NUCLEAR PHYSICS B
PROCEEDINGS
SUPPLEMENTS

ELSEVIER Nuclear Physics B (Proc. Suppl.) 73 (1999) 626–628

The structure of the gluon propagator*

D. B. Leinweber[a], C. Parrinello[b†], J. I. Skullerud[a†] and A. G. Williams[a]

[a]CSSM and the Department of Physics and Mathematical Physics,
The University of Adelaide, SA 5005, Australia

[b]Department of Mathematical Sciences, University of Liverpool,
Liverpool L69 3BX, England

The gluon propagator has been calculated for quenched QCD in the Landau gauge at $\beta = 6.0$ for volumes $16^3 \times 48$ and $32^3 \times 64$, and at $\beta = 6.2$ for volume $24^3 \times 48$. The large volume and different lattice spacings allow us to identify and minimise finite volume and finite lattice spacing artefacts. We also study the tensor structure of the gluon propagator, confirming that it obeys the lattice Landau gauge condition.

1. INTRODUCTION

The infrared behaviour of the gluon propagator is important for an understanding of confinement. Previous conjectures range from a strong divergence [1] to a propagator that vanishes in the infrared [2,3]. Lattice QCD should be able to resolve this issue by first-principles, model-independent calculations. However, previous lattice studies [4,5] have been inconclusive since they have not been able to access sufficiently low momenta. Here we will report results using an asymmetric lattice with a spatial length of 3.3 fm This gives us access to momenta as small as 400 MeV.

2. LATTICE FORMALISM

We use the 'symmetric' definition of the gluon field, given by $U_\mu(x) = \exp(ig_0 a A_\mu(x+\hat{\mu}/2))$. This gives the gluon field in momentum space,

$$A_\mu(\hat{q}) = \frac{e^{-i\hat{q}_\mu a/2}}{2ig_0 a}\left[B_\mu(\hat{q}) - \frac{1}{3}\mathrm{Tr}\,B_\mu(\hat{q})\right] \quad (1)$$

where $\hat{q}$ denotes the discrete momenta $\hat{q}_\mu = 2\pi n_\mu/(aL_\mu)$, $B_\mu(\hat{q}) \equiv U_\mu(\hat{q}) - U^\dagger_\mu(-\hat{q})$, and $U_\mu(\hat{q}) \equiv \sum_x e^{-i\hat{q}x}U_\mu(x)$. An alternative, 'asymmetric' definition of the gluon field can be provided by $U_\mu(x) = \exp(ig_0 a A'_\mu(x))$. In momentum space, this differs from $A_\mu(x)$ by a factor $\exp(i\hat{q}_\mu a/2)$.

*Talk presented by J. I. Skullerud
†UKQCD Collaboration

Table 1
Simulation parameters. The lattice spacing is taken from the string tension [6].

Name	β	a^{-1} (GeV)	Volume	$N_{\rm conf}$
Small	6.0	1.885	$16^3\times 48$	125
Large	6.0	1.885	$32^3\times 64$	75
Fine	6.2	2.63	$24^3\times 48$	223

The gluon propagator $D^{ab}_{\mu\nu}(\hat{q})$ is defined as

$$D^{ab}_{\mu\nu}(\hat{q}) = \langle A^a_\mu(\hat{q})A^b_\nu(-\hat{q})\rangle \,/\, V\,, \quad (2)$$

where $A_\mu(\hat{q}) \equiv t^a A^a_\mu(\hat{q})$. In the continuum Landau gauge, the propagator has the structure

$$D^{ab}_{\mu\nu}(q) = \delta^{ab}(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})D(q^2)\,, \quad (3)$$

At tree level, $D(q^2)$ will have the form $D^{(0)}(q^2) = 1/q^2$. On the lattice, this becomes $D^{(0)}(\hat{q}) = a^2/(4\sum_\mu \sin^2(\hat{q}_\mu a/2))$. Since QCD is asymptotically free, we expect that up to logarithmic corrections, $q^2D(q^2) \to 1$ in the ultraviolet. Hence we define the new momentum variable q by $q_\mu \equiv (2/a)\sin(\hat{q}_\mu a/2)$, and use this throughout

We have analysed three lattices, with different values for the volume and lattice spacing. The details are given in table 1. All the configurations have been fixed to Landau gauge with an accuracy $\langle(\partial_\mu A_\mu)^2\rangle < 10^{-12}$.

0920-5632/99/$ – see front matter © 1999 Elsevier Science B.V. All rights reserved.
PII S0920-5632(99)00025-0

3. TENSOR STRUCTURE

By studying the tensor structure of the gluon propagator, we may be able to determine how well the Landau gauge condition is satisfied, and also discover violations of continuum rotational invariance.

The continuum tensor structure (3) follows from the condition $q_\mu A_\mu = 0$. This translates directly to the lattice provided we use the symmetric definition of the gluon field. If we use the asymmetric definition, we will instead obtain the condition $\sum_\mu (i \sin \hat{q}_\mu + \cos \hat{q}_\mu - 1) A'_\mu(\hat{q}) = 0$.

The tensor structure may be measured directly by taking the ratios of different components of $D_{\mu\nu}(q)$ for the same value of q. The results for the small lattice are summarised in table 2, and compared to what one would expect from (3), and to what one would obtain by replacing q with $\hat{q}$ in (3). The results are similar for the two other lattices. It is clear from table 2 that our numerical data are consistent with the expectation from (3). We can also see that in general, the asymmetric definition A' of the gluon field gives results which are inconsistent with this form.

4. FINITE VOLUME EFFECTS AND ANISOTROPIES

In the following, we are particularly interested in the deviation of the gluon propagator from the tree level form. We will therefore factor out the tree level behaviour and plot $q^2 D(q^2)$ rather than $D(q^2)$ itself.

Fig. 1 shows the gluon propagator on the small lattice as a function of qa. For low momentum values on the small lattice, there are large discrepancies due to finite volume effects between points representing momenta along the time axis and those representing momenta along the spatial axes. These discrepancies are absent from the data from the large lattice, shown in fig. 2. This indicates that finite volume effects here are under control.

However, at higher momenta, there are anisotropies which remain for the large lattice data, and which are of approximately the same magnitude for the two lattices. These

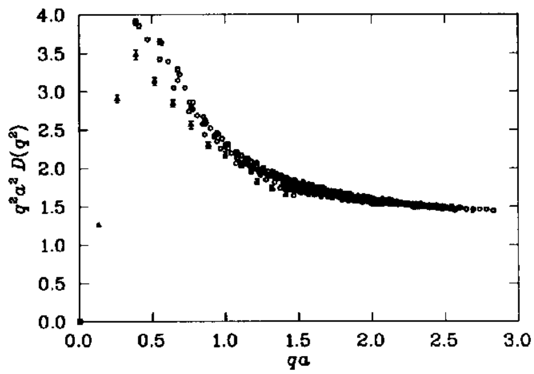

Figure 1. The gluon propagator multiplied by q^2 as a function of q for the small lattice. The filled triangles denote momenta directed along the time axis, while the filled squares denote momenta directed along one of the spatial axes.

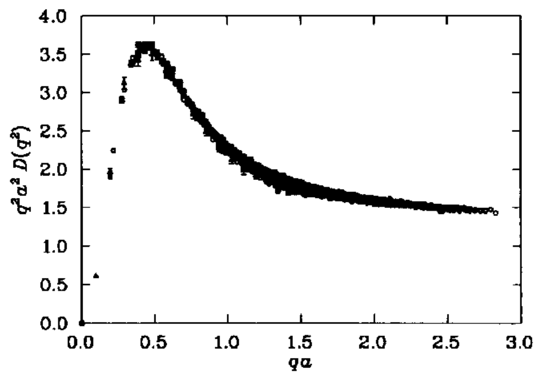

Figure 2. The gluon propagator multiplied by q^2 as a function of q for the large lattice. The symbols are as in fig. 1.

anisotropies are considerably reduced on the fine lattice, indicating that they arise from finite lattice spacing errors. In order to eliminate these anisotropies, we select momenta lying within a cylinder of radius $\Delta\hat{q}a = 2 \times 2\pi/32$ along the 4-dimensional diagonals. The result of this cut on the large lattice is shown in fig. 3. A more detailed discussion of these cuts can be found in [7].

5. CONCLUSIONS

We have evaluated the gluon propagator on three different lattices. The tensor structure has

Table 2
Tensor structure for the small lattice. $\hat{q}$ is in units of $2\pi/L_s$, where L_s is the spatial length of the lattice. The theoretical predictions are the values for the ratios one obtains from (3), and from (3) with $q \rightarrow \hat{q}$. The numbers in brackets are the statistical uncertainties in the last digit(s). Where no error is quoted, the statistical uncertainty is less than 10^{-6}.

		Theoretical prediction		This simulation	
$\hat{q}$	Components	Using $\hat{q}$	Using q	Using A	Using A'
[2,1,0,0]	(1,1)/(1,2)	-0.5	-0.509796	-0.509796	-0.519783
	(1,1)/(2,2)	0.25	0.259892	0.259892	0.259892
	(1,1)/(3,3)	0.2	0.206281	0.204(8)	0.204(8)
	(1,2)/(2,2)	-0.5	-0.509796	-0.509796	-0.5
[4,1,0,0]	(1,1)/(1,2)	-0.25	-0.275899	-0.275899	-0.331821
	(1,1)/(3,3)	0.05882	0.070736	0.076(3)	0.076(3)
	(1,2)/(2,2)	-0.25	-0.275899	-0.275899	-0.229402
[4,2,1,0]	(1,1)/(1,2)	-0.625	-0.681848	-0.678(9)	-0.743(10)
	(1,1)/(2,3)	-2.5	-2.47137	-2.3(4)	-2.5(5)
	(1,2)/(2,2)	-0.4706	-0.502914	-0.500(6)	-0.456(6)

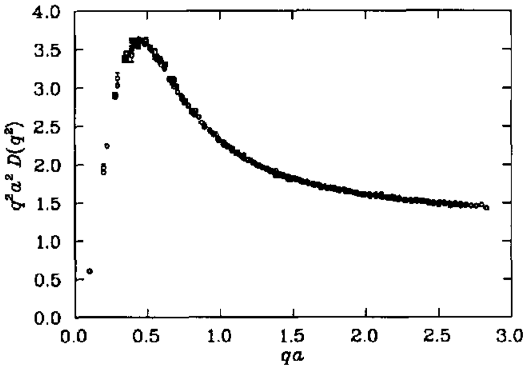

Figure 3. The gluon propagator multiplied by q^2 as a function of q for the large lattice, after the cylindrical cut.

been analysed and shown to agree with the continuum Landau gauge form. By studying the anisotropies in the data, we have been able to conclude that finite volume effects are under control on the largest of our lattices.

A clear turnover in the behaviour of $q^2D(q^2)$ has been observed at $q \sim$ 1GeV, indicating that the gluon propagator diverges less rapidly than $1/q^2$ in the infrared, and may be infrared finite. An analysis of scaling and the functional behaviour of these results is presented in [8].

ACKNOWLEDGMENTS

The numerical work was mainly performed on a Cray T3D at EPCC, University of Edinburgh, using UKQCD Collaboration time under PPARC Grant GR/K41663. Financial support from the Australian Research Council is gratefully acknowledged.

REFERENCES

1. N. Brown and M.R. Pennington, Phys. Rev. D 39, 2723 (1989)
2. V.N. Gribov, Nucl. Phys. B 139, 19 (1978)
3. M. Stingl, Phys. Rev. D 34, 3863 (1986); Phys. Rev. D 36, 651 (1987)
4. C. Bernard, C. Parrinello, A. Soni, Phys. Rev. D49, 1585 (1994)
5. P. Marenzoni, G. Martinelli, N. Stella, Nucl. Phys. B 455, 339 (1995); P. Marenzoni *et al*, Phys. Lett. B 318, 511 (1993)
6. G.S. Bali and K. Schilling, Phys. Rev. D 47, 661 (1993)
7. D.B. Leinweber, C. Parrinello, J.I. Skullerud, A.G. Williams, Phys. Rev. D 58, 031501 (1998)
8. D.B. Leinweber, C. Parrinello, J.I. Skullerud, A.G. Williams: *Modelling the gluon propagator*, these proceedings.