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Supporting student teachers' enactment of relational mathematics in the classroom: an Action Research study

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ABSTRACT

The problem of enactment describes a teacher's inability to translate effective theories of teaching into practice. This is common for student-teachers, and invariably results in instructional practices that are ineffective and inconsistent with their beliefs. The motivation for this study was based on the observation that student-teachers move away from practices linked to relational understanding of mathematics and regularly fail to enact their mathematical knowledge in the classroom, despite having proven competence in the area. Therefore, the objective was to understand the reasons for this failure and make subsequent changes in the designated module to address the problem of enactment. Action Research was chosen as the research design considering the changes into one's practice and subsequently evaluating those changes. The current study represents Cycle 1 of a larger two-cycle Action Research study. Qualitative data were gathered from student-teachers using focus groups and classroom observations and analysed using Braun & Clarkes (2006) thematic analysis and the Mathematical Quality of Instruction framework, respectively. The findings indicated that the intervention was effective in addressing the problem of enactment, but further changes needed to be made in Cycle 2 to ensure it meets the needs of student-teachers and the pupils they teach.

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Problem of enactment; student-teachers; relational mathematics understanding; mathematical beliefs

Introduction

The current study focuses on student-teachers' experiences of learning mathematics in the four-year Bachelor of Education (B.Ed.) (primary) programme in an Irish university. As part of their Initial Teacher Education (ITE), each student-teacher is required to take a content-specific module called 'maths competency' which was developed to address deficiencies in student-teachers' mathematical content knowledge (MCK). As the focus of the current study, this module concentrates on exclusively addressing these deficiencies of student-teachers by developing their procedural and conceptual mathematical knowledge relevant to the primary school mathematics curriculum.

Additionally, student-teachers learn about mathematics pedagogy in the 'maths methods' module. In this module, student-teachers have opportunities to actively learn

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about relevant and innovate methodologies appropriate to teaching mathematics. Theories of emergent numeracy, number acquisition, and mathematical teaching and learning appropriate to primary school pupils are also addressed in the methods module.

Both maths competency and maths methods modules run concurrently across the B.Ed programme. Although both modules are separate entities, the assumption was that student-teachers' MCK would transfer automatically to the practice setting via pedagogy learned during the maths methods sessions. In this context, the practice setting refers to school placement (SP) which is the fulcrum of the B.Ed. programme.

The genesis of this study was the observation that a preponderance of student-teachers were, consistently over time, not enacting basic MCK learned on the maths competency modules while on SP. This was perplexing because they were required to attain a minimum 70% pass threshold on their maths competency examinations. Despite their proven competency, it was observed that they tended to teach mathematics instrumentally (i.e. procedurally), while neglecting the more fundamental mathematical knowledge that may lead to the development of pupils' relational (i.e. conceptual) understanding. This challenged the assumption that student-teachers' MCK would transfer automatically to the practice setting. Hence, this study is about exploring why student-teachers can demonstrate adequate MCK but are unwilling/unable to enact this knowledge in practice. Action research was the chosen methodology because the study is about interrogating and changing practices within the maths competency module to address the enactment problem in student-teachers' teaching of mathematics.

Literature Review

The problem of enactment

The 'problem of enactment' is defined as situations where novice teachers demonstrate an inability to translate effective teaching theories into practice and, as a result, fail to produce effective classroom learning (Allen and Wright 2014; Kennedy 1999; Zimmerman 2017). Furthermore, student-teachers tend to enact instructional practices that are inconsistent with their beliefs (Kennedy 1999) and this is one of the primary obstacles within their practice (Darling-Hammond 2006).

Researchers have been aware of the problem of enactment in ITE for decades (Zeichner and Tabachnick 1981; Zimmerman 2017). More than forty years ago, Zeichner and Tabachnick (1981) provided overwhelming evidence that the impact of ITE on student-teacher learning was 'washed out' by school experience (7). They identified several contributing factors including the influence of co-operating teachers, the ecology of the classroom, the bureaucratic norms of the school, teacher colleagues and even pupils. Veenman (1984) identified the 'harsh and rude reality of everyday classroom life' as the cause of this abandonment of beliefs for less favourable classroom practices (143).

Student-teacher beliefs about mathematics

The fundamental issue, which drives the problem of enactment, is related to the tensions that exist between university and school contexts (Valencia et al. 2009; Farrell 2023; Young, O'Neill, and Simmie 2015). These can be addressed by contextualising student-teachers' MCK and affording them sufficient opportunities to enact their knowledge in complex settings. (Darling-Hammond 2006; Kennedy 1999).

Student-teacher beliefs also play an important role in determining the nature of mathematics they enact on SP (Philipp 2008). Their beliefs about mathematics form a sort of mathematical worldview or ideology that shapes practice and can influence decisions about their mathematics teaching including what knowledge is worth teaching, and what social and pedagogical norms and routines should be established in the classroom (Speer 2005). Philipp (2008) contends that many student-teacher beliefs are absolutist in nature and may lead to the conclusion that mathematics is primarily instrumental.

Adding to the complexity, the belief that mathematics is fundamentally relational in nature does not necessarily mean this will be enacted in the classroom. For example, Sztajn (2003) found that classroom practice is determined not only by beliefs about mathematics, but also beliefs about society, pupils and education more generally. Furthermore, Raymond (1997) found that teachers' beliefs about the nature of mathematics are not necessarily consistent with beliefs about mathematics pedagogy, which ultimately take precedence in the classroom. This contention is consistent with Veneman's (1984) original finding that the problem of enactment is about the abandonment of one's beliefs for less favourable classroom practices.

Consequently, Philipp (2008) suggests developing more nuanced beliefs about mathematics teaching, and concludes that this could be achieved by motivating student-teachers to see pupils' overall learning needs, rather than mathematics per se being the primary consideration. Similarly, Hourigan and Leavy (2012) propose that it is possible to challenge student-teachers' limiting beliefs about mathematics through the provision of opportunities to reflect on and critique their own experiences with mathematics.

Knowledge domains in ITE: limitations of the traditional cognitive model

To support student-teachers to seamlessly integrate content knowledge and pedagogical skills into the complex task of teaching, as well as opportunities for reflection to challenge beliefs, requires ITE programmes to examine their dominant model of learning. Within the traditional model of ITE, the university provides the knowledge, the school provides the placement setting, and the student-teacher provides the individual effort to implement it all (Wideen, Mayer-Smith, and Moon 1998; Farrell 2023). A core feature of this model is the separation between foundation and methods modules (Grossman, Hammerness, and McDonald 2009) which results in the separation of theoretical knowledge from practical classroom work (Chaiklin and Lave 1996; Loughran 2006). This promotes propositional knowledge as the dominant epistemology, and relegates important aspects of teaching to individual modules which puts the responsibility on student-teachers to 'integrate it all' (Korthagen 2010; Lampert 2010, 24). Therefore, the challenge for ITE is to reconcile the specific nature of methods courses with the general nature of conceptual tools.

To address this challenge, Korthagen (2010) developed a model to integrate the cognitive and situated perspectives on student-teacher learning. It is built upon Lave and Wenger's (1991) conception of learning as a process of social constructivism and allows researchers to analyse the 'friction between teacher behaviour in practice and the wish to ground teachers' practices in theory' (Korthagen 2010, 98) by using the metaphor of a gestalt. Crucially, the model relies on critical reflection for learning which is essential to address unhelpful beliefs which may impede enactment of relational mathematics. There are three levels of teacher learning presented in the model:

Level 1 is the gestalt level which describes a cohesive whole of student-teachers' past experiences which are unconsciously evoked during practical experiences (Korthagen 2010). At this level, student-teachers are unaware of, and unreflective about their classroom behaviours. Furthermore, teaching is determined by a wide range of factors including beliefs, and is largely instrumental and automatic. At this level, learning is characterised by the awareness of previously subconscious behaviours.

Through reflection, student-teachers can move to the schematic level at which several similar situations occur and they develop a generalised knowledge about the familiar situations. Then, student-teachers form a conscious network of principles, namely schemata, that help them to describe practice. This is not abstract as it is driven by a student-teachers' desire to know how to act in specific situations.

At the theory level, student-teachers demonstrate a desire to understand practice in depth by examining relationships between schemata. Eventually, a theory or schema may be reduced to a single gestalt and used in a less conscious way, resulting in the emergence of new and improved student-teachers behaviour.

A pedagogy of teacher education

Grossman and Mcdonald (2008) developed a signature pedagogy of teacher education that allows for gestalt formation through realistic and meaningful engagements. Their pedagogy is based on approximations of practice and values the integrated nature of theory and practice, while offering student-teachers opportunities to 'practice elements of interactive teaching in settings of reduced complexity' (190).

The framework is built around high-leverage teaching practices (HLTPs) that allow student-teachers to integrate skills and knowledge (Grossman 2018) by moving away from concept-centred direct instruction to the one where student-teachers are given opportunities to enact elements of practice in a controlled environment. They include, for example, modelling mathematical content and leading a mathematical discussion.

To facilitate enactment, Grossman, Hammerness, and McDonald (2009) developed representations, decompositions and approximations of practice. Representations make the work of expert teachers visible to student-teachers through videos, stories, and narratives of practice, and case studies. Representations also refer to artefacts such as lesson plans, pupil work and observations of practice. They allow student-teachers and teacher educators to jointly analyse aspects of practice and develop a deeper understanding of teaching.

Decompositions break down complex practices (i.e. representations of practice) into their constituent parts, so that they can be visible to student-teachers and enacted at an individual level (Grossman, Hammerness, and McDonald 2009). To preserve the integrity of teaching, each of these parts should eventually be recomposed to form the original complex practice.

Finally, approximations of practice refer to the enactment of complex practices in situations of reduced complexity (Grossman 2018). In recent studies, approximations were used effectively to create opportunities for the development of student-teachers' beliefs by Nic Mhuirí, Twohill, and Harbison (2019), and by Delaney (2013) to address the complexities inherent in learning mathematics for teaching.

Theoretical frame

The theoretical frame for this study is based on Freire's (1970) *Pedagogy of the Oppressed*. In his book, Freire (1970) argued that education can be used as an instrument for conformity or for the practice of freedom. Freedom, as he argued, is about nurturing students' ability to deal critically with the world – a concept Freire referred to as *conscientization*.

Freire (1970) posited that education is used as a tool for conformity when it uses the banking approach. The banking approach is an educational paradigm whereby students are conceived as empty vessels whose purpose is to be uncritically filled with information, and such a conception of education ultimately stifles conscientisation. Within this system, the more meekly, uncritically and unconsciously students allow themselves to be filled with propositional knowledge, the better they are viewed. Freire (1970) described the banking approach as static, compartmentalised and disconnected from the totality to the exclusion of critical thinking. In a more recent critique, Chomsky (2004) described it as 'mindless skills-based education' (24) that works against democratic processes and independent thought, in favour of obedience and conformity.

Freire (1970) aimed to achieve *conscientization through* the problem-posing approach that encourages respectful dialogue and simultaneous reflection between student and teacher. In this model, knowledge is not conceived as fragmented but 'interacting constituent elements of the whole' (85) and students are not docile listeners, but critical co-investigators. Hence, education becomes a practice of freedom in which learners are not independent from but considered in relation to the world. This necessitates a type of community of practice where learners' experiences are valued within a climate of mutual trust, and such a conception of education cannot exist within the banking model.

Relational mathematical understanding

Consistent with Freire's (1970) problem-posing pedagogy, mathematics should not be viewed as fragmented, unrelated parts but as a set of 'interacting constituent elements of the whole' (85). Skemp (1978) categorised mathematical understanding into relational and instrumental. Relational understanding includes knowledge at a conceptual level, including relationships between concepts (Hurrell 2021). This understanding allows learners to make connections between new and previous knowledge by building a conceptual schema (Skemp 1978). Although it is time-consuming to develop (Willingham 2009), relational understanding involves less memory-work because mathematics is understood as a connected whole (Skemp 1978).

In contrast, instrumental understanding can be characterised as the capacity to follow pre-determined steps in sequence to solve a mathematical problem (Rittle-Johnson and Schneider 2015; Willingham 2009). According to Skemp (1978), teaching for instrumental understanding limits one's ability to analyse and criticise the procedure itself. The passive and uncritical nature of this 'understanding' mirrors Freire's (1970) banking model of education and is fundamentally undemocratic because it stifles creativity, intellectual growth and independent thought (Dewey 1938; Freire 1970; Chomsky 2004). Some researchers disagree with this characterisation of instrumental understanding. For example, Baroody, Feil, and Johnson (2007) argue that there can be a relational nature to instrumental understanding because procedures are often interconnected or embedded within other procedures.

Despite the limitations of an instrumental-only knowledge base, both types of mathematical knowledge are necessary (Richland, Stigler, and Holyoak 2012), and it is now accepted that an iterative bi-directional approach is the most effective because conceptual and procedural knowledge are mutually reinforcing (Hurrell 2021). However, relational knowledge should be a priority, as it often leads to the development of procedural knowledge rather than the other way around (Pesek and Kirshner 2000).

Methodology

Study context

This study represents one cycle of a larger two-cycle action research project to examine the problem of enactment with student-teachers within their maths competency module. The research in its entirety was carried out over a two-year period for a single cohort of student-teachers. Cycle 1 began in October 2019 with some baseline data collection, followed by a teaching intervention designed to address the problem of enactment. Further data were collected at several key points in time including during the intervention, and during and after a 3-week period of SP. The data were analysed and the results were used to inform changes to the intervention going into Cycle 2. Therefore, the current study focuses on the evaluation of the intervention after Cycle 1.

Intervention

The original maths competency modules used primarily direct instruction to, more or less, 'deliver' content to student-teachers in a way that resembled Freire's (1970) banking method of education. Using Korthagen's (2010) Gestalt model as a conceptual framework, the intervention involved a move away from concept-centred direct instruction to a mode in which student-teachers were given opportunities to problematise, contextualise and enact their MCK through high-leverage teaching practices to promote relational understanding and teaching of mathematics. Planned opportunities for reflection supported student-teachers in their learning while simultaneously challenging unhelpful beliefs that may contribute to the problem of enactment.

The intervention was implemented over seven lectures at the beginning of the academic year to precede a block of SP where student-teachers would have opportunities to enact what they had learned. HLTPs formed the core of the intervention aiming for meaningful and deliberate engagement in practice while preserving the relational nature of teaching (Grossman and Mcdonald 2008). *Leading a mathematical discussion* and *modelling mathematical content* were the two HLTPs used during this phase of the intervention.

A range of representations, decompositions and approximations of practice were also used. Role-playing was an important part of approximations where student-teachers took on various roles including teacher, pupil and a dedicated feedback role. Inspired by Freire (1970), the feedback role was used to generate meaningful dialogue between student-teachers to encourage critical reflection. The final approximation involved working collaboratively with visiting pupils from a partner school.

The other component of the intervention was the Mathematical Quality of Instruction (MQI) framework (Learning Mathematics for Teaching Project 2011) which was used to guide and evaluate mathematics instruction during approximations of practice. Once student-teachers became familiar with HLTPs and the MQI framework, the researcher/lecturer was able to take a step back and assume, primarily, the role of facilitator. This involved planning and directing activities within each session, providing resources and posing critical questions, while student-teachers were given several opportunities to use the MQI framework to analyse, discuss and reflect on their practice.

Sampling

The intervention was implemented with all Year 3 students ($n = 58$) in the BEd programme. This cohort was selected because they had taken two years of the maths competency module and were familiar with its purpose, content and approach. The student-teachers were scheduled to teach senior primary classes (4th–6th classes) on SP; making the current maths competency content more suitable for their placement.

Information letters and consent forms were distributed to each ST, explaining the purpose of the study and what was expected of them if they wish to participate. Then, student-teachers were asked to agree to participate in a subgroup for SP observations with post-observation discussions and subsequent focus groups. Convenience sampling, based on SP proximity to the university, was used to select this group. From this, 14 student-teachers were selected, and subsequently invited to take part in one of two scheduled focus groups.

Focus groups

Focus groups were chosen over individual interviews because they encourage a higher level of criticality among student-teachers as they open up in a comfortable space and respond meaningfully to each other's views (Bryman 2004). In addition, because the data were collected in terms of the student-teachers' 'own words and contexts … there is a minimum of artificiality of response' (Stewart and Shamdasani 1990, 17). Two semi-structured focus groups were conducted to gather data about student-teachers' reflections on the revised module and its impact on their practice.

SP observations

One of the researchers observed the student-teachers' mathematics teaching during their placement. These observations were guided by the richness of mathematics element of the MQI framework, which captures the essence of relational mathematics. Where relevant, the remaining elements of the framework were used as part of the overall analytic process after the observations.

Data analysis

Braun and Clarke's (2006) thematic analysis was used to analyse the focus group data. A blended inductive/deductive approach to coding was used by considering the theoretical perspectives and literature while also being open to unique perspectives from the student-teachers. Occasionally, unexpected codes prompted further review of the literature in search of meaning which allowed for a more nuanced analysis of the data. Post-lesson discussions, student-teacher reflections and the researcher's reflective journal were used for triangulation purposes and analysis of these was guided by the themes generated from the focus groups.

The SP observations were analysed using the MQI richness of mathematics scale. This construct enabled us to objectively observe the following factors of mathematical quality: linking between representations; mathematical explanations; mathematical sense-making; multiple solution methods; patterns and generalisations; and mathematical language. Detailed descriptions of each observed lesson were analysed retrospectively using a broader application of the MQI framework to include task cognitive demand, pupils' errors and contributions.

Ethical Considerations

Considering the nature of action research, the research took place in the natural setting of an ITE module and focuses on the reconstruction of the module by taking care of student-teachers' needs and practices. Consent forms and information letters were distributed to all student-teachers enrolled in the maths competency module. The information letter explained that they were under no obligation to participate in the study, and there would be no penalty for not participating. It was also explained that participation referred only to collection and analysis of observation and focus group data. As such, all student-teachers were obliged to engage with the teaching intervention because this was part of their ITE programme, but they were not obliged to provide data.

To ensure confidentiality, each student-teacher was assigned a unique identification number, and the key was only available to the researchers. Furthermore, student-teachers involved in the focus groups and classroom observations were referred to by pseudonyms.

Results

The data from the focus groups were analysed and five themes were generated. These were student-teachers' experiences and motivations, mathematics modules, pupils resisting to relational mathematics and neoliberal influences.

Theme 1: Student-teachers' experiences and motivations

Seven of the 14 student-teacherss reported negative experiences of secondary school mathematics and approved the instrumental nature of their experience. They agreed that this had potentially compromised their disposition to learn mathematics relationally in ITE, and more importantly to enact this knowledge in the classroom. For example, Claire recalled: 'I remember in primary school it was the same, they just call out the answers to the homework'. She continued to explain that pupils were rarely asked to explain the reasoning behind their solutions, and that she had similar experiences of secondary school mathematics:

> *I think it kind of comes as well from the Leaving Cert. Like it's the same thing that you were kind of just taught something. You weren't told why but you knew you had to know it to pass your Leaving Cert. I think that's the same way I came in first year, the same thing for maths competency.*

These negative experiences appeared to contribute to student-teachers' concerns about engaging with mathematics professionally. For example, Sharon mentioned that her school experiences caused her current anxiety around teaching division:

> *I was terrified to go near division. Absolutely terrified when the teacher gave me the topic. But I think it all goes back [to] secondary school. … I'm still questioning myself all the time … I don't know if I can teach division.*

Similarly, Gillian revealed how she 'doesn't feel comfortable' about teaching certain topics, while the prospect of teaching mathematics relationally causes feelings of 'panic'. Furthermore, Aoife talked about her experiences of testing that contributed her negative feelings about mathematics, and mentioned that this continued into ITE with the 70% pass threshold for the maths competency examinations.

The negative feelings about mathematics can be problematic for some student-teachers because they often lead to avoidance of mathematics-related tasks, and an overreliance on instrumental mathematics in the classroom (Gresham 2018).

Student-teachers' motivations

Student-teachers appeared to be more motivated by improving the educational experiences of pupils than by some innate desire to learn mathematics. For example, the student-teachers in both focus groups discussed child-centred learning including relevant content to children's lives, catering for diverse needs of learners, inclusion, understanding children's thinking, and the impact of their attitudes on children's engagement.

Both Mary and Vicky agreed that the original maths competency module was for their MCK development, but they welcomed the intervention as an opportunity to concurrently develop their teaching knowledge and skills as part of the same module.

These findings support the idea that student-teachers' learning is socially constructed (Wenger, 1998; Korthagen, 2010), and that MCK is improved when learning is framed around the overall learning needs of pupils, rather than mathematics per se being the primary consideration (Philipp 2008).

Theme 2: mathematics modules

This theme represents student-teachers' critique of the mathematics modules offered in ITE. This data also helped to shape the intervention in Cycle 2. The sub-themes are presented below.

Original maths competency module

While there was almost unanimous agreement that the original module improved student-teachers' MCK, there was little evidence of their improved classroom performance because of the module. During the focus groups, student-teachers attributed this to a lack of specific pedagogical input as part of the maths competency module to support their enactment of MCK. For example, Mary explained how 'actually going in teaching it is a different thing altogether' and highlighted the need for accompanying pedagogical skills. Similarly, Gillian mentioned that maths competency 'feels separate to teaching … separate from … classes where you teach children'. According to Paul, student-teachers were passive and 'just sat there in rows' and watched the lecturer 'do maths for an hour'.

The student teachers also reported the challenge of connecting MCK to their pedagogical knowledge from the maths methods module. Five of the student-teachers agreed that they had the relevant MCK, but because the modules were not explicitly connected, they lacked the opportunity to develop the associated pedagogical knowledge to make mathematical concepts meaningful for pupils.

These findings suggest a an element of fragmentation within the ITE programme with respect to mathematics education. This separation of the technical from the practical does not necessarily respect the complexity of mathematics teaching (Grossman and Mcdonald 2008). Also, the reported passive role played by student-teacher was indicative of Freire's (1970) banking concept of education which arguably works against democratic processes and independent thought (Chomsky 2004).

The intervention

The data indicate that the intervention benefited student-teachers in their preparation for teaching mathematics on SP. Despite remaining disconnected from the maths methods module, the student-teachers benefitted from the intervention through linking content and pedagogy. For example, Tony stated that his pedagogical content knowledge improved mainly from approximating practice:

> *Because I actually had to think about it. And I'd done all the things like discussion, I'd done modelling … It was all stuff we did in lectures here.*

Tony expanded that because the MCK was contextualised through the visit by the pupils to the university; he was then able to authentically enact what he had learned during those approximations. Similarly, planning and enacting approximations allowed Sharon to 'see what their base level of understanding might be before you go into a class', while Gillian described how approximations enabled her 'to take that knowledge and phrase it for children'. Mary also referred to how video representations of practice improved her MCK: 'I would never have known that kind of thing, you know to multiply fractions', while Derek shared how the benefits of representations have 'stayed with me since' and positively influenced his preparation for SP.

Student-teachers also benefited from working collaboratively in a risk-free environment where mistakes were framed as learning opportunities. Derek noted how collaboration resulted in meaningful dialogue: 'Because we were using what we had learned and looking at each other's teaching, and what was good, what was bad' and claimed this approach informed his practice: 'So much more than the last two years of the plain competency'. Tony summarised the philosophy around making mistakes as: 'You said don't worry about making mistakes, make them here and then you can fix them in the classroom'.

Student-teachers also agreed that the intervention would have a positive impact on their subsequent SP. Both Helen and Aoife benefited from the representations and approximations related to fraction multiplication. Helen recalled: ‘Even with the area model … I found [helped] the children who were struggling’, while Aoife described how *modelling mathematical content* improved her practice:

> *I think modelling really helped because when you’re thinking out loud, you’re kind of asking yourself the same questions children are asking. I found that really useful.*

Furthermore, Claire used *leading a mathematical discussion* to encourage pupils to ‘think a bit more’ about the characteristics of polygons, while Mary successfully used this HLTP for problem-solving activities. Claire took the advantage of having examples of HLTPs available so she could ‘look back on that video and have … an idea of like what good teaching looks like’.

Improvements to the intervention

While there were notable gains due to the intervention, student-teachers indicated three areas for improvement. Firstly, they highlighted that they would learn more through ongoing work with the same pupils during their approximations of practice. Paul explained that ‘It would have been nice to get that same group back again and progress just to see if you can change it with the same group’.

The second recommendation was related to the sharing of MCK. Although different groups reported an improvement in MCK by focusing on one content area each, this knowledge was not shared as effectively as it should have been. Suggestions included posting lesson plans and reflections on Moodle, and sharing electronic recordings of approximations.

Finally, the student-teachers enacted MCK effectively only if it was used in a similar way during approximations. For example, Helen mentioned that because fractions were used as representations during the intervention, she was able to make links to SP because ‘the content was the same’. However, this contrasts with student-teachers’ learning about number base. Although the content knowledge was fundamental, they agreed that unless it was explicitly linked to a curriculum strand, they would be unlikely to use it to support their MCK in practice. Similarly, Brona was teaching percentages and because the relevant MCK was not addressed as part of the intervention, she reverted to ‘just presenting it to the kids without actually giving them a reason for it’.

Theme 3: pupils resisting relational mathematics

The data indicate that there was a critical mass of pupils who resisted learning mathematics relationally such that it became a burden to persist in it. Claire reported a culture of amongst pupils of calling out answers to questions without explanations, and when she asked for explanations; ‘most of them couldn’t because they’ve never been asked before’. Tony agreed that ‘some kids in the class were really annoyed that I was teaching them why it worked, and just wanted the quick way around’, while Gillian noted that ‘the whole concept of being asked why they did something; they actually kind of seemed uncomfortable with it’. When Derek asked pupils for explanations, they typically replied: ‘Why should I have to defend my answer, it’s right’. These pupil reactions made some student-teachers question the standard of their practice. Gillian, for example, felt like she was doing a 'bad job' while Jessica and Sharon discontinued teaching relational mathematics.

Moreover, pupils' resistance to relational mathematics was compounded by their perceptions that mathematics should be fast, which is at odds with understanding mathematics relationally. Derek summarised pupils' attitudes as: 'Faster is smarter … It's almost a culture among children'. Similarly, Tony reported his experience:

> *They kept asking me what is the quick way. And they were getting worked up that they had to spend so long doing the equation.*

Jenny agreed that 'they just want to get it done quick', and Sharon reported some pupils even refuse to write things down to save time. Both Vicky and Paul reported similar experiences and both agreed with Derek that it was an ingrained culture amongst primary pupils.

Finally, student-teachers reported pupils' tendency to value neatness and correct answers over messy creativity. Gillian described how this hampered pupils' relational understanding of the long division algorithm because they just wanted to get their 'neat little … long division sum … done … and they got the answer and they ticked it'. Additionally, Sharon highlighted the need for pupils to keep their copies 'pristine' and admitted 'to get them to do rough work was hard'.

It was also found that the cooperating teacher (CT) played an important role on pupils' attitudes. While Helen agreed her CT promoted relational mathematics, most of the others reported that they promoted instrumental mathematics. For example, Gillian, Derek, Vicky and Paul's CTs promoted instrumental mathematics by framing the subject as a silent, workbook-based and individual, while discouraging collaborative and discussion-based methodologies.

This theme highlighted a problematic mismatch between some pupils' preferences about instrumental mathematics versus the implicit requirement for student-teachers to teach relationally. This creates a particular context for the student-teachers that is significantly different from the intervention, which might explain why some student-teachers enacted practices inconsistent with their purported beliefs. This finding is supported by several researchers, including Raymond (1997) and Sztajn (2003), who found that context plays an important role in teachers' practice and this may appear to override their beliefs about mathematics.

Theme 4: Neoliberal influences

Within the study context, the researchers identified a neoliberal influence contributing to the problem of enactment. Whereas the Irish curriculum promotes mathematics as a 'source of fascination' through 'the exploration of patterns and relationships, [and] the satisfaction of solving problems' (National Council for Curriculum and Assessment 1999, 3), the student-teachers perceived mathematics as a procedural activity whose main purpose was testing and examinations. This neoliberal slant was identified and resulted in a re-examination of the focus group data.

Sharon, for example, reported how her 6th class pupils questioned the necessity for 'rough work' and were ultimately motivated to do this by the prospect of taking additional points in the Leaving Certificate examination for showing their work. Jenny's pupils also shared concerns about maximising the additional 25 points for passing the Leaving Certificate higher-level mathematics examination. Concerningly, the same pupils were already practicing the Leaving Certificate foundation level examination papers as part of their primary education.

This neoliberal view of mathematics education was also reflected by some CTs. Gillian, Brona and Claire all described how their CTs were anxious about meeting external demands rather than pupils' understanding. Brona explained how there was 'no value placed on understanding … because understanding doesn't get you points, it doesn't get you marks'. Both Sharon and Jessica mentioned that how demands of the CT to complete workbooks reduced opportunities for teaching mathematics relationally. Evidently, this view promotes mathematics as instrumental and conceptualises mathematics education as 'static, compartmentalised, and disconnected' (Freire 1970).

Classroom observations

To understand how the student-teachers' MCK was enacted, they were observed in their SP setting using the 4-point MQI framework. Through this framework,, each student-teacher was closely observed and detailed observation notes were taken. Shortly after each lesson, these notes were analysed using the richness of mathematics dimension of the MQI 4-point version, which captures the depth of mathematics offered to pupils.

A score was calculated for each of the student-teachers to rank them from lowest to highest. From these, the three highest (Sharon, Claire and Vicky) and the three lowest (Gemma, Jessica and Brona) were chosen for qualitative analysis. Each was analysed under the following MQI elements: richness of mathematics, cognitive demand, remediation of errors and pupils' contributions. Only six observations were chosen because the analysis was highly descriptive, paying particular attention to individual nuance which was characteristic of a complex teaching environment.

The data allowed us to conceptualise how instruction looked like from both high and low MQI ends which, in turn, gave us a sense of the impact of the intervention on student-teacher classroom behaviour.

Richness of mathematics

This dimension captured the depth of mathematics offered to pupils. We found that the student-teachers in the high category delivered lessons steeped in rich mathematical teaching including explanations of concepts, multiple representations of ideas and the use of generalisations. Mathematical language was well-defined and used intentionally to improve pupils' understanding and capacity to engage meaningfully in lessons.

For those in the low MQI category, the overall richness of mathematics was low. The student-teachers seemed unaware of aspects such as multiple procedure methods and generalisations. Where explanations were used, they were procedural and often limited to recall of steps. Furthermore, student-teachers appeared to adapt this approach at least partly because of a concern that relational mathematics would have caused confusion for pupils.

Task cognitive demand

This dimension captured pupil engagement in tasks in which they were required to think deeply and reason about mathematics. Specifically, cognitive demand was present in the high category. Examples included pupils exploring the idea of infinity, testing conjecture and justifying strategies.

In contrast, cognitive demand in the low MQI category was essentially absent. These lessons cantered primarily around recall of well-established procedures and while exploration of mathematical ideas was either absent or purely instrumental.

Remediation of errors

This dimension captured instances of remediation in which pupils' misconceptions about the content were addressed. Although remediation sparse for both low and high performing MQI, where it existed, it was at a conceptual level in high MQI lessons, and at a procedural level in low MQI lessons.

Pupil contributions

This element captured the extent to which the student-teachers used pupils' mathematical contributions to move instruction forward. There were significant differences between low and high MQI in this category. On the lower end of the MQI scale, pupils' contributions could be described as *pro forma*, with little influence on how the lesson developed because of that contribution. Typical contributions included calling out answers or listing steps in a procedure. The student-teachers' choice of language seemed to reinforce this culture. For example, Jessica told the pupils: 'I want everyone quiet and looking up … '. Brona gave the following instruction to her pupils: 'We have no excuse to talk … you're working by yourselves … if you have a question put up your hand'.

In contrast, pupils were highly encouraged to and communicated with the student-teacher and other pupils and participate in the high-level MQI lessons. In this regard, learning was often incidental and occurred more naturally. Examples included pupils in Sharon's class exploring geometry in the schoolyard and reporting back to her and other pupils for discussion, and the pupils in Vicky's class justifying their answers which were then used to engage other pupils to develop the lesson further.

Concluding remarks on observations

The classroom observations highlighted three examples of high-quality mathematics teaching for the stage these student-teachers were at their teacher education programme, which suggested the intervention had some positive impact. We also observed some poor-quality lessons which suggested there was more work to be done in terms of improving the intervention in Cycle 2. For instance, a significant feature of the student-teachers who enacted low MQI lessons was their lack of awareness of what high-quality mathematics instruction looks like.

Interestingly, the student-teachers, who demonstrated low MQI scores, did not necessarily have content knowledge deficiencies. For example, Gemma, who scored low on her MQI, was described by her peers as exceptionally able at mathematics, and her exam results confirmed this. In post-lesson discussions, the student-teachers who scored low MQI offered deep mathematical explanations, which indicated that they chose not to enact that knowledge for various reasons. At least in some cases, the student-teachers with low MQI scores believed that some pupils could not engage with relational mathematics. Reasons for this, reported by the student-teachers, included the belief that pupils lacked the ability, had fear of causing anxiety, or simply the belief that pupils did not have a desire to learn mathematics relationally. This was consistent with the findings from the focus groups.

Implications for Cycle 2

Data from the focus groups and observations suggested that one of the main challenges for student-teachers was contending with pupils' preference for instrumental mathematics. This preference, in general, was often opposed to the underlying democratic principles of the intervention and this created conflicting situations for the student-teachers. Using Freire (1970) as the theoretical position, Cycle 2 would include more opportunities for student-teachers to authentically and critically reflect on their practice. Cycle 2 would therefore introduce a new HLTP called 'norms for giving mathematical explanations', which focuses on mathematical explanations in the context of classroom dialogue, while promoting relational understanding of mathematics (TeachingWorks 2022).

The data also revealed a lack of explicit use of the MQI framework in student-teachers' practice, or indeed in their planning. Retrospectively, its use as part of the intervention was perhaps too informal which may have somewhat negated its perceived importance. To address this, a more explicit emphasis would be placed on the framework as a formative learning tool, and planning guide, in Cycle 2.

Finally, it was decided by the student-teachers that it would be more beneficial to focus exclusively on one mathematics content area at a time. Although this would reduce the breadth of MCK use, it was expected that it would allow the student-teachers to share similar experiences, thereby creating opportunities for collaboration and critical dialogue within a community of practice in a way that limiting beliefs can be challenged (Freire 1970).

Discussion

Adopting Grossman and Mcdonald's (2008) pedagogies of enactment as a core part of the intervention allowed the student-teachers in this study to contextualise MCK in a way that acknowledged and embraced the complexity of teaching. Representations, decompositions and approximations of practice provided opportunities for the student-teachers to closely examine HLTPs to support their mathematics teaching for relational understanding. The STs reported that approximations involving visiting pupils to the university introduced authentic situational learning which allowed learning to happen socially within a community of practice (Wenger, 1998). Therefore, approximations of practice should be progressive, range from low to high across the approximation scale, so that student-teachers can engage in a full and graduated range of practical experiences.

The findings also suggest an element of fragmentation within the programme with respect to mathematics education (i.e. a separation of content and pedagogy) which needs to be addressed in order to preserve the complexity of teaching. This will afford student teachers opportunities to reflect on authentic practice and subsequently challenge unhelpful beliefs which may be contributing to the enactment problem.

Lastly, enactment pedagogies need to be introduced from year 1 of the ITE programme and should be the main approach, not something that is implemented six weeks before SP, or on an ad hoc basis. Introducing this approach early is essential because it will take time for student-teachers to become accustomed to a new way of learning, compared to the passive style they may have experienced at school, and within a larger university ecosystem that often rewards transmission style learning (Loughran 2006).

Implications for teacher education

To support meaningful gestalt formation, ITE must organise 'sufficient, suitable and realistic experiences tailored to the needs and concerns' of student-teachers, while preparing for schema development by offering opportunities for reflection on suitable experiences (Korthagen 2009, 104). Suitable experiences are those that are challenging enough to 'offer opportunities for a confrontation with gestalts the educator would like to change' (104). This requires a signature pedagogy (Shulman 2005) which allows student-teachers to simultaneously pay attention to the content of what is being taught and how it is being taught (Loughran 2006). Loughran argues that it is easier for student-teachers to focus on only to what is being taught because this passive style of learning is encouraged by formal schooling experiences which reinforce the notion that learning is simple and transmissive. Therefore, a focused effort is required to change student-teachers' perceptions of what meaningful learning is (Loughran 2006). Reflection and metacognition promote awareness and inform decision-making as student-teachers construct their personal pedagogies (Hoban 2004 , 135), and facilitate progression to Korthagen's schema level and the development of situational understanding (Loughran 2006).

Consequently, teacher educators need to think about how content is taught and the underlying messages this 'pedagogical turn' conveys to student-teachers (Russell 1997). The challenge for teacher educators is to grapple with the competing agendas of teaching content while at the same time paying specific attention to how they teach that content. This necessarily involves unpacking teaching and making pedagogical decisions explicit so that student-teachers are given access to the 'pedagogical reasoning, uncertainties and dilemmas of practice that are inherent in understanding teaching as being problematic' (Loughran 2006, 7). Particularly, teacher educators need to provide opportunities for student-teachers to iteratively problematise and enact practice, followed by rich discussions and reflections to challenge unhelpful beliefs and promote professional and intellectual growth.

Disclosure statement

The authors report there are no competing interests to declare.

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