\right)$.

Follow the same procedure as shown in [14], we can first show that, for given $g \geq g_T$, $\frac{\log(z_{\max})}{\log(\log M)}$ converges in probability to 1. Then, define the event A.

$$A = \{\log M - \Lambda^* \mathcal{O}(\log(\log M)) \leq z_{\max} \leq \log M - \Lambda^* \mathcal{O}(1)\} \tag{20}$$

For M is large enough, we can then show that $\liminf_{M\to\infty} \frac{E\left[\log(z_{\max}) 1_{\{z_{\max}\geq\Lambda^*\}} \mid \mathcal{S}_1^c\right]}{\log(\log M)} \geq 1$ as follows.

$$\begin{aligned} E\left[\log(z_{\max}) 1_{\{z_{\max}\geq\Lambda^*\}} \mid \mathcal{S}_1^c\right] &\geq E\left[\log(z_{\max}) 1_A \mid \mathcal{S}_1^c\right] \\ &\geq \log(\log M) + \mathcal{O}(1) \\ \therefore \frac{E\left[\log(z_{\max}) 1_{\{z_{\max}\geq\Lambda^*\}} \mid \mathcal{S}_1^c\right]}{\log(\log M)} &\geq \frac{E\left[\log(z_{\max}) 1_A \mid \mathcal{S}_1^c\right]}{\log(\log M)} \\ &= 1 + o(1) \end{aligned} \tag{21}$$

Then, we can prove that $\lim\limits_{M\to\infty} \frac{E[z_{\max} \mid \mathcal{S}_1^c]}{\log M} = 1$, by similar procedure from the proof of Lemma 2 in [14]. Finally, $\limsup_{M\to\infty} \frac{E\left[\log(z_{\max}) 1_{\{z_{\max}\geq\Lambda^*\}} \mid \mathcal{S}_1^c\right]}{\log(\log M)} \leq 1$ can be shown as follows

$$\begin{aligned} \because\ E\left[\log(1+z_{\max}) \mid \mathcal{S}_1^c\right] &\leq E\left[\log(1+z_{\max}) \mid \mathcal{S}_1^c\right] \\ &\overset{(a)}{\leq} \log(1 + E\left[z_{\max} \mid \mathcal{S}_1^c\right]) \\ &= \log(\log M) + \mathcal{O}(1) \\ \therefore \frac{E\left[\log(z_{\max}) 1_{\{z_{\max}\geq\Lambda^*\}} \mid \mathcal{S}_1^c\right]}{\log(\log M)} &= 1 + o(1) \end{aligned} \tag{22}$$

Note that (a) in (22) follows from Jensen's inequality and the equality is due to $\lim\limits_{M\to\infty} \frac{E[z_{\max} \mid \mathcal{S}_1^c]}{\log M} = 1$. By (21) and (22), we finally have $\lim\limits_{M\to\infty} \frac{E\left[\log(z_{\max}) 1_{\{z_{\max}\geq\Lambda^*\}} \mid \mathcal{S}_1^c\right]}{\log(\log M)} = 1$.

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