Towards efficient extremum-seeking control of wave energy systems: possibilities and pitfalls

Diego Moens de Hase, Edoardo Pasta, Nicolás Faedo*, and John V. Ringwood

Abstract—**Given its success in other renewable energy domains, such a solar and wind, extremum-seeking control (ESC) would seem to be a promising candidate for the wave energy energy-maximising control domain. There are many advantages to ESC, principal among them the lack of a need for a mathematical system model, with effective wave energy hydrodynamic modelling being particularly challenging. However, a number of fundamental limitations of ESC for wave energy converters (WECs) can be found, with the panchromatic (stochastic) nature of the wave excitation a key issue, resulting in potentially long performance function evaluation times. This, combined with the desire to perform intra-wave control adjustment, creates a natural tension in the solution of the wave energy ESC problem. Motivated by this, we examine, in this paper, the fundamental opportunities and pitfalls of ESC for the wave energy control problem, providing a concise definition of WEC ESC and its characteristics. In particular, we investigate the intrinsic limitations behind ESC for the WEC control problem, and provide a potential set of future directions aiming at alleviating such disadvantages and directly contributing towards the development of efficient model-free control systems for wave energy devices.**

Index Terms—**Wave energy, model-free control, extremum-seeking, optimal control**

I. Introduction

Wave energy converters (WECs) intrinsically require advanced and tailored control system technology to operate at maximum efficiency [1], [2]: Energy extraction from ocean waves has to be optimal, while intrinsically preserving the structural integrity of the device. Successful achievement of this goal secures, in turn, a competitive levelised cost of energy (LCoE), supporting the future commercial viability and widespread installation of WEC devices [1], [3].

The control problem for WECs naturally falls under the umbrella of *optimal control theory* [4], where the control objective is formulated in terms of a specific performance function, *i.e.* energy absorption, which needs to be maximised while respecting, at the same time, the inherent limitations of device and actuator components [2]. Regardless of the specific solution method selected to compute such an optimal energy-maximising control law, the definition of the optimal control problem (OCP) is normally *model-based*, and inherently depends upon the specification of a suitable *control-oriented WEC model*. Not only is the structure of the model relevant for the definition of the associated OCP, but also its associated complexity: Given that the optimal control force needs to be computed in real-time, there is clearly a limit to the computational complexity of the WEC model employed within the control design procedure, while there is also a limit to the (analytical) complexity of mathematical models for which numerical solution methods are effectively well-posed [2], [5], *i.e.* where a globally optimal solution can be found. In other words, there is an inherent trade-off between *accuracy* and *complexity*, which need to be taken on board from the very conception of any WEC controller design procedure.

Hydrodynamic WEC models have their origin in the Navier-Stokes equations, which provide a consistent physical and mathematical description of the motion of a fluid (or fluid-structure interaction) under the most diverse conditions. Nonetheless, the level of complexity of Navier-Stokes is well-beyond any acceptable degree of complexity for control design, with consistent numerical approximation methods requiring a computational expense in the order of *thousands* of seconds *per second* of simulation [6]. This motivates researchers to impose simplifying standing assumptions, aiming at reducing the degree of complexity behind the hydrodynamic description of the WEC motion. These assumptions, in turn, generate a great deal of uncertainty, both within the final structure of the control-oriented model, and the definition of the system parameters involved [7], [8], even under fully linear modelling conditions (*i.e.* linear potential flow theory [9]). The ubiquitous existence of these sources of uncertainty can be problematic for WEC control design: Due to the energy-maximising nature of the design procedure, model-based WEC controllers can be very sensitive to modelling inaccuracies/variations [10], hence potentially presenting a degraded performance under realistic operating conditions.

Altogether, and even though both systematic model reduction techniques [11], [12], and system identification routines [13], [14] have been developed/applied to compute suitable WEC control-oriented models for specific input conditions, the ideal scenario would be to *avoid the use of a model at all*, *i.e.* develop *model-free* control systems for WECs. Within this context, a particularly well known control strategy, which effectively fully prescinds a model for design purposes, is *extremum-seeking control* (ESC).

ESC is essentially a subset of adaptive control, which can be traced back as far as 1922 [15], and denotes a class of methods which are used to locate and track a steady-state optimal performance online (see

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Diego Moens de Hase is with the Departamento de Ingeniería Civil, Universidad de Chile, Chile.
Edoardo Pasta and Nicolás Faedo are with the Marine Offshore Renewable Energy (MOREnergy) Lab., Department of Mechanical and Aerospace Engineering at Politecnico di Torino, Italy.
John V. Ringwood is with the Centre for Ocean Energy Research, Maynooth University, Ireland.
* Corresponding author. E-mail: nicolas.faedo@polito.it.

Section II for further details). This family of controllers shares the fundamental advantage of *not requiring* a model to achieve such an optimal regime, using only output feedback from available measurements to optimise the process operating condition. This, in turn, directly facilitates real-time implementation of ESC in an almost straightforward fashion, hence being especially appealing for realistic scenarios. As such, ESC is tailored for optimisation problems such as the WEC control case, where the system is either complex to model (in a control-oriented sense), or where different sources of uncertainty preclude the determination of an accurate offline prediction model. Note that, within an ideal scenario, ESC strategies would perform as optimally as a controller based upon a very precise (but potentially complex) model, with the significant advantage of not requiring explicit knowledge of the specific equations governing the dynamics of the process.

Given that the original purpose of ESC is focussed on the more general problem of optimisation (and not tracking/regulation), many applications of ESC have found their way into energy systems and, particularly, renewable energy technologies: ESC is increasingly being considered a 'silver bullet' in both solar and wind energy. In both areas the objective is that of maximum power point tracking, *i.e.* extraction of maximum feasible energy from the target system in the absence of accurate modelling of the inherent dynamics. Though this performance objective is shared with the WEC OCP, a number of fundamental differences between wind and solar applications of ESC, and the WEC control case, can be found, which are further discussed in this paper in Section II.

To the best of our knowledge, ESC within the wave energy context was first introduced in [16], followed chronologically by [17]–[20]. The early paper of [16] already spots a number of fundamental limitations of the ESC WEC formulation, specifically in terms of the evaluation of the associated performance function. As discussed in detail in Section II, standard ESC methods assume that the steady-state response of the process in the optimal performance regime is *constant*. This is clearly not the case for WEC control systems, which are driven by a stochastic uncontrollable external force, *i.e.* the wave excitation force.

Motivated by the above discussion, this paper attempts to precisely identify both advantages and disadvantages of ESC control for the wave energy case, making an explicit comparison with well-established and state-of-the-art model-based control technology. In other words, we aim to assess the feasibility of ESC for the WEC application case, highlighting any potential opportunities and pitfalls in their design and operation. By doing so, we offer the reader a timely and thorough discussion on ESC, together with a set of future potential directions, aiming at alleviating the current limitations found within the ESC literature in wave energy. We remark that, ultimately, our principal aim with this study is to directly contribute towards the development of efficient model-free control systems for a general class of wave energy devices.

The remainder of this paper is organised as follows. Section II recalls the fundamentals of standard ESC design, including a brief account of the application of ESC within solar and wind energy technology. Section III introduces the formal definition of the WEC OCP, and provides a brief summary of state-of-the-art model-based WEC controllers. Section IV is devoted to ESC for the WEC control case, highlighting opportunities and pitfalls within the available WEC ESC literature. Finally, based on the analysis of Section IV, Section VI provides a thorough discussion on the current fundamental limitations of ESC for WECs, and offers a set of future directions to alleviate the effects of such disadvantages.

II. Extremum-seeking control

As briefly described in Section I, extremum-seeking control is a subfield of adaptive control, which be traced back to the work of Leblanc [15], in 1922, and denotes a large class of online optimization methods used for steady-state *performance* optimisation of a given process. Such a performance measure is commonly written in terms of a user-defined *objective function*, so that, in essence, ESC methods target the problem of maximising (equivalently minimising) such an objective, via manipulation of one or more control inputs.

Though researchers showed a significant interest in ESC in the '50-'60s (see, for instance [21]–[23]), the exponential growth in ESC development was not until the year 2000, where the seminal game-changing work of Krstić [24] put ESC back in the map. To summarise, [24] provides the very first proof of stability for the classic perturbation-based ESC scheme for a general class of nonlinear systems, by employing tools of averaging and singular perturbation analysis. We refer the interested reader to [23], [25], [26], for further information in the historical trace of ESC, in the context of applied and theoretical control.

Regardless of the specific algorithms available to approach the ESC problem, which might be significantly different, the vast majority of the literature in ESC share two fundamental features: ESC is formulated as a *model-free* control methodology, *i.e.* these methods do not rely upon access to a model for process optimisation, and the maximisation of the objective is purely based upon feedback from online measurements. A general scheme for ESC is shown in Figure 1.

Remark 1: Though outside the scope of this study, note that so-called 'grey-box' ESC algorithms [27] also exist, where a-priori modelling knowledge about the process can be incorporated to, for instance, improve the convergence rate of the associated optimisation. Examples of this type of ESC are found in [28], [29].

It is relevant to note that, in ESC, the feedback control law is expressed in terms of a finite number of parameters, which are determined to achieve optimal performance. In other words, the objective function (which is effectively a measure of the defined process performance) is suitably parameterised. Commonly, static feedback control laws are considered, though dynamic laws can also be included via similar procedures, with an analogous set of assumptions to those listed in Section II-A.

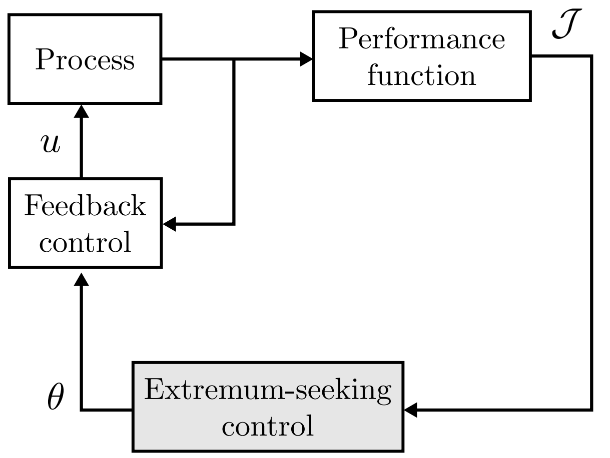

Fig. 1: Generic schematic of an ESC control system.

Due to its inherent model-free nature, ESC can be applied in cases where, for instance, modelling resources are limited to develop a sufficiently representative model for process optimisation, as well as in cases where fundamental difficulties (such as uncertainty) might arise when trying to construct control-oriented models. The latter clearly encompasses the wave energy control case (see the discussion provided in Section I), where constructing precise control-oriented models can be a daunting task, even in the fully linear case.

A. Standard ESC formulation

We consider the formulation of ESC as in [24]. In particular, let the continuous-time, finite-dimensional, single-input single-output, nonlinear system be defined as[1,2]

$$\begin{aligned} \dot{x} &= f(x,u,d), \\ y &= \mathcal{J}(x), \end{aligned} \tag{1}$$

where x is the state vector, u is the control input, d represents an uncontrollable external force, y is the plant output, and the mappings f and $\mathcal{J}$ are considered to be sufficiently smooth. Note that $\mathcal{J}$ denotes the performance (objective) function. Suppose a sufficiently smooth feedback control law

$$u = \alpha(x,\theta), \tag{2}$$

is known, parameterised in terms of a parameter (vector) θ. The closed-loop system, which can be compactly written as

$$\dot{x} = f(x,\alpha(x,\theta),d), \tag{3}$$

has equilibria parameterised in terms of θ. The following standing assumptions are now introduced and recalled from [24].

1) There exists a smooth function l such that $f(x,\alpha(x,\theta),0)=0$ if and only if $x=l(\theta)$.
2) For each θ, the equilibrium $x=l(\theta)$ of system (1) is locally exponentially stable in the Lyapunov sense.
3) There exists an optimal parameter $\theta^\star$ such that $\mathcal{J}'(l(\theta^\star))=0$ and $\mathcal{J}''(l(\theta^\star))<0$.

The first assumption listed above explicitly specifies that the equilibria of (1) are effectively parameterised in terms of θ. The second assumption implies that the control law (2) is robust with respect to its own parameter θ, in the sense that it exponentially stabilises any of the equlibria that θ may produce. The third and last standing assumption is central in the ESC method: The output equilibrium map $y=\mathcal{J}(l(\theta))$ has a *maximum* at $\theta=\theta^\star$. So, in short, the objective of standard ESC is to develop an output feedback mechanism which maximises the steady-state value of y, without requiring explicit knowledge of f, $\theta^\star$ or l. We now introduce a set of important remarks.

Remark 2: Note that the last assumption, *i.e.* Assumption 3 above, states that the performance objective has to be a convex function of the parameter θ. While this can be easily guaranteed in the WEC case for relatively simple controller parameterisations, it is not, in general, trivial to ensure. This is further discussed in Section VI.

Remark 3: The evaluation of the objective function in standard ESC is considered to be time-invariant. In other words, the steady-state output of the plant is assumed to be *constant* under optimal operating conditions. While this is effectively consistent with a number of applications, such as the case of solar photovoltaic (PV) technology (see Section II-B), it is a fundamental limitation for the wave energy case, where the response of the system in an optimal regime is effectively time-varying. Nonetheless, a number of techniques exist to circumvent this issue, which we further discuss in detail in Section IV.

Remark 4: ESC problems are virtually always formulated in terms of an unconstrained optimisation form, *i.e.* without state and input constraints. As a matter of fact, note that neither state constraints, nor input limitations on the performance function, can be generally imposed, since the state-vector of the associated system is often unknown, and the full influence of the process on the performance objective is not necessarily available *a priori*.

B. The role of ESC in wind and solar energy

As discussed in Sections I and II, though ESC is one of the oldest feedback methods, its fundamental purpose is not regulation, but optimisation. This is exactly why many applications of ESC have found their way into energy systems and, particularly, renewable energy technologies. In particular, ESC is increasingly being considered a 'silver bullet' in both solar and wind energy. In these areas, the objective is that of maximum power point tracking (MPPT), *i.e.* extraction of maximum feasible energy from the target system in the absence of accurate modelling of the inherent dynamics. Note that this objective is consistent with that of the WEC control problem, though a number of fundamental differences arise, as further detailed in the following paragraphs.

For wind energy conversion, MPPT is considered to tune the set-point for the turbine speed which

[1]The dependence on t is dropped when clear from the context.

[2]Note that the definition of 'system' provided herein is 'loosely' formulated, aiming to simplify the exposition. The interested reader is referred to, for instance, [30], for a formal treatment of the definition of a dynamical system.

optimises tip-speed ratio. For solar PV arrays, MPPT is commonly considered to tune the duty cycles of the DC/DC converters employed in the system, balancing the current-voltage relationship to maximise its product. The reader is referred to [31] and [32], [33] for further detail on the specifics behind both solar and wind energy conversion via ESC, respectively, including an account of the current state-of-the-art.

The inherent synergy between ESC and both solar and wind energy control problems relies upon the fact that the formulation of both optimal performance objectives is, in general, consistent with the assumptions posed in Section II-A, particularly in the sense of having a constant steady-state response in the optimal regime (see Remark 3). As further discussed in Section IV, this is one of the fundamental differences with the case of WECs, where the optimal steady-state output response is, effectively, time-varying due to the stochastic nature of the driving wave resource.

III. The wave energy control problem

As informally discussed in Section I, the wave energy control design entails an *energy-maximisation* criterion, where the objective is to maximise the absorbed energy from ocean waves over a time interval $\mathcal{T}$, which can be generally cast as an optimal control problem (OCP), with an objective (performance) function[3]

$$\frac{1}{T}\int_{\mathcal{T}} f_{\text{PTO}}(\tau)\dot{z}(\tau)d\tau = \frac{1}{T}\int_{\mathcal{T}} P(\tau)d\tau, \tag{4}$$

where P is the useful instantaneous power, and f_{PTO} and z denote the control (PTO) force (to be optimally designed), and the displacement of the WEC, respectively. Given that the unconstrained energy-maximising optimal control law often implies unrealistic device motion and excessively high PTO (control) forces (see [2], [35]), constraints on both the displacement and velocity of the WEC, z and $\dot{z}$, and the exerted control force f_{PTO}, have to be considered within the optimal control design, to guarantee the integrity of the device. This set of constraints can be compactly written as

$$\mathscr{C} : \Big\{ |z| \leq Z_{\max},\, |\dot{z}| \leq \dot{Z}_{\max},\, |f_{\text{PTO}}| \leq F_{\max}, \tag{5}$$

with $t \in \mathcal{T}$, and where $\Big\{ Z_{\max}, \dot{Z}_{\max}, F_{\max} \Big\} \subset \mathbb{R}^+$.

If we assume that the WEC dynamical model is given by a set of differential equations as in (1), with x the state-vector characterising the WEC behaviour, f_{PTO} the control (PTO) force in place of u, and f_{e} the wave excitation force as the uncontrollable external signal d, the constrained energy-maximising OCP can be written as

$$\begin{aligned} f_{\text{PTO}}^{\text{opt}} = \arg\max_{f_{\text{PTO}}} \; & \frac{1}{T}\int_{\mathcal{T}} f_{\text{PTO}}(\tau)\dot{z}(\tau)d\tau, \\ & \text{s.t.:} \\ & \text{WEC dynamics (1)}, \\ & \text{Constraint set (5)}. \end{aligned} \tag{6}$$

[3]Note that we consider the WEC control problem for a single degree-of-freedom (DoF) device, for simplicity of exposition, though similar arguments can be made for multi-DoF and array cases (see, for instance, [12], [34]).

Remark 5: In order to solve the OCP defined in (6), full knowledge of the wave excitation force is required for the time-interval $\mathcal{T}$, *i.e.* solving (6) implicitly requires *instantaneous* and *future* values of wave excitation to *achieve true optimality*. Such estimates of f_{e} are commonly computed via estimation and forecasting strategies, respectively. The reader is referred to [36], [37] for further detail on input-unknown estimation and forecasting techniques applied within the WEC field.

A. A brief overview of solution methods

Using a rather 'generic' categorisation, model-based wave energy control systems can be divided into two different families of controllers (see [38]): optimal-control-based, and impedance-matching-based strategies. In the case of the former family of controllers, the energy-maximising control objective is fully treated as an OCP (*i.e.* as in (6)), where both input and state variables are often discretised using different criteria, aiming to map the infinite-dimensional problem into a computationally (numerically) tractable nonlinear program, *i.e.* (6) is solved via direct optimal control methods (see [4]). In contrast, impedance-matching-based controllers do not rely on numerical routines, but are mostly based on the fundamental principle behind maximum power transfer in electric circuits: the impedance-matching principle [35], [39].

To provide a very brief account of optimal-control-based controllers, which directly attempt to maximise time-averaged power extraction from ocean waves, we note that most of the available strategies are inspired by the underlying theory behind model predictive control (MPC), with [40] one of the pioneering study in the field. More advanced direct optimal controllers are now available in the WEC literature, entailing tailored parameterisation of system and control variables, with enhanced computational efficiency. Among these strategies, one can find controllers based upon spectral methods [41], pseudospectral optimal control [42], differential flatness [43], and moment-based theory [44]–[46]. A detailed account of the state-of-the-art of optimal-control-based controllers can be found in [2], [5].

Remark 6: An immediate advantage of optimal-control-based controllers is that optimal constraint handling becomes straightforward, *i.e.* one can translate physical limits on device motion and PTO force in terms of the set of state and input constraints in (5). A clear disadvantage is that the real-time capabilities of these strategies depend on a number of factors, primarily the discretisation technique utilised to parameterise the state and input variables, and the hardware available for its implementation.

On the other hand, impedance-matching-based controllers attempt to provide a (physically implementable) realisation of the anti-causal impedance-matching condition for maximum power transfer. Such an approximation is commonly obtained in terms of linear time-invariant (LTI) structures, aiming at prioritising simplicity of implementation over 'true' optimality [35]. Note that, in general, this family of strategies does not observe any state nor input constraints, so that a different (additional) mechanism is often required in the loop to guarantee safety limitations. Examples of impedance-matching-based controllers can be found in [35], [38].

B. Robust optimal control to approach inaccurate modelling

As discussed in Section I, one of the key motivations to develop model-free controllers recognises the inherent complexity behind precise hydrodynamic modelling. Even in the fully linear case, *i.e.* under potential flow theory, a significant number of modelling inaccuracies arise almost naturally, including structured (*e.g.* parametric [7]) and unstructured (*e.g.* radiation subsystem mismatch [47]) uncertainty. This motivated a handful of researchers to approach the OCP (6) in a *robust sense*.

Examples of energy-maximising robust control approaches, *i.e.* controllers which can solve the OCP (6) robustly, include [48], [49]. Note that, as per design specifications, robust control approaches are, in general, conservative by definition. Since specifying exact bounds on the hydrodynamic modelling uncertainty is far from trivial (though some encouraging results have been recently presented [50]), robust design is commonly performed using 'sufficiently large' uncertainty sets[4], such that the designer is somewhat sure that every possible source of inaccuracy is effectively considered within the control design procedure. This, almost inevitably, results in controllers which optimise for a worst-case scenario, hence inherently having a conservative performance.

IV. ESC in wave energy: Advantages and pitfalls

To the best of our knowledge, ESC found its way to the wave energy application for the first time in [16], where a continuous-time perturbation-based ESC is presented. Subsequent key studies, applying different variations of ESC, are (in chronological order) [17], [18], [19], and [20]. These strategies are described in the following paragraphs, along with the advantages and fundamental issues characteristic of ESC methods for the wave energy application. Before getting into these details, it is important to note that the totality of the studies mentioned above consider a PTO control force parameterised in terms of either device velocity, or both velocity and displacement of the device, *i.e.*

$$f_{\mathrm{PTO}}(z,\theta_{\mathrm{P}}) = \theta_{\mathrm{P}}\dot{z}, \quad \text{or} \quad f_{\mathrm{PTO}}(z,\theta_{\mathrm{PI}}) = \theta_{\mathrm{PI}}\begin{bmatrix}\dot{z}\\ z\end{bmatrix}, \qquad (7)$$

with $\theta_{\mathrm{P}} \in \mathbb{R}$ and $\theta_{\mathrm{PI}}^{\intercal} \in \mathbb{R}^2$. Note that, if we consider velocity as the 'nominal' output of the WEC system (1), the first equation in (7) represents a standard proportional (P) controller, while the second structure is of a proportional-integral (PI) control nature.

[4]Note that this is a vague definition for the purpose of simplicity of exposition. The interested reader is referred to, for instance, [51] for a formal treatment of robust optimisation.

A. ESC algorithms considered for WECs

With respect to the specific solution methods adopted in the WEC literature to address the ESC control loop (presented in Figure 1), the pioneering study of [16] considers a continuous-time perturbation-based ESC, in spirit of that described in the seminal work [24]. The chronologically subsequent study [17] follows a similar trail, though the perturbation-based algorithm of [17] is developed in an exclusively discrete-time setting. The same approach has been followed later in the development of one of the solutions proposed in [20], where the classical continuous-time perturbation-based ESC has been adopted to successfully find the two optimal parameters of a reactive control law $f_{\mathrm{PTO}}(z,\theta_{\mathrm{PI}})$.

A different approach is taken in [18], [19], where the discrete-time ESC solution algorithm is directly tackled in terms of numerical optimisation techniques, via so-called 'flower pollination' (see [52]). The proposed technique is applied on a reactive PI control law formulation, and it is compared with a standard perturbation-based ESC, showing how the flower pollination algorithm can apparently result in an improvement in convergence rate for their case study. This meta-heuristic optimization algorithm is based on the concept of 'flowers' composed by 'pollens', similarly to the individuals inside the generations of the more known genetic algorithm. The performance of each of these pollens in terms of objective function is evaluated in series, by changing the pair of parameters related to the control law every 20 [s]. Once all the members of the flower have been evaluated, a new population is generated by means of both *cross-pollination* and *self-pollination* processes. We do note, although, that there seems to be an underlying conflict between two main assumptions/directives taken in the study: while the objective function is evaluated using steady-state motion values (ostensibly considering full knowledge of the Fourier series associated with the excitation input), the control parameters are varied periodically in a discontinuous fashion, directly generating a 'new' transient behaviour, which naturally conflicts with the steady-state evaluation of the associated cost, unless the integration time is assumed to be sufficiently long.

Finally, [20] presents a 'suite' of ESC algorithms to tackle the wave energy control case, and this is the most complete study (and best comparison paper) available in the literature to date. In particular, apart from the aforementioned classic perturbation-based ESC [24], the authors explored the use of continuous-time solutions like sliding mode ESC [53], [54] and self-driving ESC [55], and the application of discrete-time algorithms as relay ESC [56] and least-squares ESC [57]. We do note that the different algorithms discussed in this section have been applied in the optimization of the parameters of both proportional and proportional-integral control formulations described in (7). The results presented in [20] show that each of the proposed ESC approaches, apart from the self-driving ESC, is able to converge to the optimal parameters corresponding with both control structures. According to [20], the best results in terms of convergence and oscillations in the final steady-state conditions are obtained by the sliding mode ESC and the perturbation-based ESC. In contrast, the two discrete algorithms (relay and least-squares) are affected by larger (convergence) oscillations. Nonetheless, both of these discrete techniques are able to converge robustly to variations in sea conditions, having the advantage of requiring a small set of parameters to be tuned in the design phase, hence simplifying final implementation.

B. Opportunities for ESC in wave energy

Though, after the discussion provided in Section I, the advantages/opportunities of ESC might seem to be almost obvious to the reader at this point, we dedicate this brief section to highlight the main advantages of ESC for the WEC case. These are, ultimately, the main motivations behind an attempt to move towards efficient ESC technology for wave energy systems.

The first (and core) advantage of ESC for WECs is the lack of need for a dynamical model to compute the control solution. This is a feature that all the studies listed in Section IV share, regardless of the specific ESC solution, and is indeed the most appealing aspect of this family of techniques; it is well-known that precise hydrodynamic modelling can be a daunting task, even in the simplest linear (small oscillation) setting. Though model reduction [11] and system identification [13], [14] methods have been successfully developed/applied in the wave energy field, to compute control-oriented dynamical structures, these models are often representative under specific input wave conditions, hence require adjustment as a function of any change in sea-state.

The second advantage of ESC is its simplicity of implementation. In contrast to many optimal-control-based strategies attempting to solve the OCP (6) via direct optimal control techniques, real-time control is achievable in almost any off-the-shelf hardware platform with ESC, with a relatively modest effort, both in terms of design and calibration. This is a major advantage with respect to the family of optimal-control-based controllers described in Section III-B, which often struggle to achieve real-time performance even under linear modelling settings (see [2]).

C. Pitfalls of ESC in the wave energy field

Within this section, we summarise the main challenges for ESC in the wave energy control case from a critical point of view, documenting, for each case, which strategies/techniques have been considered among the ESC WEC literature to fully/partially circumvent some of these limitations.

1) On the definition of the control objective: The first fundamental issue in applying ESC control for wave energy systems was already documented in the early work of Hals et. al. [16]: The steady-state response output mapping of the WEC in optimal regime is inherently time-varying. In other words, the performance of the system, evaluated in the spirit of (4), is inherently related to a time-varying behavior, which limits the application of ESC theory, as recalled in Section II-A. Before getting into the details on how the wave energy literature addresses/circumvents this fundamental issue, we illustrate the nature of this problem in terms of the following simple approach, so that the reader can appreciate the roots of this issue. Suppose that our WEC system, controlled via a PI structure (see equation (7)), is subject to a realistic (*i.e.* irregular) sea-state, characterised by a given peak period T_p. Since evaluating (4) for future times in a model-free setting is out of the question, one might be simply tempted to 'look back' in time for a certain number of wave periods to evaluate the (raw) objective function in (4), and hope this effectively reduces the variability of the performance by extracting the fundamental (stochastic) characteristics of such a mapping. This practice will, in general, render a non-convex function of the parameters θ_{PI}, unless we look back within the corresponding integration window for a *sufficiently large period of time*. This is illustrated in Figure 2, where the evaluation of the objective function (4) for a PI controller acting on a full-scale spherical point absorber WEC is presented, for 10, 100, and 1000 peak periods, respectively.

Hals et. al. [16] attempt to circumvent this issue by proposing a measure that represents the absorbed useful power, which stays relatively constant, even in spite of variations of the incident wave power level. In particular, [16] suggests low-pass filtering of the useful instantaneous power P, to then compute its ratio with a squared low-pass filtered version of the wave excitation force. With such a strategy, ≈ 100 periods are required in order to achieve acceptable performance, which is already a significant improvement with respect to the raw evaluation presented in Figure 2. Nonetheless, note that [16] uses knowledge of the wave excitation force to adapt the control objective, hence an unknown input observer would be required to implement this technique in a realistic scenario.

For the case of the perturbation-based ESC designed in [17], the authors do not directly address this issue, and essentially integrate the objective function (4) for a sufficiently long period time. In particular, for their specific device (which is not full-scale), the evaluation of the performance function is performed using an average of ≈ 200 peak wave periods.

The numerical-optimisation-based ESC presented in [18], [19] assumes full knowledge of the wave excitation force signal, *i.e.* past, instantaneous, and future values are assumed to be available, directly limiting the application of this technique in realistic scenarios. In particular, it is assumed that the Fourier expansion of the wave excitation signal is known with an arbitrarily degree of accuracy. In other words, by assuming full knowledge of f_e, [18], [19] turn the evaluation of (4) into a deterministic problem, 'removing' any stochastic behaviour in the nature of the wave input. However, we note that the study proposed by the authors is, to the best of our knowledge, the first attempt to incorporate an additional (second) objective into the overall objective function, by also considering the minimisation of the power peak-to-average ratio.

Finally, a more sophisticated approach to circumvent the issue related to the evaluation of (4) is presented in [20], where a combination of two main techniques

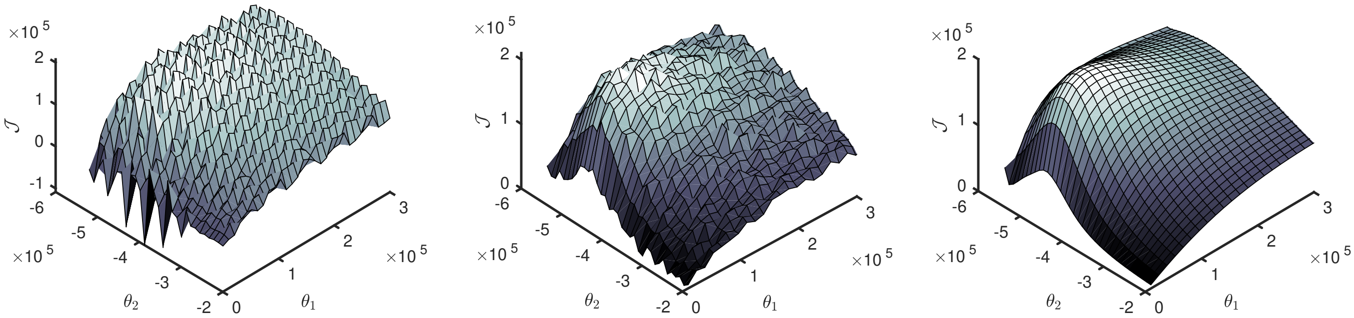

Fig. 2: Raw evaluation of the performance objective (4) ‘looking back’ 10 (left), 100 (center), and 1000 (right), peak wave periods, respectively.

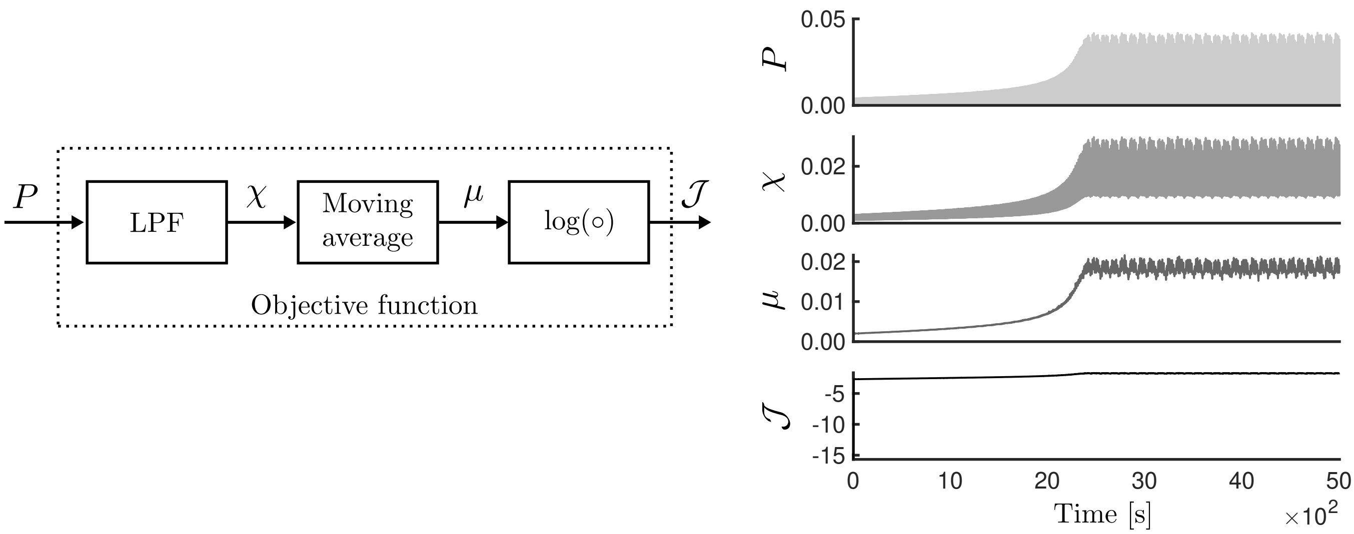

Fig. 3: Performance evaluation mapping for the WEC ESC study [20], including a low pass filter (LPF), a moving average operator, and a logaritmic function. Figure adpated from [20].

is utilised. First, in the spirit of the ESC theoretical design presented in [58] for steady-state performance optimization of general nonlinear plants with arbitrary periodic steady-state outputs, a moving-average filter is incorporated into the ESC loop, in addition to a standard low-pass filter (as also considered in [16]). Nonetheless, given that the nature of the sea-state is purely stochastic, and the wave excitation input can only be considered to be a T-periodic signal in practice only for a sufficiently large period T, the authors of [20] incorporate an additional element into the loop, following the steps performed in [59], [60], namely a logarithmic evaluation of the performance objective. This drastically reduces the sensitivity of the controller response, by rendering the steady-state performance objective almost constant *even if only a few past peak wave periods* are considered in the evaluation of (4). In particular, [20] uses only two peak wave periods to obtain satisfactory performance, which is a major improvement from the ‘raw’ evaluation approach presented in Figure 2. The interaction between components in the ESC loop of [20] is presented in Figure 3, along with a representation of the performance evaluation at different stages.

D. On the parameterisation of the control law

The origin of the second fundamental issue in ESC control lies in the inherent non-causality of the energy-maximising control solution for WECs, *i.e.* the optimal feedback controller is *anti-causal* (see also [61]), hence the parametric control law (7) computed via extremum-seeking, is inherently suboptimal with respect to solving the OCP (6) with full knowledge of instantaneous and future values of the external uncontrollable input (*i.e.* the wave excitation force).

To further elaborate on this, consider for a moment the case of a WEC described by linear potential flow theory, *i.e.* a WEC system represented by an LTI operator G. In a feedback setting, and without considering state/input constraints, it is relatively straightforward to show (see, for instance, [35], [61], [62]) that there exists an optimal input-output response T^{opt}, which can be fully characterised in terms of G. The mapping T^{opt} is what is called an *ideal filter* (see [63]), and hence is non-causal and zero-phase. With a PI controller (as in (7)), which has only two design degrees-of-freedom given by the vector $\theta_{\text{PI}}^{\intercal} \in \mathbb{R}^2$, it is only possible to interpolate the optimal condition $T^{\text{opt}}(\omega)$ *at a single frequency point*, *e.g.* at the frequency characterising the peak period of the wave excitation input. This is schematically illustrated in the Bode plot of Figure 4, where the optimal input-output mapping T^{opt} for a full-scale CorPower-like device (see [44]) is presented (solid-black), along with the frequency response of the input-output mapping obtained with a PI controller (dashed-grey), interpolating the corresponding optimal condition at a selected peak period of $T_p \approx 6.5$ [s]. Note that the PI solution is inherently narrow-banded, and presents acceptable performance in the neighborhood of the selected interpolation point only.

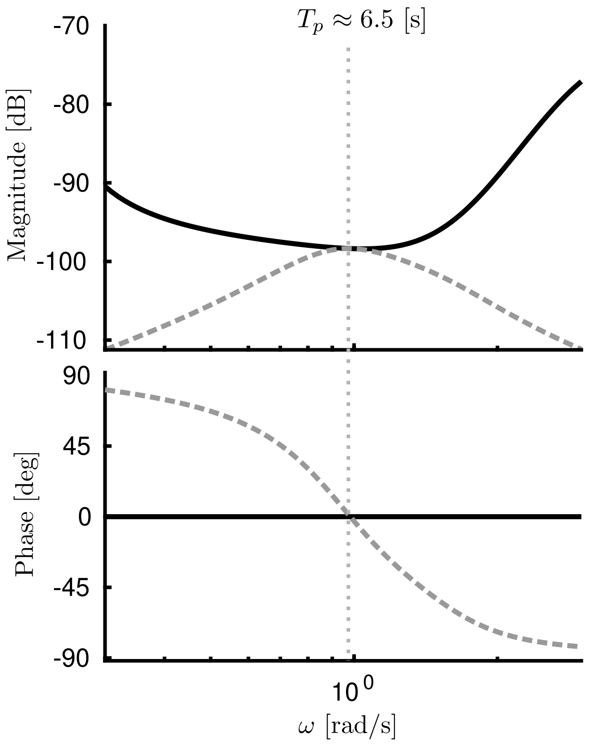

Fig. 4: Optimal input-output frequency response $T^{\text{opt}}(\omega)$ (solid-black) for a CorPower-like device, along with the input-output response arising from a PI controller (dashed-grey), tuned to interpolate $T^{\text{opt}}(\omega)$ at a specific frequency of interest.

In other words, the ESC strategies found in the WEC literature are, at their essence, model-free adaptive PI controllers, which converge to parameter values which interpolate the optimal energy-maximising condition at a single point in frequency. While this might be sufficient to have satisfactory performance in narrow-banded seas, optimal-control-based controllers can operate in fully broad-banded seas, being able to optimally extract energy considering the complete power spectrum of the wave elevation. Furthermore, even within the family of impedance-matching-based controllers, described in Section III-B, there exist model-based solutions that can approximate the maximum-power transfer condition in a broad-banded sense [35].

E. On the handling of state and input limitations

Given the inherent model-free nature of ESC, handling of constraints is not straightforward, as it is in model-based direct optimal controllers (see Remark 6). To the best of our knowledge, to date, none of the available WEC ESC algorithms consider either state or input constraints. Note that this is not a minor issue, since WECs under optimal control conditions tend to present large motion (see [2], [64], induced by the controller itself in the pursuit of maximum energy extraction. We discuss potential opportunities and solutions to this fundamental issue in Section VI.

V. Review of ES algorithms applied in WEC control

As mentioned in Section IV-A, different ES algorithms have been applied in the wave energy field.

VI. Discussion and future directions

The wave energy control problem falls under the umbrella of optimal control theory, where the performance objective departs from traditional tracking/regulation, and directly relates to maximum energy absorption. Solving the associated OCP requires a number of features that can render the problem infeasible for real-time application. In particular, the computation of *precise* control-oriented WEC models, capable of displaying high degrees of accuracy without exceeding a level of 'allowed' computational/analytical complexity (for which an optimal control input can be effectively computed in real-time), is far from being trivial. Though a family of robust control approaches, which are capable of optimising energy-absorption under the inherent presence of hydrodynamic modelling uncertainty, have been recently introduced in the literature of WEC control, quantifying exact bounds for these modelling inaccuracies is not straightforward, and hence these robust solutions are inevitably designed to be conservative.

In the light of this, and even though novel modelling and optimal control advances are consistently being presented within the wider marine and control literature, there is certainly an appetite for having *model-free* control solutions available to WEC community, which avoid the potential minefield of going through the complex world of hydrodynamic modelling for energy-maximising control design, synthesis, and implementation.

Within this context, ESC arises as a potential solution, having the following two fundamental and important advantages: control design can be performed without the requirement of having a model; and implementation is straightforward, hence being especially appealing for real-time implementation.

Despite these appealing advantages, ESC comes with its own pitfalls, some of which have been addressed (at least partially) in the WEC control literature. The first issue relates to the time-varying nature of the WEC response under optimal control conditions, which defies the underlying assumptions of standard ESC. Nonetheless, feasible solutions are arising to circumvent this issue, with [20] offering the most sophisticated strategy to date. The second issue relates to the relatively simplistic controller parameterisation utilised within the WEC application so far, which only includes P and PI control structures, in contrast to their model-based counterparts, which can include a wide variety of complex parameterisations when solving the associated OCP via direct optimal control (see Section III). This can be potentially resolved, retaining the model-free characteristic of ESC, by considering dynamic control laws instead of simply static output feedback. The downside of following this path is that there is, in general, no guarantee that a specific dynamic controller renders 1) a stable closed-loop response for every possible θ, and 2) a convex objective function of its parameters, which are both standing assumptions for ESC. In other words, one has to put specific conditions in the structure of the dynamic controller such that 1) the closed-loop is internally stable, and 2) the performance objective is effectively convex. While 1) could be tackled by, for instance, ensuring passivity of the dynamic controller, guaranteeing 2) is not straightforward in general, and needs to be addressed for each specific potential controller structure. Finally, the last fundamental issue relates to constraint handling, which appears to be completely ignored so far in the literature of ESC WEC control. We note that a suboptimal, yet simple to implement, approach would be to add soft constraints in the objective function (4) to penalise 1) control energy, *i.e.* a term proportional to f_{PTO}^2, and 2) motion energy, *i.e.* a term proportional to z^2 and/or $\dot{z}^2$. This, however, would only guarantee constraint handling in an 'average sense', since the weighting terms used to add these soft constraints are virtually always considered to be constant in time, and hence are tuned offline. Further methods worth exploring for constraint handling in ESC are those reported in [65], where a set of (convex inequality) constraints on the WEC system parameters are handled via explicitly augmenting the performance function with penalty functions. Alternatively, a projection-based method is proposed in [66], which represents another promising option for the WEC case.

In summary, we conclude that the current state-of-the-art ESC design within the wave energy case might not yet be a 'silver bullet', as in the case of solar PV and wind energy extraction. Even though it presents significant advantages with respect to model-based controllers, the current parameterisations are probably too simplistic to successfully operate in broad-banded seas at a comparable level of performance, and constraint handling without availability of a dynamical entity to predict the WEC motion is not straightforward, which compromises the optimality of ESC under safety limitations. Nonetheless, more complex parameterisations, with guarantees of closed-loop stability and a convex performance objective, can lead to improved results in terms of energy absorption, while the addition of soft constraints can partially solve the problem of constraint handling. Proposing solutions to these two issues can potentially fully turn the scales in favor of ESC, effectively transforming this model-free control technique into the 'holy grail' of wave energy control systems.

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