On the feasibility of energy-maximising controllers for an Argentinian wave energy system

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***Abstract*—Wave energy has attracted significant attention during the past decades worldwide, due to the significant amount of energy available in ocean waves. However, to date, wave energy systems have not reached commercial viability. As such, appropriate control system technology is considered a key driver to achieve commercialisation of wave energy converters (WECs). As of today, Argentina has not taken an active part in the discussion and development of suitable control techniques to harvest this energy resource, despite the geographic opportunities and the vast experience that Argentina has acquired and demonstrated in other non-carbon-based energy sources. In the light of this, we provide, in this paper, a feasibility assessment of three state-of-the-art control methodologies for wave energy harvesting systems, for a WEC prototype developed in Argentina. To this end, representative operating conditions, characteristic of the wave resource in the coastal area of Argentina, are considered. Finally, the performance of each controller is discussed, together with a set of conclusions and potential directions on the fundamental design of the WEC prototype from a control-oriented perspective.**

***Index Terms*—Wave energy converters; Energy-maximising control; Optimal control; Renewable energy**

I. Introduction

Ocean waves comprise a vast source of energy that can be captured by means of suitable wave energy converters (WECs), having the potential to play a significant role in the pathway to decarbonisation. As such, this vastly untapped resource has attracted significant attention during the past decades, both from the academic, and industrial communities. Nowadays, different WEC prototypes are constantly arising worldwide, together with business models around WEC technology, resource assessment, among others, including both public and private participation [1]. However, wave energy has not yet reached commercial viability, which can be largely attributed to the fact that harnessing the irregular reciprocating motion of the sea is not as straightforward as, for example, extracting energy from the wind (see, for instance, [2]). In such scenario, appropriate *control system technology* is considered to be a fundamental building block towards economic viability of wave energy.

Despite the global efforts to efficiently harvest energy from ocean waves, Argentina has not taken an active part in the discussion associated to this resource. This somewhat strikes as surprising, given the vast experience and technology that Argentina has produced in other non-carbon-based technologies as, for example, wind or solar, particularly from a control and dynamical systems perspectives. We do highlight that, in spite of this apparent absence of area-specific research, wave energy has been declared a strategic resource in the national research program *Argentina Innovadora 2020* [3], [4]. As a matter of fact, the energy resource in Argentina generally exhibits a low seasonality which, in spite of the relative low maximum available power, is highly convenient for efficient performance of wave energy systems [5].

Nonetheless, exceptions to this notable lack of wave-energy-specific research do exist: The studies in [6] and [7] present a WEC prototype specifically designed in Argentina. This WEC system, originally inspired by the well-known Wavestar device [8], is a hemispherical point-absorber. Such a device arises as an initiative from the *Grupo de Interés en Energías del Mar Argentino* (GEMA), a research group within *Universidad Tecnológica Nacional* (UTN). The WEC system, presented in [6], [7], can be largely considered, to date, the most substantial project regarding WEC systems in Argentina. A further study, strongly linked to this project, can be found in [9].

Appropriate control technology, capable of maximising energy extraction from ocean waves, *i.e. energy-maximising control*, plays a fundamental role in the roadmap towards commercialisation of wave energy systems. Such controllers, which face 'non-standard' design challenges, such as non-causality, marginal stability of the closed-loop in energy-maximising control conditions, constraint handling, among others (see, for instance, [10]), can be essentially separated into two relatively broad categories [11]: optimisation-based, and non-optimisation-based. Optimisation-based controllers, such as those described in [12]–[14], treat the energy-maximising control objective as an optimal control problem (OCP), which is later commonly solved via direct optimal control techniques. On the contrary, non-optimisation-based controllers, such as those described in [15], do not rely on online optimization routines, and are generally based on the fundamental principle behind maximum power transfer in electric circuits: the impedance-matching theorem [16].

Motivated by the potential of the Argentinian wave energy resource, and the pre-existence of the national WEC prototype [6], [7], this paper analyses the feasibility of state-of-the-art WEC control systems for such technology. To this end, three different control strategies for WECs are applied to the WEC device proposed in [6], [7], namely the so-called 'reactive control' methodology [17], the linear time-invariant (LTI) non-optimisation-based controller described in [18], [19], and the optimisation-based control approach formally introduced in [14], [20]. Each of these controllers is evaluated according to the operating conditions characteristic of the Argentinian resource, so as to help in assessing the local feasibility of the technology developed in [6], [7] under controlled conditions.

The remainder of this paper is organised as follows. Section I-A introduces the notation utilised throughout our study. Fundamental concepts behind WEC dynamics, and the corresponding control problem, are introduced in Section II. A description of the considered control approaches is offered in Section III, while the application of each of these controllers to the Argentinian WEC prototype [6], [7] is explicitly addressed in IV. Finally, Section V encompasses the main conclusions from our study.

A. Notation

$\mathbb{R}^+$ denotes the set of non-negative real numbers. $\mathbb{C}^0$ denotes the set of pure-imaginary complex numbers, and $\mathbb{C}_{<0}$ denotes the set of complex numbers with negative real part. $\mathbb{I}_n$ denotes the identity element in the space $\mathbb{C}^{n\times n}$. The spectrum of a matrix $A \in \mathbb{R}^{n\times n}$, *i.e.* the set of its eigenvalues, is denoted as $\lambda(A)$. The Kronecker product operator is denoted with $\otimes$, while the Kronecker sum operator (see [21]) is indicated with $\hat{\otimes}$. The convolution between two functions f and g, with $\{f, g\} \subset L^2(\mathbb{R})$, over the set $\mathbb{R}$, *i.e.* $\int_{\mathbb{R}} f(\tau)g(t-\tau)d\tau$ is denoted as $f * g$. If $z \in \mathbb{C}$, then we use $z^\star$ to denote the complex-conjugate of z. Finally, the Laplace transform of a function f (provided it exists), is denoted as $F(s)$, with $s \in \mathbb{C}$.

II. WEC DYNAMICS AND CONTROL OBJECTIVE

A. WEC Model

This section provides a brief summary of the fundamentals behind WEC modelling. The interested reader is referred to [22] for an exhaustive description of WEC dynamics. The vast majority of control-oriented WEC models (see [10], [23]) are based on a common theoretical framework, known as *linear potential flow theory* (see [22]). Under such assumptions, the equation of motion for a single degree-of-freedom WEC can be expressed as[1]:

$$(M + m_\infty)\ddot{x}_p + \dot{x}_p * h_r + s_h x_p = f_{ex} - u, \tag{1}$$

with x_p, $\dot{x}_p$, and $\ddot{x}_p$ the WEC displacement, velocity, and acceleration, respectively. The notation $M \in \mathbb{R}^+$ is used for the mass of the oscillating body, while m_∞ is the so-called added mass at infinite frequency[2]. The mapping $h_r \in L^2(\mathbb{R})$ is the so-called radiation force impulse response, which accounts for the fluid memory effects acting on the device, while $s_h \in \mathbb{R}^+$ is the hydrodynamic stiffness, linked to buoyancy effects. Additionally, in equation (1), f_{ex} is the wave excitation force, *i.e.* the force experienced by the device arising from incoming waves, while u represents the control force applied through the so-called power take-off (PTO) system.

Though we do not perform an explicit derivation (for economy of space), we do note that, by an appropriate replacement of the convolution term in (1) (see, for instance, [24]), equation (1) can be characterised in state-space form, as

$$\begin{aligned} \dot{x} &= Ax + B(f_{ex} - u), \\ v &= Cx = \dot{x}_p, \end{aligned} \tag{2}$$

with a suitably defined (minimal) triple of matrices (A, B, C), $\lambda(A) \subset \mathbb{C}_{<0}$ (*i.e.* (2) is asymptotically stable), and where the device velocity $\dot{x}_p$ is set as output v.

B. Control problem definition

Let us consider, without any loss of generality, a time interval $[0, T] \subset \mathbb{R}^+$. The useful absorbed energy by the WEC device can be calculated as follows:

$$J(u) = \int_0^T v(\tau)u(\tau)d\tau, \tag{3}$$

where the mapping $u \mapsto J(u)$ represents the control *objective function*. With the definition of J in (3), and the WEC dynamics described by (2), the energy-maximising control problem consists in finding the control law u^{opt} such that

$$\begin{aligned} u^{\text{opt}} = \arg\max_u &\ J(u), \\ &\text{subject to:} \\ &\text{WEC dynamics (2).} \end{aligned} \tag{4}$$

Typically, the OCP defined in (4) is solved via direct optimal control techniques, where both system and input variables are discretised accordingly, transcribing the infinite-dimensional problem (4) into a finite-dimensional numerically-tractable nonlinear program (NP). This is precisely the case for the control technique discussed herein in Section III-C.

C. Frequency-domain optimality

Let $G : \mathbb{C} \to \mathbb{C}$, $s \mapsto G(s)$, be the transfer function associated with (2). An alternative approach to solving (4) is based upon the so-called impedance-matching condition for maximum power transfer. In particular, let $Z = G^{-1}$ be the *intrinsic impedance* of the WEC system. It is possible to show that the optimal control law, which maximises (3) under linear

[1]From now on, the dependence on t is dropped when clear from the context.

[2]The interested reader is referred to, for instance, [22], for a formal definition of this quantity.

modelling conditions, can be characterised in the frequency-domain [15], [23] as

$$U^{\text{opt}}(j\omega) = Z^{\star}(j\omega)V(j\omega). \tag{5}$$

There is, although, an immediate problem with (5): As a direct consequence of the fact that G is strictly proper, and the nature of the complex-conjugate operator, $Z^{\star}$ exhibits anti-causal behavior. A family of controllers, particularly the so-called impedance-matching-based solutions, attempt to find a (commonly LTI) causal controller capable of approximating condition (5) in a suitably defined sense. This is the case for both control techniques discussed in Sections III-A, and III-B. We do note that, while the pursue of this route delivers controllers which are simple and straightforward to implement, it almost inevitably produces controllers which are suboptimal with respect to those explicitly solving (4). This becomes evidently clear, for the particular WEC prototype considered, in Section IV.

III. Controllers

This section briefly discusses the characteristics of the controllers considered for the corresponding feasibility assessment. In particular, Section III-A discusses the so-called reactive controller [17], Section III-B the non-optimisation-based strategy [18], [19], and, finally, Section III-C discusses the moment-based optimal control solution [14], [20]. The interested reader is referred to each of the references given immediately above for further detail.

A. Reactive control

The term 'reactive control', coined by researchers in the field of hydrodynamics, essentially refers to a proportional-integral (PI) output feedback control strategy. In particular, the corresponding control law u is defined, in the Laplace domain, simply as

$$U(s) = \left(\theta_1 + \frac{\theta_2}{s}\right) V(s), \tag{6}$$

where, differently from PI techniques adjusted for tracking/regulation purposes, the set of values $\Theta = \{\theta_1, \theta_2\} \subset \mathbb{R}$ is optimised to maximise the absorbed energy, *i.e.* J in (3). This is commonly done in the literature via exhaustive search procedures, though a well-defined analytical solution exists if one desires to fulfill optimal energy absorption for a single input frequency. In other words, the set Θ can be used to uniquely interpolate (5) at a specific (suitably selected) input frequency, though this interpolation does not, in general, guarantee a stable closed-loop system.

B. LiTe-Con

The energy-maximising controller proposed in [18], termed 'LiTe-Con', directly aims to approximate, in a broadband sense, the optimal frequency-domain condition of Section II-C. Unlike the feedback PI controller described in Section III-A, LiTe-Con adopts a feedforward structure, hence avoiding potential closed-loop stability issues when approximating the optimal frequency-domain conditions.

In particular, [18] explicitly uses the closed-loop frequency-domain mapping associated with the control condition (5), *i.e.*

$$T^{\text{opt}}(j\omega) = \frac{G(j\omega)}{1 + G(j\omega)Z^{\star}(j\omega)} = \frac{G(j\omega)G^{\star}(j\omega)}{G(j\omega) + G^{\star}(j\omega)}, \tag{7}$$

and re-writes the control loop in terms of a feedforward structure $U^{\text{opt}} = H^{\text{opt}}F_{ex}$, with the mapping H defined as (see [18]):

$$H^{\text{opt}}(j\omega) = \frac{G(j\omega)}{G(j\omega) + G^{\star}(j\omega)}. \tag{8}$$

Based upon such a feedforward frequency-domain condition, the LiTe-Con framework makes use of black-box system identification routines (*e.g.* [24]–[26]) to compute an *causal and stable* approximation of (8) in terms of a LTI system H^{LT}, for a suitably selected frequency range $\mathcal{W} \subset \mathbb{R}^+$, *i.e.*

$$H^{\text{LT}}(s) = \arg \min_{H^{\text{LT}}(s)} \|H^{\text{LT}}(j\omega) - H^{\text{opt}}(j\omega)\|_2, \tag{9}$$

for every $\omega \in \mathcal{W}$. We do note that, although beyond the analysis presented in this study, the controller developed in [18] provides a LTI constraint handling mechanism, which is used in practice to maintain the device motion within safety limitations (see [18] for further detail).

C. Moment-based control

The moment-based energy-maximising control strategy, presented in [14], [20], provides an efficient and convenient pathway to transcribe the infinite-dimensional energy-maximising OCP, defined in (4), to a finite-dimensional NP. In particular, [14], [20] uses the mathematical notion of a *moment* (see [27]), allowing for a finite-dimensional parameterisation of (4) in terms of the steady-state response of a suitably defined interconnected system. Though we briefly summarise key concepts in this section, the reader is referred to [14], [20] for a formal treatment of the subject.

Given the harmonic nature of ocean waves, and inspired by standard results in nonlinear output regulation (see [28]), [14], [20] defines both excitation force, and control inputs, as the solution of a ν-dimensional exogeneous system, *i.e.*

$$\dot{\xi} = S\xi, \quad f_{ex} = L_{ex}\xi, \quad u = L_u\xi, \tag{10}$$

where the matrix S is such that $\lambda(S) = \{jp\omega_0\}_{p=1}^{\nu/2} \subset \mathbb{C}^0$, with ω_0 the so-called *fundamental frequency* associated with the input variables f_{ex} and u, *i.e.* $\omega_0 = 2\pi/T$.

Using the set of differential equations (10), and the state-space WEC system in (2), [14], [20] maps the OCP in (4) to a concave quadratic program (QP), *i.e.* the optimal control input $u^{\text{opt}} = L_u^{\text{opt}}\xi$, which maximises energy absorption, can be computed as the global solution of the following QP:

$$L_u^{\text{opt}} = \arg \max_{L_u^{\intercal} \in \mathbb{R}^{\nu}} -\frac{1}{2}L_u\Psi L_u^{\intercal} + \frac{1}{2}L_{ex}\Psi L_u^{\intercal} \tag{11}$$

where the matrix $\Psi \in \mathbb{R}^{\nu\times\nu}$ is uniquely characterised both in terms of the WEC dynamics, and the corresponding input description, as:

$$\Psi^{\intercal} = (\mathbb{I}_{\nu} \otimes C)\left(S \hat{\otimes} A\right)^{-1}(\mathbb{I}_{\nu} \otimes -B). \tag{12}$$

Though beyond of the scope of the presented analysis, we note that one can naturally add both state and input constraints to (11) within the same moment-based framework, to guarantee operation under safe conditions.

IV. APPLICATION CASE

A. WEC system

The setup considered for this application case is fully inspired by the Argentinian WEC prototype [6]. The device is of a point-absorber type, with an axis-symmetric cylindrical geometry, and a hemispherical bottom. The schematic of the considered WEC is shown in Figure 1, while the main characteristics are summarised in Table I.

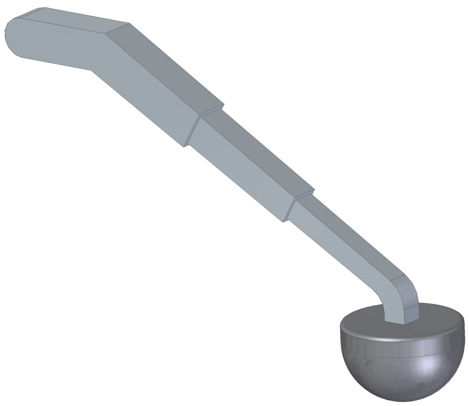

Fig. 1. Schematic of the considered WEC.

TABLE I
MAIN CHARACTERISTICS OF THE CONSIDERED DEVICE.

Characteristic	Value
Radius hemisphere	1.5 [m]
Radius cylinder	1.5 [m]
Height cylinder	0.5 [m]
Arm length	12 [m]
Mass	60 [kg]

The characterisation of each hydrodynamic effect described in Section II, and effectively present in the equation of motion (1), is computed via so-called boundary element methods (BEMs), in particular with the open-source software NEMOH [29]. The resulting frequency-response, $G(j\omega)$, for the device of Figure 1, can be appreciated in the Bode plot of Figure 2

B. Wave characteristics in the South-Atlantic ocean

Waves in the South-Atlantic ocean are characterised by peak periods T_p lying between approximately 9 [s], and 14 [s], with predominant periods in the coastal area of Buenos Aires within the range $[12, 13]$ [s] (see [30]). The corresponding significant wave height H_s is relatively constant along the complete coastal area of Argentina, with a mean value of approximately 2 [m]. To assess the controllers, designed in Section IV-C, fully covering the wave characteristics corresponding to Argentina, we (numerically) generate representative ocean waves according to a stochastic process characterised by the so-called JONSWAP spectrum (see [31]), with peak periods in the range $[6, 15]$ [s], and a fixed significant wave height of 2 [m]. The so-called peak-enhancement factor is set to 3 (see [31]), while the simulation length for each wave is set to $T = 200$ [s].

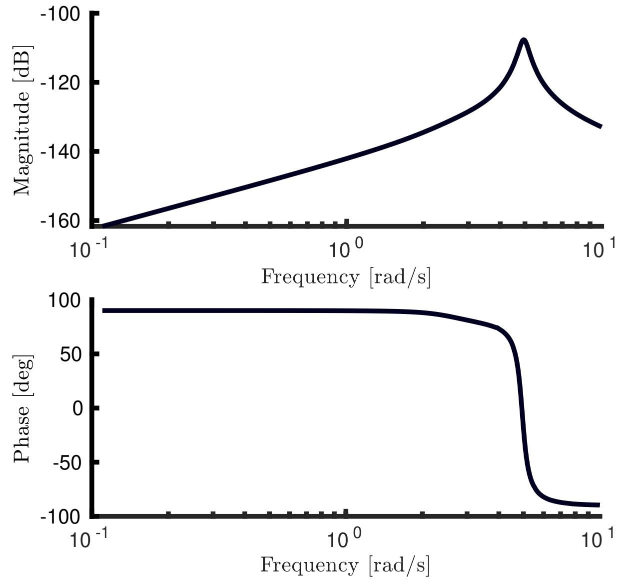

Fig. 2. Frequency response mapping for the selected WEC.

C. Controller design

We begin this section by discussing the PI control structure of Section III-A. Given the relatively large range of peak periods selected, we choose to design the PI controller to be optimal at the middle point in the range, *i.e.* $T_p \approx 11$ [s]. In other words, we design the set of parameters Θ in (6) such that it interpolates the optimal frequency-domain condition (5) at $\omega = 2\pi/11 \approx 0.57$ [rad/s], resulting in the set of values:

$$\theta_1 = 8.45 \times 10^3 \quad \theta_2 = -13.12 \times 10^6, \tag{13}$$

which produces the (stable) closed-loop response presented in Figure 3 (dotted). Note that, as expected from the interpolation condition adopted, the closed-loop response of the PI-controlled system interpolates the optimal energy-maximising frequency-domain response in (7) (denoted with a solid line in Figure 3) at ≈ 0.57 [rad/s].

The LiTe-Con structure is computed based upon the approximation condition in (9), by means of moment-matching frequency-domain identification [32], with a frequency range of $\mathcal{W} = [0.3, 5]$ [rad/s], which thoroughly covers the operating conditions described in Section IV-B. The resulting feedforward controller is a LTI system with 12 poles and 11 zeros. The corresponding input-output frequency-response mapping for the LiTe-Con controller is presented in Figure 3 using a dashed line, showing a satisfactory approximation of the optimal response (7) within the range $\mathcal{W}$.

Finally, the moment-based controller, described in III-C, is designed based upon an input description with 200 harmonics, which translates to a value of $\nu = 100$ in (10). The fundamental frequency is set according to the simulation length, *i.e.* $\omega_0 = 2\pi/200$ [rad/s]. The resulting (concave) QP problem is solved via interior-point methods [33].

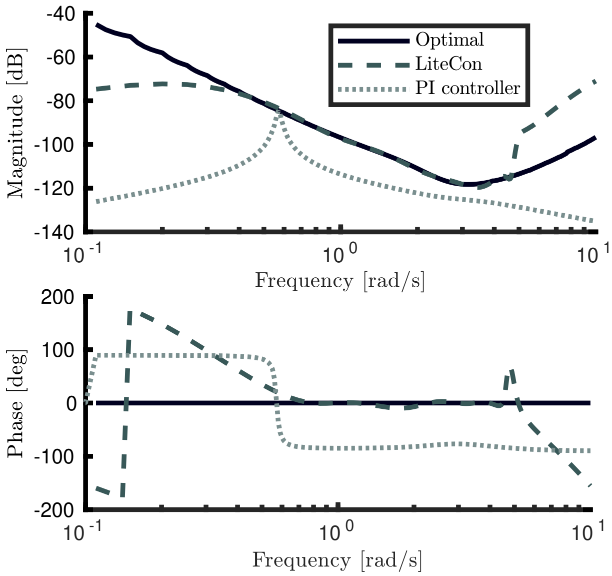

Fig. 3. Optimal input-output frequency response (solid), along with the controlled response with a PI structure (dotted), and LiTe-Con (dashed).

D. Performance assessment

Figure 4 shows absorbed energy, *i.e.* J in (3), for each of the controllers designed in Section IV-C, for waves generated in the range of peak periods described in Section IV-B. To provide statistically consistent performance results, the values presented throughout this section are averaged over 50 random realisations of each sea-state. Note that there is, effectively, a one-to-one relation between the performance results, presented in Figure 4, and the frequency-domain behavior of Figure 3: The PI controller, which is only able to interpolate the optimal energy-maximising condition at a single point in frequency, occupies the last position in terms of performance, while the LiTe-Con design effectively approaches the energy absorbed by the optimal moment-based controller for the range of frequencies (correspondingly periods) where the approximation of T^{opt} is satisfactory. Around $T_p = 11$ [s], the LiTe-Con clearly starts to behave suboptimally, while the moment-based controller, which is based upon directly solving the energy-maximising OCP (4), progressively takes the lead in terms of performance.

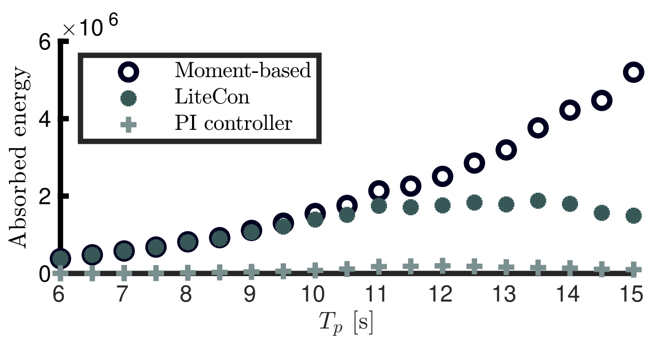

Fig. 4. Performance results for the analysed controllers.

Finally, Figure 5 presents the characteristics of the applied control force for each of the controllers designed in Section IV-C. Note that, while both the LiTe-Con design, and the moment-based controller, share similar root mean square (RMS) and maximum control force values, the latter effectively extracts significantly more energy for $T_p > 11$ [s], so that the control effort is exploited more efficiently.

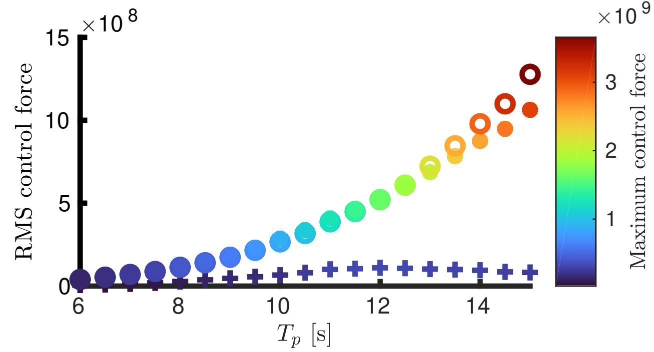

Fig. 5. Characteristics of control forces.

E. Discussion

Though up until this point we have successfully shown that the analysed controllers are feasible from a design/synthesis viewpoint, we note that both rms and maximum values for the control forces involved in Figure 5 are substantially large, particularly for those two controllers with the best overall performance. Such forces are likely to require a proportionally large actuator, which potentially translates to a high cost in practice. The requirement of such large control actions, to achieve maximum energy absorption, can be explained in terms of Figure 6, where both the frequency-response (in magnitude) of the optimal input-output behavior (7) (solid), and the WEC dynamics (dashed), are explicitly shown. In particular, note that the WEC prototype is operating significantly closer to the corresponding optimal conditions around its own resonant frequency (about 6 [rad/s] for this case), while its substantially 'further away' from optimality in the frequency range characterising the wave resource along the Argentinian coast. This translates in the necessity of large forces to 'move' the response of the system towards optimality (see [34]).

Note that, though seemingly simplistic, the result offered in Figure 6 provides valuable information on how to improve the design proposed in [6], to harvest, with a minimum possible effort (from a control perspective), the wave resource along the Argentinian coast, hence contributing in the pathway towards developing a device tailored for the region.

V. Conclusion

Argentina has the potential to extract a vast wave energy resource, which, up to date, remains untapped. Aiming to contribute in the pathway towards effective conversion of the Argentinian ocean energy potential, we assess, in this paper, the feasibility of three state-of-the-art energy-maximising control techniques for an Argentinian WEC prototype. We show that, though feasible from a design and synthesis viewpoint, the controllers require a significantly large control effort to operate optimally, which can likely translate into high costs

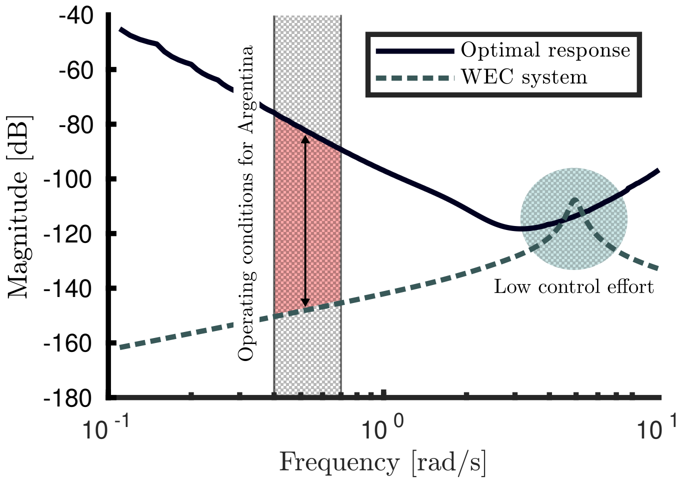

Fig. 6. Frequency response (in magnitude) of the analysed WEC system (dashed), along with the corresponding optimal energy-maximising behavior (solid).

in practice. We discuss the rationality behind this issue, and provide control-oriented tools to analyse future prototypes, hence contributing in the trajectory towards successful design of a device tailored for the Argentinian resource.

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