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Current renormalization constants with an $O(a)$-improved fermion action

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Using chiral Ward identities, we determine the renormalization constants of bilinear quark operators for the Sheikholeslami-Wohlert action lattice at $\beta = 6.2$. The results are obtained with a high degree of accuracy. For the vector current renormalization constant we obtain $Z_V = 0.817 \pm 2 \pm 8$, where the first error is statistical and the second is due to mass dependence of Z_V. This is close to the perturbative value of 0.83. For the axial current renormalization constant we obtain $Z_A = 1.045 {}^{+10}_{-14}$, significantly higher than the value obtained in perturbation theory. This is shown to reduce the difference between lattice estimates and the experimental values for the pseudoscalar meson decay constants, but a significant discrepancy remains. The ratio of pseudoscalar to scalar renormalization constants, Z_P/Z_S, is less well determined, but seems to be slightly lower than the perturbative value.

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I. INTRODUCTION

In a recent paper [1], we reported values for the decay constants f_π and f_K from lattice calculations that were considerably lower than the experimental values. This has been a persistent feature of lattice calculations of pseudoscalar decay constants [2,3]. However, it has not been possible to attribute this discrepancy to any particular systematic error in the calculations (e.g., quenching), because of uncertainties in the renormalization of the current operators involved.

A fully nonperturbative determination of renormalization constants for finite operators can be achieved using chiral Ward identities [4,5], thereby bypassing the need for perturbative calculation of these quantities. A preliminary calculation at $\beta = 6.0$, presented in [5], indicated that the axial-vector current renormalization, in particular, differs significantly from its perturbative value. In this paper, we present results for the vector, axial-vector, pseudoscalar, and scalar renormalization constants at $\beta = 6.2$, and update the values for decay constants given in [1].

II. LATTICE WARD IDENTITIES AND RENORMALIZATION CONSTANTS

We define the (nonconserved) lattice vector current, the axial current, and the pseudoscalar and scalar densities as follows:

$$V_\mu^{La}(x) = \overline{\psi}(x)\gamma_\mu \frac{1}{2}\lambda^a \psi(x), \tag{1}$$

$$A_\mu^{La}(x) = \overline{\psi}(x)\gamma_\mu \gamma_5 \frac{1}{2}\lambda^a \psi(x), \tag{2}$$

$$P^a(x) = \overline{\psi}(x)\gamma_5 \frac{1}{2}\lambda^a \psi(x), \tag{3}$$

$$S^a(x) = \overline{\psi}(x)\frac{1}{2}\lambda^a \psi(x). \tag{4}$$

In the continuum limit, these operators are related to operators obeying the correct current algebra by multiplicative renormalization constants Z_V^L, Z_A^L, etc., so that $V_\mu = Z_V^L V_\mu^L$, etc. [5].

The renormalization constants for V_μ^L can easily be determined by evaluating

$$Z_V^L = \frac{\sum_{\vec{x}} \langle P^\dagger(\vec{0},0) P(\vec{x},T)\rangle}{\sum_{\vec{x},\vec{y}} \langle P^\dagger(\vec{0},0) V_4^L(\vec{y},t) P(\vec{x},T)\rangle}. \tag{5}$$

Inserting a complete set of states and noting that the matrix element of $V_4^L(0)$ between a degenerate pseudoscalar meson state P_n is $\langle P_n \mid V_4^L \mid P_n\rangle = 2E_n/Z_V^L$, we see that this should give a precise estimate provided the effect of the off-diagonal matrix elements $\langle P_m \mid V_4^L \mid P_n\rangle$ can be neglected.

For the axial case, there is no conserved current, or any other "easy" way of determining the renormalization constants, but they can be obtained using chiral Ward identities. Using the arguments of [4,5], we obtain the following identities for the Sheikholeslami-Wohlert (SW) [6] action:

$$\begin{aligned} 2\rho \sum_{x,\vec{y}} \langle P^a(x) A_\nu^{Lb}(y) V_\rho^{Lc}(0)\rangle \\ = -i\left(\frac{Z_V^L}{Z_A^{L\,2}} - \rho r a\right) f^{abd} \sum_{\vec{y}} \langle V_\nu^{Ld}(y) V_\rho^{Lc}(0)\rangle \\ -i\left(\frac{1}{Z_V^L} - \rho r a\right) f^{acd} \sum_{\vec{y}} \langle A_\nu^{Lb}(y) A_\rho^{Ld}(0)\rangle \end{aligned} \tag{6}$$

and

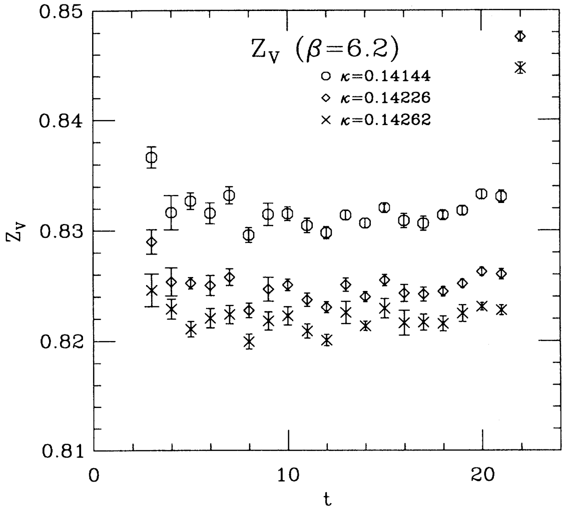

FIG. 1. Z_V^L as a function of t.

$$2\rho \sum_{x,\vec{y}} \langle P^a(x) S^b(y) P^c(0) \rangle$$

$$= \left(\frac{Z_P}{Z_A^L Z_S} - \rho r a \right) d^{abd} \sum_{\vec{y}} \langle P^d(y) P^c(0) \rangle$$

$$+ \left(\frac{Z_S}{Z_A^L Z_P} - \rho r a \right) d^{acd} \sum_{\vec{y}} \langle S^b(y) S^d(0) \rangle, \tag{7}$$

where

$$\rho = \frac{\langle 0 \,|\, \partial_4 A_4^{La} \,|\, P^a \rangle}{2\langle 0 \,|\, P^a \,|\, P_a \rangle} \tag{8}$$

and d^{abc} is defined by

$$\{\lambda^a, \lambda^b\} = 2 d^{abc} \lambda^c + \frac{4}{N_f} \delta^{ab}. \tag{9}$$

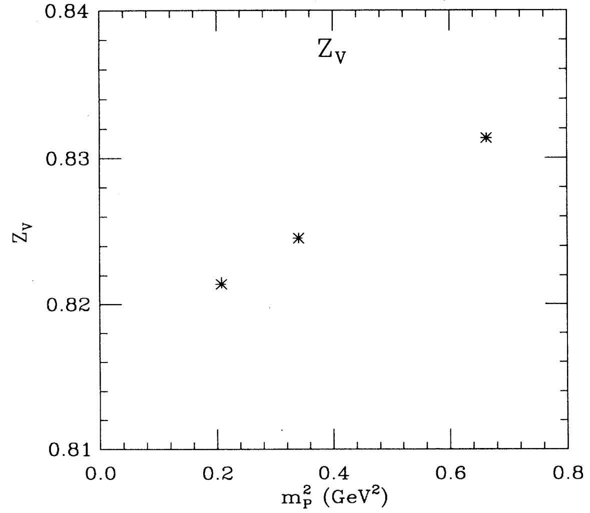

FIG. 2. Z_V^L as a function of the mass of the pseudoscalar meson. The scale is taken from the string tension [7].

TABLE I. Values of the renormalization constant Z_V^L as a function of the quark mass. The first set of errors are the statistical errors, while the second set are the errors due to the variation between the time slices.

κ	m_{PS}^2	Z_V^L
0.14262	0.208	$0.82139^{+41}_{-12}{}^{+25}_{-25}$
0.14226	0.341	$0.82453^{+24}_{-22}{}^{+24}_{-23}$
0.14144	0.663	$0.83136^{+23}_{-16}{}^{+22}_{-23}$

III. COMPUTATION AND RESULTS

We have performed simulations at $\beta = 6.2$ on a $24^3 \times 48$ lattice. We have generated propagators using the SW action, with two quark masses, corresponding to $\kappa = 0.141\,44$ and $0.142\,62$, where $\kappa = 1/2(m_0 + 4r)$ and $r = 1$. Sixty configurations have been analyzed at $\kappa = 0.141\,44$, and 26 configurations at $\kappa = 0.142\,62$ (for details, see [7]). The statistical errors are calculated with a bootstrap procedure, using 100 bootstrap samples.

Z_V was determined from Eq. (5), using 10 configurations, at three values for the quark mass (corresponding to $\kappa = 0.141\,44$, $\kappa = 0.142\,26$, and $\kappa = 0.142\,62$). The results are presented as a function of t in Fig. 1. We see that the values for Z_V are roughly independent of t. Our best values, obtained by fitting to time slices 5–19, are given in Table I. The errors from the variation between the time slices are obtained from fits to 100 bootstrap samples of time slices within the fit range.

The results are plotted as a function of the square of the mass of the pseudoscalar meson (proportional to the quark mass) in Fig. 2. We see that the results show a clear (linear) dependence on the quark mass, consistent with the expectation that the leading corrections to our calculations should be of $O(\alpha_s m_0 a)$. Perturbation theory at one-loop level [8,9] with a "boosted" coupling constant [10] gives $Z_V^L \approx 0.83$, which is quite close to our nonperturbative values.

We have also used Eq. (6), with $\nu = \rho = 0$, to check the consistency of our results at $\kappa = 0.141\,44$, using the value for Z_V^L quoted in Table I and the value for Z_A^L given in Table II as input. This gives $Z_V^L = 0.817^{+8}_{-10}$, which is within 2σ of the result obtained from Eq. (5).

The axial-vector renormalization constant Z_A^L is determined using Eq. (6), with $\nu = \rho = i$ and summing over $i = 1, 2, 3$, using the values for Z_V^L quoted in Table I as input. The results for $\kappa = 0.141\,44$ are plotted against t in Fig. 3. We see that, apart from the effect of the contact terms on the first few time slices, they show vir-

TABLE II. Values of the renormalization constants Z_A^L and Z_P/Z_S as functions of the quark mass.

κ	m_{PS}^2	Z_A^L	Z_P/Z_S
0.14262	0.208	1.040^{+10}_{-9}	0.693^{+28}_{-40}
0.14144	0.663	1.047^{+8}_{-8}	0.649^{+9}_{-8}

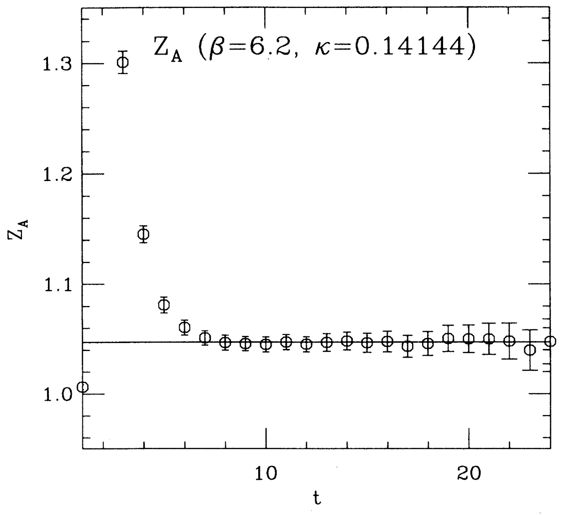

FIG. 3. Z_A^L as a function of t for $\kappa = 0.141\,44$.

TABLE III. Values of decay constants in physical units, using perturbative and nonperturbative values for the renormalization constants. The second set of errors in the vector meson decay constants are systematic uncertainties due to the quark mass dependence of Z_V.

	Old estimates [1]	Updated estimates	Experiment
f_π	102^{+6}_{-7} MeV	110^{+7}_{-8} MeV	132 MeV
f_K	123^{+5}_{-6} MeV	133^{+7}_{-7} MeV	160 MeV
$1/f_\rho$	0.316^{+7}_{-13}	$0.311^{+7}_{-13}{}^{+1}_{-1}$	0.28
$1/f_{K^*}$	0.298^{+5}_{-9}	$0.293^{+5}_{-9}{}^{+1}_{-1}$	
$1/f_\phi$	0.280^{+3}_{-6}	$0.276^{+3}_{-6}{}^{+1}_{-1}$	0.23
f_π/m_ρ	0.138^{+6}_{-9}	0.149^{+6}_{-10}	0.172
f_K/m_ρ	0.160^{+7}_{-8}	0.172^{+8}_{-9}	0.208
f_K/m_{K^*}	0.144^{+4}_{-6}	0.155^{+5}_{-7}	0.179

tually no dependence on t. Our best estimates are given in Table II.

Within the statistical errors, these results show no dependence of Z_A on the quark mass. The comparison with results from perturbation theory is more interesting: one-loop calculations with a boosted coupling constant give $Z_A \approx 0.97$, which is considerably lower than our nonperturbative results. The discrepancy is higher at lower β, as expected; in [5] Z_A at $\beta = 6.0$ was found to be 1.09.

In Table III we show how the values for the decay constants reported in [1] change when we use the results given above for the renormalization constants. For Z_V, we have extrapolated the values in Table I to the limit of zero quark mass, giving $Z_V = 0.817 \pm 2$, with an additional uncertainty due to the quark mass dependence of Z_V of ± 0.008, which corresponds to the difference between the value at our largest quark mass and the value for zero quark mass. For Z_A, we have taken a best estimate, combining our results at the two κ values, of $Z_A = 1.045^{+10}_{-14}$, with the errors corresponding to the spread between the highest and lowest estimate. We see that all the decay constants move closer to the experimental values, but that a significant discrepancy still remains, especially for f_ϕ and f_{K^*}. f_π turns out to be about 3σ away from its experimental value. The APE Collaboration has found $f_\pi/(m_\rho Z_A) = 0.186(20)$ at $\beta = 6.2$ [11], which gives a value for f_π/m_ρ compatible with experiment.

The ratio of pseudoscalar to scalar renormalization constant is determined using (7). The results are given in Table II. The uncertainty in these results is too large to determine whether there is any dependence on the quark mass here. Perturbative calculations with a boosted coupling constant give $Z_P/Z_S = 0.68$. As can be seen, our result for the heavier quark mass (which is the more accurate) is slightly lower than this, while the lighter quark mass gives a value compatible with perturbative results (although the errors here are still quite large). Comparison with the result reported in [5] at $\beta = 6.0$ shows that, as in the case with Z_A, the discrepancy decreases with increasing β.

IV. CONCLUSIONS

In this paper we have reported the determination of lattice renormalization constants using chiral Ward identities with the Sheikholeslami-Wohlert action. Our results are obtained with good accuracy, yielding values for Z_V close to the values from perturbation theory (but increasing with increasing quark mass), while the value for Z_A is considerably higher than perturbative results. For Z_P/Z_S the uncertainties are larger, but the results we can have confidence in lie slightly lower than perturbative values. The result for Z_A brings our estimates for f_π and f_K considerably closer to experimental values — within 3σ for f_π.

Note added. The value quoted for f_π in Ref. [1] is 130(8) MeV, in apparent contradiction to our statement that this provides an example of persistently low values for pseudoscalar decay constants from lattice calculations. However, this result is obtained using the nonperturbative value for Z_A. If the perturbative value had been used instead, it would have resulted in a value for f_π lower than experiment, as in the other papers quoted.

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