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Exploiting the Rounding Mode of Floating-point in the Simulation of Chua's Circuit

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Abstract

The Chua's circuit has been considered as one of the most important paradigms for nonlinear science studies. Its simulations is usually undertaken by means of numerical methods under the rules of IEEE 754-2008 floating-point arithmetic standard. Although, it is well known the propagation error issue, less attention has been given to its consequences on the simulation of Chua's circuit. In this paper we presented a simulation technique for the Chua's circuit, it exhibits qualitative differences in traditional approaches such as RK3, RK4 and RK5. By means of the positive largest Lyapunov exponent we show that for the same initial condition and same set of parameters, we produce a periodical and a chaotic solution.

1 Introduction

We live in a complex world, where the majority of dynamical systems is nonlinear. According to Aguirre [1], the choice of these models brings with it an inevitable increase in the complexity. One of the most studied examples of nonlinear dynamics is the Chua's circuit, which has been studied by many researchers and one of the most robust practical implementation is given by Kennedy [2].

The Chua's circuit, developed by Chua [3], is known to exhibit chaotic behaviour similar to the system proposed by Lorenz [4]. It is a simple electronic network which exhibits a variety of phenomena, such as strange attractors and bifurcations. This circuit consists of two capacitors, an inductor, a linear resistor, and a nonlinear resistor, which consists of the most well known implementation using operational amplifier, as presented by Kennedy [2].

To study nonlinear dynamics, one of the most important tool is the numerical computation. According to Goldberg [5], the numerical computation uses floating-point arithmetic in most computers under the IEEE 754-2008 standard. This standard is a systematic approximation of the real arithmetic and it is represented by a finite subset of the real numbers. As a result, Institute of Electrical and Electronics Engineers (IEEE) [6], some properties of the arithmetic of real numbers are not guaranteed for the floating-point. Hence, it is necessary to pay attention to limitations of computers with respect to scientific computing.

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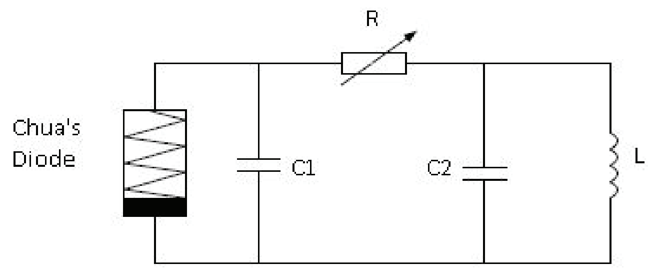

Fig. 1 Chua's circuit.

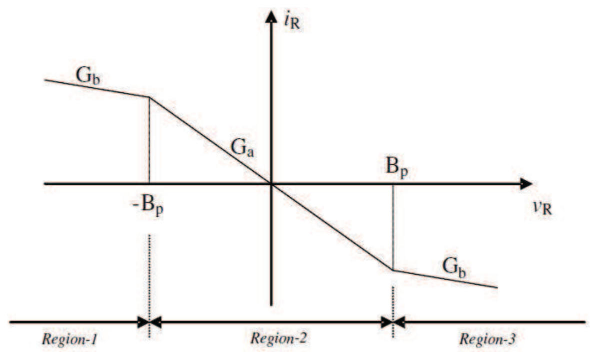

Fig. 2 Kiliç [13], curve of Chua's diode, current-voltage characteristic.

Nepomuceno [7], uses a recursive function that mathematically converge to a fixed point and confirms that arithmetic floating-point possess limitations, because his simulations demonstrated that a recursive function converge to a period 2-orbit. This work agrees with Lozi [8], who said that there are many published works whose reliability of the numerical results is not carefully evaluated.

In a recent work [9], two equivalent mathematical realizations presents a difference in simulations as time goes by. Thus, by Moore [10], the inequalities presented in the Chua's circuit model are addressed in parallel with interval analysis of the circuit. Taking the average of round mode towards $+\infty$ and $-\infty$, for the same initial conditions and paramenters, we produce a qualitative change on the dynamics of the system. The method has been applied using the order 3, 4 and 5 from Runge-Kutta approach.

In order to ensure the method presented in this paper, we calculate the largest Lyapunov exponent [11] and we proved favorably, that the use of computer numerical round modes gives us a periodic result.

2 The Chua's circuit

The Chua's circuit is one of the most used systems as an example to study the nonlinear dynamics and chaos [12] and is represented by Equation 1. The circuit shown in Figure 1 is composed of linear elements, except the Chua's diode, which shows nonlinear behaviour as presented in Figure 2.

$$\begin{cases} C_1 \dfrac{dv_{c_1}}{dt} = \dfrac{v_{c_2} - v_{c_1}}{R} - i_d(v_{c_1}), \\ C_2 \dfrac{dv_{c_2}}{dt} = \dfrac{v_{c_1} - v_{c_2}}{R} - i_d, \\ L \dfrac{di_L}{dt} = -v_{c_2}. \end{cases} \tag{1}$$

Since V_{C_1} is the voltage across the capacitor C_1, V_{C_2} is the voltage across the capacitor C_2, i_L is the current through the inductor and the current in the diode is:

Table 1 Values of components and constants used in the simulations.

Components	Values
C_1	10 nF
C_2	100 nF
L	19 mH
R	1978.5 Ω
m_0	-0.37 mS
m_1	-0.68 mS
B_p	1.1 V

Table 2 Values of initial conditions and the constants of time.

Components	Values
V_{C1}	$0.7V$
V_{C2}	$0V$
i_L	$0A$
Integration step	$1\mu s$
Time of simulation	$75ms$

$$i_d(v_{C_1}) = \begin{cases} m_0 v_{C_1} + B_p(m_0 - m_1) \text{ for } v_{C_1} < -B_p, \\ m_1 v_{C_1} \text{ for } |v_{C_1}| \leq B_p, \\ m_0 v_{C_1} + B_p(m_1 - m_0) \text{ for } v_{C_1} > B_p. \end{cases} \quad (2)$$

2.1 Simulation methods

The simulations of Chua's circuit is based in the component values and constants presented in Table 1, the values of the initial conditions, integration step and the time of simulation are presented in Table 2. Using the *Matlab* software we did the Algorithms 1 and 2, applied in this work and displayed in Appendix.

Matlab uses the rounding mode to nearest, that according to the standand of IEEE 754-2008 [6] is the best way to garantee the lower error in the simulation, but there is no rounding mode that ensures the lower error in a function with many math operations.

Using the Matlab's function system_dependent to round towards $-\infty$ and other towards $+\infty$, this is round mode to minus an plus infinite, in each time step. Consequentely, we make a procedure to decrease the error propagation, this method is presented in in Algorithm 1. And to show the reliability of the methodoly applied we use the order 3, 4 and 5 from Runge-Kutta approach.

Algorithm 2 shows that we only make the average, that is, we get the mid position for each state variable from the interval established by the two round modes. This seems quite simple and naive, but it is not. When we make the average, we are in some way approaching the round to the nearest, but from the point of view of functions, what we may call as round to the function. If a computer simulation was perfect, making the average result in the same value that the round to nearest does. Additionally, as the error coming from rounding off and truncation can be as random (or at least pseudo-random), the average can be also seen considered as mode to reduce the total error, that is, it acts as a kind of filter that reduces the influence of system error. This algorithm caused a really abrupt change in the results, causing a periodic output. There are many related works that study numerical methods to make a qualitative difference in the results, as [14, 15].

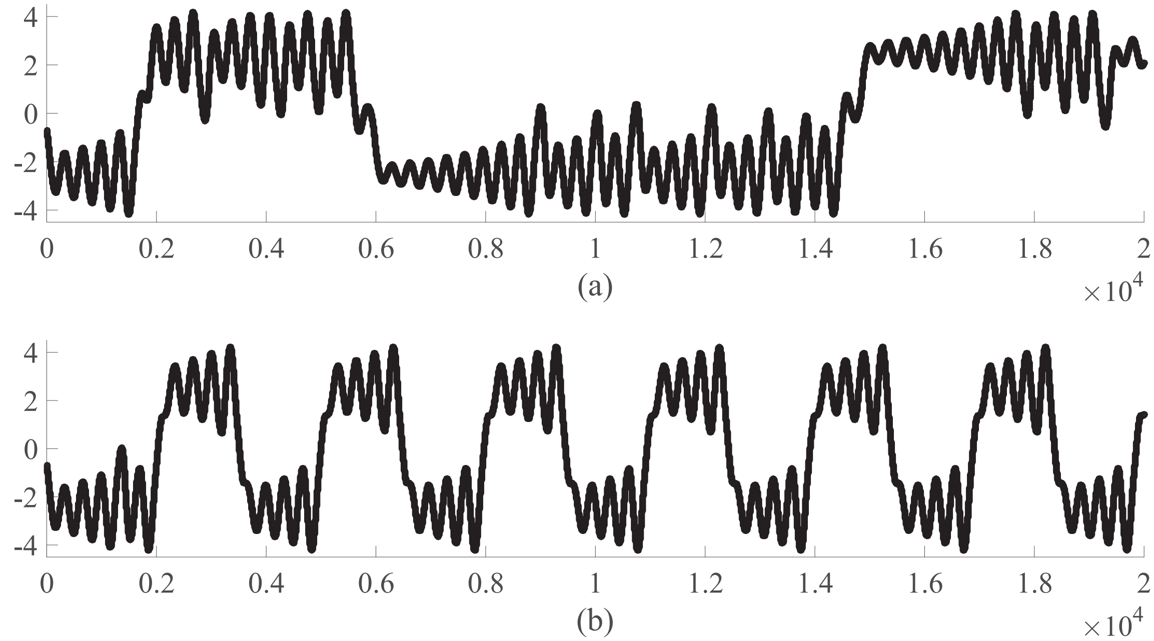

Fig. 3 Iterations-voltage in Chua's diode characteristic by means of RK3. (a) Voltage in Chua's diode when the simulation was traditionally performed using by standard IEEE 754-2008. (b) Voltage in Chua's diode when the simulation was performed by the average of the two round modes towards to $+\infty$ and $-\infty$.

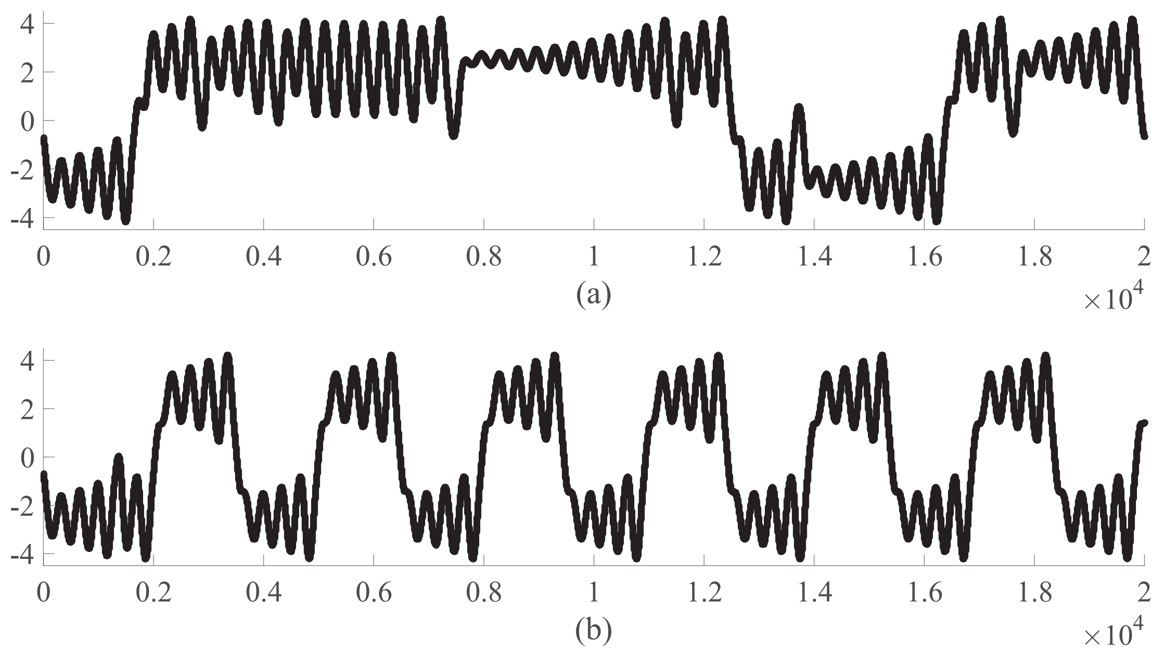

Fig. 4 Iterations-voltage in Chua's diode characteristic by means of RK4. (a) Voltage in Chua's diode when the simulation was traditionally performed using by standard IEEE 754-2008. (b) Voltage in Chua's diode when the simulation was performed by the average of the two round modes towards to $+\infty$ and $-\infty$.

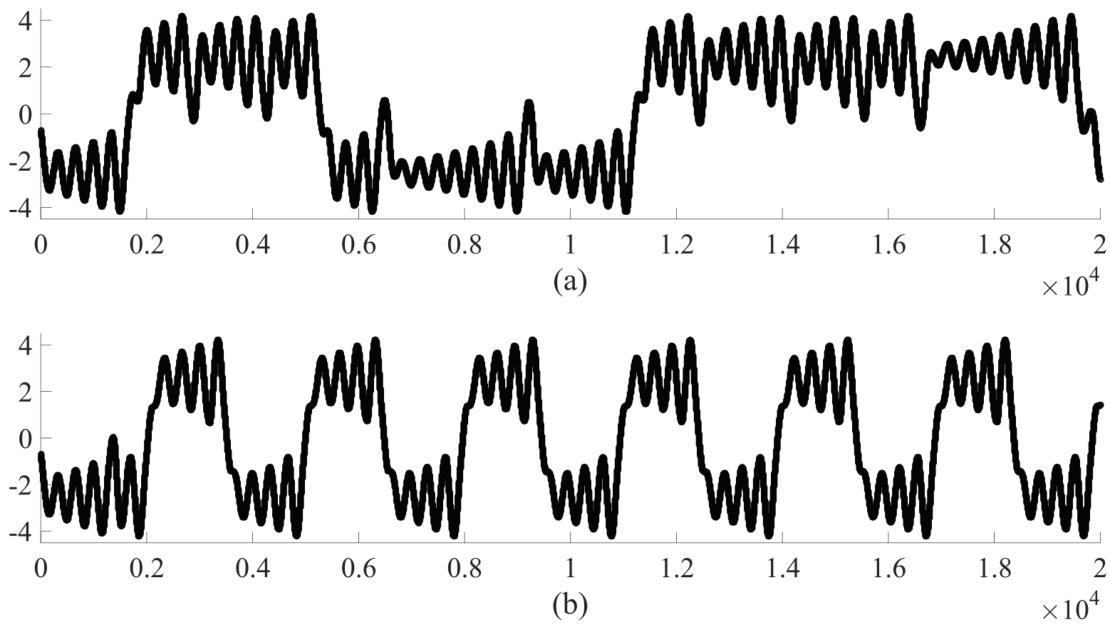

Fig. 5 Iterations-voltage in Chua's diode characteristic by means of RK5. (a) Voltage in Chua's diode when the simulation was traditionally performed using by standard IEEE 754-2008. (b) Voltage in Chua's diode when the simulation was performed by the average of the two round modes towards to $+\infty$ and $-\infty$.

Table 3 Values of Lyapunov exponents for the Chua's circuit.

Simulation Methods	Lyapunov Exponents		
	RK3	RK4	RK5
Traditional simulation by IEEE 754-2008	0.041958	0.024295	0.128948
Simulation proposed in this paper	-0.025003	-0.025001	-0.025006

3 Results

The results of Chua's circuit simulation according to the values in Table 1 and Table 2, using the IEEE 754-2008 standard for floating point are shown in Figures 3-8a. In these first results we used the traditional approaches through RK3, RK4 and RK5. However, the results from the solution proposed by Algorithm 1, using the same parameters and initial condition, are presented in Figures 3-8b.

We know that the diode curve, Figure 2, is given by the characteristic voltage *versus* current and in Figures 3-5 has the diode voltage. Thus, it is found by applying the intervals analysis Chua's circuit, the voltage begins to show a periodic behavior, Figures 3-5b. Therefore the dynamics of the system is affected, as shown in Figures 6-8b.

Taking the time series shown in Figures 3-5, we calculate their largest Lyapunov exponent using the method proposed by Wolf *et al.* [11] and obtained the results shown in Table 3.

According to Alligood et al. [16] and Devaney [17], the orbit of a chaotic dynamic system is sensitive to initial conditions and aperiodic. However, for the same set of initial conditions and parameters, Figures 3-5b show a periodic result, as verified by the largest Lyapunov exponent.

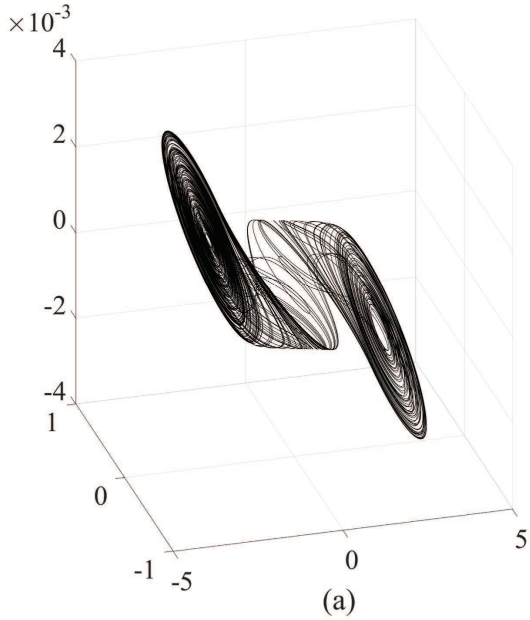

Fig. 6 Scroll of Chua's circuit by means of RK3. (a) Double attractor traditionally generated. (b) Periodical result by the proposed algorithm.

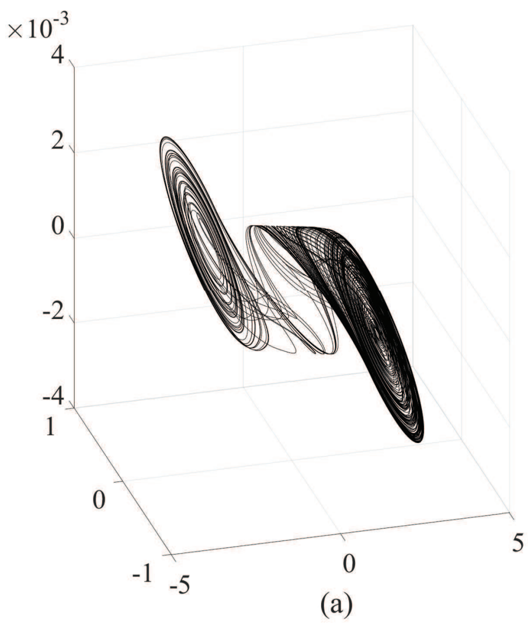

Fig. 7 Scroll of Chua's circuit by means of RK4. (a) Double attractor traditionally generated. (b) Periodical result by the proposed algorithm.

4 Conclusion

We observe that even though infinitesimal errors along the simulation can compromise the result. Interval analysis were employed as the round mode was adapted to set the mid value of an interval composed by the round modes towards to $-\infty$ and $+\infty$. Hence, what it could be noticed is that the system simulated by RK3, RK4 and RK5 and IEEE-754-2008 round mode standard, that is, round to the nearest, presents a strange attractor with a

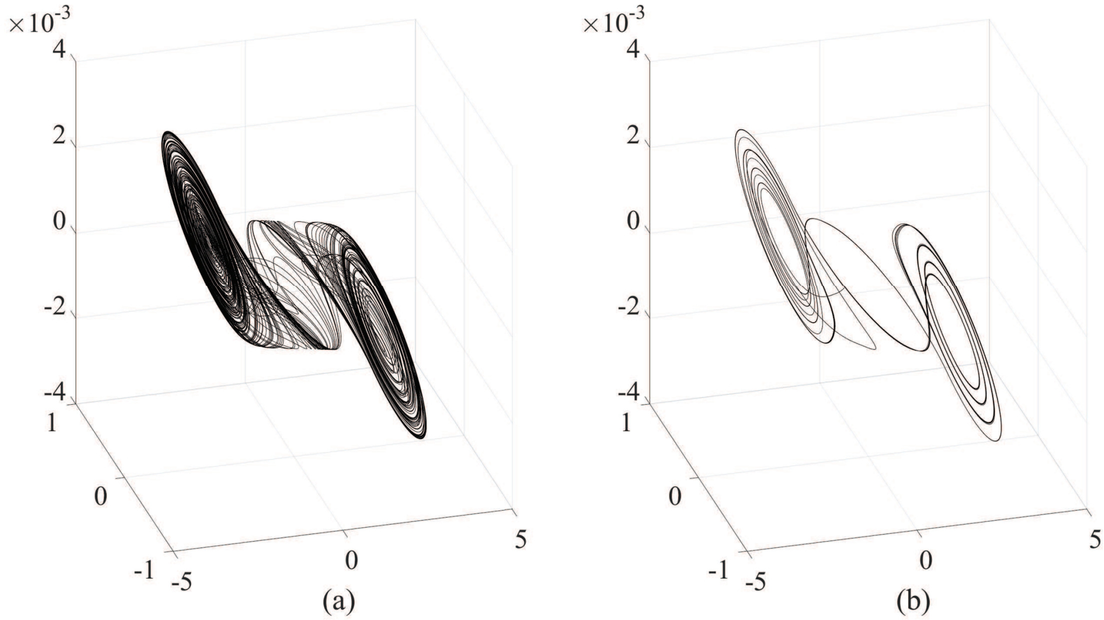

Fig. 8 Scroll of Chua's circuit by means of RK5. (a) Double attractor traditionally generated. (b) Periodical result by the proposed algorithm.

chaotic behaviour, because the largest Lyapunov exponents are positives. On the contrary, the proposed algorithm furnishes beautiful periodics results for three Runge-Kutta methods and the largest Lyapunov exponents are negatives.

Which one is the correct? We do not know for sure, but the simplicity of periodic solutions were the reasons given for using an average of the round methods, and the negative Lyapunov exponent helps us to believe in the veracity of this solution. We intend to build Chua's circuit and compare this simulated results with experimental results. Although, this procedure seems a way to give a final answer, we should keep in mind that there is no way to implement a circuit that matches perfectly the set of equations and parameters given in this paper.

Thus, using the strategy proposed in this work, we believe that we adopted a procedure that filters some of error such that a system before considered chaotic passed to have no chaotic behavior. The method proposed in this work requires a further investigation to see if what happened can also be seen as a stabilization of an unstable orbit.

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APPENDIX

Algorithm 1

```
%Initial Conditions
clear all
system_dependent('setround',-Inf);
ym=[-0.7 0 0];
system_dependent('setround',Inf);
yp=[-0.7 0 0];
tf=0.075;
h=1e-6;
tspan = 0:h:tf;
N=length(tspan);
for k=1:N-1
system_dependent('setround',-Inf);
aux = ode(@chua,tspan(k:k+1), ym(k,:),yp(k,:)); % We use here the Runge-Kutta
%approuches of 3, 4 and 5 order, that is, ode3, ode4 and ode5.
ym(k+1,:)=aux(2,:);
system_dependent('setround',Inf);
aux = ode(@chua,tspan(k:k+1), yp(k,:),ym(k,:)); % We use here the Runge-Kutta
%approuches of 3, 4 and 5 order, that is, ode3, ode4 and ode5.
yp(k+1,:)=aux(2,:);
end
%Figures
figure(1)
plot(1:N,ym(:,1),1:N,yp(:,1),'k')
figure(2)
plot3(ym(:,1),ym(:,2),ym(:,3),'k')
view(-16,24);grid;
```

Algorithm 2

```
function out = chua(t,in,in2)
x = in(1); y = in(2); z = in(3);
x2=in2(1);y2=in2(2);z2=in2(3);
L = 19.2*10^(-3);
C1  = 10*10^(-9);
C2  = 100*10^(-9);
R = 1978.5;
G = 1/R;
m0=-0.37*10^(-3);
m1=-0.68*10^(-3);
Bp=1.1;
%Average +Inf and -Inf
x=(x+x2)/2;
y=(y+y2)/2;
z=(z+z2)/2;
%Diode
if  x >Bp
g=m0*x+Bp*(m1-m0);
elseif  (x >= -Bp)&(x <= Bp)
g=m1*x;
else
g=m0*x+Bp*(m0-m1);
end
% Chua's Circuit Equations
xdot = (1/C1)*(G*(y-x)-g);
ydot = (1/C2)*(G*(x-y)+z);
zdot  = -(1/L)*y;
out = [xdot ydot zdot]';
```