Efficent Estimation of the Non-linear Volatility and Growth Model

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ABSTRACT

Ramey and Ramey (1995) introduced a non-linear model relating volatility to growth. The solution of this model by generalised computer algorithms for non-linear maximum likelihood estimation encounters the usual difficulties and is, at best, tedious. We propose an algebraic solution for the model that provides fully efficient estimators and is elementary to implement as a standard ordinary least squares procedure. This eliminates issues such as the 'guesstimation' of initial values and multiple runs. Our approach also facilitates testing the validity of the Ramey and Ramey (1995) model. We illustrate our approach by reanalysing the R&R data, demonstrating virtually identical results.

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I INTRODUCTION

Ramey and Ramey (1995) proposed and implemented a non-linear model to analyse the relationship between volatility and growth when utilising cross-country data. The model is

$$y_{ij} = a + \beta_1 x_{1i} + \beta_2 x_{2i} + \ldots + \beta_m x_{mi} + \lambda \sigma_i + e_{ij}\,, \qquad (1)$$

with e_{ij} assumed $N(0, \sigma_i^2)$, The dependent variable y_{ij} is growth rate of output per capita in country i in year j. The m explanatory variables x_{ki} are constant over years and differ only between countries, Variances differ between countries and the non-linear nature of the model arises from assuming the standard deviation of growth occurs in the model for the mean as well as in that for the variance. Ramey and Ramey had m = 4, However the many subsequent authors who have employed the model sometimes added other variables. These authors include Van der Ploeg and Poelhekke (2007), Aizenman & Marion (1998), Barlevy (2002), and Akai et al (2007).

Ramey and Ramey and the other authors used econometric package software for maximising likelihood functions from non-linear models to obtain the MLE estimates for the coefficients of model (1). As with most non-linear maximisation, obtaining solutions identifiable with global optima can be very tedious, requiring experimentation with guesstimated values, repetitive checking for local rather than global optima and constant adjustment of convergence criteria. Greene, (2008, p.1061) who reviews the complications and difficulties, says it can require a "balanced mix of art and science". Some authors, for example, Kroft and Lloyd-Ellis (2002) choose to view model (1) as a special case of ARCH in the mean models, currently popular in financial econometrics[1], and employ the corresponding package routines. However, these ARCH-M routines employ the same software, with the same difficulties, as described above[2].

Since model (1) is not terribly complicated as non-linear models go, it is worth looking for a semi-analytic solution to it and we will derive one that is simpler and computationally faster than general non-linear optimisation or ARCH-M. So one

[1]ARCH-M is hardly a natural generalisation of model (1). It also requires variation in standard deviation to identify a coefficient in the mean equation, but obtains that by presuming autoregressive evolution of σ_t^2 in a time series, while model (1) uses cross-country variation in σ_i^2. Developments of model (1) might consider factors affecting σ_i^2, but not analogously to Arch-M.

dimension of the "efficient" in our title refers to computational efficiency. However, this is achieved without losing any of the asymptotically optimal properties attached to ML estimation, justifying the usual interpretation of "efficient" as attaining minimum (asymptotic) variance. In addition we will show that our approach lends itself to easy assessment of the validity of model (1) itself.

The log likelihood function corresponding to model (1) is

$$-\frac{rn}{2}\log 2\pi - \frac{n}{2}\sum_{i=1}^{r}\log\sigma_i^2 - \sum_{1}^{r}\frac{1}{2\sigma_i^2}\sum_{1}^{n}(y_{ij} - a - \beta_1 x_{1i} - .. - \beta_m x_{mi} - \lambda\sigma_i)^2, \quad (2)$$

where there are r countries and n years of data per country. Note again that x_{ki} is constant within country i. So model (1) is nested within the family of models

$$y_{ij} = \alpha_i + e_{ij}. \qquad (3)$$

Model (1) makes the r country means α_i equal $a + \beta_1 x_{1i} + \beta_2 x_{2i} + .. + \beta_m x_{mi} + \lambda\sigma_i$, so depending on $m+2$ parameters in addition to the σ_i parameters. An unrestricted model (3) would allow each country its mean and variance with a total of 2r parameters.

II. THE UNRESTRICTED MODEL

Model (3) is always 'correct' as a fit to the data, although it is over parameterised if (1) is the true model. The ML estimators of α_i and σ_i^2 are, of course,

$$\tilde{\alpha}_i = \bar{y}_i \quad \text{and} \quad \tilde{\sigma}_i^2 = \frac{1}{n}\sum_{j=1}^{n}(y_{ij} - \bar{y}_i)^2$$

respectively and the estimated log likelihood is then

$$-\frac{rn}{2}\log 2\pi - \frac{n}{2}\sum_{i=1}^{r}\log\tilde{\sigma}_i^2 - \frac{rn}{2}. \qquad (4)$$

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[2] Indeed, we failed to get a solution from the ARCH-M routine in our particular econometric package.

If (1) is the true model $\tilde{\sigma}_i^2$,while still a consistent estimator, may not be the efficient estimator obtainable under the model. But comparison of the two, remembering the variance of the difference between an efficient estimator and an inefficient one is the difference of the variances can provides a Hausman (1978) type test of the validity of model (1) and we will return to this theme later.

III ML ESTIMATORS OF THE NON-LINEAR MODEL

Differentiating the log likelihood (2) with respect to a and equating to zero gives

$$n\sum_{i=1}^{r}(\bar{y}_i - a - \beta_1 x_{1i} - \beta_2 x_{2i} - \ldots - \beta_m x_{mi} - \lambda\sigma_i)/\sigma_i^2 = 0$$

or

$$\sum\frac{\bar{y}_i}{\sigma_i^2} = a\sum\frac{1}{\sigma_i^2} + \beta_1\sum\frac{x_{1i}}{\sigma_i^2} + \ldots \beta_m\sum\frac{x_{mi}}{\sigma_i^2} + \lambda\sum\frac{1}{\sigma_i}. \tag{5}$$

Similarly, differentiating with respect to β_k gives m equations k=1, 2, .., m

$$\sum\frac{x_{ki}\bar{y}_i}{\sigma_i^2} = a\sum\frac{x_{ki}}{\sigma_i^2} + \beta_1\sum\frac{x_{ki}x_{1i}}{\sigma_i^2} + \ldots \beta_m\sum\frac{x_{ki}x_{mi}}{\sigma_i^2} + \lambda\sum\frac{x_{ki}}{\sigma_i} \tag{6}$$

While differentiating with respect to λ gives

$$\sum\frac{\bar{y}_i}{\sigma_i} = a\sum\frac{1}{\sigma_i} + \beta_1\sum\frac{x_{1i}}{\sigma_i} + \ldots \beta_m\sum\frac{x_{mi}}{\sigma_i} + r\lambda \tag{7}$$

Now treating the σ_i just for the present as known parameters, equations (5), (6) and (7) are the same as the OLS equations that would result from regressing $\bar{y}_i/\sigma_i$ on x_{ki}/σ_i and the $1/\sigma_i$ variable and including an intercept term[3]. Furthermore, as taking second derivatives of (2) to obtain the Hessian matrix shows, the OLS variance formulae from such a regression match the asymptotic ML variances as given by

[3] There are evident resemblances to the weighted regression procedure except that the constant is not suppressed.

$$\Theta^{-1} = \left[E\left\{ -\frac{\partial^2 \log L}{\partial\theta\,\partial\theta'} \right\} \right]^{-1} \tag{8}$$

where $\theta' = (\lambda, \beta_1, \ldots \beta_m, a)$.

$$\Theta = n\begin{bmatrix} r & \sum\frac{x_{1i}}{\sigma_i}\ldots & \sum\frac{x_{4i}}{\sigma_i} & \sum\frac{1}{\sigma_i} \\ \sum\frac{x_{1i}}{\sigma_i} & \sum\frac{x_{1i}^2}{\sigma_i^2}\ldots & \sum\frac{x_{1i}x_{4i}}{\sigma_i^2} & \sum\frac{x_{1i}}{\sigma_i^2} \\ \sum\frac{x_{4i}}{\sigma_i} & \sum\frac{x_{1i}x_{2i}}{\sigma_i^2}\ldots & \sum\frac{x_{4i}^2}{\sigma_i^2} & \sum\frac{x_{4i}}{\sigma_i^2} \\ \sum\frac{1}{\sigma_i} & \sum\frac{x_{1i}}{\sigma_i^2}\ldots & \sum\frac{x_{4i}}{\sigma_i^2} & \sum\frac{1}{\sigma_i^2} \end{bmatrix}$$

Of course, the σ_i are actually unknown parameters. If we possessed the ML estimates $\breve{\sigma}_i^2$ of σ_i^2 they could be inserted into (5), (6) and (7) giving ML estimates $\breve{\lambda}, \breve{\beta}_1, \ldots \breve{\beta}_m$ and $\breve{a}$. They could also be inserted into (8), but when the σ_i^2 have to be estimated the variance matrix of θ is no longer exactly (8) unless parameter orthogonality holds between θ and the vector of variances, Σ say. If

$$E\left\{ -\begin{bmatrix} \frac{\partial^2 \log L}{\partial\theta\,\partial\theta'} & \frac{\partial^2 \log L}{\partial\theta\,\partial\Sigma'} \\ \left(\frac{\partial^2 \log L}{\partial\theta\,\partial\Sigma'}\right)' & \frac{\partial^2 \log L}{\partial\Sigma\,\partial\Sigma'} \end{bmatrix} \right\} = \begin{bmatrix} \Theta & \Gamma \\ \Gamma' & \Omega \end{bmatrix}$$

then the variance matrix of θ is $\left[\Theta - \Gamma'\Omega^{-1}\Gamma\right]^{-1}$. Differentiating (2) with respect to σ_i^2 gives

$$\begin{aligned} &-\frac{n}{2}\frac{1}{\sigma_i^2} + \frac{1}{2\sigma_i^4}\sum_{j=1}^{n}(y_{ij} - a - \beta_1 x_{1i} - \ldots - \beta_m x_{mi} - \lambda\sigma_i)^2 \\ &\qquad + \frac{n\lambda}{2\sigma_i^3}(\bar{y}_i - a - \beta_1 x_{1i} - \ldots - \beta_m x_{mi} - \lambda\sigma_i) = 0. \end{aligned} \tag{9}$$

And differentiating this with respect to the elements of θ and Σ and taking expectations shows

$$\Gamma = \frac{n\lambda}{2}\begin{bmatrix} \frac{1}{\sigma_1^2} & \frac{1}{\sigma_2^2} & .. & \frac{1}{\sigma_r^2} \\ \frac{x_{11}}{\sigma_1^2} & \frac{x_{12}}{\sigma_2^2} & .. & \frac{x_{1r}}{\sigma_r^2} \\ . & . & . & . \\ \frac{1}{\sigma_1^3} & \frac{1}{\sigma_2^3} & .. & \frac{1}{\sigma_r^3} \end{bmatrix}$$

and Ω to be a diagonal matrix with terms

$$\frac{n(1+\frac{\lambda^2}{2})}{2\sigma_i^4}. \qquad (10)$$

Then the variance matrix of the estimates of θ proves to be

$$\Theta^{-1}(1+\frac{\lambda^2}{2}) \ . \qquad (11)$$

So standard errors of coefficients from the OLS solutions ought to be multiplied by

$\sqrt{1+\lambda^2/2}$, although it may make little difference.

We have estimators $\tilde{\sigma}_i^2$ of the nuisance parameters σ_i^2 and can insert these into (5), (6) and (7) giving estimates $\hat{\lambda},\hat{\beta}_1,..\hat{\beta}_m$ and $\hat{a}$. Are these efficient estimators, that is, asymptotically equivalent to $\breve{\lambda},\breve{\beta}_1,..\breve{\beta}_m$ and $\breve{a}$ so that variance formula (11) applies?

Before demonstrating they are, it is worth showing that $\tilde{\sigma}_i^2$ is not an efficient estimator

of σ_i^2 under model (1) and observing how dependent an efficient estimator is on the exact truth of that model. The estimator $\tilde{\sigma}_i^2$ does extract all the information about σ_i^2 from within country i, but if (1) is true

$$\sigma_i^+ = (\bar{y}_i - a - \beta_1 x_{1i} + .. - \beta_m x_{mi})/\lambda$$

also estimates σ_i once estimates of $a,\beta_1,..\beta_m$ are available. Asymptotically, σ_i^+

has variance $\sigma_i^2/(n\lambda^2)$ and so $(\sigma_i^+)^2$ has variance $4\sigma_i^4/(n\lambda^2)$ while $\tilde{\sigma}_i^2$ has variance $2\sigma_i^4/n$. Combining inversely by variances

$$\frac{\tilde{\sigma}_i^2 + \frac{1}{2}\lambda^2(\sigma_i^+)^2}{1+\frac{1}{2}\lambda^2} \tag{12}$$

is efficient. Its variance is easily seen to be

$$\frac{2\sigma_i^4}{n(1+\frac{1}{2}\lambda^2)},$$

which of course is the reciprocal of (10). But unlike $\tilde{\sigma}_i^2$ the consistency of (12) is totally dependent on the truth of model (1).

Returning to whether the $\hat{\theta} = (\hat{\lambda}, \hat{\beta}_1, .. \hat{\beta}_m, \hat{a})'$ are efficient estimators, the fact that $\tilde{\sigma}_i^2$ is not efficient does not preclude $\hat{\theta}$ from being so. Discussion of asymptotic properties clearly supposes the number of observations becoming large and we will presume that r and n increase in proportion. This is convenient since $O_p(n^s)$ is then also $O_p(r^s)$, but it is probably the most sensible approach anyway. Assuming n increases much faster than r would imply that when r is large enough to apply asymptotic arguments $\tilde{\sigma}_i^2$ has become σ_i^2, θ is estimated accordingly and (8) is its variance. Assuming r increases much faster than n could perhaps invalidate assumptions underlying likelihood inference given that there are r of the σ_i^2 parameters, so that the number of nuisance parameters increases indefinitely[4].

A standard result in asymptotic inference is that is an estimator which differs from the maximum likelihood estimator only by a term of $O_p(1/t)$ where t is the

[4] The Neyman and Scott (1948) example of inconsistent estimation arose in such a situation.

sample size has the same asymptotic properties and is efficient. However, the parameter vector *6* is estimated only from the r cross country values of $\bar{y}_i$ and the explanatory variables.

So we need to show that the coefficients obtained by regression of $\bar{y}_i / \tilde{\sigma}_i$ on the $x_{ki} / \tilde{\sigma}_i$ and $1/\tilde{\sigma}_i$ will differ from the coefficients obtained by regression of $\bar{y}_i / \breve{\sigma}_i$ on $x_{ki} / \breve{\sigma}_i$ and $1/\tilde{\sigma}_i$ only by terms of $O_p(1/r)$. Of course the actual variances of coefficient are also inversely proportional to n through the variance of $\bar{y}_i$.

Equating (9) to zero and substituting ML estimates of parameters gives

$$\begin{aligned} -\frac{n}{2}\frac{1}{\breve{\sigma}_i^2} + \frac{1}{2\breve{\sigma}_i^4}\sum_{j=1}^{n}(y_{ij} - \breve{a} - \breve{\beta}_1 x_{1i} - .. - \breve{\beta}_m x_{mi} - \breve{\lambda}\breve{\sigma}_i)^2 \\ + \frac{n\lambda}{2\breve{\sigma}_i^3}(\bar{y}_i - \breve{a} - \breve{\beta}_1 x_{1i} - .. - \breve{\beta}_m x_{mi} - \breve{\lambda}\breve{\sigma}_i) = 0. \end{aligned} \tag{13}$$

where $\breve{\theta}$ denotes an ML estimate of θ. Noting

$$\sum_{j=1}^{n}(y_{ij} - \breve{a} - \breve{\beta}_1 x_{1i} - .. - \breve{\beta}_m x_{mi} - \breve{\lambda}\breve{\sigma}_i)^2 = \sum_{j=1}^{n}(y_{ij} - \bar{y}_i)^2 + nz_i^2 ,$$

where

$$z_i = \bar{y}_i - \breve{a} - \breve{\beta}_1 x_{1i} + .. - \breve{\beta}_m x_{mi} - \breve{\lambda}\breve{\sigma}_i ,$$

(13) implies

$$\breve{\sigma}_i^2 = \tilde{\sigma}_i^2 + z_i^2 + \lambda\breve{\sigma}_i z_i .$$

Now:

$$\begin{aligned} z_i &= \bar{y}_i - a - \beta_1 x_{1i} + .. - \beta_m x_{mi} - \lambda\sigma_i - (\breve{a} - a) - (\breve{\beta}_1 - \beta_1)x_{1i} - .. - (\breve{\beta}_m - \beta_m)x_{mi} \\ &\quad - \lambda(\breve{\sigma}_i - \sigma_i) - (\breve{\lambda} - \lambda)\sigma_i - (\breve{\lambda} - \lambda)(\breve{\sigma}_i - \sigma_i) . \end{aligned}$$

The ML estimates differ from true values by terms of $O_p(1/\sqrt{rn}) = O_p(1/r)$ and the $\bar{y}_i - (a + \beta_1 x_{1i} + .. + \beta_m x_{mi} + \lambda\sigma_i)$ has mean zero so it is $O_p(1/\sqrt{n}) = O_p(1/\sqrt{r})$. So

z_i is $O_p(1/\sqrt{r})$ while z_i^2 is $O_p(1/r)$. Also, since $\breve{\sigma}_i = \sigma_i + O_p(1/\sqrt{n})$

$$\breve{\sigma}_i^2 = \tilde{\sigma}_i^2 + \lambda \sigma_i z_i + O_p(1/r).$$

and

$$\frac{1}{\breve{\sigma}_i^2} = \frac{1}{\tilde{\sigma}_i^2} - \lambda \frac{\sigma_i z_i}{\tilde{\sigma}_i^4} + O_p(1/r)$$

$$\frac{1}{\breve{\sigma}_i^2} = \frac{1}{\tilde{\sigma}_i^2} - w_i + O_p(1/r)$$

So

$$\sum_1^r \frac{1}{\breve{\sigma}_i^2} = \sum_1^r \frac{1}{\tilde{\sigma}_i^2} - \sum_1^r w_i + O_p(1) \qquad (14)$$

and since $\bar{y}_i$ and $\tilde{\sigma}_i^2$ are independent Σw_i is also $O_p(1)$ [5]. The term on the left hand side of (14) and the first on the right hand side are $O_p(r)$. Similarly we can show that terms such as

$$\sum_1^r \frac{1}{\breve{\sigma}_i}, \quad \sum_1^r \frac{x_{ki}}{\breve{\sigma}_i} \text{ and } \sum_1^r \frac{x_{ki} x_{mi}}{\breve{\sigma}_i^2}$$

equal corresponding terms with $\breve{\sigma}_i^2$ replaced by $\breve{\sigma}_i^2$ plus terms of $O_p(1)$. So the effect of using $\tilde{\sigma}_i^2$ rather than $\breve{\sigma}_i^2$ in the right hand sides of (5), (6) and (7) is just to add terms of $O_p(1)$. For the left hand side of (5)

$$\sum_1^r \frac{\bar{y}_i}{\breve{\sigma}_i^2} = \sum_1^r \frac{\bar{y}_i}{\tilde{\sigma}_i^2} + \sum_1^r \bar{y}_i w_i = \sum_1^r \frac{\bar{y}_i}{\tilde{\sigma}_i^2} + O_p(1)$$

because although the expectation of $\bar{y}_i z_i$ is non-zero and

[5] Because each $\bar{y}_i - (a + \beta_1 x_{1i} + .. + \beta_m x_{mi} + \lambda \sigma_i)$ has mean zero, their sum over countries is not $O_p(r/\sqrt{n})$ but $O_p(\sqrt{r}/\sqrt{n}) = O_p(1)$ and since an ML estimator minus true value is $O_p(1/r)$ the sum over countries is $O_p(1)$.

$$E[\bar{y}_i(\bar{y}_i - a - \beta_1 x_{1i} - .. - \beta_m x_{mi} - \lambda\sigma_i)] = \sigma_i^2 / n = O_p(1/r),$$

while the covariance of $\bar{y}_i$ and an ML estimator is $O_p(1/r)$. The same result follows for the left hand sides of (6) and (7). So the equations (5), (6) and (7) with $\breve{\sigma}_i^2$ in place of σ_i^2 are just the equations with $\tilde{\sigma}_i^2$ in place of σ_i^2 plus terms of $O_p(1)$. Solving the equations multiplies the $O_p(r)$ vector

$$(\sum_1^r \frac{\bar{y}_i}{\tilde{\sigma}_i^2}, \sum_1^r \frac{x_{1i}\bar{y}_i}{\tilde{\sigma}_i^2}, .. \sum_1^r \frac{x_{4i}\bar{y}_i}{\tilde{\sigma}_i^2}, \sum_1^r \frac{\bar{y}_i}{\tilde{\sigma}_i})'$$

by the inverse of the matrix

$$\begin{bmatrix} r & \sum \frac{x_{1i}}{\tilde{\sigma}_i} .. & \sum \frac{x_{4i}}{\tilde{\sigma}_i} & \sum \frac{1}{\tilde{\sigma}_i} \\ \sum \frac{x_{1i}}{\tilde{\sigma}_i} & \sum \frac{x_{1i}^2}{\tilde{\sigma}_i^2} .. & \sum \frac{x_{1i}x_{4i}}{\tilde{\sigma}_i^2} & \sum \frac{x_{1i}}{\tilde{\sigma}_i^2} \\ \sum \frac{x_{4i}}{\tilde{\sigma}_i} & \sum \frac{x_{1i}x_{2i}}{\tilde{\sigma}_i^2} .. & \sum \frac{x_{4i}^2}{\tilde{\sigma}_i^2} & \sum \frac{x_{4i}}{\tilde{\sigma}_i^2} \\ \sum \frac{1}{\tilde{\sigma}_i} & \sum \frac{x_{1i}}{\tilde{\sigma}_i^2} .. & \sum \frac{x_{4i}}{\tilde{\sigma}_i^2} & \sum \frac{1}{\tilde{\sigma}_i^2} \end{bmatrix}$$

In which all terms are $O_p(r)$,giving an $O_p(1)$ vector, and the vector of $O_p(1)$ terms by the same inverse giving an $O_p(1/r)$ vector. So the coefficients obtained using $\tilde{\sigma}_i$ rather than $\breve{\sigma}_i$ differ only by terms of $O_p(1/r)$ and are asymptotically equivalent.

IV The Ramey and Ramey 1995 data analysis

Ramey and Ramey (1995) are interested in developing the hypothesis that growth is negatively related to volatility. The explanatory variables included in their analysis are the average investment fraction of GDP, initial log GDP per capita, initial human capital and the average growth rate of population. They use a panel structure and measure volatility as the standard deviation of the residuals in the growth regression to find a negative relationship between volatility and growth. Ramey and Ramey

(1995) use a sample of ninety-two countries from 1960-1985[6]. The results of the Ramey and Ramey (1995) analysis is presented below alongside the results of the Byrne and Coniffe (2009) analysis[7]. Using maximum likelihood, they find a significant, negative effect of volatility on growth for the ninety-two country sample.

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Table 1
Results of Ramey and Ramey (1995) analysis and Byrne and Coniffe (2009) Analysis

Independent variable	Ramey and Ramey 92-country sample (2,208 observations)	Byrne and Coniffe 92-country sample (2,208 observations)
Constant	0.077	0.076
	(3.72)	(3.18)
Volatility (σ)	-0.211	-0.217
	(-2.61)	(-2.21)
Average investment share of GDP	0.127	0.124
	(7.63)	(5.98)
Average population growth rate	-0.058	-0.062
	(-0.38)	(-0.32)
Initial human capital	0.0008	0.0009
	(1.18)	(1.17)
Initial per capita GDP	-0.0088	-0.0093
	(-3.61)	(-3.10)

Numbers in parentheses are t statistics.

It is clear from the results that the estimates of coefficients are effectively identical, as are the standard errors.

There have been differing opinions among economists on whether volatility is harmful or beneficial to growth although it is now generally accepted that volatility has a negative effect (Aghion and Banerjee (2005), Pritchett (2000), Jerzmanowski

[6] Ramey and Ramey also analysed a sub-sample of OECD countries.
[7] Our computations were carried out using the Shazam package. The program is available from the authors.

(2005)). Since the introduction of the Ramey and Ramey (1995) analysis, many subsequent authors have used and adapted the model to further analyse the link between growth and volatility. Barlevy (2004) follows the R&R model but adds the standard deviation of the log of the investment output ratio as an explanatory variable in an attempt to capture the non-linearities of the investment function while Van der Ploeg & Poelhekke (2009) use the R&R framework to examine the natural resource curse. The R&R methodogoly is also followed by Akai et al (2007) to examine the impact of fiscal decentralisation on the fifty US states. Kroft & Llyod-Ellis (2002) adapt the R&R framework by decomposing volatility into two components; within regime volatility and between regime volatility. Using this methodology, they find that the source of volatility matters. Examining developing countries, Aizenman & Marion (1998) use the R&R method and find a significant negative relationship between innovation volatility and private investment in developing countries.

V Concluding remarks

Having estimated the θ parameters using $\tilde{\sigma}_i^2$ for σ_i^2 it would be easy to calculate (12) and repeat the estimation process with (12) replacing σ_i^2 in (5), (6) and (7). But it hardly seems worthwhile in view of the efficiency of $\hat{\theta}$. Not only have they equal asymptotic variances, but the variance of the difference between two efficient estimators is of second order. So with large samples the actual estimates will be very close as the analysis of Ramey and Ramey in section IV showed.

Also, should model (1) be at all inaccurate (12), unlike $\tilde{\sigma}_i^2$, is not a consistent estimator, so the iteration could make matters worse. Indeed, observed differences between (12) and $\tilde{\sigma}_i^2$ can be used to test the validity of model (1) through a Hausman type test. The difference is

$$\frac{\lambda^2}{2+\lambda^2}\left[(\sigma_i^+)^2 - \tilde{\sigma}_i^2\right]$$

The variance of the difference, remembering that (12) is efficient, is

$$\frac{\lambda^2\sigma_i^4}{n(1+\frac{\lambda^2}{2})}.$$

So

$$\frac{n\lambda^2}{4(1+\lambda^2/2)}\sum_1^r \frac{\left[(\sigma_i^+)^2 - \tilde{\sigma}_i^2\right]^2}{\tilde{\sigma}_i^4}$$

is asymptotically χ_r^2 if model (1) is correct.

No model is ever exactly the correct one and alternatives to model one are certainly conceivable. However, it is not our purpose in this paper to test or criticise the Ramey and Ramey model. We have developed an algebraic approach to estimation of the model that provides efficient estimators and is far less computationally demanding than econometric package algorithms for maximising likelihood functions from non-linear models.

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