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Student perceptions of a guided inquiry approach to a service-taught ordinary differential equations course

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ABSTRACT

This paper reports on data obtained during semi-structured interviews with students who had recently experienced a guided inquiry approach to learning ordinary differential equations (ODEs) in a service-taught module. Previous research identified the strengths and weaknesses of similar cohorts of students. The results from that research informed the design of an intervention (15 guided inquiry tutorials) which sought to improve the outcomes for students taking a subsequent version of the module. This paper reports on the data gathered during interviews with these students that describe their perceptions of the guided inquiry approach used. The students noticed the change in emphasis toward conceptual understanding that the intervention was trying to instil. They cited the change in questioning style most frequently as being where they saw this change, but also noted the prevalence of group work and change in interaction pattern as keys to its success. Thus, by probing students' opinions in this way, we find validation for a guided inquiry approach to teaching ordinary differential equations in third level that emphasizes active learning and lateral interactions among students. The students' personal goal orientation and the goal structure of the learning environment are also discussed.

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KEYWORDS
Ordinary differential equations; guided inquiry; student perceptions; service teaching

1. Introduction

This paper, which is part of a larger, multi-stage investigation (Hyland, 2018) reports on an analysis of data related to the perceptions of two successive cohorts of students following a successful intervention in relation to the teaching and learning of ODEs. Throughout the project we used an intervention mixed methods design (Creswell et al., 2009) to investigate students' learning of differential equations. The initial stage of the research described students' concept image of ODEs upon completion of the module prior to our involvement. We reported that the students possessed fragmented concept images and exhibited shortcomings in instrumental understanding (Hyland et al., 2017). We then designed and implemented an intervention that resulted in significant enrichment of students' concept images in addition to modest improvements in instrumental understanding (Hyland,

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2018). The intervention comprised 15 one-hour guided inquiry tutorials that included pre-tests, worksheets students tackled in small groups with periodic guidance from a tutor, and post-tests. The tutorials were delivered in conjunction with traditional lectures and replaced the previous tutorials, which would be described as recitation sessions.

The intervention we implemented comprises guided inquiry tutorials patterned after the successful method developed by McDermott and co-workers (see e.g. Shaffer & McDermott, 1992); similar methods have previously been implemented at our university (Doughty, 2013). Our conception of guided inquiry is in keeping with the vision of Inquiry-Based Mathematics Education (IBME) put forth by Laursen and Rasmussen (2019). They detailed four pillars of IBME: student engagement in meaningful mathematics, student collaboration for sensemaking, instructor inquiry into student thinking, and equitable instructional practice to include all in rigorous mathematical learning and mathematical identity-building.

Throughout the project, we have probed students' concept image as an indicator of their conceptual learning, as well as their instrumental understanding. The term 'instrumental understanding' refers to students' ability to display procedural competence without necessarily understanding why the procedure is effective, a situation Skemp (1976, p. 2) calls 'rules without reasons'. When we refer to concept image, we are referring to Tall and Vinner (1981) who defined the concept image as 'the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes' (Tall & Vinner, 1981, p. 152). The concept image evoked depends on the context. Tall and Vinner note that students are more likely to use the ideas they form from their experience with a concept (i.e. their concept image) than the formal concept definition. Adopting Tall and Vinner's work is also in keeping with our constructivist paradigm. When describing concept image, they state that:

> It is built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures. (Tall & Vinner, 1981, p. 152)

Within this research, we also gathered data on students' perceptions of the guided inquiry approach adopted in tutorials, which is the focus of this paper. The research question we address in this paper is as follows:

> How is the intervention perceived by its participants?

A need for this type of research, both nationally and internationally, has recently been identified in the literature. O'Sullivan et al. (2015) conducted a comprehensive analysis of teaching and learning research in Ireland across a 25-year period which included 2275 entries. The authors organized each of the entries into one of four themes. The theme of *student experience* had the fewest publications, and the authors concluded that there is a 'clear gap' with respect to research characterized by student involvement in inquiry. Bakker et al. (2021) conducted an international survey of researchers in mathematics education seeking to identify the areas for research to focus on in the next ten years. Eight themes were identified, of which two (*approaches to teaching* and *affect*) are relevant to this research. *Approaches to teaching* was the theme most frequently mentioned in the study with two observations that are pertinent to this research: first, the need for further 'design and evaluation of teaching approach' studies was noted; also, the question of how teaching can be engaging for students was raised. There is significant overlap between *affect* and other themes particularly *approaches to teaching*, with student engagement and involvement recurring throughout.

A recent study on students' perceptions of teaching approaches found that the degree to which students considered their class to be teacher-centred had a detrimental effect on their disposition toward maths (Pampaka & Williams, 2016). Ciftci (2015) found that student perceptions of the quality of their mathematics education had an effect on their performance and anxiety. Though research on perceptions of IBME has been conducted, it remains underrepresented in the literature relative to studies about the efficacy of such approaches.

We used semi-structured interviews to gather information on the students' perceptions of the guided inquiry tutorials. In particular, we explored whether and to what extent students enjoyed[1] the intervention, what elements of the intervention contributed to enjoyment, how the intervention differed from other tutorials they attended, and what changes they would make to the intervention moving forward.

1.1. Context: the module Introduction to Ordinary Differential Equations

To provide context for this study, we will now describe in some detail the mathematics module Introduction to Ordinary Differential Equations which was at the centre of our research. It is taken by a variety of students in our university, including those enrolled on BSc degrees in Applied Physics, Physics with Astronomy, and Science Education (a concurrent mathematics and science teacher education programme). The module is taken by students in the third year of their degree: they have previously studied modules on differential and integral calculus of one and several variables, linear algebra and (in the case of Science Education students) probability and statistics.

The learning outcomes for the module state that students will encounter some definitions and theorems, several solution techniques, and problems involving ODEs in context during the module. The module could be described as a methods course, however, with the focus very much on the latter two outcomes. Following a brief recap on differential and integral calculus, students encounter the separation of variables and integrating factor methods in the opening weeks of the twelve-week module. They use these techniques to solve first order ODEs in a variety of contexts (for example, Newton's Law of Cooling), including many initial value problems. Students are then introduced to second order linear constant coefficient and Euler-Cauchy equations (both homogeneous and inhomogeneous). These equations are also presented in context (damped oscillators), where students use the method of undetermined coefficients or variation of parameters.

The module is delivered in a $2 + 1$ structure, meaning that there are two lectures and one tutorial each week of semester. Attendance is typically optional, and students attend in groups of 30 or fewer. Tutorials at our institution could more often than not be described as recitation sessions, where students are led through a problem sheet by a tutor. This was the case with the module prior to our involvement in the project. Students typically achieved well in the terminal exam, but concerns about conceptual understanding were reflected by a survey of academic staff in the School of Physical Sciences, which found that students were unable to apply their learning in other modules. Thus, the students' instrumental understanding was not accompanied by sufficient conceptual understanding to allow them to formulate, solve, and interpret the solutions of differential equations in new contexts. Hoban et al. (2013) have discussed how a greater depth of conceptual understanding can underpin the successful application of mathematical knowledge in different contexts. Reflection by the research team identified that the module was strongly focussed on (dissociated) instrumental understanding. This was corroborated by a diagnostic survey given to outgoing students, and a decision was made to implement an intervention, with the aim of enriching the students' concept image of ODEs.

The intervention which took the form of 15 one-hour tutorials, included a number of conceptually focussed tasks in addition to tasks devoted to the various procedures that appear throughout the module. The intervention essentially retained the $2 + 1$ structure, but the nature of the tutorials was changed significantly as part of our intervention. The first week of teaching included two extra tutorials – one which reviewed calculus and one on the meaning of the derivative – because this content is a prerequisite of learning ODEs. Among the conceptual aspects of the intervention was a clear emphasis on what an ODE is, what is meant by a solution to an ODE, modelling with ODEs, and multiple representations (for example, direction fields) of differential equations and their solutions.

In changing the nature of the tutorials from recitation sessions toward IBME, significant changes were made to the types of tasks the students encountered and environment in which the students worked. Previously, the tasks focussed exclusively on the students' procedural fluency and instrumental understanding of solution techniques to various ODEs which is in keeping with a recitation-style session. For the IBME approach, however, we designed new worksheets that comprise highly scaffolded activities that all students work on together in facilitated groups to develop their understanding of key concepts in ODEs. These worksheets, which contained intermittent breaks to discuss progress with a facilitator, encapsulate the four pillars of IBME outlined in the introduction. The change in learning environment resulted in changes in the role of the student and the role of the tutor. Where before, students worked individually, now they work in small groups. Similarly, the facilitator, who previously practiced exposition exclusively from the top of the classroom, now focuses on fostering the students' mathematical thinking and inquiry. This was done mainly by an approach involving questioning and prompting (Watson & Mason, 1998). This allowed the students to build their understanding by being guided towards correct answers to the tutorial questions, and also ensure that they had opportunities to articulate their understanding of relevant mathematical concepts. The change in role for the tutor should not be under-estimated, and Rasmussen et al. (2017) detail various techniques tutors can adopt (e.g. revoicing) which promote student explanation and justification. Figure 1 (below) contains a sequence of tasks from early in the intervention, where students consider what constitutes a differential equation (Activity 3), and what is meant by a solution to a differential equation (Activity 4). Both activities are illustrative of the type of task students encounter as part of the intervention, which we assert puts into practice each of the pillars of IBME, as detailed by Laursen and Rasmussen (2019) (above).

Laursen and Rasmussen (2019) further outline a variety of traits of IBME classrooms: 'working in small groups on unfamiliar and challenging problems'; 'explaining their own thinking' (Yackel & Cobb, 1996); 'making and justifying conjectures'. Another feature of these activities is the explicitly planned tutor-student interactions, which allows for the tutor to listen to and interpret the students' reasoning at regular and significant stages throughout the activity (Laursen & Rasmussen, 2019).

Tutorial 3: Differential Equations and their Solutions

Activity 3

1. Given that $y = e^x + 2x$, show that:

   a. $\dfrac{dy}{dx} = e^x + 2$;

   b. $\dfrac{dy}{dx} - y = 2(1 - x)$.

2. Are either of the above differential equations? Justify your answer.

3. What does $y = e^x + 2x$ represent in this context?

Discuss your answers with a tutor.

Definition

A solution to a differential equation is any function $y(x)$ that satisfies the differential equation in question. That is, when we calculate the derivatives of y, and substitute for y and its derivatives, the left hand side is equal to the right hand side for all values of x in the domain of y.

Activity 4

1. Examine the following functions. Decide in your groups whether or not they are solutions to the differential equations that accompany them, and justify your decision. Throughout these problems, primes denote derivatives with respect to x.

   a. Is the function $y = x^3 + 7$ a solution to the differential equation $y' = 3x^2$?

   b. Is the function $x = 7t^2 + 3t$ a solution to the differential equation $\dfrac{dx}{dt} - 4t = x - 1$?

   c. Is the function $y = -e^{-3x}$ a solution to the differential equation $y'' = 3y'$?

   d. Is the function $y = (1 + x^2)^{-1}$ a solution to the differential equation $y' + 2xy^2 = 0$?

2. For what values of r are the following statements true?

   a. The function $y = x^2 - rx$ is a solution to the differential equation $y' = 2x + 10$.

   b. The function $y = Ce^{x^r}$ is a solution to the differential equation $\dfrac{dy}{dx} = 2xy$.

Discuss your answers with a tutor.

CASTeL, Dublin City University *Winter 2017* *Contact Diarmaid Hyland, Paul van Kampen, Brien Nolan*

Figure 1. Illustrative examples of tasks students encounter throughout the intervention.

2. Research method

This section begins by outlining the theoretical and conceptual frameworks used throughout the project. A profile of the participants in the study is then provided. Finally, the collection, analysis, and presentation of the data gathered to answer the research question are described.

2.1. Theoretical Framework

Gresalfi and Lester (2009) emphasized the importance of subscribing to an educational paradigm as a guide to what happens in the classroom, and to educational design generally. We have adopted a social constructivist approach to teaching and learning, as we seek to investigate and support the learning of individuals through small group discussions. In doing so, we follow in the footsteps of many researchers and educators in physics and mathematics education (Arnon et al., 2013; Doughty et al., 2014; Rasmussen et al., 2017; Shaffer & McDermott, 1992). The instructional design and resulting activities maximized the opportunity for students to construct their own knowledge in a classroom environment supporting communication, especially discourse and cooperation. Crotty (1998) has highlighted the link that exists between one's worldview, theoretical lens, methodological approach, and methods of data collection, and explained how these elements should be consistent with one another. The semi-structured interviews and concept image we used to gain insight into our students' learning are consistent with (though by no means exclusively applicable to) a social constructivist approach, which added a level of consistency and transparency that strengthens the research and its findings.

2.2. Conceptual framework

In this paper, we situate the discussion of results in achievement goal theory, highlighting evidence of the students' personal goals and their perceptions of the goal structure of the intervention. Previous work has examined interview data through a similar lens with respect to student perceptions (Urdan, 2004), as well as the importance of student's perceptions of their learning environment on disposition in mathematics (Pampaka & Williams, 2016).

Wolters (2004, p. 236) explains that '*Achievement goal theory* proposes that students' motivation and achievement-related behaviours can be understood by considering the reasons or purposes they adopt while engaged in academic work (Ames, 1992; Dweck & Leggett, 1988; Urdan, 1997)'. Since its inception in the 1980s, achievement goal theory has grown to become one of the most popular theories of motivation (Anderman & Wolters, 2006; Pintrich, 2000). In particular, mastery goals and performance goals have dominated research in achievement goal theory, which Meece et al. (2006) describe as follows:

> A mastery goal orientation is defined in terms of a focus on developing one's abilities, mastering a new skill, trying to accomplish something challenging, and trying to understand learning materials … By contrast, a performance goal orientation represents a focus on demonstrating high ability relative to others, striving to be better than others, and using social comparison standards to make judgments of ability and performance. (Meece et al., 2006, p. 490)

Meece et al. (2006, p. 490) elaborate on this by explaining how learners with each goal focus derive success. With mastery '[s]uccess is evaluated in terms of self-improvement, and students derive satisfaction from the inherent qualities of the task, such as its interest and challenge', whereas with performance goals '[a] sense of accomplishment is derived from doing better than others and surpassing normative performance standards'.

Beyond personal goals are goal structures, which concern the messages contained in the learning environment that give prominence to certain goals above others (Ames, 1992). It has been shown that learners perceive a stronger mastery goal structure when instructors highlight the importance of understanding and the value of learning (Stipek et al., 1998; Urdan et al., 1999). The learning environment in our tutorials emphasizes mastery goals through (among other things) construction and assignment of challenging tasks that are meaningful to the students and emphasizing improvement and growth over competition when interacting with the learner, which Ames (1992) suggested would create such a goal structure. Learning environments with a mastery goal structure have been shown to promote similar personal goal focuses in learners of various ages and in a number of contexts (Anderman & Midgley, 1997; Roeser et al., 1996; Ryan et al., 1996).

2.3. Participants

This paper reports findings that form part of a larger research project (Hyland, 2018) that investigated how students enrolled in a typical service module learned ODEs; identified gaps in students' attainment; developed an intervention to try to address these gaps; and assessed its effects. As part of the last aspect of the project, we conducted semi-structured interviews with a subset of the participants. The interviews with students took place across two successive years. The students were pursuing an undergraduate degree in physics or mathematics teaching and were in the third year of their four-year degree.

2.4. Data collection

We used semi-structured interviews to gather data to answer the research question. The interviews followed the design guidelines described by Jääskeläinen (2010). We chose to conduct interviews because of the richness of the data obtained. Interviews facilitate more open-ended questions than pen and paper tests and allow for spontaneous lines of enquiry to be followed by the interviewer. With respect to the research question, the interview setting allowed students to express themselves more freely than written post-tests. The interviews lasted approximately 20 minutes and covered a number of mathematical questions initially, before discussing the intervention and students' perceptions of it (typically for 5 minutes). Only data related to the questions about perceptions is discussed in this paper, though data related to differential equations is reported elsewhere (Hyland, 2018).

In total, 40 students (33% of the cohort) took part in interviews. Nine individual or small group interviews were carried out with 19 students from the 2016-17 cohort of students. Individual interviews were carried out with 21 students from the 2017-18 cohort. These students self-selected from students who participated in the intervention. Each interview across both years was conducted one week after the intervention was completed. The interviews were conducted by an independent researcher, and one of the authors. Our intention was to have an independent interviewer for all interviews but the number of students who opted to be interviewed (particularly in the second year) was far larger than anticipated. Hence, they were shared between the independent researcher and one of the authors. The independent researcher carried out seven of nine interviews in the first year and the author carried out the remaining two interviews that year and all interviews the following year.

The semi-structured interviews were based on the following 'core' questions that were initially posed to students. Depending on the students' response, the interviewer chose how to advance the conversation, either through probing further or beginning a new line of enquiry.

(1) Did you enjoy the tutorials for MS225?
(2) What aspects of the tutorials did you like/dislike?
(3) Were they similar to your other mathematics tutorials?
(4) If not, in what ways did they differ?
(5) You attended X/15 tutorials for MS225. What caused you to attend this many? Is this typical for you with mathematics tutorials?
(6) Do you feel more or less prepared to encounter ODEs in subsequent modules as a result of the manner in which you learned them during tutorials?
(7) What changes would you make to the tutorials if given the chance?

The choice of 'core' questions was dictated by a desire to elicit as much information about student perceptions as possible (e.g. Q1-6), but also to generate ideas for revisions to the intervention for future iterations (e.g. Q7). McDermott (2001) explains that 'use of the findings to guide the development of curriculum' is a component of the development of their instructional design.

Where interview excerpts are used in the results and discussion section, different letters are used at the beginning of each line to differentiate between speakers. Each interview is identifiable by a code provided at the end of the excerpt. The code details the year in which the interview was conducted and the interview number, separated by a hyphen. Thus, the interviews from the 2016-17 academic year are labelled 16-1 to 16-9 and the interviews from the 2017-18 academic year are labelled 17-1 to 17-21.

2.5. *Data analysis*

The qualitative data gathered during the interviews was analysed as described by Thomas (2006), who details a general inductive approach to qualitative data analysis. Thomas (2006) provides an outline of this approach for qualitative data analysis with the primary purpose of allowing 'research findings to emerge from the frequent, dominant, or significant themes inherent in raw data, without the restraints imposed by structured methodologies' (2006, p. 283). A general inductive analysis uses coding to develop categories. The coding process Thomas (2006) describes aims to condense the raw data and to establish transparent and defensible links between the research objectives and the summary findings. We adhered to the procedures described by Thomas (2006) to assess the trustworthiness of the category system, such as independent coding, coding consistency check, and stakeholder checks. The final stage in the process (reporting the findings) emphasizes the top-level categories and uses detailed descriptions and quotations to illustrate the meaning of the findings.

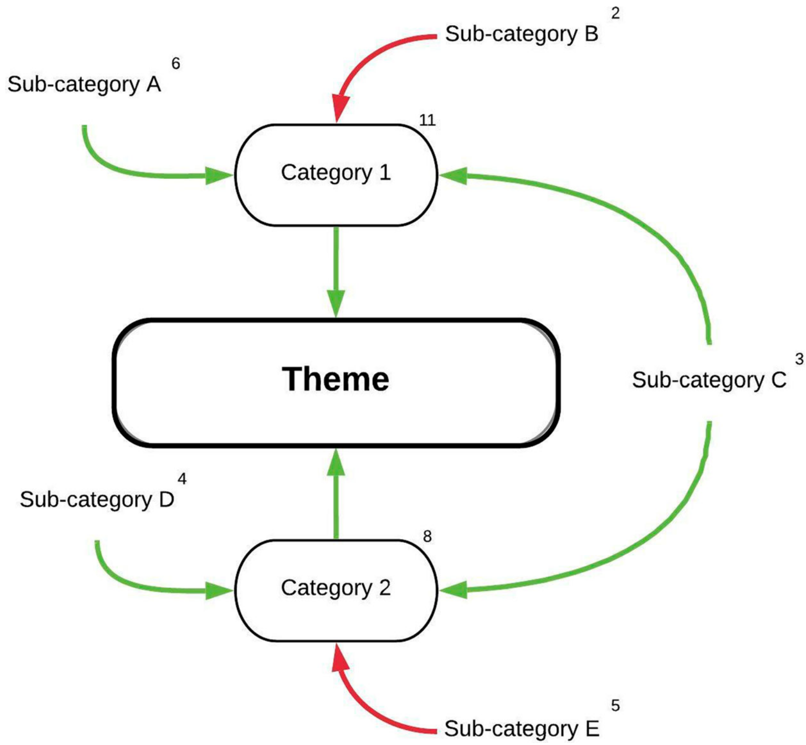

Figure 2. Example of visual representation of data analysis where green denotes a perceived strength and red denotes a perceived weakness between labels.

In this study, we analysed all responses from the two cohorts to open-ended questions in the interviews. The interview transcripts (raw data) were read closely several times before segments of student answers (utterances) were highlighted and grouped together (coded) under a common heading. The initial coding resulted in a large number of headings by each member of the research team who coded the raw data independently. The research team then met and discussed their lists of headings before refining and reducing them. This resulted in a set of subcategories which were then organized into a hierarchy depending on relatedness. In the case of this data, seven to nine subcategories mapped onto three categories, all of which had a primary theme in common (see Figure 2 for an illustrative example).

2.6. Data representation

Given the detailed nature of the data, reporting them in a clear and concise way requires care and creativity. This may be done through the integration of the qualitative and quantitative data. We have chosen to use *joint displays* as visual representations of the interview data and its analysis. A joint display is defined as a way to 'integrate the data by bringing the data together through a visual means to draw out new insights beyond the information gained from the separate quantitative and qualitative results' (Fetters et al., 2013). Joint

Table 1. Description of marks and attributes.

Mark	Meaning
Labels	Each label results from an iterative coding process and represents the name of a subcategory, category, or theme.
Numbers	The numbers beside each label represents the frequency with which that label was mentioned explicitly.
Arrows	Represent links between the primary theme and categories, and categories and subcategories.
Borders	The primary theme is contained within a square border, the categories have an elliptical border, and subcategories have no borders.
Attribute	**Meaning**
Label font and border format	Both font (size and boldness) and border distinguish subcategory (small font, no border), category (normal font, elliptical border) and theme (bold large font, square border).
Arrow direction	Represents the bottom-up approach
Arrow colour	Colour is used to indicate whether a given subcategory is a strength or a weakness of the category to which it is connected. Green is used for perceived strengths and red is used for perceived weaknesses.

displays provide a convenient way to represent large amounts of complex data of different forms.

Our joint displays show the primary theme and various categories and subcategories emerging from the interview data, along with their interrelatedness. We integrated the qualitative data from the interview transcripts with a tally of the frequency with which they were mentioned by students. This results in a visual representation that conveys information about the primary theme, the categories and subcategories, their frequency, and their relationship to each other. An illustrative example is provided in Figure 2 along with a description of the information contained within it.

Figure 2 is an example of data representation that uses text, numbers, and colour to convey information. These are just some of the ways Kirk (2016) mentions for encoding data, all of which derive from combinations of two properties: *marks* and *attributes,* defined by Kirk (2016) as follows:

> Marks are visible features like dots, lines and areas. An individual mark can represent a record or instance of data. (Kirk, 2016, p. 151)

> Attributes are variations applied to the appearance of marks, such as the size, position, or colour. They are used to represent the values held by different quantitative or categorical variables against each record or instance. (Kirk, 2016, pp. 151–152)

Our use of a general inductive analysis results in the creation of a primary theme, categories, and subcategories that can overlap. The result can be quite complicated, and Figure 2 provides a visual representation of the same data that can help the reader understand the complex nature of the data. Table 1 describes the marks and attributes used in Figure 2 and the information they convey.

3. Results and discussion

The presentation and discussion of results are combined in this section. The section comprises three distinct parts. Firstly, we illustrate how we established the subcategories, categories, and the primary themes from the raw data. We then discuss the similarities and differences between the two cohorts we investigated. Finally, we profile a typical student who participates in an intervention such as this. This profile is built from the results obtained during the interviews and is followed by a discussion of the results in the context of achievement goal theory.

3.1. Interview data

In the case of the interview data, the general inductive analysis approach was used to identify the subcategories, categories, and primary theme described in the following paragraphs. To exemplify the initial coding process, we discuss the following excerpt[2]:

1	Interviewer: Did you enjoy the MS225[3] tutorials?	Uncoded
2	R: Like I did.	Uncoded
3	A: Yes definitely.	Uncoded
4	R: They were interactive.	Interaction
5	A: Yes, they take a different approach than what an ordinary lecture would be,	Different from lecture
6	instead of just everybody sitting in silence	Active learning
7	you kind of interact with people and get their thoughts on different things	Peers as a resource
8	and you get to discuss some just freely	Peer discussion
9	instead of just being silent and listening to somebody in a lecture	Active learning & different from lecture
10	I think it's far more helpful.	Beneficial
11	R: Yes, you're actually learning by doing it	Active learning
12	and talking to someone beside you	Peer interaction
13	or if you're walking around obviously you're going to help as well. (16-9)	Interaction with tutor

Initially, we coded each line in this excerpt as shown above. Note that almost every line gets a single unique coding, and that some lines may remain uncoded. In the excerpt above lines 2 and 3 remained uncoded because the responses therein are of the yes/no type without conveying any further information.

The second stage of the process started by listing and tallying all initial codes. We then refined the list in two ways: we looked for entries with very low tallies, and attempted to bring these in under a similar, more frequently identified coding we termed a *label*. For example, the codings 'peer discussion' and 'peers as a resource' occurred rarely, but we found that they could be subsumed within the 'peer interaction' label. In other cases, this was not possible: a distinct label remained where required. While these are clearly of some interest, they are unlikely to reflect the perceptions of the general population and were therefore disregarded in what follows.

With 14 labels established in this way, we then sought to establish a relationship between them. In two cases, we found that pairs of these labels were identically linked to more than one other label, which led us to combine these pairs into a single new label. In one case, the labels 'less pressure' and 'good atmosphere and friendly environment' both linked to 'group work' and 'interaction pattern' in the same way; this caused us to group these two together under a single label 'atmosphere'. Likewise, 'peer instruction' and 'interaction with tutor' could be grouped together under the label 'interaction pattern'.

Thus, we established a final list of 12 related labels. For example, we found that the labels 'tutor is not the lecturer' and 'vulnerable to large numbers' both related to 'interaction pattern'. This led us to designate the latter label as a *category* while the two former labels were labelled *subcategories*. Note that categories and subcategories both started as the same label

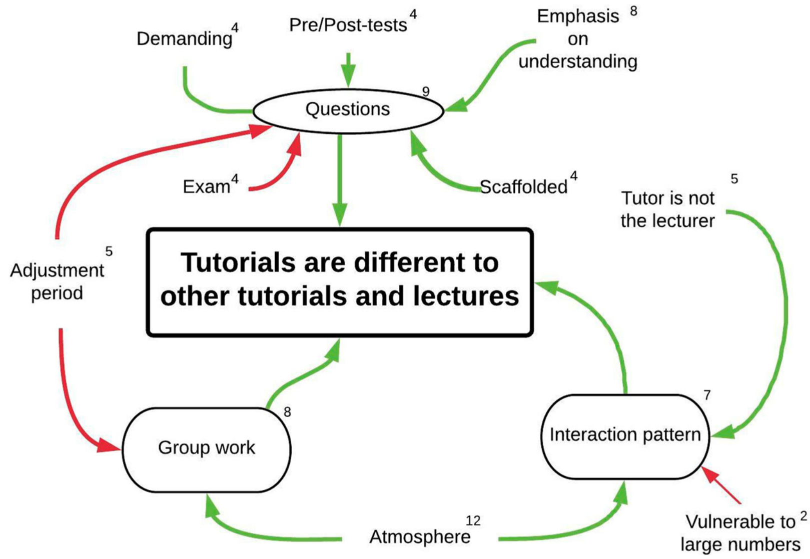

Figure 3. Visualization of 2016–17 data analysis. Green denotes a perceived strength and red denotes a perceived weakness between labels.

used to uniquely code (part of) a student utterance; the distinction arises purely from how we connected the labels, and there is no relationship between the number of occurrences of each label.

In total, we identified nine subcategories, three categories, and one primary theme across both years of interviews. The primary theme identified was *tutorials in the intervention are different to other tutorials and lectures*. The 'difference' referred to in the primary category has multiple layers. Primarily it speaks to the experience students had during tutorials and how they contrast that with their experience in lectures and other tutorials they have experienced during their course.

3.2. Emerging themes

The one primary theme and three categories that emerged from analysis of the semi-structured interviews were the same for both cohorts. The primary theme that emerged is that the tutorials delivered as part of the intervention are different from other tutorials and lectures; the three categories can be described under the headings of (1) the kinds of questions posed; (2) group work; and (3) interaction pattern. The joint displays of Figures 3 and 4 show the differences at the subcategory level. We discuss at the level of individual cohorts, before discussing the differences between two cohorts.

In both cohorts, students expressed an appreciation of the prevalence of group work, which led to an environment that students felt was interactive (See Appendix A for illustrative excerpts from the interviews). This point was mentioned in 16 of the interviews, with the previous excerpt from an interview with two students (R and A) from 2016-17 being typical.

These students also reported that the tutorials were far more interactive and engaging than they are used to (as did 15 others), and while five interviewees in 2016-17 said this took

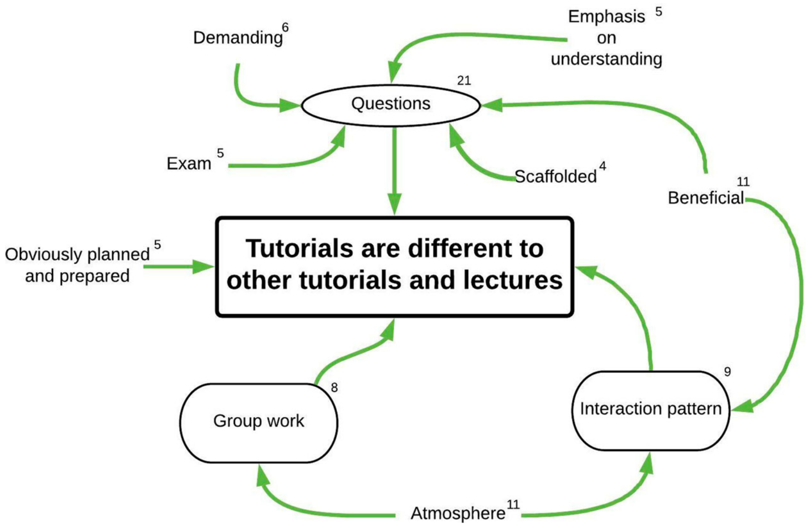

Figure 4. Visualization of 2017–18 data analysis. Green denotes a perceived strength between labels.

adjusting to, they all said this was a positive aspect of their experience. The students stated that they felt less pressure in relation to tutorials, due in part to the relaxed atmosphere that developed but also because they felt there was no preparatory work required in order to attend. Six students noted that they found it easier to ask questions at these tutorials than other mathematics tutorials. This point is clearly made when a student is comparing the intervention to other tutorials:

> H: … in tutorials the lecturer will just sort of stand up on the board and just write out solutions but you wouldn't really have a chance to like ask a lot of questions or say 'I can't understand this can you go back over it again?' because you'd be holding up like 34 people. (16-4)

A difference in the type of questions asked of the participants during the intervention was apparent to the students and was mentioned in some form in every interview. The participants saw a clear change in the emphasis of questioning toward understanding the concepts (mentioned in 13 interviews) and they appreciated that the questions were scaffolded or 'built up from basics' (mentioned in 14 interviews). While the idea of a change in emphasis toward understanding was appreciated by students, it caused a small number to feel anxious (mentioned in five interviews). This point is well articulated by a student as follows:

> C: There was a little bit of anxiety on my side from seeing that I hadn't been doing exam questions or I hadn't been doing more concrete problems up until a certain point. Now, I know that's how it's structured and it did work well, but it's probably no harm to have a direction of, these are tutorial sheets you could be doing in your own time in the MLC[4] taking over, and to just kind of keep you working. Just, I'm sure I'll fly the exam in two days, but it would help when … just set your mind at ease knowing that you have done the work. (17-5)

In this instance, the anxiety is related to their performance on the terminal exam, and how it may be adversely impacted by not spending all their time on 'exam questions' (17-5). They appreciate the manner in which they have learned ODEs during the intervention but cannot disregard the study skills that they have relied on until now.

A positive finding relating to the category of *questions* is that students reported, as a good thing, the questions being more demanding. In addition to this, they saw the guided inquiry approach as a way of learning that has a longer lasting impact and that will stand them in better stead in their future studies and careers. When asked whether they found the guided inquiry approach frustrating, one student said the following:

> C: Oh no, that's key. It's whenever … whenever you're just given or presented with the answer, that's absolutely no use to you. You don't … it's never going to stick in my head if I'm just given the answer, this is how you do it. Whereas, you know that kind of thing where if you eventually stumble across the answer yourself, you're more likely to remember it, more likely to … (17-5)

Their desire to learn in a more meaningful manner is in line with the design and intentions of the module and contrasted with the behaviours of other students described by Skemp (1976, p. 21) 'whose goal is to understand instrumentally'. Evidence of the existence of this type of student is provided during our comparison of both cohorts, but the fact that the former is seen in greater numbers is reassuring.

3.3. Summary of the cohort analyses

We used a general inductive approach to analyse the interview data from each year they were conducted. Both times, *the tutorials are different to other tutorials and lectures* emerged as the primary theme and *questions, group work,* and *interaction pattern* emerged as categories. With respect to questions, students appreciated the switch in emphasis towards understanding, and scaffolding questions. Some students grew frustrated at the apparent disconnect between questions of this nature and their terminal exam, but conceded that it would be more beneficial to their long-term learning. A friendly environment and a feeling of reduced pressure led to a good atmosphere. The interaction pattern also encompassed the tutor and lecturer being different people, which students described as positive. The overall nature of the feedback given by students was overwhelmingly positive. They were eager to learn and more than willing to apply themselves when they saw a benefit attached to the work. A direct comparison between the 2016-17 and 2017-18 cohorts is made next.

3.4. Interview data: comparison of 2016-17 and 2017-18

Drawing comparisons between data sets allows us to identify relative changes in feedback. Much can be learned through comparing successive cohorts and any differences identified may be evidence of the impact of the cycles of revision the intervention has undergone, with the proviso that other factors can also contribute to changes in feedback (for example a change in interviewer and differences between cohorts).

There were seven notable differences between the data sets after analysis. Two subcategories arose for the first time in 2017-18, one subcategory took on a new meaning, and four terms that appeared in 2016-17 did not reappear in 2017-18. Table 2 provides a summary of the differences between the cohorts. We discuss each of these differences in turn, beginning with the subcategories that only appeared in 2016-17 before discussing the ones that emerged for the first time in 2017-18. In each case, we describe what the subcategory means before explaining why we think it appeared in one year and not the other.

Table 2. Differences in feedback between both cohorts who have participated in the intervention.

No.	Subcategory	Description
No longer appearing		
1	*Adjustment period*	It took students time to adjust to the change in pedagogy
2	*Pre/post-tests*	The pre/post-tests were cited as a way to track progress
3	*Tutor and lecturer are different*	The tutor was a different person to the lecturer
4	*Vulnerable to large numbers*	Interaction time with the tutor is dependent on the number of students attending the tutorial.
Additions		
1	*Beneficial*	The tutorials were beneficial to the learners, which drove attendance. Guided inquiry was also recognized as beneficial for longer-term learning.
2	*Obviously planned and prepared*	Students noticed and appreciated that the tutorials were well prepared and were planned with their backgrounds in mind.
Changes		
1	*Exam*	The relationship between the intervention and the exam was viewed negatively and positively by students in each cohort. The balance changed from 3–4 to 5–1 (positive to negative).

3.4.1. Subcategories that appeared in 2016-17 only

No *adjustment period for the change in pedagogy* was reported by the 2017-18 cohort. This was in contrast to the 2016-17 cohort, where a minority of students expressed difficulty adjusting to the change in pedagogical approach. An excerpt from Interview seven is representative of this point, which was mentioned in five interviews in total.

> Z: It's a bit frustrating when you're being asked do you understand that, and every other maths module we've ever done is like, just show me how to do it and that's it. I don't really care after that. (16-7)

Braun et al. (2017) caution practitioners of active learning methods to expect resistance from students, particularly at the beginning. They give the example of students who have had success with traditional methods potentially feeling threatened by the new environment. We posit that the thinking exhibited by the students described by Braun et al. (2017) is also shown by the students represented by the excerpt from Interview 7. This concern was not raised in the 2017-18 interviews, most likely due to a portion of the continuous assessment (CA) marks for the module being assigned to tutorial attendance and participation.

While the reallocation of CA marks was successful in incentivizing high achieving students to attend, it undoubtedly motivated students who are driven primarily by their final grade to attend, with one student from 2017-18 stating simply that

> D: Well if they contribute to CA I'll go. (17-16)

The *vulnerability of the pedagogical approach to large numbers* did not emerge as a subcategory from the analysis of the 2017-18 data. In 2016-17, student feedback highlighted a reduction in student-tutor interaction time when there are a high number of students at the tutorial. Two of the nine groups of students interviewed cited this when asked about areas of potential improvement, with one of the groups acknowledging

> Z: But that's the type of tutorial, you need a smaller group. (16-7)

Certainly, a large student-tutor ratio (between 18 and 30 to 1 across the intervention) has a greater effect on classrooms that adopt an inquiry-based approach than ones that use a lecture format but we believe this shortcoming is more than made up for in other areas, as illustrated in Figures 3 and 4. Also, both of the interviews referred to the same instance: a one-off situation at the beginning of semester where logistical constraints required all ($N = 70$) of the students to attend the same tutorial. The presence of the lecturer meant that the facilitator-student ratio did not increase, but the atmosphere – as reported in both interviews – was adversely affected none the less. The reason this issue was not cited by students during the 2017-18 is a consequence of better planning (an extra tutor was present for this one tutorial) and a smaller student cohort in 2017-18.

The subcategory *the tutor and the lecturer are different* did not emerge in 2017-18**.** This point is largely self-explanatory and was mentioned in 2016-17. The fact remained the same across both years, but details of the interviews changed. In 2016-17, a member of academic staff unattached to the project conducted the interviews whereas in 2017-18 the tutor conducted the interviews. We believe that the students who did mention this felt it necessary to do so to the 'outside' interviewer during the first set of interviews but not to the tutor, who conducted all interviews in the following year.

The final point that appeared in 2016-17 but not the following year was that the pre/post-tests allowed students to observe their progress across tutorials. This point was cited by four of the nine groups during 2016-17 and described by one group as follows:

> X: Post-test and pre-test, we would have never done that in a tutorial before. If you actually look at them and before you know something, like if you actually know something, and it's kind of good then, and it's actually kind of useful. (16-7)

Even though the occurrence of pre/post-testing was an obvious difference with other tutorials, it was not mentioned in the second year.

The four subcategories that appeared in 2016-17 but not in 2017-18 were identified as: *no adjustment period for the change in pedagogy*; *the vulnerability of the pedagogical approach to large numbers*; *the tutor and the lecturer are different*; and *the pre/post-tests*. With the exception of the point on pre/post-testing, the difference between cohorts can be attributed to a change in interviewer (the tutor and the lecturer are different) or a revision to the intervention (remaining subcategories). In the case of the points on pedagogy, we consider the nature of the revision and the outcome of its implementation to be an improvement to the intervention. We will now discuss the emergence of four new subcategories during the analysis of the 2017-18 interview data.

3.4.2. Subcategories that emerged in 2017-18

In 2017-18, a positive aspect of the link between the intervention and exams was mentioned during student interviews**.** Analysis from the 2016-17 interviews revealed an apparent *dis*connect between the tutorials and the exam that caused anxiety for some students. While they appreciated the content and mode of delivery of the tutorials, and even preferred them to other tutorials, they struggled to see how they related to the exam as exemplified by the excerpt from interview 17–5 reproduced earlier in this section.

Analysis of data from 2017-18 revealed a reduction of this concern (from four of nine interviews to one of twenty-one interviews) and also yielded a positive link between tutorials and exams (mentioned during five of twenty-one interviews). An example of this is the following response:

E: Yeah, no definitely, it was towards an understanding, that you look at what's in the exam eventually and you know what you're doing not reaming off a method that you just learned off. (17-8)

Some students explained how the tutorials have left them in a strong position heading into their exam and that they would even use them as a study aid. Possibilities for the reduction in student dissatisfaction at the perceived disconnect between the exam and the tutorials include the reallocation of CA (10% of the final grade was attached to tutorial attendance and participation) and the delayed post-test question appearing on the previous year's exam. As described previously, the reallocation of CA acts as motivation for some students to participate in tutorials. This, coupled with students seeing a 'typical' tutorial question on the terminal exam from the previous year, may have shown students how the intervention relates to their final grade.

The idea of the tutorials being beneficial appeared repeatedly in 2017-18, resulting in its appearance in Figure 4. It conveys two meanings here. Firstly, it relates to *Interaction Pattern* because students reported that their interactions with peers and with the tutor were of benefit to their learning, which factored into their decision to attend as outlined in this comment

Q: But I felt like this tutorial, it felt more like kind of even going back into secondary school, like he was really like, very engaging. And I felt like I should go because I know I'm going to actually do something in this. Important yeah. And very engaging, and like I knew like when I'd go in there I knew I'd get an hour's work done. Every time I was in. (16-3)

Secondly, students saw the type of questioning used throughout the intervention as beneficial. This is closely related to how the manner in which students learn may affect how long their understanding lasts (Kogan & Laursen, 2014). The point is made in this section that six of the twenty-one students believe learning ODEs through guided inquiry equips them well for any subsequent ODEs they meet in their studies. The long-term benefits associated with learning through guided inquiry (Kogan & Laursen, 2014) were supported on a small scale using an additional delayed post-test as part of the larger research project (Hyland, 2018). This delayed post-test was given to a subset of students one year after they participated in the intervention. All of the nine respondents to this delayed post-test said that the tutorials were beneficial to them in a subsequent mechanics module (taken one year after MS225) where ODEs are prevalent, and seven of the nine students referenced a different approach as the key to this (Hyland, 2018).

The same point can be made specifically of physics students. Each physics student that was questioned about how the approach affected their long-term learning agreed that both the content of tutorials and the manner in which they are delivered is of more benefit to them moving forward than traditional instruction. This was in reference both to subsequent modules and to eventual careers, highlighted by this student response:

M: Yeah, I think so, I suppose going back to the Vibrations and Waves[5], I think had I known what I know now about them, it would have been easier to understand where all the equations where coming from. I think last year we kind of just said, this is a formula, we're going to use it, but I can't remember exactly them saying, this is a differential equation. And say even just a quick introduction as to why it's cosine, sine, whatever. And then we'll say the exponentials for damping and stuff like that. So yeah, I think it's good. (17-14)

In fact, one student even suggested increasing the amount of modelling in tutorials when asked for potential improvements. The quote below was made by one of the students from the 2017-18 cohort in response to a call for potential improvements:

> D: I think just have more examples to do with, like give them a sheet of like scenarios give them like do them in a way like an experiment was conducted or something like these things and that the differential equation involved or find the differential equation like we did that in one of the tutorials I remember it was kind of like I think it was baking a cake or something and the temperature going down, and that kind of stuff but I think just having something like that and just like if this was the situation that happened, because we did labs for the and we understand like things kept constant other stuff, like the whole like tutorials based on that one. (17-16)

The final subcategory to emerge from the 2017-18 interview data was an acknowledgement from students that the worksheets were *obviously prepared and personalized*. This is closely linked to our previous point on how beneficial the intervention was to students. In one instance, students describe the tutorials as follows:

> J: We definitely look forward – well, not look forward – but it's one of our favourites to go to where you'd be, not as if you'd be sitting there. You'd be actually doing something, you'd be trying to encourage and-
>
> K: You'd be concentrating the whole time.
>
> J: Yeah. And the 45 minutes you're like, yeah I learned something today. (16-5)

There was a variety of reasons attributed to the students' perception that the worksheets were obviously planned. They referred to the questions and structuring of the worksheets, the pre/post-tests, and the recognition of the prior learning of the students. In the following exchanges, four students (eight in total) expressed an appreciation for the intervention incorporating their primary subject:

> K: Because we're physics, population. Necessarily, it mightn't be on our exam but it's beneficial to learning rather than just, here are the numbers, here are the formulas, do it. You don't know what you're doing it kind of for, so … (16-5)
>
> F: Yes, and like a lot of the stuff that we were doing is like relevant to us as physicists as opposed to last year where it was like linear mathematics and I still don't have a clue about any of that stuff. (16-4)
>
> J: Rather than just telling you, he spent of lot of time on modelling. (16-5)
>
> Y: A way of explaining things and bringing – like to the population or certainly the first question you did there, he was really relating it to us. (16-7)

Whether tutorials will be of benefit to students appeared repeatedly during interviews and is the strongest intrinsic motivator we found driving tutorial attendance.

When we compared the data that informed the research question between both cohorts, we saw strong agreement. The primary theme (*tutorials are different to other tutorials and lectures*) remained the same, as did the three categories (*group work*, *questions*, *interaction pattern*). There were changes at the lowest level, however. In total, four subcategories did not reappear (*adjustment period*, *pre/post-test*, *tutor is not lecturer*, and *vulnerable to large numbers*), two new subcategories (*beneficial* and *obviously prepared and planned*) emerged, and one subcategory (*exam*) took on a new meaning. We described each of these changes in this section, which are also summarized in Table 2. With the exception of the disappearance of the subcategory *pre/post-tests* from the 2017-18 data, each of these changes are explained by elements of revision or circumstances surrounding the interviews.

We will now describe the characteristics of a typical student who engages with the intervention. This profile is based on the results described in this section.

3.5. Student profile

Analysing the interview data was very informative. The research question was designed to inform the research team on the perceptions of students who completed the intervention, with the expectation that this feedback could improve aspects of its content and delivery. We believe that it has been successful in doing so and will continue to provide insight that will improve the intervention as a product. We found the candour of the students during the interviews particularly useful and note that their contributions have influenced most, if not all, the revisions since we began.

We reported on many aspects of the intervention that emerged during the analysis of interview data. Within this, there was a significant amount of data that contributed to our understanding of our students. This allows us to attempt a description of what we consider to be typical students. Of these students we suggest the following:

- For many students, attendance is tied to how beneficial they deem a tutorial to be to their learning. They are willing to grapple with 'more demanding' questions as a result.
- Students appreciate when content is tailored to them as learners with regard to difficulty level and relevance to study programme. This point is expressed more strongly by the physics students. ODEs are of direct relevance to their programme of study, and content is more readily adapted to their needs from this perspective. Alignment of mathematics tasks with core areas of study should go beyond using a word paragraph to (for example) hide initial conditions for a given initial value problem. Students consider many end-of-chapter 'word problems' to be situated in unfamiliar contexts. In many instances, they learn to scan the text and extract the necessary information before solving the problem without ever grappling with the context. Our students (both those studying physics and education) reacted very well to the modelling tutorials they completed and some of the physicists even requested a more sophisticated version of the modelling tutorials to be developed for second order ODEs.
- All of the students we interviewed noticed the shift in emphasis toward understanding and saw the role of scaffolded questions in this. Some students needed time to adjust to the change in approach, however.
- Students recognized the benefit in partaking in guided inquiry. In this case, they agreed that the understanding they have of ODEs (which used guided inquiry during tutorials) will last far longer, and is far more accessible than their other mathematics modules (which used traditional instruction during tutorials).
- Students noted that it was easier to ask questions during the tutorials than other mathematics tutorials. This is due at least in part to the prevalence of group work which is in line with the findings of Walker (2015).

- Students reacted to the environment. The nature of these tutorials required students to work closely with their peers and the tutor. This required relationships to form and grow and took time for some students, but once it did it resulted in an excellent learning environment, one in which students asked and responded to questions within their group and with the tutor; one where they gradually gained control and eventually autonomy over their learning (as it pertains to *delegation of authority* (Cohen et al., 1997)). Interaction patterns and environment were major themes of the interviews, as was the notion of pressure and anxiety. These positives were amplified and negatives were mitigated to an extent when students developed stronger relationships with the other tutorial participants.
- Even students who appreciated the intervention and saw long-term benefits retained a focus on the final exam. We saw this happen far less frequently for students who engaged with the intervention vs. students who attend recitation sessions, but it was still apparent in the 2017-18 data. There were several factors that reduce the focus on the final exam, but it does not disappear completely. Students who 'buy in' to inquiry learning believed that there will be a long-term effect on their understanding (17-11) which gave them confidence and reduced their focus on the objective of passing the module. Reallocating CA marks as described above required students whose primary motivation is achievement to engage with the intervention in a meaningful way. Finally, the appearance of the delayed post-test on the past exam paper gave students a tangible link between the intervention and succeeding in the terminal exam. Despite this, one student out of twenty-one who were interviewed in 2017-18 wanted a more visible link between the worksheets and the exam. At this level, we believe only a cultural shift will eliminate this type of thinking; certainly, a twelve-week intervention for third-year university students will not make enough of a difference in isolation.

The above points are intended to describe the outcomes of typical students who participated in the tutorials we designed. These points are supported by a representative quote and tally from each year (Appendix A).

3.6. Achievement goal theory

We will now consider the results through the lens of achievement goal theory. We consider whether the students perceived the learning environment as having a mastery goal structure and sought evidence in support of or against this in the interview data. We also consider the students' personal goal focus (mastery or performance), which has been shown to correspond with the goal structure of the classroom setting (Anderman & Midgley, 1997; Roeser et al., 1996; Ryan et al., 1996). This analysis resulted in the following:

3.6.1. Goal structure

When describing our conceptual framework, we said that the learning environment in our tutorials has a mastery goal structure partly because it contained three elements that Ames (1992) suggested would foster such a focus.

We present these elements along with data from student interviews in the form of labels from the joint displays and individual excerpts, all of which are included in the previous section and are representative of our students' perceptions of the intervention.

Ames (1992) first feature is the design and use of challenging tasks that carry meaning for students: a feature that informed the design of the intervention. The labels *Demanding* and *Scaffolded,* which appear in the joint displays for both years of data, are evidence of such tasks being noticed throughout the intervention. In addition, the label *Obviously planned and prepared* (explained above) carries evidence of this feature. Ames (1992) also stressed the importance of how the teacher interacted with students, noting that growth and improvement should be emphasized (above competition with peers). We see evidence that this was perceived by the students across several labels (for example, *Beneficial, Emphasis on understanding, Demanding,* and *Pre/Post-tests*) which all have roots in a deeper, longer-lasting form of learning. Finally, Ames (1992) suggested that increasing autonomy and allowing more opportunity for choice would foster a mastery-oriented goal structure. Though no labels exist that exclusively convey this meaning, there is evidence from the larger research project (Hyland, 2018) that demonstrate the students' acknowledgement of, and appreciation for the autonomy and choice afforded to them during tutorials. The clearest example of this is during the modelling tutorials, mentioned above, where problems of varying difficulty in multiple contexts are presented to the groups to work on. The students use take this opportunity to choose which problem they want to work on first and are free to proceed as they saw fit.

A similar study to Ames (1992) was carried out by Meece (1991), who identified four key aspects of instruction that appear common to mastery-oriented students: promoting meaningful learning, tailoring instruction to level and to personal interest, supporting autonomy and peer collaboration, and emphasizing the intrinsic benefit of learning. Meece's four traits share a lot of similarities to Ames' work, and evidence in support of them are among the most frequently occurring labels (*Emphasis on understanding, Scaffolded, Group work*) across both cohorts.

Similarly, Stipek et al. (1998) and Urdan et al. (1999) reported that students perceive a shared mastery goal where teachers emphasize the importance of understanding the information presented in the classroom and the value of learning. There is evidence of each of these features being alluded to by students in the labels (for example, *Emphasis on understanding*) that appear in our joint displays across both years of interviews. Finally, enjoyment, which appears implicitly throughout the intervention (Hyland, 2018) and is mentioned above is positively related to mastery goals (Daniels et al., 2009).

3.6.2. Personal goal focus

Although the students' personal goals correspond with the goal structure of the classroom (Anderman & Midgley, 1997; Roeser et al., 1996; Ryan et al., 1996), they do not necessarily align perfectly. We have argued that the data demonstrates a mastery goal focus among students who took part in the intervention, however some evidence of performance goals was also found.

There were three pieces of data that represent a performance goal focus: the negative association between the type of questions on exams and the questions in the intervention; the adjustment period for students to the teaching approach; and the acknowledgement that continuous assessment drives attendance. The first two points can be seen in the initial cohort of students and are represented by red connections on the joint display. Each concern (mentioned during four and five interviews respectively) did not recur in the second year of the intervention as outlined above. Interview excerpts (D: Well if they contribute to CA I'll go. (17-6)) in the results section support the final point on performance goals. Each of these facets of the intervention can cause anxiety among students, which Daniels et al. (2009) found to correlate positively with performance goals (discussed below). We acknowledge the presence of each type of personal goal focus but suggest that the ratio would be significantly different were it not for the goal structure of the intervention.

A final point on individual goals is whether students adopt multiple goals for learning simultaneously, which Midgley et al. (2001) acknowledged as a possibility. We offer to the following excerpt from an interview to this discussion:

> C: There was a little bit of anxiety on my side from seeing that I hadn't been doing exam questions or I hadn't been doing more concrete problems up until a certain point. Now, I know that's how it's structured and it did work well, but it's probably no harm to have a direction of, these are tutorial sheets you could be doing in your own time in the MLC taking over, and to just kind of keep you working. Just, I'm sure I'll fly the exam in two days, but it would help when… just set your mind at ease knowing that you have done the work. (17-5)

This student's entire interview transcript contains multiple instances where a clear mastery goal is reflected in their responses. This response, however, acknowledges the anxiety the student experiences from not preparing explicitly for the exam, which is a characteristic of a performance goal focus. Further to this, Daniels et al. (2009) reported a positive relation between performance goals and anxiety, which is also present.

4. Conclusions

We have investigated the perceptions of students who participated in a set of tutorials designed to improve their conceptual understanding of ODEs, while also developing their instrumental understanding. Interview data showed that students in both cohorts held similar views of the tutorials, which had a positive effect on the students' instrumental understanding and concept image (Hyland, 2018). The students noticed the change in emphasis toward understanding the intervention was trying to instil. They cited the change in questioning style most frequently as being where they saw this change, but also noted the prevalence of group work, and change in interaction pattern as keys to its success. Together, these three categories formed the basis of the analysis, with all other points falling under these headings and under the overarching theme of *the tutorials in the intervention are different to other tutorials and lectures*. Overwhelmingly their perceptions of the tutorials were positive, and in line with the research outlined in the opening sections of this paper. The feedback validates that the intervention did indeed provide a means of enacting an active learning approach, that was noticed and engaged with by the students. Beyond this, the intervention designed for MS225 has since been adapted for other service modules at DCU, with similar improvements noted.

Viewing the results through the lens of achievement goal theory was very insightful. We found evidence that the students perceived our intervention as having a mastery goal structure, and that a large majority of the students possessed a personal mastery focus, though some evidence also existed of personal performance goals.

Notes

1. We initially worded this as 'how students experienced the intervention' but the word 'enjoy' recurred in interviews, leading us to reword our line of enquiry.
2. The letters A and R are used to distinguish between respondents within the interview excerpt.
3. MS225 is the code for the service-taught module on ODEs
4. The MLC (Maths Learning Centre) is a drop-in mathematics support centre located in the University Library that is free for students to use. In this interview, the student is referring to the MLC as a place where they would study mathematics in their own time – another service offered by the MLC.
5. A module taken by students during the second year of their degree.

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Appendix A . Additional interview excerpts.

Point	Quote	2016-17 tally (9)	2017-18 tally (21)
Tutorials are beneficial (mentioned in previous section) and questions are more demanding.			
More demanding	Interviewer: Would you … are they more challenging because you're working for the full hour in that way? C: Oh, a hundred per cent. You can't … you know yourself, the only way I ever learn in maths is by making mistakes and by trying to work my way through things. If somebody just pops up the board and says, oh you can't do Q3, here's exactly how it's done, that's no benefit to me. Whereas, when you are scattered around the room and you're just given just enough information to steer us in the right direction, or to get us back and focussed towards a more linear … . that was brilliant. (17-5)	4	2
Tutorials are personalized, within this modelling (for example) is mentioned.			
Well prepared and personalized	Y: A way of explaining things and bringing – like to the population or certainly the first question you did there, he was really relating it to us. (16-7) Z: It was well-prepared. (16-7)	4	4
Modelling	J: Rather than just telling you, he spent of lot of time on modelling. Because we're physics, population. Necessarily, it mightn't be on our exam but it's beneficial to learning rather than just, here are the numbers, here are the formulas, do it. You don't know what you're doing it kind of for, so … (16-5)	3	3
Shift towards understanding, scaffolded questions, and adjusting.			
Emphasis on understanding	Interviewer: So is it that you know the material now, or do you understand it? Do you think you understand it as well as just knowing how to do them? H: I think – I understand. I don't understand it completely, but I – yeah, I don't understand it completely but it's a lot better than most modules because 95% of modules to be honest we don't understand. A lot of it, do you know. To be honest, I need to say it. O: It's pure – for a lot of them, it's pure rote learning. It's just layers and layers of notes, learn this, do it this way, don't ask why, just do it. H: But there's like – compared to other modules, definitely, yeah. (16-3) M: There was a lot of thinking. Interview: So you don't think maths is thinking? M: No, but a lot of it is procedure, a lot of this is a lot of deep thinking. (16-1)	8	5
Scaffolded Qs	H: I'd say the types, because they're very building. They're very like scaffolding questions. They won't just throw you into the deep end. And then like, if you kind of half knew it and then your friend didn't have a clue then you'd be like, well I know this. (16-3)	5	9
negative adjustment period	Z: It's a bit frustrating when you're being asked do you understand that, and every other maths module we've ever done is like, just show me how to do it and that's it. I don't really care after that. (16-7)	5	0
The long-lasting effect of guided inquiry			
Guided inquiry results in longer lasting understanding	V: When asked whether the manner in which they learned ODEs will better prepare them for using ODEs in the future 'Oh yeah, honestly I would, I'm not saying that the question has tempted to be set up, but you know that, because say for … like I was saying, calculus or something, I'd my exam the other day and I thought it went really well. If I was to go back … if I was to do calculus even in May, the exam, I'd have to put in just as much study as I did for this one, maybe a tiny bit less, whereas if I'm … after I used differential, if I was to do it again in May, the amount of study I'd have to do would be definitely a lot less than what I have to do for this, because I'd remember things more. (17-11)	4	6

	Interviewer: Is the 'just enough information' approach does that get frustrating over a while or … ? C: Oh no, that's key. It's whenever … whenever you're just given or presented with the answer, that's absolutely no used to you. You don't … it's never going to stick in my head if I'm just given the answer, this is how you do it. Whereas, you know that kind of thing where if you eventually stumble across the answer yourself, you're more likely to remember it, more likely to. (17-5)		
The effects of the learning environment.			
An appreciation for the prevalence of group work, leading to a pleasant working atmosphere	Interviewer: Did you enjoy the MS225 tutorials? R: Like I did. A: Yes, definitely. R: They were interactive. A: Yes, they take a different approach than what an ordinary lecture would be instead of just everybody sitting in silence you kind of interact with people and get their thoughts on different things and you get to discuss some just freely instead of just being silent and listening to somebody in a lecture I think it's far more helpful. R: Yes you're actually learning by doing it and talking to someone beside you or if you're walking around obviously you're going to help as well. (16-9)	8	8
Less pressure: atmosphere	Interviewer: And did you always know more after? M: Yeah, we wanted more. T: It actually worked in the class, then if you needed a hand, he was there to help. But at the same time, if you wanted to bop on by yourself. (16-1)	4	2
Less pressure so easier to ask questions	Comparing the intervention to other tutorials H: … tutorials the lecturer will just sort of stand up on the board and just write out solutions but you wouldn't really have a chance to like ask a lot of questions or say 'I can't understand this can you go back over it again?' because you'd be holding up like 34 people. (16-4) Q: You weren't afraid to ask a question, or help with something that maybe we'd moved on from, but coming back to it. Or whatever, yeah. (16-3)	4	5
Less pressure: no prep work	Z: You went with a pen and a piece of paper. (16-7)	4	1