A Simple and Effective Excitation Force Estimator for Wave Energy Systems

Nicolás Faedo, Ulises Bussi, Yerai Peña-Sanchez, Christian Windt, and John V. Ringwood, *Senior Member, IEEE*

***Abstract*—Wave energy converters (WECs) need to be optimally controlled to be commercially viable. These controllers often require an estimate of the (unmeasurable) *wave excitation force*. To date, observers for WECs are often based upon 'complex' techniques, which are counter-intuitive in their design, additionally requiring an explicit model to describe the excitation as part of an (augmented) system. The latter imposes strong assumptions on the design of each observer, while also implying an additional computational burden associated with the necessity of augmenting the WEC model to include the dynamics of the input. We propose a *simple* and *effective* excitation force estimator based on linear time-invariant (LTI) theory, *without the need* for an explicit model of the input. In particular, we re-formulate the unknown-input estimation problem as a tracking control-loop, so that a wide-variety of LTI design techniques (arising from either classical or modern control theory) can be used to compute an estimate of the excitation force. We demonstrate performance, simplicity, and intuitive appeal of the proposed observer, by means of a case study based on a realistic computational fluid dynamics simulation, comparing the technique against a large set of WEC observers, showing that the approach is able to outperform available estimators.**

***Index Terms*—Energy-maximising control, optimal control, wave energy, wave excitation force, WEC.**

I. Introduction

DESPITE the fact that ocean waves represent a vast clean energy resource, estimated (worldwide) to be around 3.7 [TW] and about 32000 [TWh/yr] in [1] and [2], respectively, wave energy converters (WECs) have not yet been successfully commercialised. The main reason for the lack of proliferation of wave energy can be attributed to the fact that harnessing the irregular reciprocating motion of the ocean waves is not as straightforward as, for example, extracting energy from the wind [3], [4]. Consequently, the current levelised cost of wave energy (LCoE) is substantially higher compared to other renewable energy sources.

Appropriate control system technology can impact WEC design and operation, by maximising energy extraction from waves, and optimising energy conversion in the power take-off (PTO) actuator system. In particular, the central problem in WEC control is to find a technically feasible way to 'act' on the device (via the PTO) so that energy absorption from waves is maximised while minimising the risk of component damage. It is already clear that control technology can enhance WEC performance over a wide range of ocean conditions, substantially reducing the associated LCoE [3], and achieving a key stepping-stone towards commercialisation of WEC technology.

Regardless of the specific control strategy selected, *true* optimal energy absorption under irregular wave motion can only be achieved by having instantaneous knowledge of the wave excitation force, *i.e.* the uncontrollable external force generated on the device by the presence of ocean waves (see [5], [6]). Unfortunately, for the WEC case, *i.e.* a moving body, the excitation force is, in general, an unmeasurable quantity [7]. Consequently, unknown-input state-estimation strategies are required to provide instantaneous values of the wave excitation force (see [8] for a comprehensive review).

Given the inherent analytical complexity and computational burden associated with the energy-maximising control problem, an increasing number of researchers aim to find simple and intuitive solutions to the WEC control design procedure, by using the fundamental theory behind maximum energy absorption, *i.e.* the so-called impedance-matching principle. In particular, this family of *simple* controllers, recently reviewed and compared in [9], attempts to provide a (physically implementable) realisation of the impedance-matching condition for maximum power transfer, by proposing 'simple' systems, mostly characterised by well-known techniques from linear time-invariant (LTI) theory. As such, these controllers have mild computational requirements, and their actual implementation can be performed in real-time with almost any physical hardware platform, including

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Nicolás Faedo is with the Marine Offshore Renewable Energy Lab, Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Turin 10129, Italy (e-mail: nicolas.faedo@polito.it).

Ulises Bussi is with the Laboratorio de Instrumentación, Automatización y Control, Departamento de Ciencia y Tecnología, Universidad Nacional de Quilmes, Buenos Aires B1876BXD, Argentina, and also with the Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina (e-mail: ulises.bussi@unq.edu.ar).

Yerai Peña-Sanchez and John V. Ringwood are with the Centre for Ocean Energy Research, Department of Electronic Engineering, Maynooth University, W23 Maynooth, Ireland (e-mail: Yerai.Pena.2017@mumail.ie; john.ringwood@mu.ie).

Christian Windt is with the Leichtweiß-Institute for Hydraulic Engineering and Water Resources, Technische Universität Braunschweig, 38106 Braunschweig, Germany (e-mail: c.windt@tu-braunschweig.de).

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low-cost microcontrollers, hence being especially suitable for real environments.

Nonetheless, the vast majority of these simple controllers effectively require an estimate of the wave excitation force input which, to date, can only be computed in terms of 'complex' unknown-input estimation strategies (see [8]), often relying upon either time-varying, optimisation-based, or nonlinear design procedures. These estimators are often inherently computationally demanding, and counter-intuitive in their design/calibration. In addition, virtually every available unknown-input estimation technique in the WEC literature requires an explicit dynamical (internal) model [10] to describe the wave excitation force input as part of an (augmented) state vector. The latter not only imposes strong assumptions on the design of the observers (and hence naturally limits their range of applicability in practical scenarios), but also implies an additional computational burden generated by increasing the dimension (order) of the WEC model to include the internal dynamics of the input force in the state-space description.

To summarise, as effectively reported in [9], the bottleneck of any simple controller, in terms of simplicity, intuitive appeal, and computational efficiency, is the lack of a simple wave excitation force estimator counterpart. Motivated by this, we present, in this paper, a *simple* and *effective* wave excitation force estimator for WEC applications, fully based upon an LTI framework. In particular, we re-formulate the unknown-input estimation problem as a reference tracking control-loop, to later use well-established LTI control design techniques to compute an estimate of the wave excitation force input. Through the Youla-Kŭcera parameterisation [11], we demonstrate performance, simplicity, and intuitive appeal of the proposed observer, by means of a comprehensive case study, where we compare the technique against state-of-the-art unknown-input observers currently reported in the WEC literature [8], showing that the proposed approach is able to consistently outperform all other strategies. Crucially, the evaluation is performed with a realistic nonlinear computational fluid dynamics (CFD) device simulation.

The remainder of this paper is organised as follows. Section I-A introduces (potentially non-standard) notation used throughout our paper. Section II recalls the fundamentals behind control-oriented linear modelling for WECs. Section III outlines the underpinning methodology of the proposed observer, reformulating the unknown-input estimation problem as a tracking control design procedure. Section IV presents a comprehensive case study based on the same nonlinear CFD simulation set-up utilised in [8], allowing for direct comparability in terms of performance with the proposed observer. Finally, Section V encompasses the main conclusions of this study.

A. Notation

$\mathbb{R}^+$ ($\mathbb{R}^-$) denotes the set of non-negative (non-positive) real numbers. $\mathbb{C}_{<0}$ denotes the set of complex number with negative real part. The symbol 0 stands for any zero element, dimensioned according to the context. The convolution between two functions f and g, with $\{f,g\} \subset L^2(\mathbb{R})$, over the set $\mathbb{R}$, *i.e.* $\int_{\mathbb{R}} f(\tau)g(t-\tau)d\tau$ is denoted as $f * g$, and where $L^2(\mathbb{R}) = \{f : \mathbb{R} \to \mathbb{R} \mid \int_{\mathbb{R}} |f(\tau)|^2 d\tau < +\infty\}$ is the Hilbert space of square-integrable functions in $\mathbb{R}$. The Laplace transform of a function f (provided if exists), is denoted as $F(s)$, $s \in \mathbb{C}$. With some abuse of notation,[1] the same is used for the Fourier transform of f, written as $F(\omega)$, $\omega \in \mathbb{R}$. Finally, the notation $\lambda(F(s)) \subset \mathbb{C}$, with F the Laplace transform of f, is used to denote the set of poles of $F(s)$.

II. WEC Modelling Fundamentals

We recall, in this section, well-known fundamentals behind control-oriented linear modelling for WECs. In particular, we assume a 1-degree-of-freedom (DoF) wave energy device; However, we note that multiple DoFs can be straightforwardly considered. Further detail on control-oriented WEC modelling can be found in, for instance, [12], [13].

Following linear potential flow theory, the motion of such a device can be written in terms of a dynamical system Σ, for $t \in \mathbb{R}^+$, given by the set of equations[2]

$$\Sigma : \begin{cases} \ddot{x} = \frac{1}{m}\left(f_r + f_{re} + f_{ex} - f_{PTO}\right), \\ y = \dot{x} = v, \end{cases} \tag{1}$$

where x is the device excursion (displacement), v is the device velocity, f_{re} the hydrostatic restoring force, f_r the radiation force, f_{PTO} the (controllable) force supplied by means of the PTO system, and m is the mass of the device. Finally, the notation f_{ex} is used for the wave excitation force, *i.e.* an external uncontrollable input due to the incoming wave field, which is to be estimated in this study by means of our proposed simple and effective observer.

To give a precise mathematical description of (1), note that the linearised hydrostatic force can be written as $f_{re}(t) = -s_h x(t)$, where $s_h \in \mathbb{R}$ denotes the hydrostatic stiffness, which depends on the device geometry. The radiation force f_r is modelled based on linear potential theory and, using the well-known Cummins' equation [14], can be written, for $t \in \mathbb{R}^+$, using the expression

$$f_r(t) = -m_\infty \ddot{z}(t) - \int_{\mathbb{R}^+} h_r(\tau)v(t-\tau)d\tau, \tag{2}$$

with $h_r \in L^2(\mathbb{R})$ the (causal) radiation impulse response function containing the memory effect of the fluid response, and $m_\infty = \lim_{\omega \to +\infty} A_r(\omega) \in \mathbb{R}$, where A_r is the so-called radiation added-mass (see, for instance, [12]).

Remark 1: The impulse response mapping h_r fully characterises a continuous-time LTI system Σ_r describing the dynamics associated with radiation effects. Some of the fundamental properties of system Σ_r include bounded-input bounded-output (BIBO) stability, strict properness, minimum phase, and passivity. The reader is referred to [15], [16] for further detail on these dynamical characteristics.

Remark 2: h_r is commonly computed numerically, employing so-called Boundary Element Method (BEM) solvers, such as the open-source software NEMOH [17], which produce a

[1] The use of the capitalised letter for Laplace or Fourier transforms is always clear from the context.

[2] From now on, we omit the dependence on t when it is clear from the context.

finite set of datapoints characterising the Fourier transform of the impulse response function h_r. In other words, a closed-form expression for h_r (and hence for the radiation system Σ_r) is not readily available. This inherently requires the use of parameterisation techniques (see [15], [16]) to compute an approximating dynamical model $\tilde{\Sigma}_r \approx \Sigma_r$, which must preserve the physical properties discussed in Remark 1.

Finally, the equation of motion of the WEC, in the time-domain, is given by

$$\Sigma : \begin{cases} \ddot{x} = \mathcal{M}\left(-h_r * v - s_h x + f_{ex} - f_{PTO}\right), \\ y = v, \end{cases} \tag{3}$$

where $\mathcal{M} = (m + m_\infty)^{-1}$.

Since the observer proposed in this paper is based on a classical (transfer function) approach to control design, it is convenient to introduce the Laplace-domain equivalent of (3). A direct application of the Laplace transform (which is always well defined for (3) [12]), considering zero initial conditions, yields[3]

$$s\mathcal{M}^{-1}V(s) + H_r(s)V(s) + \frac{s_h}{s}V(s) = F_{ex}(s) - F_{PTO}(s), \tag{4}$$

for all $s \in \mathbb{C}$. Without any loss of generality, given the LTI nature of the radiation impulse response mapping h_r, let H_r be written as $H_r(s) = H_r^N(s)/H_r^D(s)$. Then, the input-output (force-to-velocity in this case) dynamics associated with (4) are

$$V(s) = G_0(s)\left[F_{ex}(s) - F_{PTO}(s)\right], \tag{5}$$

where the mapping G_0, defining the input-output dynamics $f_{ex} - f_{PTO} \mapsto v$, is given by

$$G_0(s) = \frac{H_r^D r(s)s}{\mathcal{M}^{-1}H_r^D(s)s^2 + H_r^N(s)s + H_r^D(s)s_h}. \tag{6}$$

Considering the dynamic characteristics of the radiation system Σ_r (see Remark 1), the following fundamental properties of G_0 can be directly derived:

- G_0 is BIBO stable.
- G_0 is a strictly proper transfer function.
- G_0 has relative degree 1.
- G_0 is minimum phase, *i.e.* the zeros of G_0 are always contained in $\mathbb{C}_{<0}$ as a consequence of the BIBO stability of the radiation system Σ_r.

Remark 3: The set of properties listed immediately above is explicitly used to guarantee stability and performance specifications of the proposed observer (see Section III).

III. Observer Design

The underpinning idea behind the method proposed in this paper is to re-formulate the excitation force input estimation problem in terms of a standard reference tracking control design procedure, where well-established techniques from both modern and classical control theory can be directly considered[4]. We describe in detail the fundamental concepts behind this approach in the following paragraphs.

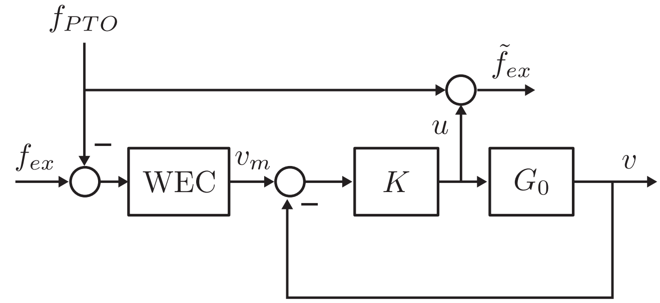

Fig. 1. Schematic representation of the proposed observer in terms of a reference tracking control loop.

Let v_m be the *measured* velocity of the WEC system (*i.e.* process) under analysis and let v be the output of the WEC model G_0 (see Section II). Suppose we can design a controller $K : \mathbb{C} \to \mathbb{C}$, $s \mapsto K(s)$, which supplies a control signal u, such that

$$\lim_{t\to\infty} \|v - v_m\| = 0, \tag{7}$$

i.e. we design K to achieve asymptotic tracking of v_m. The associated control loop is schematically depicted in Fig. 1.

If the WEC system is effectively described by the LTI WEC model G_0, it is straightforward to show that (7) holds if and only if

$$\lim_{t\to\infty} \|u - (f_{ex} - f_{PTO})\| = 0, \tag{8}$$

i.e. both the actual WEC system and model G_0 *must be* affected by the same input signal to produce the exact same output as $t \to \infty$. In other words, if (7) holds, the reference tracking control signal u can be directly used to approximate the wave excitation input by means of a simple linear mapping, *i.e.*

$$f_{ex} \approx \tilde{f}_{ex} = u + f_{PTO}. \tag{9}$$

Remark 4: Unlike the vast majority of the unknown-input observers proposed in the wave energy field (see Section I), the estimation of the wave excitation force $\tilde{f}_{ex}$, presented in (9), *does not require* a dynamic model for f_{ex}.

Remark 4 highlights a potential advantage of the presented approach, for two reasons. Firstly, accurate modelling of the underlying dynamics of f_{ex} is often impossible to achieve in terms of a deterministic system, given the intrinsic stochastic nature of ocean waves. This, in turn, can potentially impact the design of WEC unknown-input observers based upon this internal model, in particular in terms of its closed-loop dynamics. In other words, if the implicit model adopted to describe the input is 'far' from the actual process, the bandwidth of the (closed-loop) estimator will have to be, in general, large. Given that real measurements are always affected by noise, this has the potential to generate a compromise between effective input estimation and noise rejection. We refer the reader to [8] for further detail on performance of these type of observers under noisy (measurement) conditions. Secondly, the use of an internal model for f_{ex}

[3]From now on, we omit the dependence on s when it is clear from the context.

[4]Note that a similar approach is considered within a completely different application field, in [18], [19], where forces experienced by the car tires are estimated in terms of a tracking control-loop.

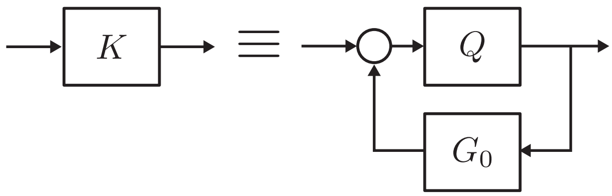

Fig. 2. Schematic representation of the Youla-Kŭcera parameterisation of all stabilising controllers.

automatically requires the definition of an augmented system, hence directly impacting on the computational burden associated with the observer strategy. The latter is especially true for the case where estimation of wave excitation is required in more than one operating degree-of-freedom.

Remark 5: Consistently with the literature in unknown-input estimation for WECs, from now on, we assume that f_{PTO} is measurable for all $t \in \mathbb{R}^+$. In other words, we assume that the force control system, associated with the PTO, achieves asymptotic tracking of the reference control signal, so that the user-supplied control law f_{PTO} can be used directly as a surrogate for force. With this assumption, the effectiveness of the estimator, which is the main focus of our study, can be analysed independently in Section IV.

A. Youla-Kŭcera Parameterisation

Given the dynamical characteristics associated with the WEC model listed in the final paragraph of Section II, we propose to design the reference tracking controller K based on the so-called Youla-Kŭcera parameterisation of all stabilising controllers (see, for instance, [11]). This approach can systematically fulfill performance specifications while consistently guaranteeing BIBO stability. Nonetheless, and by virtue of the re-formulation of the unknown-input estimation problem in terms of a tracking control loop, we note that a wide-variety of techniques can be considered to design the associated controller K.

Let $Q : \mathbb{C} \to \mathbb{C}, s \mapsto Q(s)$, be a proper rational transfer function. The family of all stabilising controllers K parameterised in Q for the WEC plant G_0 (as desribed in Section II) can be written as

$$K(s) = \frac{Q(s)}{1 - Q(s)G_0(s)}. \quad (10)$$

We make the controller parameterisation in (10) explicit (schematically) in Fig. 2.

Remark 6: The stability condition of Q, *i.e.* $\lambda(Q(s)) \subset \mathbb{C}_{<0}$, is both necessary and sufficient to guarantee closed-loop stability for the tracking control system illustrated in Fig. 1 [11]. In other words, choosing a stable rational function Q in (10), directly guarantees well-posedness of the proposed excitation input estimation technique. Note that guarantees of stability are not always provided in the literature for unknown-input WEC estimators. For instance, this is the case for the observers designed in [20], [21], which are compared with our simple and effective estimator in Section IV, and do not provide any explicit guarantees of closed-loop stability.

A common choice for Q, to successfully achieve reference tracking, relates with the principle of plant inversion, *i.e.*

$$Q(s) = F_Q(s)G_0^{-1}(s), \quad (11)$$

where the mapping F_Q, commonly referred to as *shaping filter*, is directly used to shape the closed-loop behaviour, *i.e.* to specify performance characteristics.

Remark 7: It is straightforward to show that the closed-loop response, under the parameterisation in (11), can be written as $T_0 = K/(1 + KG_0) = F_Q$. Hence the frequency response of the shaping filter needs to be such that $F_Q(\omega) \approx 1$, $\omega \in \mathbb{R}$, in the range of frequencies which characterises the reference input (*i.e.* measured velocity).

Remark 8: As indicated in Remark 6, Q needs to be stable to guarantee closed-loop stability. This directly implies that the parameterisation in (11) is well-posed as long as G_0 does not have any non-minimum phase zeros, which is always the case for the WEC model arising from the physical principles described in Section II.

Remark 9: To have a proper controller, *i.e. implementable*, Q must be proper. In other words, the shaping filter F_Q needs to have relative degree at least equal to the negative of that of G_0^{-1}. Note that this can always be achieved by including factors of the form $(\tau s + 1)^{n_Q}$, $\tau \in \mathbb{R}^+$, as part of the denominator of F_Q. The value of n_Q has to be chosen such that Q is at least biproper, and τ should be selected to meet any necessary design trade-offs (further discussed in Section IV).

In the following, and to summarise the fundamental concepts posed in this section, we proceed to highlight the main features of this *simple and effective* observer:

- There is no need to assume an internal model for the wave excitation force input. This avoids the need to augment the WEC model to include an extra differential operator to describe the dynamics of f_{ex}.
- Closed-loop stability can be guaranteed straightforwardly.
- The approach is inherently computationally efficient and entirely composed of LTI systems.
- Due to its LTI nature, real-time implementation can be performed using virtually any commercially available hardware platform.
- The design is intuitive, based upon traditional and well-known control techniques.

IV. Case Study and Performance Assessment

To assess the performance of the proposed observer, we consider a spherical heaving point absorber WEC with a diameter of 5 [m]. The sea state considered for the subsequent motion simulation and performance assessment is chosen to be representative of real sea conditions, *i.e.* irregular. In particular, we generate a wave train based on a JONSWAP spectrum [22], with a significant wave height of $H_s = 1.5$ [m], peak period of $T_p = 8$ [s], and a peak enhancement factor $\gamma = 3.3$. The simulation (time) length, *i.e.* duration of the wave input, is set to 160 [s].

Aiming to compare the performance of the estimator proposed in this paper with state-of-the-art unknown-input observers applied in the WEC field, we consider a numerical simulation set-up identical to that presented in [8]. This allows for direct performance comparison with the set of 11 estimators evaluated in [8] (see Section IV-D for further detail). In particular, the numerical simulation is performed within a *high-fidelity* environment: We use a CFD-based numerical wave tank (NWT) to obtain precise computation of the motion of the chosen WEC device. Such a NWT provides both a realistic and reliable environment to assess the performance of the proposed observer. Resistive control, *i.e.* $f_{PTO} = b_{PTO}\dot{x}$, with a damping factor of $b_{PTO} = 170$ [kNs/m], is employed as a benchmark WEC control strategy [4], giving realistic WEC operating conditions.

The actual wave excitation force f_{ex}, *i.e.* the quantity to be estimated, can be computed with the WEC fixed in its equilibrium position [8]. For this diffraction-only case, since the WEC device is not allowed to move, radiation and hydrostatic forces are zero, and the total hydrodynamic force measured on the device is[5] effectively f_{ex}. Since this total force is the integral of the pressure over the wetted surface of the WEC, it can be readily measured using a finite number of numerical pressure probes, set at specific locations on the hull of the device. This effectively mimics a realistic test setup.

Using the exact same wave (as for the above described diffraction test), simulations are performed with the device free to move in heave. This allows for the computation of the (reference) velocity of the WEC, *i.e.* v_m, which is used by the observer to provide an estimate of the wave excitation input in terms of the tracking structure proposed in Section III.

We provide a detailed description of both the CFD-based NWT, used as a high-fidelity simulation environment, and the WEC dynamical model G_0, used to design and synthesise the proposed simple and effective estimator, in Sections IV-A and IV-B, respectively. Finally, the design and performance of the proposed observer, for this case study, are detailed in Sections IV-C and IV-D, respectively.

A. CFD-Based NWT

The CFD-based NWT model utilised in this study is that employed in the wave estimation comparison paper presented in [8]. The model is based on the open-source CFD software OpenFOAM [23]. The hydrodynamics in the CFD-based NWT are captured by solving the incompressible Reynolds Averaged Navier-Stokes (RANS) equations, describing the conservation of mass and momentum (see [24]), commonly applied in the WEC literature [25]. The Volume of Fluid (VoF) method [26] is used to capture the wave advection. Turbulence is modelled using a standard $k - \omega$ SST turbulence model with wall functions for the turbulent kinetic energy, the turbulence frequency, and the eddy viscosity. The numerical wave maker, used to generate and absorb waves in the CFD-based NWT, is based on the relaxation zone method, as in the *waves2FOAM* toolbox [27]. The relaxation zone is depicted in Fig. 3 (a).

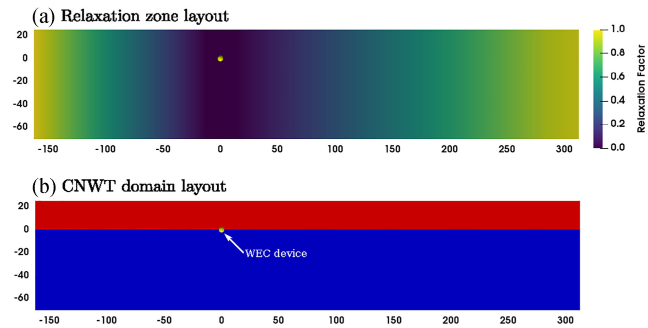

Fig. 3. CFD-based NWT domain definition with all relevant dimensions given in [m] (figure adapted from [8]).

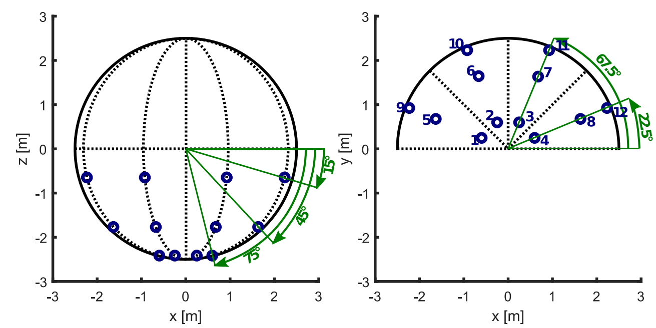

Fig. 4. Numerical pressure probes, used to compute the target f_{ex} (figure adapted from [8]).

The symmetry of the test case is exploited by implementing a symmetric boundary condition in the domain, reducing the computational burden, while retaining the accuracy of the results. A schematic of the CFD-based NWT domain, with all relevant dimensions[6], is depicted in Fig. 3 (b). For the pressure measurements, used to compute the target excitation force f_{ex}, 12 numerical pressure probes are positioned at specific locations on the WEC hull, as illustrated in Figure 4. Note that the use of discrete locations, and their specific distribution on the hull is inspired by physical WEC systems which may be equipped with pressure sensors (*e.g.* see [28]). It is apparent that such sensors can also only measure pressures and, thus, the excitation force at discrete locations. We show, in the following, that the distribution of sensors utilised in our study is effectively able to provide an accurate measure of the target wave excitation force signal, used in Section IV-D as a benchmark for the compared estimators. In particular, Fig. 5 shows the normalised root mean square error (NRMSE) between the excitation force obtained by integrating the pressure over the complete wetted surface of the sphere, and by using a different discrete number of pressure sensors along the hull. The location of the pressure sensors is evenly distributed over the wetted surface and different for each case presented in Fig. 5.

[5]Linear conditions are required to use the f_{ex} signal obtained from the diffraction tests as a reference case for the WEC. The linearity of the case study considered herein has been verified in [8, Section IV-C].

[6]For more details on the spatial and temporal problem discretisation, including results of convergence studies, as well as the mesh layout, the interested reader is referred to [25]

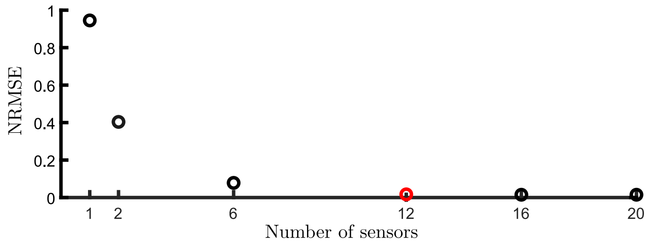

Fig. 5. NRMSE between the excitation force obtained by integrating the pressure over the complete wetted surface of the sphere, and by using a different discrete number of evenly distributed pressure sensors. The NRMSE for the sensor configuration used in our study is highlighted with a red color.

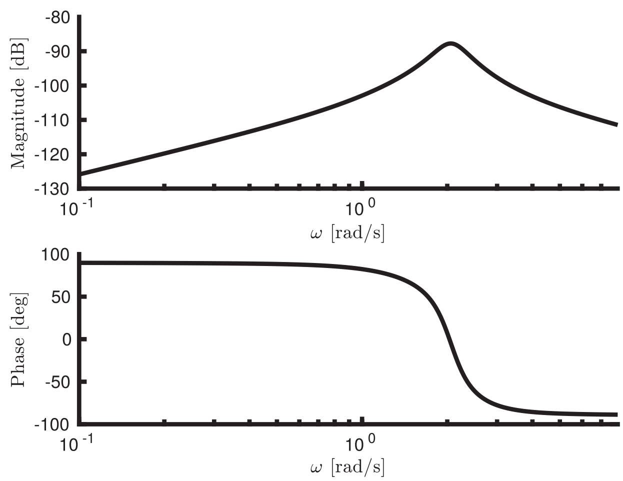

Fig. 6. Bode plot of the WEC model G_0 for the heaving point absorber device considered in this case study.

B. WEC Dynamical Model

The WEC dynamical model G_0 (as described in Section II) used to design the proposed observer, is characterised via BEM methods. In particular, a frequency-domain characterisation is computed using the open-source software NEMOH [17], which provides a description of $H_r(\omega)$ in terms of a finite set of datapoints (frequencies) in $\mathbb{R}^+$. Given the inherently non-parametric nature of the result arising from BEMs (see also Remark 2), we parameterise H_r in terms of a continuous-time, finite-dimensional system $\tilde{\Sigma}_r$. Such an approximating system is computed via moment-matching [15], [16], using two interpolation points, resulting in a BIBO stable, minimum phase, strictly proper, passive (*i.e.* it respects all the associated radiation physical properties, see Remark 1) approximation structure of order (dimension) 4. The set of poles of G_0, *i.e.* $\lambda(G_0(s))$, has a cardinality of 6, while its associated set of zeros, *i.e.* $\lambda(G_0^{-1}(s))$, has a cardinality of 5. The bode diagram associated with G_0 is presented in Fig. 6. We note that this model has been validated against results from the CFD-based NWT model, under the same sea-state conditions considered in this paper, in [8].

C. Observer Design

For the design of the observer, the shaping filter F_Q is chosen such that the frequency response of the mapping $v_m \mapsto v$ is ≈ 1

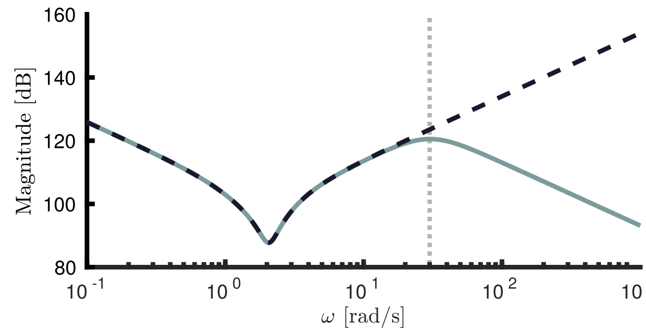

Fig. 7. Magnitude of the frequency response of both the Youla parameter Q (solid line), and the WEC inverse model G_0^{-1} (dashed line). The cut off frequency ω_c is denoted with vertical dotted line.

for the range of frequencies characterising v_m (see also Remark 7). In particular, we write F_Q as

$$F_Q(s) = \frac{1}{\left(\frac{1}{\omega_c}s + 1\right)^{n_Q}}, \tag{12}$$

where $\omega_c \in \mathbb{R}^+$ is the so-called cutoff frequency.

Remark 10: The selection of a suitable F_Q is, naturally, non-unique. The specific filter selected in (12) is mainly motivated by its simplicity and intuitive appeal: One can easily control the trade-off between, for instance, tracking performance (*i.e.* estimation performance) and noise attenuation, by simply changing the values of ω_c and n_Q. Nevertheless, we note that other selections of F_Q are possible.

Remark 11: F_Q in (12) is such as $\lambda(F_Q(s)) \subset \mathbb{C}_{<0}$, *i.e.* it guarantees closed-loop stability (see Remark 8).

In particular, for this case study, we choose ω_c to be $\omega_c = 30$ [rad/s], which provides a suitable trade-off between performance and noise attenuation, as demonstrated in Section IV-D. Following Remark 9, note that n_Q needs to be selected such that $n_Q \geq 1$ in order to have an implementable controller. For this case, we select $n_Q = 2$, so that the Youla parameter $Q = F_Q G_0^{-1}$ effectively attenuates any potential high frequency components, located beyond ω_c. We make this evident in Fig. 7, where the magnitude of the frequency response of both Q (solid line) and G_0^{-1} (dashed line), are explicitly illustrated. Note that Q effectively behaves as the inverse of the WEC model for low frequencies, and attenuates any component beyond ω_c (cutoff frequency denoted with a vertical dotted line). In addition, and for the sake of clarity, the Bode plot of the closed-loop response, *i.e.* the shaping filter F_Q, is explicitly illustrated in Fig. 8.

D. Performance Assessment

To assess the performance of the proposed observer, and to have a direct comparability with state-of-the-art estimators applied within the WEC field reported in [8], we use a set of metrics identical to those proposed in [8], *i.e.:*

Estimation accuracy: In order to quantify the accuracy of the observer, the estimated excitation force $\tilde{f}_{ex}$ is compared to the f_{ex} measured with the fixed body (computed using the CFD-based NWT), in terms of the Normalized Root Mean

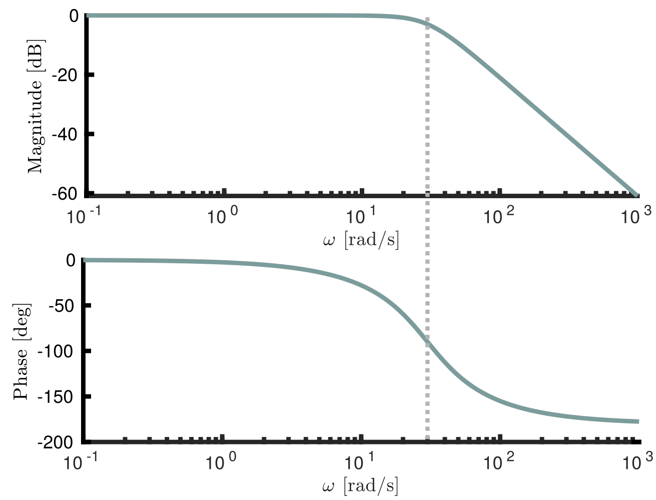

Fig. 8. Bode plot of the shaping filter F_Q. The cut off frequency ω_c is denoted with vertical dotted line.

TABLE I
NUMERICAL APPRAISAL OF ESTIMATION PERFORMANCE FOR THE PROPOSED SIMPLE AND EFFECTIVE OBSERVER. NOTE THAT, IN ADDITION, THE SET OF 11 OBSERVERS, PRESENTED IN [8], IS INCLUDED IN THIS TABLE FOR THE SAKE OF COMPARISON (DATA EXTRACTED FROM [8, TABLE II]). NOTE THAT THE ACRONYMS USED IN [8] ARE ALSO PRESERVED FOR RAPID COMPARABILITY

Estimator	*NRMSA* Clean	Noisy	*Delay* [s]	*C. time*
Simple and effective	0.920	0.908	0	23
[30] $\text{CPWE}_{\text{past}}$	0.857	0.854	-0.03	1
[30] CPWE_{up}	0.872	0.871	-0.06	1
[31] KFRW	0.891	0.865	0.04	25
[32] KFHO	0.907	0.902	0	28
[20] EKFHO	0.903	0.837	0	4×10^3
[31] RHE	0.847	0.828	0.07	30
[33] FAUIE	0.813	0.688	0.16	1×10^3
[34] UIO	0.896	0.856	0.09	117
[35] ULISE	0.903	0.901	0	98
[30] PADE	0.793	0.792	-0.01	29
[21] EKFPS	0.872	0.822	0.03	9×10^3

Square Accuracy (NRMSA):

$$\text{NRMSA}(\tilde{f}_{ex}) = 1 - \sqrt{\frac{(f_{ex} - \tilde{f}_{ex})^2}{f_{ex}^2}}. \quad (13)$$

Estimation accuracy under measurement noise: To assess performance in an even more realistic scenario, the numerical measurement of device velocity v_m is artificially polluted in a post-processing step, by adding a normally distributed, zero mean, white noise, comparable to real measurements from physical sensors. The standard deviation σ of the noise is chosen as in [8], *i.e.* $\sigma = 0.005$. The same NRMSA measure, defined in (13), is used for the estimated signal arising from this noise-contaminated scenario.

Delay: Any delay present in the estimated excitation force can be detrimental for the performance of energy-maximising controllers, as demonstrated in a number of studies (see, for instance, [29]). Motivated by this, any time delay between f_{ex} and $\tilde{f}_{ex}$ is herein reported as in the comparison paper [8], computed via cross-correlation.

Normalised computational time: Time required by the observer to compute an estimation sample, relative to that required by the fastest observer reported in [8] (fastest time: 1×10^{-6} [s]). It is worth noting that the same PC,[7] as that used in [8], is employed to measure the computational demand of the proposed observer, to have a direct comparability with the set of estimators presented in [8].

The results for the simple and effective estimator, proposed in this paper, are explicitly presented in Table I. In addition, and since we use the exact same case study, we include the results for the set of 11 unknown-input estimators compared in [8], allowing for a direct assessment of the performance of our novel observer against a large set of well-established techniques. Note that, to be consistent with [8], we use the same acronyms identifiers for each competing estimator. The reader is referred to [8, Section II] for a detailed description of each technique and associated acronym.

Direct analysis of Table I elucidates a strong conclusion: Though slightly more (computationally) demanding than the fastest observer, the proposed simple and effective estimator outperforms each of the 11 state-of-the-art observers, analysed in [8], in terms of estimation accuracy (even in a noise-polluted scenario, having a very similar performance to the KFHO case), with *zero* estimation delay. Note that the latter feature is indeed fundamental to guarantee satisfactory performance of any energy-maximising controller for WEC applications. To provide a graphical illustration of the performance associated with the proposed observer, Figs. 9 and 10 show time traces associated with velocity and wave excitation force estimation performance, respectively. From Fig. 9, it can be appreciated that the closed-loop design posed in Section IV-C is effectively capable of following the measured velocity, exhibiting an almost indistinguishable behaviour between v_m (dashed line) and v (solid line). Finally, Fig. 10 illustrates the actual (dashed line) and estimated (solid line) wave excitation force obtained with the proposed technique, directly showcasing the accuracy provided by the simple and effective observer.

Remark 12: While v_m and v are virtually indistinguishable (see Figure 9), actual and estimated wave excitation force present slight differences at certain points in time, as can be seen from Fig. 10. It is then clear that this mismatch between f_e and $\tilde{f}_{ex}$ does not arise as a consequence of a 'tracking error,' but rather as the result of potentially unmodelled dynamics: Though the WEC model G_0 has been validated against the corresponding CFD-based NWT, the dynamic response of the high-fidelity environment is *never exactly the same as that of the LTI WEC model* G_0, which inherently leads to differences in the corresponding estimated input. Note that the requirement of having a sufficiently accurate description of the underlying system is standard in the WEC unknown-input estimation literature, being this necessary to 'discriminate' between inner-dynamics, and input behaviour.

[7]Processor Intel CORE i7 with 8 GB of RAM (DDR3).

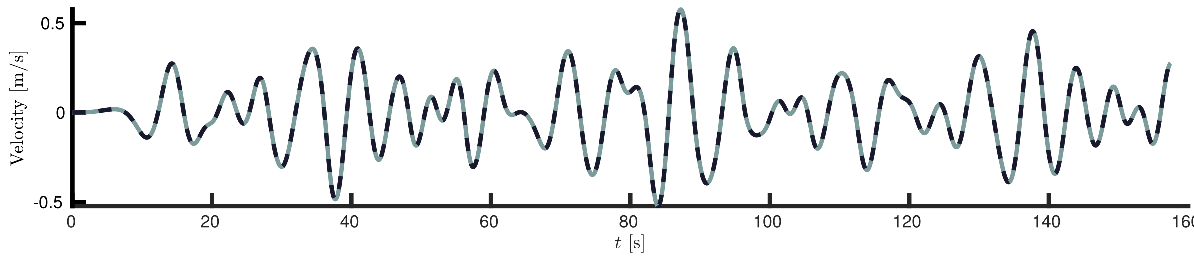

Fig. 9. Time traces of measure velocity v_m (dashed line), computed within the CFD-based NWT, and estimator output velocity v (solid line), arising from the corresponding tracking loop design.

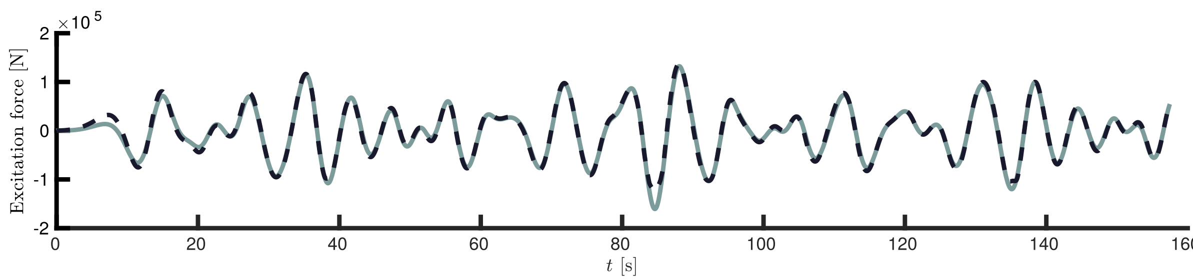

Fig. 10. Time traces of actual wave excitation force f_{ex} (dashed line), computed within the CFD-based NWT, and estimated excitation force $\tilde{f}_{ex}$ (solid line).

Remark 13: In terms of computation time, the proposed simple and effective observer computes in the same order of time magnitude as those based upon (linear) Kalman filtering techniques.

V. Conclusion

In this paper, we introduce a simple and effective excitation force estimator for wave energy applications. The observer is based on the novel redefinition of the unknown-input estimation problem in terms of a traditional reference tracking control loop, permitting the use of standard and well-established control techniques (from either classical or modern control theory viewpoints) to solve the resulting tracking problem. In particular, we use the Youla-Kŭcera parameterisation to address the control design procedure, capable of trading-off, for instance, estimator tracking performance and noise attenuation, in a straightforward fashion.

The proposed estimator presents a number of key advantages with respect to state-of-the-art unknown-input observers for wave energy systems. Firstly, there is no need to assume a specific internal model for the wave excitation force signal, which avoids the requirement of augmenting the WEC model, and a fundamental parametric error in the representation of f_{ex}. Secondly, given the dynamical characteristics of the WEC model, closed-loop estimator stability is always guaranteed, *i.e.* the observer design process is always well-posed, which is not always the case in the literature of unknown-input WEC estimators. Thirdly, the design and synthesis of the observer is based on LTI system theory, which both facilitates real-time implementation in readily available hardware platforms, and allows for intuitive design of the estimator.

Finally, throughout this paper, we show that our estimator is able to provide an accurate approximation of the wave excitation force input, by testing the proposed technique with a high-fidelity simulation platform (CFD-based NWT). In particular, we show that the proposed estimator outperforms the entire set of unknown-input techniques currently reported in the literature, hence not only being *simple*, but also *effective*.

VI. Acknowledgment

The results of this publication reflect only the author.s view and the European Commission is not responsible for any use that may be made of the information it contains.

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Nicolás Faedo was born in Buenos Aires, Argentina, in 1991. He received a degree in automation and control engineering from the National University of Quilmes, Buenos Aires, Argentina, and the Ph.D. degree in electronic engineering from the Centre for Ocean Energy Research Group, Maynooth University, Kildare, Ireland, in 2015 and 2020, respectively, with a focus on optimal control and model reduction for wave energy converters from a system-theoretic perspective.

In 2017, he joined the Centre for Ocean Energy Research Group, Maynooth University. In 2018, he was a Visiting Researcher multiple times with the Control and Power Group, Imperial College London, London, U.K.

He is currently a Postdoctoral with the Department of Mechanical and Aerospace Engineering, Politecnico di Torino, Turin, Italy. His current research interests include nonlinear optimal control theory and data-driven model reduction with special emphasis in applications involving renewable energy systems.

He was the recipient of the 2019 ISOPE Best Student Paper Award and the Exxonmobile Prize, finalist for the 2018 IFAC CAMS Young Author Award, and has been selected as an Outstanding Reviewer of the IEEE TRANSACTIONS ON SUSTAINABLE ENERGY, 2020. He was also the recipient of a Marie Sklodowska-Curie Actions Individual Fellowship (MSCA-IF 2020).

Ulises Bussi was born in Buenos Aires, Argentina, in 1990. He received a degree in automation and control engineering from the National University of Quilmes, Buenos Aires, Argentina, in 2016. He is currently working toward the Ph.D. degree in science and technology within the Department of Science and Technology, National University of Quilmes, with a focus in applied sensor fusion, filtering, and estimation.

Yerai Peña-Sanchez received the Diploma in renewable energies engineering from the University of the Basque Country, Eibar, Basque Country, and the Ph.D. degree in electronic engineering from the Centre for Ocean Energy Research (COER) group, Maynooth University, Kildare, Ireland, in 2016 and 2020, respectively, with a thesis entitled hydrodynamic excitation force estimation and forecasting for wave energy applications.

He is currently a Postdoctoral with Centre for Ocean Energy Research (COER), Maynooth University, Maynooth, Ireland, focused on design and development of test benches for performance evaluation of hydrodynamic optimal controllers of wave energy systems.

Christian Windt received the M.Sc. degree in energy systems from the Hamburg University of Technology, Hamburg, Germany, in 2015, during which he gained experience in the field of ocean wave energy conversion as a Visiting Research Student with Queen's University Belfast, Belfast, U.K., and the University of California at Berkeley, Berkeley, CA, USA, and the Ph.D. degree from Maynooth University (Center for Ocean Energy Research), Kildare, Ireland, in 2020 with a thesis on the "High-fidelity numerical modelling of ocean wave energy systems".

He is currently a Postdoctoral Researcher with the Leichtweiβ-Institute for Hydraulic Engineering and Water Resources, Braunschweig, Germany, where he is coordinating the MarTERA ERA-NET CoFund project NuLIMAS numerical modelling of liquefaction around marine structures.

John V. Ringwood (Senior Member, IEEE) received the Diploma in electrical engineering from the Dublin Institute of Technology, Dublin, Ireland, and the Ph.D. degree in control systems from Strathclyde University, Glasgow, U.K., in 1981 and 1985, respectively.

He was with the School of Electronic Engineering, Dublin City University, Dublin, Ireland. He has held visiting positions with Massey University, Palmerston North, New Zealand, and The University of Auckland, Auckland, New Zealand. He is currently a Professor of electronic engineering and the Director of the Centre for Ocean Energy Research, Maynooth University, Kildare, Ireland. He is the Co-Author of the monograph Hydrodynamic Control of Wave Energy Devices (with Umesh Korde). His research focuses on control systems and its applications, including renewable energy systems (and wave energy in particular), physiology, and exercise physiology.

He was the Co-Recipient of the IEEE 2016 Control Systems Outstanding Paper Award. He is also an Associate Editor for the IEEE TRANSACTIONS ON SUSTAINABLE ENERGY, *Journal of Ocean Engineering and Marine Energy*, and *IET Renewable Power Generation*. He is a Chartered Engineer and a Fellow of Engineers Ireland.