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Estimating Optimal Synchronization Parameters for Coherent Chaotic Communication Systems in Noisy Conditions

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ABSTRACT It is known, that coherent chaotic communication systems are more vulnerable to noise in the transmission channel than conventional communications. Among the various methods of reducing the noise impact, such as extended symbol length and various digital filtering algorithms, the optimization of the synchronization coefficient may appear as a very efficient and simple straightforward approach. However, finding the optimal coefficient for the synchronization of two chaotic oscillators is a challenging task due to the high sensitivity of chaos to any disturbances. In this paper, we propose an algorithm for finding the optimal synchronization parameter K_{opt} for a coherent chaos-based communication system affected by various noises with different signal-to-noise ratios (SNR). It is shown, that under certain conditions, optimal K provides the lowest possible bit error rate (BER) during the data transmission. In addition, we show that various metrics applied to the message analysis and demodulation task propose different noise immunity to the overall system. In the experimental part of the study, we simulated and physically prototyped two chaotic communication systems based on well-known Rössler and Lorenz chaotic oscillators. The microcontroller-based prototype of a wire chaotic communication system was developed to investigate the influence of noise in the physical data transmission channel. The experimental results obtained with the designed hardware testbench are in good correspondence with the theoretical propositions of the study and preliminary simulation results. The suggested evaluation metrics and optimization algorithms can be used in the design of advanced chaos-based communication systems with increased performance.

KEYWORDS
Chaotic synchronization
Coherent chaotic communication
Covert communication
MCU
Chaos with noise

INTRODUCTION

Dynamical chaos and chaotic synchronization are essential phenomena in nonlinear dynamics. Recently, many chaos applications such as chaotic encryption (Volos *et al.* 2013), chaos-based sensors (Karimov *et al.* 2021a), and chaos-based communication systems were proposed. One of the most promising applications of chaotic synchronization is chaos-based communication systems, which possess a broadband channel for concealed data transfer. Chaotic communication systems (CCS) based on chaotic synchronization are called coherent (Kaddoum 2016).

Recent works include studies on optical coherent chaotic communication systems (Wang *et al.* 2020; Yang *et al.* 2020), digital data transfer systems for Internet of Things (IoT) applications (Babajans *et al.* 2023, 2022; Cirjulina *et al.* 2022), area- and power-efficient implementation of chaotic oscillators for coherent secure systems

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(Hedayatipour *et al.* 2022).

Coherent chaos-based communication systems assume achieving chaotic synchronization on the receiver side, which has prompted the development of various synchronization methods. Initially, this phenomenon was discussed in studies (Fujisaka and Yamada 1983) and (Afraimovich *et al.* 1986). However, the topic of chaotic synchronization gained great attention after the introduction of Pecora and Carroll's method. In their famous paper, these authors proposed to use a simple synchronization method in secure communications (Pecora and Carroll 1990). The Pecora-Carroll synchronization suggests a master-slave system architecture, represented by two identical chaotic oscillators with the same parameter set, where the signal from the master system is driving the other system dynamics (Pecora and Carroll 1990).

To transform the meaningful signal into chaotic carrier changes, various modulation and demodulation techniques were developed. The most popular modulation methods used in chaos-based communications are chaotic shift keying (CSK) (Dedieu *et al.* 1993; Dmitriev and Panas 2002), parametric modulation (PM) (Yang and Chua 1996; Koronovskii *et al.* 2009), and chaotic symbolic dynamics (Kaddoum 2016). Although some other modulation techniques have been proposed recently, they mainly involve variations or combinations of the aforementioned approaches (Kharel 2011). In the current study, we focus on parametric modulation (PM), as an easy-to-implement and highly secure common approach.

In classical studies, such as (Carroll and Pecora 1995) and (Willsey *et al.* 2011), as well as in more recent works, e.g. (Abib and Eisencraft 2015), scholars proposed the application of analog circuits to generate chaotic signals in communication systems. However, this approach possesses some disadvantages, e.g. the limited precision of the chaotic circuit components, conditional parameter drift, etc. In addition, as is known from several recent studies (Minati *et al.* 2017; Karimov *et al.* 2023; Emiroglu *et al.* 2022; Alexander *et al.* 2023), the behavior of analog circuits may significantly differ from the original mathematical models and vary between different implementations, which in the field of coherent chaotic communications may lead to difficulties in matching between transmitter and receiver parameters. Therefore, one can consider a completely digital communication system based on direct digital synthesis (DDS) as a prospective technology.

The DDS is a method of generating an analog signal using a digital-to-analog converter (DAC) and data from a digital processing unit (Liu *et al.* 2007). DDS can generate highly precise and stable waveforms, allowing one to use it in a wide variety of applications, such as telecommunications, signal processing, test, and measurement equipment (Cordesses 2004a,b). The DDS was also reported to be used for noise radar (Willsey *et al.* 2011), as well as for covert messaging in noisy conditions (Lukin and Zemlyaniy 2016) - tasks that are very closely related to chaotic messaging.

Latest developments have shown the opportunity for chaotic communication DDS systems to use modulation schemes where the discretization operator is varied to obtain different finite-difference equations. An example of such a technique is the symmetry coefficient modulation (SCM) (Karimov *et al.* 2021b). SCM operates by manipulating the numerical method parameter, called symmetry coefficient, and can be used to construct the digital chaotic messaging signal. Several papers (Rybin *et al.* 2022a, 2023) show that SCM may provide more covert messaging than traditional PM techniques.

As an alternative to coherent chaotic communication systems, so-called non-coherent systems have been researched and developed. Such systems do not use synchronization but are based on correlation or other matching methods. Recent works on the subject include digital underwater communication systems (Bai *et al.* 2019, 2018), systems based on chaotic oscillators with special properties for general applications (Rajagopal *et al.* 2018), and other techniques (Moysis *et al.* 2020; Lyu *et al.* 2015). Non-coherent communication systems are considered more resistant to noise while being also less resistant to attacks (Kaddoum *et al.* 2010; Kaddoum 2016). It should be noted, that low resistivity to noise is one of the key shortcomings of coherent CSS. Thus, considering their high-security level, especially when using DDS technology, it seems promising to develop some methods of improving their noise immunity.

Such methods can include denoising techniques for chaotic signals (Voznesensky *et al.* 2022), as well as various techniques for improving the reliability of communications with noisy input signals. One natural idea here is to find the optimal synchronization coefficient K (the proportion, in which the transmitter signal is mixed via the receiver signal to force its synchronization) for the given channel conditions. Our previous studies have shown that the synchronization coefficient value provides a noticeable impact on the quality of messaging in coherent systems without noise in the communication channel(Rybin *et al.* 2021, 2022b). Therefore, the current study aims to investigate the possibility of finding optimal K comprehensively. We consider different chaotic oscillators, various signal-to-noise ratios in the channel, and symbol lengths in order to experimentally validate the obtained results.

The main contributions of this study can be summarized as follows:

1. A model of a coherent chaos-based communication system with parameter modulation (PM) under various signal-to-noise (SNR) levels is considered. We choose classical Lorenz and Rössler systems for the CCS prototypes due to their well-known chaotic properties, and Gaussian white noise as typical interference noise.

2. Various metrics for synchronization error analysis were used to distinguish binary characters '0' and '1' in demodulating algorithms of the prototyped systems.

3. It was discovered that different metrics for synchronization error analysis at the receiver side provide different noise immunity to the overall system. The most effective metrics were found to be root-mean-square (RMS) and mean value calculation.

4. The suggestion that the optimal synchronization coefficients K depend on the noise level in channel and symbol length, and the form of this dependence is unique for a particular chaotic oscillator, is confirmed experimentally. For practical applications, the optimal K value can be approximated by a simple expression with sufficient accuracy, and then dynamically selected during communication based on an estimate of the noise level and the data transfer rate, which advances the architecture of the chaotic communication system.

5. A new algorithm for finding an array of optimal synchronization coefficients for an arbitrary chaotic oscillator under given parameters and certain conditions is proposed.

Summarizing, the reported research makes a significant step toward solving the problem of finding the optimal synchronization coefficient and further improving the design of coherent CCS. Being a simple and efficient technique, this approach could be

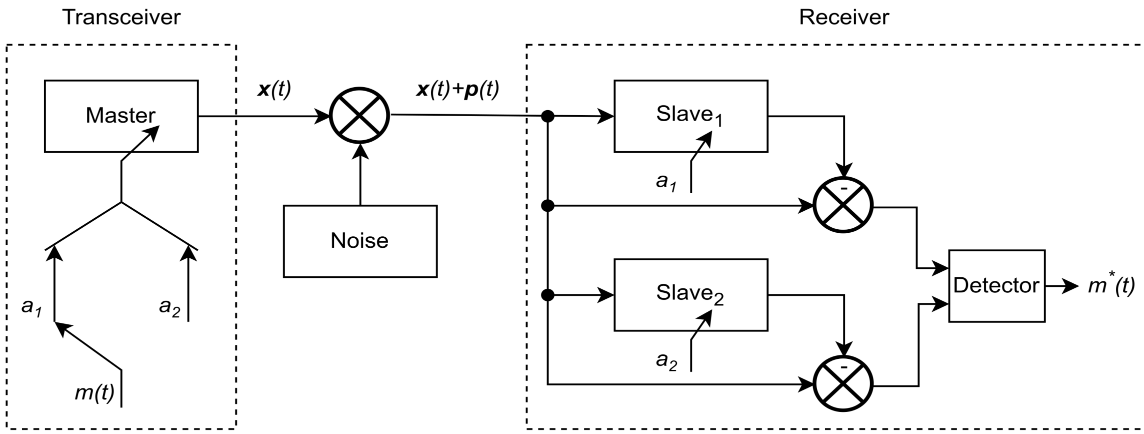

Figure 1 The scheme for a chaotic communication system based on parameter modulation.

considered alongside other noise-reducing methods to advance the development of robust and reliable communication systems.

The rest of the paper is organized as follows: in Section 2, the investigated chaotic systems and the architecture of the chaos-based communication system are described. The experimental setup, as well as the results of the experimental investigation, are presented in Section 3. Section 4 discusses the obtained results considering their practical applications, and Section 5 concludes the paper.

MATERIALS AND METHODS

This section provides a brief description of the chosen chaotic oscillators and the communication system under investigation, as well as the methods of analyzing the synchronization error on the receiver side.

Investigated chaotic systems

In previous works, we attempted to determine the optimal synchronization coefficients for chaotic communication systems based on Lorenz and Rössler systems (Rybin *et al.* 2021, 2022b). These canonical systems are used as chaos generators in the current study as well, providing a basis for chaotic communication systems under investigation.

The well-known Lorenz chaotic system (Liao 1998) is described by the following system of ordinary differential equations:

$$\begin{aligned} \dot{x} &= \sigma(y - x), \\ \dot{y} &= x(r - z) - y, \\ \dot{z} &= xy - bz, \end{aligned} \tag{1}$$

where $\sigma = 10$, $r = 28$, $b = \frac{8}{3}$.

Let us apply the Pecora-Carroll (Pecora and Carroll 1990) synchronization to system (1) to obtain the slave (receiver) oscillator system:

$$\begin{aligned} \dot{x} &= \sigma(y - x), \\ \dot{y} &= x(r - z) - y + K(y_M - y), \\ \dot{z} &= xy - bz, \end{aligned} \tag{2}$$

where K is the coupling strength coefficient, and y_M is the second variable of the master system. It is known (Rybin *et al.* 2021, 2022b) that the preferred synchronization variable for the Lorenz system is y and the approximate value of coupling strength $K \approx 40$ provides the fastest synchronization.

The Rössler system (Gaspard 2005) is described by the following system of ordinary differential equations:

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay, \\ \dot{z} &= b + z(x - c), \end{aligned} \tag{3}$$

where $a = 0.2$, $b = 0.2$ and $c = 5.7$. Applying the Pecora-Carroll synchronization to system (3), one can obtain the equations of the slave oscillator:

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay + K(y_M - y), \\ \dot{z} &= b + z(x - c), \end{aligned} \tag{4}$$

where K is a synchronization coefficient and y_M is the second variable of the master system, which is also the optimal synchronization variable for Rössler system. The value of coupling strength $K \approx 1.93$ provides the rapid synchronization process (Rybin *et al.* 2021, 2022b).

Chaotic communication system design

In the experimental part of the study, we considered a conventional coherent chaotic communication system with parametric modulation. It should be noted that all the methods used in this research can be used for improving the chaotic communication systems based on other modulation methods, for example, the recently proposed symmetry coefficient modulation (Karimov *et al.* 2021b; Tutueva *et al.* 2022). The investigated scheme for a chaotic communication system based on parameter modulation is shown in Figure 1.

The operating principle of the CCS based on parametric modulation can be described as follows. The desired digital signal $m(t)$ modulates parameter a of the chaotic oscillator on the transceiver side. Next, the signal passes through the simulated communication channel, where white Gaussian noise is added along with the noise of the DAC (digital-to-analog converter) and ADC (analog-to-digital converter) units. In our experiments, the bit depth of the DAC and ADC was 12 bits. One can observe that the generalized chaotic synchronization occurs on the receiver side, depending on the transmitted bit of the message. The message recovery $m^*(t)$ is performed by determining the lowest value of the synchronization error. The example of message transmission is shown in Figure 2.

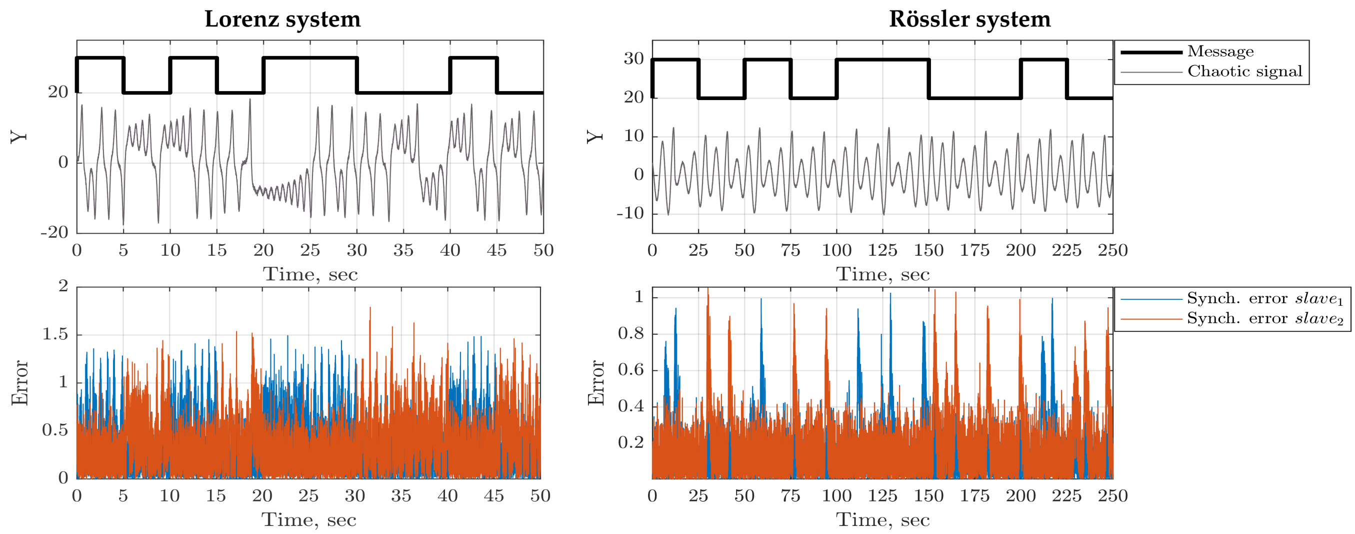

Figure 2 Transmission of message "1010110010" by chaotic communication system with parameter modulation based on Rössler and Lorenz system.

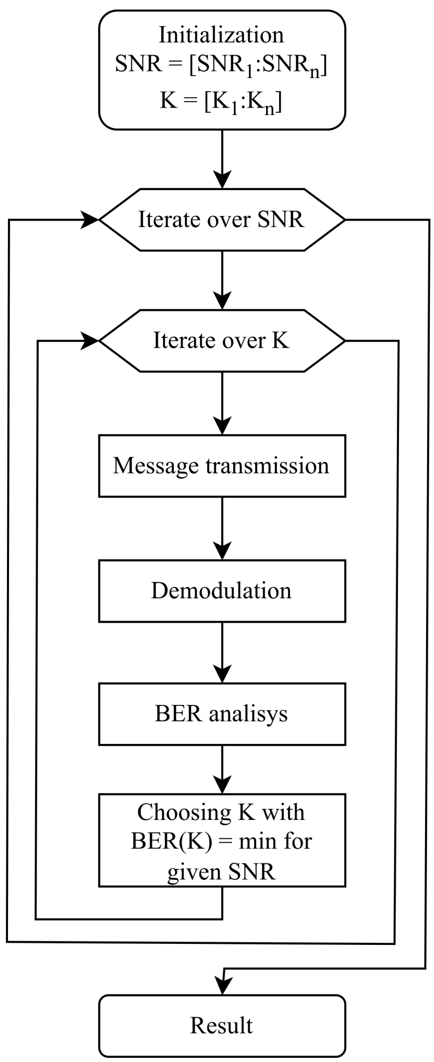

Figure 3 Scheme of an algorithm for the synchronization coefficient investigation.

As one can see from the Figure 2, the chaotic signal possesses no visible correlation with the transmitted message in both cases, especially being compared with the behavior of the synchronization error.

In some studies, the transmission of certain informational bits is determined by threshold (Rybin *et al.* 2023; Kaddoum 2016) and assumed post-processing of the resulting synchronization error. However, in the presence of noise in the communication channel, threshold detection can be inefficient. Thus, in the current paper, we have chosen and evaluated several comparative methods for estimating the difference in synchronization error.

Finding the optimal synchronization coefficient

To determine the optimal values of K for an arbitrary SNR range, one should set the optimization criterion first. Let us call the synchronization coefficient K optimal if the bit error rate (BER) of the communication system is minimal. The following algorithm was developed (see Figure 3) to solve the optimization task. First, the SNR range and the synchronization coefficient K are initialized. Then, an iterative enumeration of the SNR is performed, and K values for which the transmission process takes place are determined. Finally, the received message is demodulated and analyzed. The number of errors in the message at a chosen K and SNR is calculated, and the BER for a chosen K is compared to the minimum error value for a given SNR. The smallest of these two values is selected. Then the K value is iteratively increased, and all the abovementioned steps are repeated until the investigation range ends. Then, the SNR is iteratively increased, and the same process continues until all K values are determined for all SNR values. One can find optimal values of $K_{opt} = f(SNR)$ when BER = $f(K)$ is minimal. The final goal, namely, to get the minimal BER for the given SNR, can be achieved via selected $K = K_{opt}$.

RESULTS

Experimental setup

All numerical experiments were performed using the National Instruments LabVIEW 2021 environment. In all numerical experiments, the explicit Runge-Kutta method of accuracy order 2

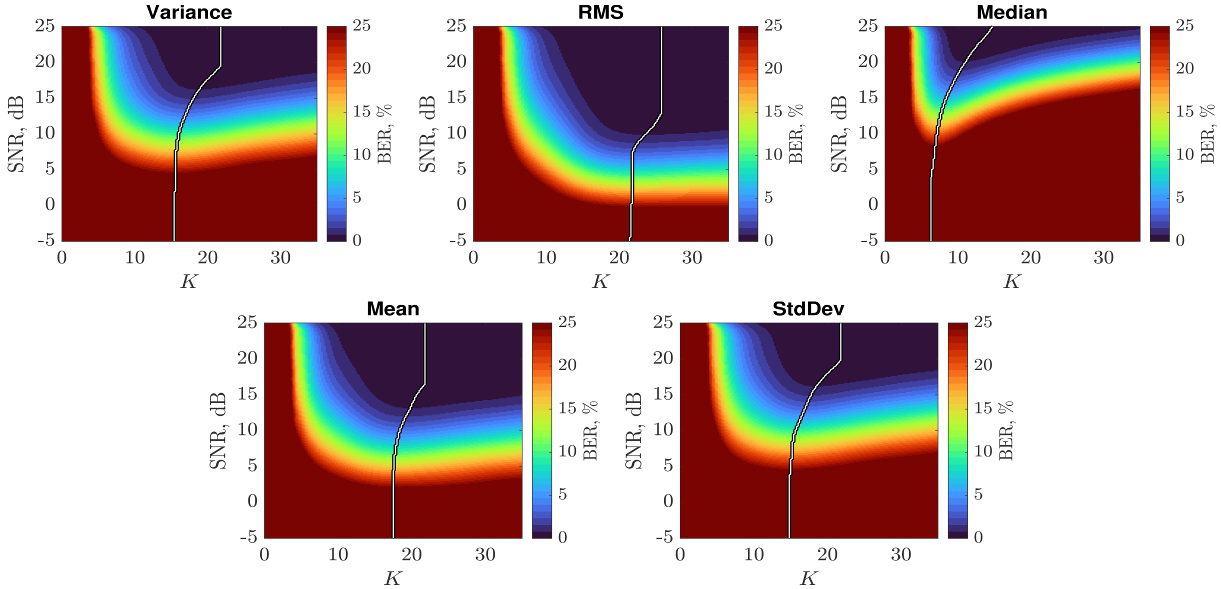

Figure 4 The dependence between BER, synchronization coefficient K, and SNR for Lorenz system. The black-and-white line corresponds to the synchronization coefficient value where BER is minimal for certain SNR values.

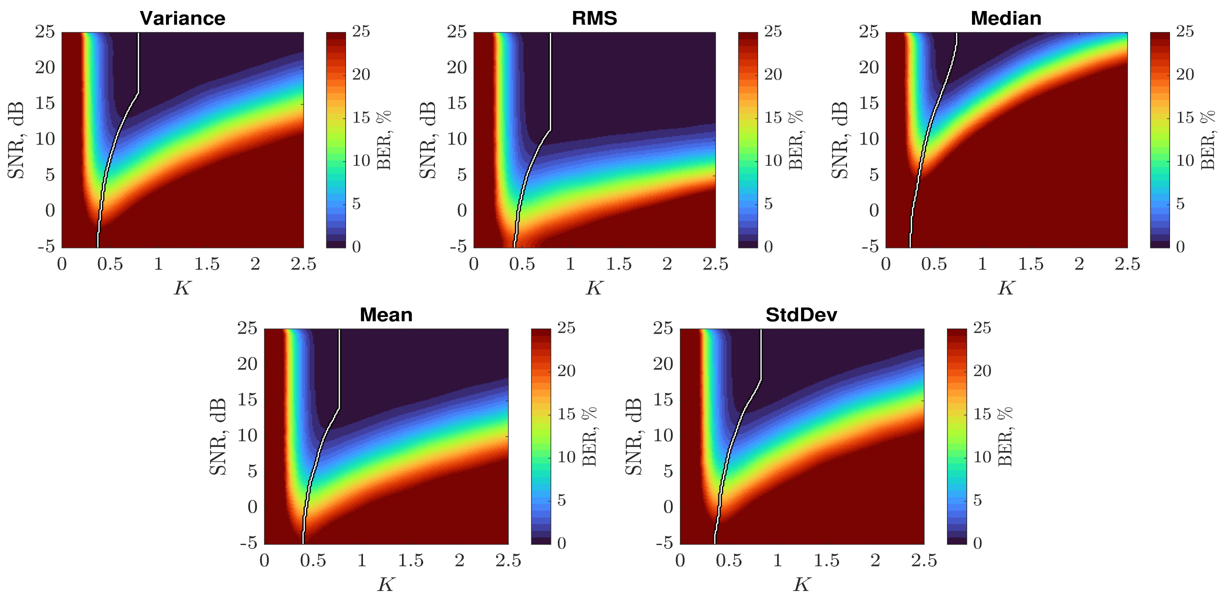

Figure 5 The dependence between BER, synchronization coefficient K, and SNR for Rössler system. The black-and-white line corresponds to the synchronization coefficient value where BER is minimal for certain SNR values.

(RK2) was used to solve the ODEs of the investigated system. The choice of the RK2 method can be explained by the fact, that obtaining the highly accurate solution of ODE is not required for CCS construction purposes. In fact, one can use almost an arbitrary finite-difference model of the chosen continuous chaotic system because both sample systems are dissipative and do not require special geometrical integration procedures for long-term simulation. The benefit of choosing an explicit second-order integration method is the simplicity of its hardware implementation. To summarize, the RK2 method ensures the stable long-term generation of the chaotic signal, and switching the bifurcation parameters does not cause a stability loss.

The integration stepsize for the Lorenz system was chosen as $h = 0.005$ and for Rössler system was set as $h = 0.025$. The initial conditions for both systems were set as $(0.1, 0.1, 0.1)$. One should note, that in a real CCS, the initial conditions may be a part of the security key.

In this study, we use relative time units to characterize the length of the transmitted symbol for the Lorenz and Rössler systems. The reason is that these systems have different dynamics and variable change speeds when presented in a natural timescale given in seconds. The Lorenz system dynamics is faster than Rössler systems. Therefore, let us introduce pseudo-periods $N_{\tilde{T}}$ as time units for this study:

$$N_{\tilde{T}} = \vartheta \cdot T_s, \tag{5}$$

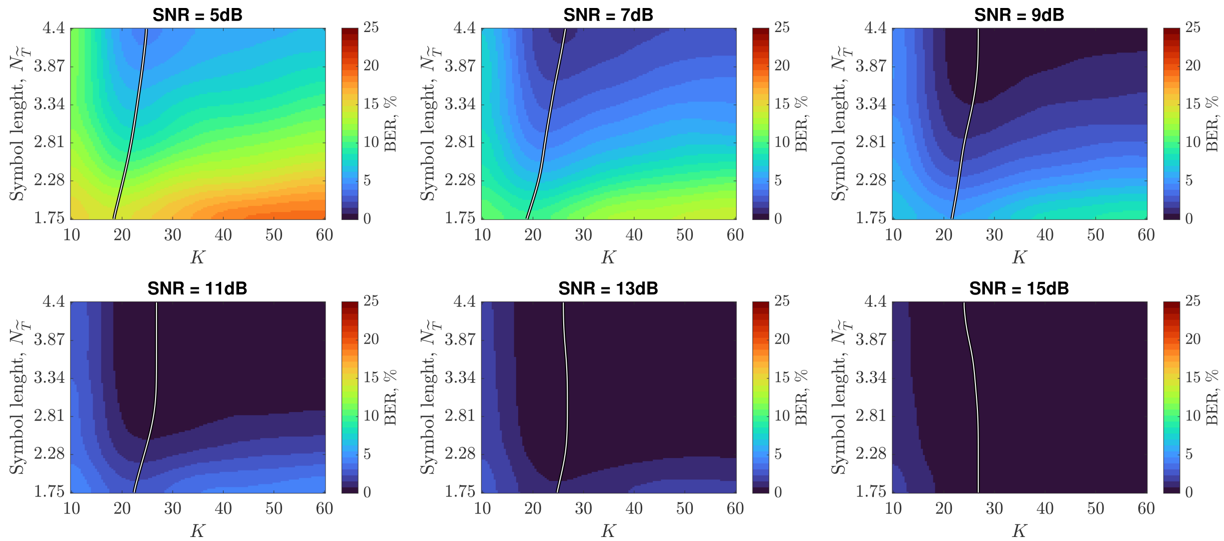

Figure 6 The dependence between BER, synchronization coefficient K, and symbol transmission length for various SNR values of Lorenz-based CCS system. The black-and-white line corresponds to the synchronization coefficient value where BER is minimal for certain SNR values.

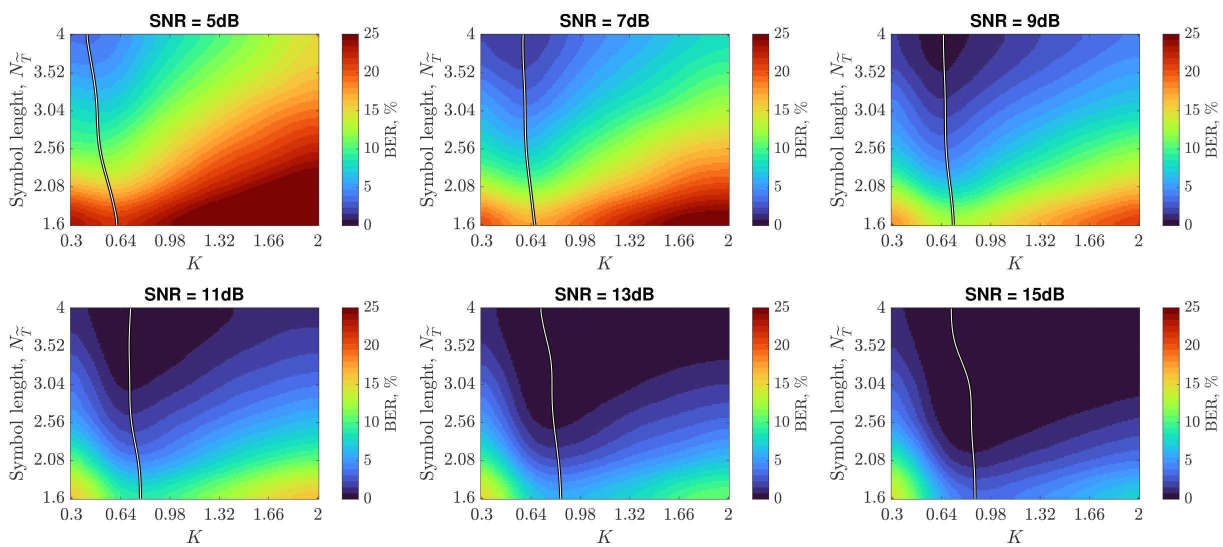

Figure 7 The dependence between BER, synchronization coefficient K and symbol transmission length with various SNR values for Rössler system. The black-and-white line corresponds to the synchronization coefficient value where BER is minimal for certain SNR values.

where ϑ is the median frequency of the chaotic signal (Hz), and T_s is the symbol transmission time in seconds.

Obtaining the optimal synchronization coefficient using BER and SNR metrics

Let us vary the synchronization coefficient under different noise conditions and calculate the corresponding bit-error rate. Figure 4 shows the experimental results for the CCS based on the Lorenz system with parameter σ ($\sigma_1 = 9.5$ and $\sigma_2 = 10.5$) modulation.

Figure 5 shows the experimental results for the investigated CCS based on Rössler system with parameter a ($a_1 = 0.18$ and $a_2 = 0.22$) modulation.

Following the experimental results, one can conclude that the most efficient metric for estimating the synchronization error in coherent communication systems is root-mean-square (RMS) as it provides the lowest BER for a given SNR value in comparison to other metrics, which makes it a perfect candidate for optimization function. Thus, our further experiments on the chaotic communication systems' optimization will be performed using this metric as a key evaluator of the design quality.

Investigating the dependence between K and $N_{\tilde{T}}$

Let us estimate the dependence of BER on the synchronization coefficient and the length of the transmitted symbol for different levels of noise present in the communication channel. Figures 6 and 7 show the experimental results for Lorenz and Rössler systems, respectively.

One can see from Figures 6 and 7, that while the SNR in the communication channel increases, the symbol transmission time may be reduced by preserving the same BER level using the proper choice of the synchronization coefficient. Thus, it can be concluded, that it also can be used for increasing the data transfer rate while

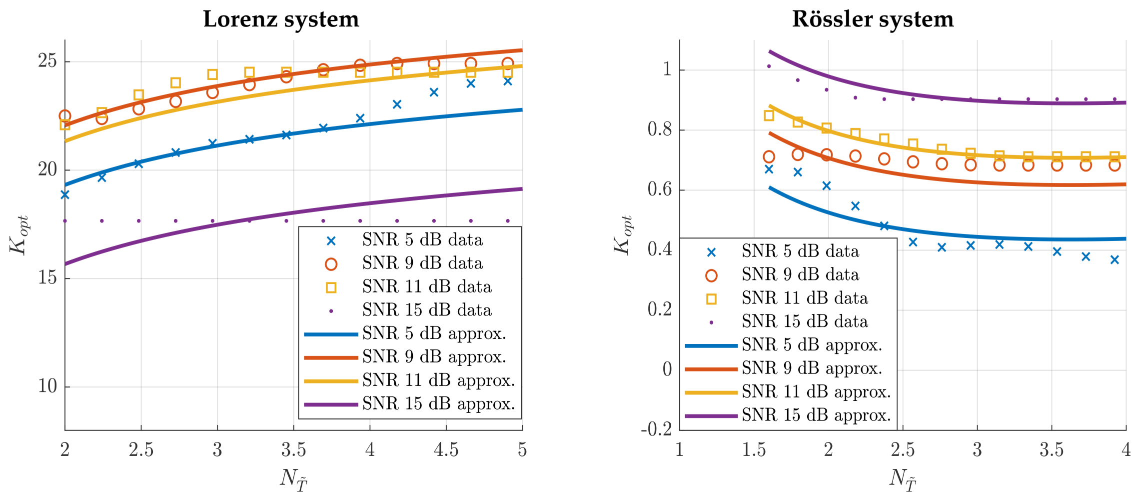

Figure 8 The behavior of optimal synchronization coefficient K while varying the symbol transmission length with various values of SNR for Lorenz and Rössler

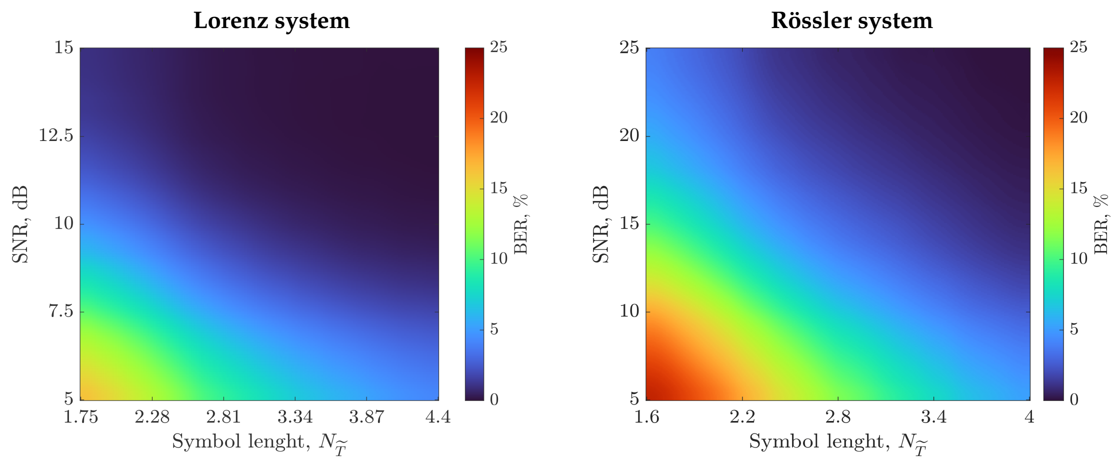

Figure 9 The dependence between BER, symbol transmission length, and SNR with variable synchronization coefficient K for Lorenz and Rössler systems.

preserving the same level of BER.

Approximation of optimal synchronization coefficient K

Using the results obtained in the previous subsection, let us obtain the equations for calculating the optimal synchronization coefficient depending on the symbol length and SNR.

The equation for optimal synchronization coefficient calculation for the Lorenz system is as follows:

$$K_L(N_{\tilde{T}}, \varsigma) = a + bN_{\tilde{T}} + c\varsigma + d\varsigma^2 + e/N_{\tilde{T}}; \quad (6)$$

where $N_{\tilde{T}}$ is a length of transmitted symbol in pseudo-periods, ς is a signal-to-noise ratio in dB, $a = 12.61$, $b = 0.1665$, $c = 3.141$, $d = -0.1753$, $e = -9.892$.

The equation for calculating the optimal synchronization coefficient for Rössler system is as follows:

$$K_R(N_{\tilde{T}}, \varsigma) = a + bN_{\tilde{T}} + c\varsigma + d/(N_{\tilde{T}})^g; \quad (7)$$

where $N_{\tilde{T}}$ is a length of transmitted symbol in pseudoperiods, ς is signal-to-noise ratio in dB, $a = -0.273$, $b = 0.06711$, $c = 0.04534$, $d = 0.886$, $g = 1.018$.

Figure 9 shows the estimation of BER in the proposed CCS with various symbol transmission lengths and SNR values, while K varies being obtained by equation 6 and 7. Experimental results confirm the linear dependence of the symbol's transmitted length on the various SNRs while maintaining the same BER value. Generally, this dependence is in direct correspondence with the Shannon theorem: when increasing the data transfer rate, the BER level will also increase (Shannon 1984).

Influence of the different CCS parameters on K

Table 1 shows the experimental results for the prototype chaotic communication systems based on Rössler and Lorenz systems, where modulated parameters are $a_0 = 0.18$ and $a_1 = 0.22$ for Rössler system, and $\sigma_0 = 9.5$ and $\sigma_1 = 10.5$ for Lorenz system.

Table 2 shows the experimental results for the CCS based on Lorenz and Rössler system with short symbol transmission length and parameter σ and a modulation, where $a_0 = 0.18$ and $a_1 = 0.22$,

Table 1 The optimal synchronization coefficient K value which provides a minimum SNR for BER = 0%

System	Rössler					Lorenz				
Method	Var	RMS	Med	Mean	StdDev	Var	RMS	Med	Mean	StdDev
Optimal K	0.6384	0.6816	0.5933	0.6394	0.6369	16.0754	22.1078	9.5615	19.1313	15.3134
Min SNR	12.7108	9.2169	17.0482	10.7831	12.8313	16.2371	10.5155	19.0206	13.7629	16.3918

Table 2 The optimal synchronization coefficient K value, providing a minimum SNR for BER = 0% for Rössler and Lorenz system for a short message.

System	Rössler					Lorenz				
Method	Var	RMS	Med	Mean	StdDev	Var	RMS	Med	Mean	StdDev
Optimal K	0.7541	0.8296	0.7148	0.7719	0.7694	14.4131	17.7306	7.8008	15.3437	14.1714
Min SNR	18.8285	14.5607	22.8452	16.318	18.7029	20.3614	14.4578	23.9759	17.4699	20.3614

Table 3 The optimal synchronization coefficient K value which provides the minimum SNR for BER = 0% while modulating a third parameter.

System	Rössler					Lorenz				
Method	Var	RMS	Med	Mean	StdDev	Var	RMS	Med	Mean	StdDev
Optimal K	0.5454	0.612	0.5104	0.552	0.5772	13.5907	19.0163	7.17294	15.3663	13.8238
Min SNR	11.2651	8.9759	15.4819	9.9398	11.506	10.7831	5.8434	13.5542	8.3735	10.6627

and $\sigma_0 = 9.5$ and $\sigma_1 = 10.5$.

Table 3 shows the experimental results for CCS based on Rössler and Lorenz system with parameter c modulation, where $c_0 = 5.7$ and $c_1 = 6.2$ for Rössler, and $b_0 = 2.3$ and $b_1 = 2.7$ for Lorenz system.

Note that in all experiments the RMS showed the highest performance, and the arithmetic mean performed slightly worse. However, the arithmetic mean can also be a potential candidate for coherent CCS implementation because of its computational simplicity, which is vital for such hardware as microcontrollers and FPGAs.

Another important conclusion from the repo experiments is that the value of optimal coefficient K depends also on the choice of parameters used for transmitting binary symbols '0' and '1'. Thus, the algorithm 3 should be executed for each parameter set in the CCS design.

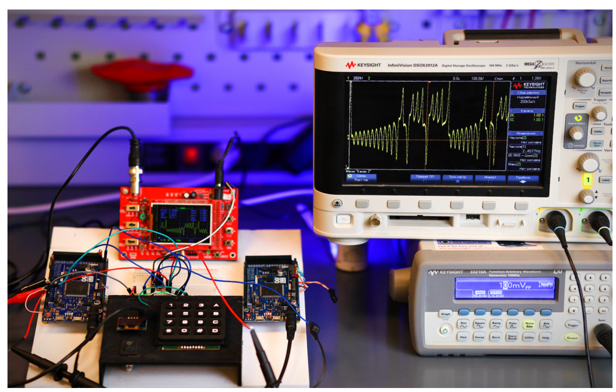

Figure 10 Experimental bench implementing MCU-operating chaotic communication system with a noisy channel based on Lorenz chaotic oscillator.

Experiments with hardware prototype

BER values obtained by numerical simulation were validated using a physical prototype of a Lorenz-based chaotic communication system. We performed this experiment using the experimental CCS testbench developed by our team for research purposes (Rybin *et al.* 2023). The appearance of the experimental bench is shown in Figure 10.

The suggested testbench consists of two microcontrollers (Arduino DUE) serving as transmitter and receiver, a wired communication channel with additive noise provided by a signal generator and op-amp-based mixer, a couple of oscilloscopes for acquiring and visualizing the signals, and a simple keyboard to input messages. In this experimental study, we used SNR levels of 5, 10, and 15 dB, and symbol lengths 2, 3, and 4 $N_{\tilde{T}}$. The optimal K values were calculated using the equation (6). The obtained results show that the numerical simulation allows us to predict most of the effects observed in the real CCS with high accuracy. The difference between the simulation and experiments using BER metrics appeared not to exceed 5%. This slight difference can be explained by statistical errors. For example, considering a data transfer rate of 6 bps, we transmitted only approximately 1000 symbols (bits) for each set of parameters. In addition, the experimental study is challenging in setting the required SNR level, as the noise admixing was performed in an analog way.

DISCUSSION

One may ask, is there a possibility that several optimal values of K exist? The Figures 4 and 5 clearly indicate that for all metrics for synchronization error analysis, the value of the optimal synchronization coefficient is unique in mathematical terms (note: this stands if the value of BER is greater than zero). In other words, if we consider K as a function of SNR, it is unimodal. For the cases with zero BER, one may find and choose the optimal synchronization coefficient which will provide the maximal transfer rate in the designed CCS.

Equations (6) and (7) may be combined with other noise estimation algorithms. Being a critical performance parameter that affects the reliability and throughput of both wire and wireless communications, the level of SNR is often estimated to dynamically adjust transmitter and receiver parameters. Many classical and recent works on the SNR estimation algorithms for communication systems indicate the high importance of the subject (Arslan and Reddy 2003; Hasan and Shongwe 2017; Khan *et al.* 2017; Türkben and Al-Akraa 2022). Having information about the current SNR level, the expression for calculating K_{opt} may be used for both selecting the symbol length at the transmitter side and for adjusting K at the receiver side.

CONCLUSION

The application of coherent chaotic communication systems is currently hampered by their insufficient performance when noise is present in the transmission channel. In the current study, we stepped towards solving this problem by analyzing test chaotic communication systems and finding an approach to estimating the optimal synchronization parameter K that allows researchers to significantly improve the noise immunity of CCS. We explicitly show that it is possible to find the optimal synchronization coefficient for an arbitrary coherent chaotic communication system when the minimum bit error rate (BER) will be achieved at the desired SNR level ς. This procedure requires taking into account other CCS parameters, such as the pair of modulation parameters for binary '0' and '1' representation (p_0 and p_1) and length of the symbol transmission $N_{\tilde{T}}$. Reducing the $N_{\tilde{T}}$, as expected, leads to a decrease in noise resistivity, and influences the value of the optimal synchronization coefficient as well.

In this study, we proposed the practically applicable algorithm for finding the optimal value of K, which takes into account all of the abovementioned factors, and constructed an empirical equation for the calculation of $K_{opt} = f(N_{\tilde{T}}, \varsigma)$ for a given modulation parameter set in a practical system.

We also investigated the efficiency of different techniques for analyzing synchronization errors that are commonly used in CCS design for distinguishing '0' and '1' symbols at the receiver side. We discovered that using arithmetic means and RMS allows us to achieve the lowest BER values. Besides, the arithmetic mean is easier to implement in microcontrollers and FPGAs, while the RMS makes it possible to choose a larger value of the synchronization coefficient, which potentially provides a higher data transfer rate.

As a practical result, we managed to increase the noise immunity of the test coherent communication system without changing its communication structure and without using any denoising or filtering algorithms. It is shown, that by choosing the proper K values and $N_{\tilde{T}}$, it is possible to achieve zero BER at a certain SNR value, while the non-optimal choice of K leads to bit errors at higher SNR levels. For both considered chaotic communication systems, we achieved nearly zero BER using K_{opt} at an SNR level of 3-5 dB, which is significantly lower in comparison to the CCS architectures with fixed synchronization coefficient values known from the literature.

As the direction of future research, we will consider noise level and noise color estimation algorithms for practical CCS implementation in FPGA, as well as combine the suggested approach with digital signal processing techniques.

Author contributions

Conceptualization: Denis Butusov and Vyacheslav Rybin; Formal analysis: Dmitriy Kvitko and Erivelton Nepomuceno; Funding acquisition: Denis Butusov; investigation: Ivan Babkin, Dmitry Kvitko and Vyacheslav Rybin; Methodology: Denis Butusov and Vyacheslav Rybin; project administration: Denis Butusov, Erivelton Nepomuceno and Vyacheslav Rybin; resources: Lucas Nardo and Timur Karimov; software: Dmitriy Kvitko, Ivan Babkin and Timur Karimov; supervision: Denis Butusov; validation: Timur Karimov and Lucas Nardo; visualization: Ivan Babkin and Vyacheslav Rybin; writing – original draft: Vyacheslav Rybin, Timur Karimov and Denis Butusov; writing – review and editing: all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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Availability of data and material

The data collected in this study are available from the corresponding author upon reasonable request.

Conflicts of interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical standard

The authors have no relevant financial or non-financial interests to disclose.

APPENDICES

Methods for synchronization errors analysis during messaging

The presence of noise in the communication channel makes it difficult to use coherent chaotic communication systems (Rybin *et al.* 2023). Therefore, it is of interest to determine the most efficient way to analyze the synchronization error. In this study, we evaluate the effectiveness of variance, root mean square, median mean, and standard deviation values.

Variance The variance is a measure of the spread of numbers in a data set relative to the mean. Using variance, we can evaluate how stretched or squeezed a distribution is. If the variance value is small then the values are close to each other, if the values are large then it means the values are far away. The variance (σ^2) is quantified as:

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N}, \tag{8}$$

where $\overline{x}$ stands for mean and x_i is the i^{th} data point.

Root mean square (RMS) the RMS (σ) is a measure of the dispersion of numbers in a data set relative to the mean value. It usually means the square root of the variance. It is calculates as follows:

$$\sigma = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2}{N}}, \tag{9}$$

where N represents the number of data points.

Median value The median ($\tilde{x}$) of a finite list of numbers is the "middle" number when those numbers are listed in order from smallest to greatest. In general, with this convention, the median can be defined as follows: for a data set x of n elements, ordered from smallest to greatest, if n is odd:

$$\tilde{x} = x_{(n+1)/2}, \tag{10}$$

if n is even:

$$\tilde{x} = \frac{x_{(n/2)} + x_{((n/2)+1)}}{2}. \tag{11}$$

Arithmetic mean The arithmetic mean ($\overline{x}$) is the simplest and most widely used measure of a mean or average. It simply involves taking the sum of a group of numbers, then dividing that sum by the count of the numbers used in the series. The equation for a data set x of n elements is

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{x_1 + x_2 + \cdots + x_n}{n}. \tag{12}$$

Standard deviation Standard deviation (S) is a statistic that measures the dispersion of a data set relative to its mean and is calculated as the square root of the variance by determining each data point's deviation relative to the mean. The equation for a data set x of n elements is

$$S = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}. \tag{13}$$

Experimental results for parameter b and c for Lorenz and Rössler system, respectively;

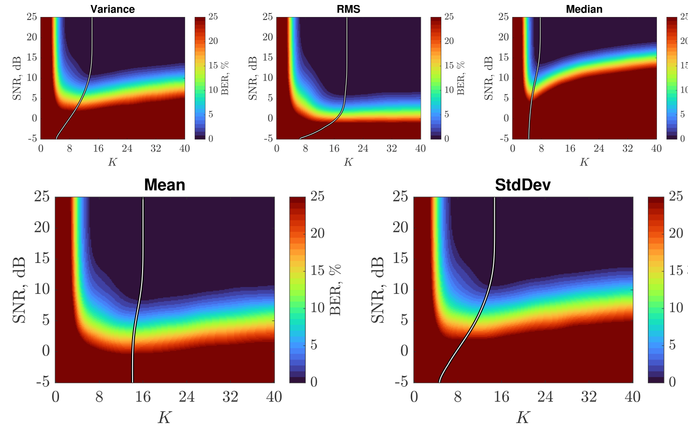

Figure 11 The dependence between BER, synchronization coefficient K, and SNR for Lorenz system with parameter b ($b_1 = 2.3$ and $b_2 = 2.7$). The black-white line corresponds to the synchronization coefficient value where BER is minimal for certain SNRs.

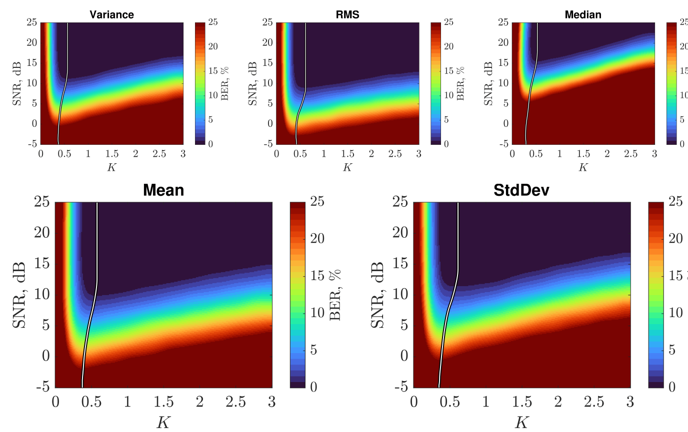

Figure 12 The dependence between BER, synchronization coefficient K, and SNR for Rössler system with parameter c ($c_1 = 5.7$ and $c_2 = 6.2$). The black-white line corresponds to the synchronization coefficient value where BER is minimal for certain SNRs.

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