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‘Disturbed’ by Euclid: Thomas Fincke and the reading of Ramist mathematics in sixteenth-century Germany

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Abstract

This article presents evidence of the transmission and reception of Petrus Ramus’s mathematical pedagogy, as witnessed in a multi-volume *Sammelband* constructed and used in late sixteenth-century Germany. It considers how the methodological influence of Ramus was transmitted to students by the mathematical work of Thomas Fincke, before suggesting that idiosyncratic users of the *Sammelband* tangled with authoritative interpretations of Euclid by incorporating their own reading and notational practices. Whilst Fincke’s work was aimed toward students familiar with Ramist teaching, marginalia found within the Sammelband reinterpret these intentions, demonstrating the pedagogical relationships shared between Fincke, his predecessors, and the later readers of the volume.

Zusammenfassung

In diesem Artikel wird die Übertragung und Rezeption von Petrus Ramus’ mathematischer Pädagogik anhand eines mehrbändigen Sammelbands gezeigt, der im späten 16. Jahrhundert in Deutschland zusammengestellt und verwendet wurde. Es wird untersucht, wie der methodologische Einfluss von Ramus durch die mathematische Arbeit von Thomas Fincke auf die Schüler übertragen wurde, bevor darauf hingewiesen wird, dass eigenwillige Benutzer mit maßgeblichen Interpretationen von Euklid verwickelt sind, indem sie ihre eigenen Lese- und Schreibpraktiken einbeziehen. Während sich Finckes Arbeit an Schüler richtete, die mit der Ramistenlehre vertraut waren, interpretierten marginale Beiträge im Sammelband diese Absichten neu: Sie zeigten die pädagogischen Beziehungen, die zwischen Fincke, seinen Vorgängern und den späteren Lesern des Bandes bestanden.

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In 1583, the precocious Dane Thomas Fincke (1561–1656) announced his mathematical ability with the publication of *Geometriae rotundi libri XIIII*. Comprised of 14 chapters, this textbook guided readers through a novel presentation of spherical geometry, moving from the form of the circle and sphere, to the relationships between their radii, diameters, and sines, before fully explicating the 'law of tangents' later developed algebraically by François Viète.[2] Twelve years before Bartholomaeus Pitiscus (1561–1613) presented a new name for the study in *Trigonometria* (1595), Fincke introduced the words 'tangent' and 'secant' to the study of triangles, offering a new means of conceptualising trigonometric functions. Having drawn extensively upon the works of Ptolemy and Regiomontanus, the author advised studious readers to follow his lead by profiting from the lessons of the German mathematician in particular.[3]

Acknowledging his debt to Regiomontanus as uppermost amidst a clutch of mathematical authorities old and new, Thomas Fincke identified himself as part of a rich lineage of practitioners motivated by the revival and improvement of classical mathematics.[4] Yet it should be noted that the Dane also took his lead from the period's most important pedagogical reformers. A pupil of the Strasbourg Academy of Johannes Sturm (1507–1589), and heavily influenced by the works of Petrus Ramus (Pierre de la Ramée, 1515–1572), Fincke ensured that his own textbook bore many of the hallmarks of Northern European humanism in which he had been schooled: one radiating outwards from the Wittenberg of Philip Melanchthon (1497–1560).[5]

Amid ecclesiastical and educational reform and counter-reform, further pedagogical territory was yet to be won. With this in mind, Fincke made Petrus Ramus's methodical presentation of mathematical theory central to *Geometria rotundi*. Combining the lessons of a nomadic education undertaken in Strasbourg, Wittenberg, and elsewhere with Ramist dialectic, Fincke promulgated a new model for mathematical pedagogy. 'Disturbed' by the Euclidean presentation of geometry,[6] the author sought to recover his discipline's classical foundations by expunging in part the supposedly artificial and abstruse syllogistic structures erected in the *Elements*. *Geometriae rotundi* was thus designed to help new generations of mathematical readers break free from theorems and from deductive reasoning, presenting instead dichotomies of definition and proposition as a means to more effectively teach geometry.

Readers' responses to Fincke's efforts—and to those of his predecessor, Petrus Ramus—litter the pages of a unique, multi-volume artefact today held in the Science Museum, London's Rare Books Collection. *Geometriae rotundi* is one of three printed quartos bound up in what I have termed the 'Wittenberg *Sam-*

[2] For a summary of the mathematical relationship between Viète's and Fincke's trigonometry, see Enrique A. Gonzalez-Velasco, *Journey through Mathematics: Creative Episodes in Its History* (New York: Springer Science + Business Media, 2011), particularly pp. 74–76. For the key sections in Fincke's work, see Thomas Fincke, *Thomae Finkii Flenspurgensis Geometriae rotundi libri XIIII* (Basel: Sebastian Henric-Petri, 1583), particularly pp. 73–76.

[3] Fincke, *Geometriae rotundi*, p. 295: 'Regiomontanus aliquot casus in secondo libro de triangulis collegit (. . .) Cujus certe libri à studiosus avidè legi debent; & cum fructu legi possunt.' On Fincke's use of the terms 'tangent' and 'secant', see Augustus de Morgan, 'On the first introduction of the words tangent and secant', in David Brewster, Richard Taylor, Richard Phillips, and Robert Kane, eds., *The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, vol. 28, January–June 1846*. (London: Richard and John E. Taylor for the University of London, 1846), pp. 382–387.

[4] For Regiomontanus's famed 'Padua Oration' of 1464, see Regiomontanus, 'Oratio Iohannis de Monteregio, habita in Patavii in praelectione Alfragani' in *Opera collectanea*, ed. Felix Schmeidler (O. Zeller: Osnabruk, 1972), pp. 43–53. Regiomontanus's oration has been situated both in the humanist educational culture of the fifteenth century and within a wider historiography of mathematics from a mathematician's perspective; see James Steven Byrne, 'A Humanist History of Mathematics? Regiomontanus's *Padua Oration* in Context', *Journal of the History of Ideas*, 67 (2006), pp. 41–61, and Michela Malpangotto, *Giovanni Regiomontano e il rinnovamento del sapere matematico e astronomico nel Quattrocento* (Cacucci Editore: Bari, 2008).

[5] Peter Mack, *A History of Renaissance Rhetoric, 1380–1620* (Oxford: Oxford University Press, 2011), p. 104.

[6] Fincke, 'Praefatio ad Lectorem', *Geometriae rotundi*, f. 1 v: 'methodum vero in ubertate tanta nullam, aut vix ullam videre potui Quam id me perturbarit'.

melband', named for the University linking the authors, teachers, and students found within its pages. Although evidence found within the *Sammelband* demonstrates that its first owners and users were located in Leipzig rather than Wittenberg, I believe that the volume was constructed to serve the needs of students who intended to attend the university or similar institutions. For Petrus Ramus, Wittenberg, thanks to the erudite leadership of Philip Melanchthon, was the jewel in the crown of German mathematics.[7] His autodidactic reader, Thomas Fincke, author of the second of the texts found in the volume, followed both his father and uncle in attending the august institution prior to returning to Flensburg to write *Geometriae rotundi* in 1583.[8] The topics of the texts match the early modern curriculum as undertaken at the university, and it appears that the annotator who constructed the volume's notes on sexagesimal notation was aware of the works of figures affiliated to Wittenberg. Finally, as we shall see, there is clear evidence of the volume being utilised (and, in all likelihood, shared) by students who attended the University of Wittenberg in the early 1590s.

Taking Thomas Fincke as a point of departure, this essay explores the interplay between the author's *Geometriae rotundi*, his pedagogical predecessors, and a second layer of readers—the owners and annotators of the *Sammelband* which contained the works of both Ramus and Fincke—to more clearly illuminate the adoption of Ramist methods in early modern mathematical teaching and learning. The contents and materiality of the *Sammelband* simultaneously allow us to witness early modern processes behind mathematics' changing transmission and reception; crucially, analysis of the volume's authors and their readers provides direct access to the socio-historical processes of diffusion and standardisation underpinning the spread of a new mathematical pedagogy. In this way, the artefact represents an 'instrument of knowing' mathematics through its opening texts' expressly Ramist pedagogy.[9]

1. The Wittenberg *Sammelband*: provenance and construction

Three-quarter bound in panel-stamped vellum on painted wooden boards and bearing the bookbinder's initials 'M K G – 1586' to its front cover,[10] the Wittenberg *Sammelband* comprises Petrus Ramus's *Arithmeticae libri duo, Geometriae septem et viginti libri* (1580), Fincke's aforementioned *Geometriae rotundi* (1583) and a contemporary edition of John Peckham's optical text *Perspectiva communis libri tres* (1580).[11]

7 For a helpful summary of Ramus's admiration of Wittenberg (and German mathematics more generally), see Robert S. Westman, *The Copernican Question: Prognostication, Skepticism, and Celestial Order* (Berkeley, Los Angeles, and London: University of California Press, 2011), pp. 168–170.

8 Jürgen Schönbeck, 'Thomas Fincke und die *Geometriae rotundi' NTM Zeitschrift für Geschichte der Wissenschaften, Technik und Medizin*, 12.2 (2004), pp. 80–89, p. 82 and p. 83.

9 This phrase is taken from Walter J. Ong, quoting Johann Heinrich Alsted's 1609 dictum from *Clavis artis Lulliane*, 'Ergo dialectica est ars tradens modum sciendi et per consequens docens instrumentum sciendi'. Ong argued that Alsted's assimilation of the methods of Aristotle, Ramon Lull, and Petrus Ramus marked something of a victory for the Ramist art of pedagogy in particular. Walter J. Ong, *Ramus, Method, and the Decay of Dialogue: From the Art of Discourse to the Art of Reason*, 2nd edn. (Chicago and London: University of Chicago, 2004; originally Cambridge, MA: Harvard University Press, 1958), p. 160.

10 Though rare, similar stamps have been discovered on books bound in England, Germany and Northern Europe in this period. J. Basil Oldham, *English Blind-Stamped Bindings* (Cambridge: Cambridge University Press, 1952), particularly pp. 33–37; Edith Diehl, *Bookbinding: Its Background and Technique*, Volume 1 (New York: Dover, 1980) (originally New York: Rinehart and Co., 1946), p. 132. Oldham points out that English and German binders of the late sixteenth-century (and other agents in the binding process) often 'signed' their work in this fashion, whilst Diehl also refers to the German market's predilection for 'rolls with a pattern divided by segments (...) on (which) were frequently engraved the initials of the bookbinder'. Earlier evidence of panel stamped bindings surrounding texts printed in Basel is discussed in Ernst Kyriss, 'Parisian Panel Stamps between 1480 and 1530', *Studies in Bibliography*, 7 (1955), pp. 113–124, particularly p. 116 and p. 123.

11 Petrus Ramus, *P. Rami Arithmeticae libri duo: Geometriae septem et viginti* (Basel: haer. Nikolaus II Episcopius, 1580) Science Museum Library Shelfmark O. B. RAM RAMUS 30209019362784; Thomas Fincke, *Thomae Finkii Flenspurgensis Geometriae rotundi libri XIIII* (Basel: Sebastian Henric-Petri, 1583), Science Museum Library Shelfmark O. B. RAM RAMUS

Following these printed works is a contemporary manuscript summary entitled *De Logistice Astronomica seu sexagenaria*. Written as a series of axioms covering the importance of sexagesimal arithmetic to the study of astronomy, these papers appear to have been lecture notes cribbed from sources such as Caspar Peucer's *Logistice Astronomica Hexacontadon et Scrupulorum Sexagesimorum* (1556), Edo Hildericus von Varel's *Logistice Astronomica* (1568), and Lazarus Schöner's *De Logistice sexagenaria liber* (1569), the first two of whom were Wittenberg professors. Perhaps the most closely related of these sexagesimal texts to the end notes of the Wittenberg *Sammelband* is that of Lazarus Schöner, the Nuremberg mathematician and Wittenberg alumni. Schöner edited and further popularised Ramus's mathematical works from a position of authority:[12] initially as a teacher in the *gymnasia* of Schmalkalden and Marburg, and then as Rector of the Korbach grammar school.[13]

To all intents and purposes, the print and manuscript contents collated in the *Sammelband* made for an enlarged type of textbook. Its printed works were ordered by their increasing complexity, and by the progress students might be expected to make as they ascended the *quadrivium*: moving from arithmetic, to geometry, to astronomy. Likely bound together in 1586, as the volume's cover stamp suggests, each quarto appears to have been uncirculated prior to this date. The texts retain their title pages and colophons, with no leaves found to be wanting. Pages are consistently trimmed, with the exception of the manuscript leaves, all of which are slightly larger than those of the printed works. This minor difference notwithstanding, there is little to suggest that *De Logistice astronomica seu sexagenaria* was not included in the original binding of 1586: tellingly, its fore-edges feature the same contemporaneous, blue-green paint washing as the printed works. When compared with like examples from late sixteenth-century Northern Europe, it is clear that the *Sammelband*, with its ornate binding and decorated fore-edges, was an object upon which time, money, and effort had all been spent.

The *Sammelband*'s earliest owners can be traced with some precision. Whilst in Leipzig, Nicholas Hommer of Copenhagen signed and dated its front pastedown to 17 November, 1587. A salutation written to Hommer by Johannes Coppius, of Leisnig, Saxony, suggests that the former was still in possession of the volume in 1589:

> (To the) Most decorated and learned young master Nicolao Hommero, of Copenhagen, writing with love and goodwill. Leipzig, M(aster) Johannes Coppius of Leisnig, 17 January (15)89.[14]

The manuscript notes of *De Logistice astronomica* appear to be largely in Coppius's hand, with occasional commentaries from Hommer interposed. The inscription above is suggestive of the relationship between a tutor and his student, or of evidence of a friendship between two students with a shared interest in mathematics, written in the fashion of the *album amicorum* popular in the period. While mottoes fell in and out

30209019362777; John Peckham, *Perspectivae communis libri tres* (Cologne: Arnold Birckmann, 1580), Science Museum Library Shelfmark O. B. RAM RAMUS 30209019362791. This copy is hereafter referred to as Wittenberg *Sammelband*, with individual texts referenced according to their shelfmark.

12 Schöner's *De Logistice sexagenaria liber* of 1569 was re-issued as part of the author's edited version of Petrus Ramus' *Petri Rami Arithmetices Libri Duo, et Algebrae totidem* (1586). The notes on sexagesimal astronomy appended to the Wittenberg *Sammelband* bear a relatively high degree of similarity to *De Logistice sexagenaria liber;* given that Schöner's work was re-issued in the same year in which the *Sammelband* was bound, I believe that the notes on sexagesimal astronomy are an owner's inexact paraphrase predominantly of Schöner's work and of other sources. See Lazarus Schöner, *De Logistice sexagenaria liber*, in Petrus Ramus, *Petri Rami Arithmetices libri duo, et Algebrae totidem* (Frankfurt: Andreae Wechelus, 1586), pp. 364–406.

13 Alastair Hamilton, *William Bedwell the Arabist, 1563–1632* (Leiden: Published for the Sir Thomas Browne Institute by E.J. Brill and The University of Leiden Press, 1985), p. 61. Howard Hotson, *Commonplace Learning: Ramism and its German Ramifications, 1543–1630* (Oxford: Oxford University Press, 2007).

14 The original text reads: 'Ornatiss(im)o et doctiss(sim)o juveni Domino Nicolao Hommero Hafniensi amoris et benevolentiae ergo scribebat Lipsiae M. Joh. Coppius Leisnicensis, 17 Jan. 89.'

of use and fashions changed, a fellow scholar's autograph or heraldry was a stamp of loyalty and of lasting friendship on journeys criss-crossing European institutions.[15]

The volume had by 1593 passed through two further sets of hands in just this fashion: those of another Dane, David Johannes Klynaeus, of Copenhagen, and those of his contemporary, Johannes Lobhartzberger of Kłodzko (then Glacio, in Lower Silesia), both of whom are registered in manuscript on the title page of Ramus's introductory work.[16] Each name was scored widely through, though not entirely obscured, and we need not necessarily assume that Nicholas Hommer had lost ownership of his *Sammelband* by this point. The palaeographic similarities shared between these inscriptions suggest that Hommer may have loaned his annotated text out to friends and countrymen at university, marking borrowers and scoring their names out upon the volume's successful return. Both Klynaeus and Lobhartzberger can be placed at the University of Wittenberg in the early 1590s, though the trail of Hommer has petered out by this point. David Johannes Klynaeus featured as one subject of the ribald verse penned by Friedrich Taubmann (1565–1613), a Wittenberg professor, poet, and something of a jester at the court of the Duke of Saxe-Weimar, Friedrich Wilhelm I (1562–1602).[17] Johannes Lobhartzberger, meanwhile, was a companion to Daniel Sennert (1572–1637), and a dedicatee of Sennert's *Templum Mnemosynes* (Wittenberg, 1599), a poem lauding the Brunian application of the art of memory.[18]

Ultimately, the precise identities of these annotations' authors are of less importance than what they suggest about the volume's transmission between users. The youthful Nicholas Hommer was reading, copying, and rearranging the mathematical texts of Petrus Ramus and Thomas Fincke in Leipzig, perhaps under the tutelage or in the company of Johannes Coppius. That Johannes Lobhartzberger and David Klynaeus then had cause to utilise the compendium whilst at Wittenberg suggests the continuing value of the collection to university students, and perhaps even to those preparing for disputations. The Renaissance teaching of 'mathematics for astronomy' popular from the mid-fifteenth century onwards has been characterised by reference to a quadripartite hierarchy, consisting of the use of *fractiones physicae*, or sexagesimal positional fractions; the arithmetic of large numbers; theories of proportions as applied to plane and spherical trigonometry, and to fractions; and a particular interest in trigonometric canons and tables of sines.[19] Studies of later Wittenberg textbooks have demonstrated that materials authored by university professors were bound together with those on related subjects, and that the mathematical teachings of these professors on topics such as Copernican heliocentrism could treat both rudimentary and complex mathematics without engaging in cosmological controversy.[20] As a consequence, the *Sammelband*'s combinations of introductory and more complex mathematics, coupled with Thomas Fincke's spherical trigonometry and canons,

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[15] Margaret F. Rosenthal, 'Fashions of Friendship in an Early Modern Illustrated Album Amicorum: British Library, MS Egerton 1191', *Journal of Medieval and Early Modern Studies*, 39.3 (2009), pp. 619–641, p. 622.

[16] Petrus Ramus, *Arithmeticae Libri Duo, Gemoetriae Septem et Viginti*, title page. Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362784.

[17] Friedrich Taubmann, *Melodaesia sive Epulum Musaeum* (Leipzig: Thomas Schurer, 1597), pp. 453–454. For a summary of Taubmann's life and endeavours, see H.C. Erik Midelfort, *A History of Madness in Sixteenth-Century Germany* (Stanford, CA: Stanford University Press, 1999), pp. 270–275.

[18] Daniel Sennert, *Templum Mnemosynes* (Wittenberg: M. Henkel, 1599), title page. See also Christoph H. Lüthy and William R. Newman, 'Daniel Sennert's Earliest Writings (1599–1600) and Their Debt to Giordano Bruno', *Bruniana and Campaneliana*, 6 (2000), pp. 261–279, particularly pp. 274–275.

[19] Grazyna Rosínska, '"Mathematics for Astronomy" at Universities in Copernicus' Time: Modern Attitudes toward Ancient Problems', in Mordechai Feingold and Victor Navarro-Brotons, eds., *Universities and Science in the Early Modern Period* (Dordrecht: Springer, 2006), pp. 9–28, p. 11.

[20] Pietro Daniel Omodeo, *Copernicus in the Cultural Debates of the Renaissance: Reception, Legacy, Transformation* (Leiden and Boston: Brill, 2014), p. 68; Stefan Kirschner and Andreas Kühne, 'The Decline of Medieval Disputation Culture and the 'Wittenberg Interpretation' of the Copernican Theory', in Wolfgang Neuber, Thomas Rahn and Claude Zittel, eds., *The Making of Copernicus: Early Transformations of the Scientist and his Science* (Leiden and Boston: Brill, 2015), pp. 13–41, p. 16. For a discussion of the methodological outlook shared among astronomers at the University of Wittenberg in the sixteenth century and

and manuscript materials on sexagesimal astronomy, appear to be entirely in keeping with the mathematical curriculum as experienced at the University of Wittenberg.

By the time Klynaeus and Lobhartzberger encountered the *Sammelband* in 1593, Thomas Fincke had begun to move away from mathematical study, gravitating instead toward a career in medicine.[21] A mere three years after its original publication, his *Geometriae rotundi* was bound up alongside the works which had so influenced its author and was used with these texts to advance the ideas of a pedagogical coterie inspired by ideas concerning the utility of mathematics. Nevertheless, Fincke's brief authorial career should not distract attention from the fact that he was a near-contemporary of the owners of the *Sammelband* that featured his work, and nor should it go unrecognised that the author was just as much a product of the self-same Northern European educational system as Nicholas Hommer and his peers. The pre-eminence of Ramist method shared by the first two titles of the *Sammelband*—Petrus Ramus's own *Arithmeticae libri duo, Geometriae septem et viginti* and Thomas Fincke's Ramus-inspired *Geometriae rotundi*—is significant for understanding both the authorship of the texts and their subsequent use and transmission; the promotion of dialectic was central to the educational philosophies of Melanchthon, Sturm, and Ramus and, ultimately, to the paratexts of the Wittenberg *Sammelband*.

2. Fincke's Educational Lineage: The Influence of Melanchthon, Sturm and Ramus

It is unsurprising to see Petrus Ramus's textbooks being put to use by readers including Thomas Fincke and Nicholas Hommer in the Northern Europe of the late sixteenth century. One leading authority has claimed that Germany was very much the 'seedbed' for Ramist philosophy following the murder of its creator;[22] indeed, the philosophical works of Ramus and his ally Omer Talon (ca. 1510–1562) gained their strongest foothold in German-speaking regions (including Switzerland and Alsace) between 1570 and 1600.[23] By the 1590s, the teachings of the French pedagogue had become entrenched in the schools of Northern Europe, despite continued attempts to halt their diffusion.[24] Even as the universities of Leipzig (1591), Rostock (1592), and, eventually, Wittenberg (1602) clamped down on Ramism and its methods, pupils nonetheless arrived expecting to continue their education in such a manner.[25] In 1597, the decree of Duke Julius of Helmstedt, intended to vet and ultimately expunge Ramist teaching, acknowledged that students came with little else in their heads. Allowances were therefore made to allow tutors to use Ramist materials, albeit privately.[26]

the impact of this outlook on the transmission of Copernican theory, see Robert S. Westman, 'The Melanchthon Circle, Rheticus, and the Wittenberg Interpretation of the Copernican Theory', *Isis*, 66.2 (1975), pp. 164–193.

21 Between 1583 and 1601 Fincke published works on mathematics and astronomical calculation, including *Theses de constitutione philosophiae mathematicae* (Copenhagen: Mads Vingaard, 1591) and *Horoscographia* (Schleswig: Nikolaus Wegener, 1591). By 1594, he had taken charge of the University of Copenhagen's curriculum, publishing *Theses Logicae* (Copenhagen: Mads Vindgaard, 1594), *Theses Philosophicae* (Copenhagen: Mads Vindgaard, 1594), as well as Bachelor's and Master's programmes—*Programma universitatis Hafniensis in promotionem baccalaureorum 15.3.1594* (Copenhagen: Mads Vingaard, 1594) and *Programma universitatis Hafniensis in promotionem magistrorum* (Copenhagen: Mads Vingaard, 1594)—all in the same year.

22 Ong, *Ramus, Method and the Decay of Dialogue*, p. 298.

23 Joseph S. Freedman, 'The Diffusion of the Writings of Petrus Ramus in Central Europe, c.1570 – c.1630', *Renaissance Quarterly*, 46.1 (1993), pp. 98–152, p. 100. The Latin edition of Ramus's *La Dialectique*, titled *Dialecticae libri duo* (1556) is included in Freedman's wider analysis.

24 Ong, *Ramus, Method and the Decay of Dialogue*, p. 299. Ong claimed that attempts to ban Ramism in the 1590s led to the development a group of scholars who syncretised Ramist, Aristotelian, and Melancthonian thought. According to Ong, this group were somewhat confusingly known to contemporaries as either 'mixts', 'Philippo-Ramists', and/or 'Systematics'.

25 For the Aristotelian backlash against Ramism in Germany, and specifically in Helmstedt, see Pietro Daniel Omodeo, 'Hoffmanstreit' in Pietro Daniel Omodeo and Karin Friedrich, eds., *Networks of Polymathy and the Northern European Renaissance* (Leiden and Boston: Brill, 2016), pp. 82–85.

26 Hotson, *Commonplace Learning*, p. 94.

Accordingly, teachers and pupils affiliated to large-scale *gymnasia* turned to works of a Ramist bent in almost every branch of the curriculum in growing numbers.[27] Though arithmetic and geometry ostensibly belonged to the medieval *quadrivium*, and were intended to be studied at university level, in practice introductory mathematical studies joined the trivium of grammar, rhetoric and dialectic on the pre-university curriculum of Protestant *scholae triviales* where Ramist materials made steady progress.[28] It is readily apparent why: Ramus's mathematical works start from perhaps the most introductory position possible, famously stating that arithmetic 'is the art of numbering [counting] well';[29] geometry, 'that of measuring well'.[30] As one early English translator had it, these pedagogical texts were 'most conuenient and fit for the master to teach and the scholler to learn, not only the art, but also the use of the art.'[31] Those who had previously read Ramus's works on mathematics to develop their own disciplinary expertise often retained an admiration for the texts' methodological lucidity as well as the step-by-step definitions offered by the author. In one such instance, John Napier opened the second book of *Mirifici logarithmorum canonis descriptio* (1614) by directly praising Ramus's succinct definition of geometry, before incorporating a number of lessons taken from his *Geometriae septem et viginti libri* on magnitude and on the figure of the triangle.[32]

The first of *Arithmeticae libri duo*'s two books moved from basic instruction on the numeration and notation of addition, subtraction, multiplication and division of whole numbers, to an explication of compound numbers and the numerators and denominators of fractions. The second, meanwhile, commenced with arithmetical and geometrical proportion before gradually presenting more complex examples of arithmetical and geometrical progression. Practical examples for the calculation of compound interest over time, involving multiplication and addition of whole numbers and fractions, are representative of its content. Following on from *Arithmeticae libri duo*, Ramus's *Geometriae septem et viginti libri* provided a cursory introduction to the foundations of geometry before devoting ever more attention to the discipline's practical application. Its goal was to inspire the reader to unite their natural faculties with the many worthwhile pursuits improved by geometrical knowledge: Ramus listed the praxis of astronomers, navigators, surveyors and architects as the fruits of geometry's vines.[33] The author's desire to kindle a love of practical application in his students is reminiscent of a comment made in his 1545 translation of Euclid's *Elements*; the student who plays at imitating the construction of geometric figures by first drawing them in the dust would, in Ramus's view, be more worthy of praise than one simply gazing at printed figures.[34]

[27] Hotson, *Commonplace Learning*, p. 115. Hotson's analysis of the Ramus and Talon inventory shows that more than 80 per cent of Ramus's works on grammar, mathematics, physics, metaphysics and theology between the author's death in 1572 and 1620 originated in Germany or Basel. See also Walter J. Ong, *Ramus and Talon Inventory: A Short-Title Inventory of the Published Works of Peter Ramus (1515–1572) and Omer Talon (Ca. 1510–1562) in Their Original and Variously Altered Forms* (Cambridge, MA: Harvard University Press, 1958).

[28] Joseph S. Freedman, *Diffusion of the Writings of Peter Ramus*, p. 123. See also Joseph S. Freedman, 'Philosophy Instruction within the Institutional Framework of Central European Schools and Universities during the Reformation Era', *History of Universities*, 5 (1985), pp. 117–166.

[29] Petrus Ramus, *Arithmeticae libri duo* (Basel: haer. Nikolaus II Episcopius, 1569), p. 1: 'Arithmetice est doctrina bene numerandi'.

[30] Petrus Ramus, *Geometriae septem et viginti* (Basel: haer. Nikolaus II Episcopius, 1569), p. 1. 'Geometria est ars bene metiendi'.

[31] Petrus Ramus, *The Art of Arithmeticke in Whole Numbers and Fractions*, trans. William Kempe (London: Richard Field for Robert Dexter, 1592), f. a iiij.

[32] John Napier, *Mirifici logarithmorum canonis descriptio* (Edinburgh: Andreae Hart, 1614), p. 21: 'Quum Geometria sit ars bene metiendi, dimensio sit magnitudinum propositarum, magnitudines figuram (potentia saltem) constituent, figura sit Triangulum, at triangulatum'.

[33] Ramus, *Geometriae septem et viginti*, p. 1: 'hic sinis geometriae usu atque opera geometrico multo splendidior apparebit, quám praeceptis, cum animadvertes astronomos, geographos, geodetas, nautas, mechanicos, architectos'.

[34] Petrus Ramus, *Euclides* (Paris: apud Lud. Grandinum, e regione gymnasij Mariani sub signo galli, 1545), p. 4: 'quódque ad figuras attinet, magis laudabo discipulum in abaco et pulvere figuras sibi demonstratas imitantem, quàm ociose et inutiliter alienas

The methodical rigour of Ramus's pedagogic style is met with early in *Geometriae septem et viginti libri*'s second chapter. With the necessary treatment of points and magnitude dealt with, the author delineates his method for the rest of the text: the common properties of magnitudes are defined, then the species are dichotomised accordingly. For Ramus, this model applied to all discursive enquiry: definition *was* demonstration.[35] Hence the diction of *Geometriae septem et viginti* was brusque and immediate, resulting in the reader being given little more in each definition than was deemed absolutely necessary. The pedagogue subjected classical authorities to this process of reduction, setting his abbreviated reading of their works against each other in his texts and so rearranging more detailed treatments of mathematical theory into what he perceived as a more expeditious, bite-sized selection of materials, with proofs eschewed for illustrative examples. Upon these squat foundations, more definitions could be heaped, and, once the definitions had been clarified, Ramus expected that his mathematical rules would be understood, piece by piece. The effect of this was to present what has been termed an observational geometry: one which encouraged pupils to witness the construction of the art and then methodically practice and repeat its rules for their own education and later application.[36]

First issued together in 1569,[37] Ramus's arithmetical and geometrical textbooks were constructed in such a way as to guide readers to the easy and immediate truth of their contents. In this manner, they were envisioned as part of the reorganization of a discipline with which Petrus Ramus had himself struggled. Between 1551 and 1555, the author had suffered a debilitating crisis of confidence in his own mathematical capabilities. Unable to comprehend the tenth book of Euclid's *Elements*, Ramus admitted to being literally crippled by the difficulties of the discipline; at least, mathematics as they were expressed in Euclidean form.[38] The pedagogue's original view that mathematics exhibited a perfect form of logical dialectic was irrevocably altered;[39] in its place rose the idea that authors such as Euclid had so obscured the truth of mathematics that a new method—one more in keeping with natural reason—had, in this telling, become essential.

Ever the logician, Petrus Ramus's efforts in popularising mathematical disciplines were tied to pedagogical and philosophical reforms following his debarment from teaching in 1544.[40] Prevented from lecturing or writing on philosophy in the wake of his attacks on Aristotle, Ramus turned to mathematics, publishing

picturas aspectantem'. Originally cited in Peter Sharratt, 'La Ramée's Early Mathematical Teaching', *Bibliothèque d'Humanisme et Renaissance*, 28.3 (1966), pp. 605–614, p. 608.

[35] An example of this is found early in Ramus, *Geometriae septem et viginti*, p. 10, where magnitude is defined before being dichotomised: 'Communes affectiones magnitudinis expositae sunt: sequitur dichotomia, quae adhuc nobis occurrit'. Ong stated that, for Ramus, 'to demonstrate something is to define it. [...] As Ramus's textbook on the subject shows, even geometry will consist not of demonstrations, but of definitions, or "rules"'. Ong, *Ramus, Method and the Decay of Dialogue*, pp. 188–189.

[36] Marta Menghini, 'From Practical Geometry to the Laboratory Method: The Search for an Alternative to Euclid in the History of Teaching Geometry' in Sung Je Cho, *Selected Regular Lectures from the 12th International Congress on Mathematical Education* (Heidelberg, New York, Dordrecht, London: Springer International Publishing, 2015), pp. 561–587, p. 565.

[37] Walter J. Ong, 'Christianus Ursitius and Ramus's New Mathematics', *Bibliothèque d'Humanisme et Renaissance*, 36.3 (1974), pp. 603–610, pp. 608–609. The 1569 edition of *Arithmeticae libri duo, Geometriae septem et viginti* was the first to combine Ramus's works on arithmetic and geometry; two previous editions of his *Arithmeticae libri duo* had already been published by this point.

[38] Robert Goulding, 'Method and Mathematics: Peter Ramus's *Histories of the Sciences*', *Journal of the History of Ideas*, 67.1 (2006), pp. 63–85, p. 74. Ramus's apologia *Oratio de professione sua* (written in 1563), relates how the pedagogue, reading Euclid, 'felt all the muscles in my back seize up', and moments later 'burst out in a rage against mathematics'.

[39] Goulding, 'Method and Mathematics: Peter Ramus's *Histories of the Sciences*', *Journal of the History of Ideas*, 67.1 (2006), p. 76. For Ramus's reconstruction of the history, identity, and use of mathematics in service of his wider goals, see Robert Goulding, *Defending Hypatia: Ramus, Savile and the Renaissance Rediscovery of Mathematical History* (Berlin and New York: Springer, 2010).

[40] For additional background on Ramus's debarment and the Ramus-Gouveia debate, see Ong, *Ramus, Method and the Decay of Dialogue*, pp. 21–24.

the first of his adaptations of Euclid's *Elements* in 1545.[41] Though the pedagogue had previously lectured on Euclid and on the sphere at the Parisian Collèges de Mans and de l'Ave Maria,[42] his interest in mathematics was primarily a product of a desire to promote the learning and use of any art according to his own proposed method; itself a mélange of humanist reading and Aristotelian analysis. To this end Ramus wished to see the *artes mechanicae*—including agriculture, architecture, trade, tailoring and the military—achieve equal standing with the liberal arts of the *trivium* and *quadrivium* in theory and in practice. His celebration of the mechanical arts was motivated by a long-held belief that the application of *any* given art—liberal or mechanical—was key to both the user's practice of that art, and to the intrinsic identity of the art itself. For Reijer Hooykaas, this belief is to be set against Ramus's rationalistic metaphysics as outlined in *Dialecticae institutiones* (1543), in which the pedagogue argued that human knowledge was predicated on reason, the *artifex* exercising our innate abilities to speak, count, measure, and so forth.[43] It was essential that reason not be obstructed by improper or incorrect teaching: any 'artificial' material obscuring this faculty natural reasoning was unnatural or, in Ramus's most cutting term, 'fabricated', and to be swept away.[44]

As a corollary of this, the presentation of propositions without synthetic demonstration afforded Ramus the space to present mathematical results expeditiously and, in his view, as they might be best grasped by the mind. The visio-spatial organisation of dichotomies was the engine driving this progress, with the end destination improvements in the clarity and order of language-orientated dialectic via the inculcation of a more mathematically-guided thought process. This outlook was then adapted in participation with the four key constants of Ramus's philosophical tenets: method, practicality, simplicity, and accessibility.[45] Mathematics was therefore prized by Ramus for its theoretical utility to the liberal arts, in which it aided the innate abilities of counting and measuring, and for its comparability to logical dialectic as a tool for the application of natural reasoning. At the same time, the use of mathematical practice as applied to the more mechanical elements of commerce, architecture, the military arts, and so on, was further proof of the discipline's worth.

Themselves readers of Petrus Ramus, Thomas Fincke and the late sixteenth-century pupils who studied the Wittenberg *Sammelband* encountered mathematics as part of an educational culture transformed by early modern humanism. Though Ramus's philosophical reforms were inarguably presented as improvements to the teaching of figures such as Rodolphus Agricola (ca. 1444–1485), the particulars of the Ramist way of thinking have most recently been characterised as more a loose and shifting assemblage of 'largely commonplace ideas and techniques' than one coherent and consistent philosophical phenomenon grafted *en bloc* onto a range of early modern intellectual cultures.[46] The French pedagogue's desire to promote the mathematical disciplines of the *quadrivium* in both theory and practice was comparable to his appreciation for the value of dialectic and its role within the *trivium*. In Ramus's interpretation, the latter's discursive arts were tangled, their constituent parts intertwined and too often overlapping. To pare back their untended vines, Ramus insisted upon pruning these arts to their essential and most logical components: grammar to

41 Sharratt, 'La Ramée's Early Mathematical Teaching', p. 608.

42 Peter Sharrat, 'Nicolaus Nancelius, "Petri Rami vita". Edited with an English Translation', *Humanistica Lovaniensia*, 24 (1975), pp. 161–277, p. 199. The intellectual and mathematical cultures of Paris which so influenced Ramus are expertly treated in Alexander Marr, ed., *The Worlds of Oronce Fine: Mathematics, Instruments and Print in Renaissance France* (Donnington: Shaun Dyas, 2009), and Richard J. Oosterhoff, *Making Mathematical Culture: University and Print in the Circle of Lefèvre D'Étaples* (Oxford: Oxford University Press, 2018).

43 Petrus Ramus, *Dialecticae institutiones* (Paris: Jacobus Bogardus, 1543), ff. 3r – 3v; Rejier Hooykaas, 'Humanities, Mechanics and Painting (Petrus Ramus; Francisco de Holanda)', *Revista da Universidade de Coimbra*, 36 (1991), pp. 1–31, p. 3.

44 Goulding, 'Method and Mathematics', p. 65.

45 Timothy J. Reiss, 'From Trivium to Quadrivium: Ramus, Method and Mathematical Technology' in Timothy J. Reiss and Jonathan Sawday, eds., *The Renaissance Computer*: *Knowledge Technology in the First Age of Print* (London: Routledge, 2000), pp. 43–56, pp. 47–48; Skalnik, *Ramus and Reform*, p. 57.

46 Hotson, *Commonplace Learning*, p. 16.

syntax and etymology, rhetoric to style and delivery, and dialectic to invention, arrangement, and judgement. Doing so would remove rhetoric from its false position at the peak of the trivium (as taught by classical authors such as Quintilian, and agreed upon by Agricola), its place taken instead by dialectic, the art which could most effectively divine the truth of a given statement.

With this reorganisation realised, the Ramist student could philosophise more effectively by arranging and comprehending terms through grammar, using the dichotomous branches common to Ramist method (in part appropriated from Johannes Sturm) to organize material for effective recognition and thereafter delivery. By then utilising dialectic to attain and judge the logic and validity of statements, students would arrive at truth, or, more correctly, philosophical certainty.[47] A devotee of Erasmian and Agricolan forms of humanism, Ramus based his ideas on the more general practices of hypercritical close reading, of allying logic to theoretical knowledge, and of inculcating the use of text and arts for a *vita activa*. Each of these were practices which Johannes Sturm and Philip Melanchthon had previously inherited and altered, with the latter educator advocating the particular value of mathematical study for intellectual, civic, and social use. It is in this Northern European, and particularly Germanic, context that the roots of Ramus's dialectical philosophy truly belonged.

Students matriculating at the University of Wittenberg in the second half of the sixteenth century—including Fincke, Lobhartzberger, and Klynaeus—therefore entered an environment rooted in the edicts of the Lutheran reformer Philip Melanchthon yet brought with them their experience with (and, perhaps, preference for) materials presented in Ramist format. Early in his career, Melanchthon had seen dialectic and rhetoric as intertwining subjects, each essential to the other. His youthful vision of dialectic was espoused in *Compendiaria dialectica ratio* (1520), where the worth of the study to pedagogy was made clear:

> (Dialectic) shows the nature and parts of any subject simply and describes the proposed subject in such clear words that the audience cannot fail to understand what it contains, whether it is true or false.[48]

This understanding of dialectic made it ideally suited for educational purposes. Although Melanchthon would himself later return to scholastic logic, motivated at least in part by the need for the ideas of the Reformation to triumph in ongoing theological debates,[49] the idea that dialectic was a foundational educative element remained influential among the pedagogues who succeeded him; his marriage of rhetoric and humanist dialectic duly influencing Martin Crusius (1526–1607), Johannes Sturm (1507–1589) and, ultimately, Ramus and his ally Talon.[50] For the *Praeceptor Germaniae*, the dual mobilization of dialectic for learning and rhetoric for oratorical persuasion was deployed so as to win hearts and minds. Mathematics became part of this programme; taught with specific emphasis to learners working toward careers in medicine, law, and theology, its study was propaedeutic to the acquisition of higher types of knowledge.[51]

In 1545, the Reformation's most influential scholar had written new statutes for the teaching of natural philosophy, calling for two lecturers to deliver lessons on mathematics. One tutor was to instruct on arithmetic and Euclid's *Elements*; the other, preparing students for the master's degree, on Sacrobosco's

47 Thomas M. Conley, *Rhetoric in the European Tradition*, 2nd edn. (Chicago and London: University of Chicago Press, 1994), pp. 128–133.

48 Mack, *History of Renaissance Rhetoric*, p. 109; originally, Philip Melanchthon, *Compendiaria dialectica ratio*, *Libri XX* (Wittenberg: Melchior Lotther Junior, 1520). The original text reads: 'Simpliciter enim cuiusque thematis naturam et partes ostendit, et quod proponitur, adeo certis verbis praescribit, ut non possit non deprehendi, quicquid inest, sive veri, sive falsi'.

49 Mirella Capozzi and Gino Roncaglia, 'Logic and Philosophy of Logic from Humanism to Kant', in Leila Haaparanta, ed., *The Development of Modern Logic* (Oxford: Oxford University Press, 2009) pp. 78–158, particularly p. 92–93.

50 Mack, *History of Renaissance Rhetoric*, p. 123, p. 129, and pp. 136–153.

51 For Philip Melanchthon's use of mathematics in the Lutheran reform of natural philosophy, see Sachiko Kusukawa, *The Transformation of Natural Philosophy: The Case of Philip Melanchthon* (Cambridge: Cambridge University Press, 1995), pp. 134–144.

De Sphaera and Ptolemy's *Almagest*.[52] Preceded by intensive study of philosophy, Latin, rhetoric, and dialectic, students were to synthesise these mathematical lessons into a more complete understanding of the Gospel as the Word of God. Melanchthon saw the study of mathematics as one of several ways to encourage recognition of the orderliness supplied by divine providence, and he supported this idea with frequent appeals to Platonism; indeed, the reformer's predilection for Plato's supposed apothegm 'God always geometrizes' and its variants is well attested in modern scholarship.[53] In his preface to Johannes Vogelin's 1536 book on geometry, the Lutheran humanist went further still, informing readers that they would be 'admonished by the voice of Plato' when turning the pages of Vogelin's work.[54]

Elsewhere, the overall goal of Melanchthonian mathematics was rarely made clearer than in a preface to Georg Peurbach's *Theoricae novae planetarum* (Wittenberg, 1535). Taking the form of a letter to the printer Simon Grynaeus, this preface was initially intended for an earlier edition of the work, published in 1532. Comparing the turbulence of recent upheavals to a long and bitter civil war in ancient Greece, Melanchthon told of an entreaty to the Apollonian oracle at Delos. According to the oracle, lasting peace would be secured only by the building of a cubic altar in greater dimensions than that which currently existed. Baffled, the Delians sought the help of Plato, who resolved the mathematical problem before interpreting for them the true meaning of the oracle's words.[55]

By making the civic and spiritual values of Platonic geometrizing apparent in his introductory epistle, Philip Melanchthon first of all highlighted the discipline's practical utility to the measurement and correct construction of the altar. Beyond this, the more tacit property of geometry was to create a lasting and intangible value far beyond that of its original use, with its practitioners sowing peace and moderation instead of disharmony and discord. In doing so, they come closer to acknowledging the thoughts of the divine. Undoubtedly, Melanchthon wished for university students to incorporate the lessons of Plato (subservient to the reformers' theological instruction) into their own practices so as to heal the Europe of the 1500s. Conceived of as part of an ideal curriculum—the definitive goal of which was a greater understanding of God and His works—Melanchthon's use of mathematics contributed to the reformative process which underpinned Lutheran education, whilst remaining propaedeutic to the higher (and even more curative) studies of medicine and theology.

In his elevation of the position of mathematical study in the early modern curriculum through a recasting of its relationship to philosophy and theology, Philip Melanchthon helped to stimulate a growth in mathematics that was replicated elsewhere in Germany. His influence spread outwards, guiding the precepts of the universities of Tübingen, Leipzig and Heidelberg, and the newer institutions founded in the sixteenth century, such as Marburg and Helmstedt.[56] Citing Johannes Sturm as a key disciple of Melanchthonian

52 Kusukawa, *The Transformation of Natural Philosophy: The Case of Philip Melanchthon* (Cambridge: Cambridge University Press, 1995), p. 176.

53 Friedrich Ohly, 'Deus Geometra: Skizzen zur Geschichte einer Vorstellung von Gott' in Norbert Kamp and Joachim Wollasch, eds., *Tradition als Historische Kraft: Interdisziplinäres zur Geschichte des Früheren Mittelalters* (Berlin: Walter de Gruyter, 1982), pp. 1–41. See also Charlotte Methuen, 'Interpreting the Books of Nature and Scripture in Medieval and Early Modern Thought: An Introductory Essay' in Jitse M. van der Meer and Scott Mandelbrote, eds., *Nature and Scripture in the Abrahamic Religions: Up to 1700*, Volume 1 (Brill: Leiden and Boston, 2008), pp. 179–218, p. 206.

54 Philip Melanchthon, 'Preface to Johannes Vogelin's Book on the Elements of Geometry (1536)' in Sachiko Kusukawa, ed., and Christine F. Salazar, trans., *Orations on Philosophy and Education* (Cambridge: Cambridge University Press, 1999), pp. 98–104, p. 99.

55 Philip Melanchthon, *Preface*, Georg Peurbach, *Theoricae Novae Planetarum* (Wittenberg: Joseph Klug, 1535), f. aij r: 'ad Platone, qui docet qua in re sit erratum, videlicet nescisse eos cubi et quadranguli discrimen, nec ex cubo fuisse quadrangulum faciendem, monstrat qua ratione cubus duplicandus sit. Caeterunt admonet hac oraculi sententiam esse, ita demum Graeciam futuram tranquillam, si se ad Philosophiam Graeci convertissent, quia haec studia animos ab ambitione, et caeteris cupidatibus, ex quibus bella et caetera mala existent, ad amorem pacis, et moderationem in ombinus rebus abducerent.'

56 Westman, 'The Melanchthon Circle', p. 169. Westman develops the ideas of both Pierre Duhem and Lynn Thorndike; see Pierre Duhem, *To Save the Phenomena: An Essay on the Idea of Physical Theory from Plato to Galileo*, trans. Edmund Dolan

doctrine, Kees Meerhoff has convincingly argued that Ramus's dialectic was the product of a combination of method, practice, and humanist study of classical texts as taught by Melanchthon and Sturm, and itself a distinctive form of Northern Humanism.[57] In keeping with the educational currents of his time, Johannes Sturm's pedagogical model conditioned pupils to amass inexhaustible troves of mnemonic material, from individual words upwards, which were later to be analysed and combined stylistically (often via imitation of approved authorities) before being practised and delivered to prove mastery of a given subject.[58]

Sturm encouraged students to employ the commonplace books familiar to the era from an early age, and advised masters to dictate commentaries contextualising and explaining single key words found in classical texts.[59] Once more, the educator was faced with a dilemma: how to educate students swiftly, reliably, but comprehensively? Sturm settled on three methods, culled from reading Galen, Aristotle, and Plato: an analytic method, which moved from the perception of objects to the principles guiding an art; a synthetic method, moving from principles to specifics; and, finally, a logical method characterised by definition and division, in which an art was divided and subdivided into constituent parts, with each part defined so as to demonstrate its relevance to the whole.[60] If such a methodology sounds familiar, Petrus Ramus certainly left little doubt as to the inspirational effect of his colleague's teaching. Ramus's report on Sturm's time in Paris from 1529 to 1536 recalled that the educator

> excited in the (Collège Royal) an incredible ardour for the art [of logic] whose utility he revealed. It was at the lessons of this great master that I first learned the use of logic, and since then I have taught in an entirely different spirit from that of the sophists (...).[61]

It is easy to imagine that Ramus was particularly taken with Sturm's lauding of a tripartite approach to teaching focussing on the simplification of terms, on brevity and on *diaresis*, or the division of definitions into smaller parts.[62] However, Johann Sturm was rather less enamoured of Petrus Ramus. Though the latter was invited to visit the Strasbourg academy (then under Sturm's aegis) in 1569, his texts were not introduced to the curriculum.[63]

Sturm was still in post as the director and a *de facto* leading light of Reformation education when Thomas Fincke arrived at Strasbourg some eight years later.[64] The Sturmian methodical fixation on textual analysis, oratory, and practice were communicated in no uncertain terms to the teachers working in his institu-

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and Chaninah Masler (Chicago: University of Chicago Press, 1969), particularly pp. 75–78 and pp. 88–98, and Lynn Thorndike, *A History of Magic and Experimental Science*, 8 Volumes (New York: Columbia University Press, 1923–1958), Volume 5, p. 378.

57 Kees Meerhoff, 'International Humanism' in Winifred Bryan Horner and Michael Leff, eds., *Rhetoric and Pedagogy: its History, Philosophy and Practice: Essays in Honour of James J. Murphy*, (New York: Routledge, 1995), pp. 213–226, particularly pp. 216–219. Walter Ong and John Monfasani have separately argued for Rhenish and Dutch influences as central to the Northern European humanists' fascination with 'method'. See Ong, *Ramus, Method and the Decay of Dialogue*, p. 232; John Monfasani, *George of Trebizond: A Biography and Study of his Rhetoric and Logic* (Brill: Leiden, 1976), pp. 325–326.

58 Pierre Mesnard, 'The Pedagogy of Johann Sturm (1507–1589) and its Evangelical Inspiration', *Studies in the Renaissance*, 13 (1966), pp. 200–219, particularly pp. 209–211.

59 Anja-Silvia Goeing, *Storing, Archiving, Organizing: The Changing Dynamics of Scholarly Information Management in Post-Reformation Zurich* (Leiden: Brill, 2017), p. 207.

60 Mack, *History of Renaissance Rhetoric*, pp. 133–134. See also Lewis W. Spitz and Barbara Sher Tinsley, *Johann Sturm on Education* (St Louis, MO: Concordia Publishing House, 1995), particularly pp. 45–58.

61 Petrus Ramus, Preface to *Scholae in Liberales Artes*: 'academiam academiarum principem incredibili tam insperatae utilitatis desiderio inflammavit: tum igitur tanto doctore logicam istam ubertatem primum degustavi, didicique longe alio fine consilioque juventuti proponendam esse (...).' I have taken this translation from Skalnik, *Ramus and Reform*, p. 31, fn. 72.

62 Monfasani, *George of Trebizond*, p. 326.

63 Hotson, *Commonplace Learning*, p. 22.

64 Fincke was enrolled at Strasbourg from 1577 to 1582. See Schönbeck, 'Thomas Fincke und die *Geometriae rotundi*', p. 83.

tion, including the mathematics instructor Conrad Dasypodius (1532–1600).[65] Of further significance is the fact that Sturm, unlike Melanchthon and, indeed, the pedagogues of Zurich and Wittenberg who followed him, believed that subjects such as mathematics, jurisprudence and medicine could be introduced to pupils from a young age.[66] Writing to Dasypodius in 1569, Sturm earnestly confirmed his wish that mathematics be taught to the two eldest classes, using the first mathematical textbook approved for use in the Strasbourg Academy: Dasypodius's 1567 collection of lectures on geometry, astronomy, and geography.[67] Along with this, Sturm counselled that pupils should read the *Elements*, and Sacrobosco's *De Sphaera*; should Dasypodius wish, he could also furnish pupils with examples from Ptolemy, Proclus, Hipparchus, and Theodosius.[68] For Sturm, mathematics offered certainty beyond the phenomenological world:

> things that our senses cannot even count, nor grasp, our spirit can nevertheless embrace, like the whole world, the sky, the seas, land. [. . .] Did Euclid describe the finite or the infinite? Euclid's problems and mathematicians' axioms are finite, but how many propositions one can deduce from them that have not been dealt with by the doctors![69]

Dasypodius, himself a graduate of the Strasbourg *gymnasium*, undoubtedly shared many of Sturm's ideas and at least some of his pedagogical zeal. It is notable his return to Strasbourg from Louvain in 1562 kickstarted a period of his life that was characterised by a significant increase in his publishing efforts. In particular, Dasypodius, in conjunction with his predecessor as professor of mathematics, Christian Herlin, was responsible for the translation and publication of a number of Greek works, including those of Euclid, Aristotle, Theodosius, and Autolycus.[70]

On the face of things, the motivations for this were simple: students needed textbooks, and towns possessing large schools and universities soon became breeding grounds for printing-presses. Conrad Dasypodius prefaced his Euclidean works by drawing attention to longstanding curricular regulations, and to the fact that all students were to learn the *Elements* from their first classes onward. The teacher argued that it would be best, then, if pupils were to have access to a small though complete treatment of Euclid's work; a position few could disagree with.[71] By happy accident, any such translation would also advance Conrad Dasypodius as a mathematician and a humanist: the recovery of classical Greek texts remained a feather in the cap for humanist educators of any stripe. At a deeper philosophical level, however, Dasypodius's efforts were part of a more general movement to present Euclidean proofs as Aristotelian syllogisms:[72] a movement that would ultimately be rejected as abstruse and unnecessary by Ramus and the Oxford mathematician Henry Savile, amongst others.[73]

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65 Pierre Mesnard, 'The Pedagogy of Johann Sturm (1507–1589) and its Evangelical Inspiration', *Studies in the Renaissance*, 13 (1966), pp. 210–219, p. 212.

66 Goeing, *Storing, Archiving, Organizing*, p. 129.

67 Conrad Dasypodius, *Volumen primum mathematicum. Prima, et simplicissima mathematicarum disciplinarum principia complectens: Geometriae. Logisticae. Astronomiae. Geographiae. Per Cunradum Dasypodium in utilitatem academiae Argentinensis collectum* (Strasbourg: Josias Rihel, 1567). See also Anton Schindling, *Humanistische Hochschule und freie Reichsstadt: Gymnasium und Akademie in Strassburg 1538–1621* (Wiesbaden: Steiner, 1977), pp. 206–207.

68 Letter from Johann Sturm to Conrad Dasypodius, 1569, in Spitz and Tinsley, *Johann Sturm on Education*, p. 321.

69 Letter from Johann Sturm to Conrad Dasypodius, March 1565 in Spitz and Tinsley, *Johann Sturm on Education*, p. 295.

70 C. Doris Hellman, *The Comet of 1577: Its Place in the History of Astronomy* (New York: Columbia University Press, 1944), p. 237. For an extensive list of Dasypodius's publishing endeavours, see Hellman, *The Comet of 1577*, p. 238, fn. 24.

71 Neal Ward Gilbert, *Renaissance Concepts of Method* (New York: Columbia University Press, 1960), p. 84.

72 Gilbert, *Renaissance Concepts of Method* (New York: Columbia University Press, 1960), p. 89. On the sixteenth-century debates concerning the certainty of mathematics, see Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (Oxford: Oxford University Press, 1996), particularly pp. 10–15 and pp. 25–28.

73 Giuliano Mori, 'Mathematical Subtleties and Scientific Knowledge: Francis Bacon and Mathematics, at the Crossing of Two Traditions', *British Journal for the History of Science*, 50.1 (2017), pp. 1–21, p. 11.

3. Text and paratext: Fincke's *Geometriae rotundi* and its goals

Existing one after the other as part of a pedagogical network influenced by the educational reforms of Philip Melanchthon and located at Johannes Sturm's Strasbourg *Gymnasium*, Conrad Dasypodius and Thomas Fincke nonetheless differed in their appreciation of how geometry might best be presented dialectically. For Dasypodius (and his teacher and now colleague, Herlin), Euclid's *Elements* would be improved by transforming proofs into syllogisms, bringing the axiomatic structure of the text into agreement with the first principles of Aristotelian logic.[74] For Fincke, little more than a methodical redrawing of Euclidean geometry would do. The Dane therefore chose not to syllogize Euclid, but sought instead to ally himself with Petrus Ramus's more radical departure from the Greek mathematician by means of a methodological presentation and an extensive use of other mathematical texts, following largely in the process the French pedagogue's methodological presentation. The contrasting positions taken by Dasypodius and Ramus were a reflection of ongoing debates on the relationship of dialectic to mathematical certainty, and on the very idea of mathematical certainty more generally.[75]

At the same time, Thomas Fincke's reasons for writing *Geometriae rotundi* were tied to more personal goals. As an introductory, Ramist textbook, Fincke's work offered students and teachers an expeditious yet complete guide to spherical geometry. At a higher level, the tables presented as improvements to Georg Joachim Rheticus's (1514–1574) astronomical canons were a vehicle for the author to demonstrate his own abilities to an expert audience. Having proven his command of geometrical theory, Fincke's intention was to use his more precise calculation of sines, tangents and secants as a platform from which to definitively evaluate the computations of a range of prior mathematical authorities. Reared as he was on the pedagogical traditions of Melanchthon and Sturm, the young author introduced his volume with learned oratory and intensive textual analysis so as to participate in an ongoing philological process of mathematical reconstruction. Doing so would, in his estimation, allow him to reformat the presentation of geometry according not to the received wisdom of Euclid, but rather to a new approach popularised by Petrus Ramus.

It is in this context that *Geometriae rotundi*'s paratextual materials—a dedicatory epistle to Frederick II of Denmark (1534–1588); the *Praefatio ad Lectorem* addressed to the English mathematician Thomas Digges and to a host of Fincke's near contemporaries; and, at the threshold of the text, the branching, diagrammatic visualisations specifying the division of topics into their composite parts common to 'Ramist' texts of the early modern period—helped to mediate readers' understandings of what followed.[76] Fincke began by citing a number of prestigious mathematical authorities as crucial to his own intellectual development, before offering readers his own synthesis of a methodological presentation of spherical geometry from within a vision of mathematics predicated on dichotomies and definitions.[77] These introductory epistles took their place within the author's humanist adherence to persuasive oratory, and it should be noted that the tone of each letter was altered according to their intended audience. A consequence of this is that the paratextual elements of Fincke's text are as much a part of his mathematical presentation as any other page of print: as Brian Vickers has counselled, the discipline of rhetoric taught in the Renaissance and early modern periods encouraged would-be orators and authors to be sensitive to their reader's likely intellect,

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74 Vincenzo de Risi, 'The development of Euclidean axiomatics. The systems of principles and the foundations of mathematics in editions of the *Elements* in the Early Modern Age', *Archive for History of Exact Sciences* 70.6 (2016), pp. 591–676, p. 598. See also H.D.P. Lee, 'Geometrical Method and Aristotle's Account of First Principles', *The Classical Quarterly*, 29.2 (1935), pp. 113–124.

75 For a detailed summary of these debates, see Chikara Sasaki, *Descartes's Mathematical Thought* (Dordrecht, Boston, and London: Kluwer Academic Publishers, 2003), pp. 333–358.

76 Ong, *Ramus, Method and the Decay of Dialogue*, particularly pp. 307–314. Ong has consistently and, at times persuasively, argued for the conceptualisation of Ramism as a visual methodology: one diffused as part of an 'aural to visual shift' made possible by the printing press.

77 Fincke, 'Epistola Nuncupatoria', *Geometriae rotundi*, f. 4 r.

emotions, judgement, and response when framing their epistles.[78] Addressing (and celebrating) Frederick II, Fincke's prose was marked by a pomposity rarely seen elsewhere in his text.[79] Treating Digges and his fellow mathematicians as intellectual equals, the Danish mathematician's praise for his ideal readership was more understated, if occasionally still fulsome.

Thomas Fincke's prefatory letter to his ideal readership highlights the manner in which he utilised the rhetorical writing strategies and intensive collation taught as part of his humanist education to marry together two key issues which had inspired the creation of *Geometriae rotundi*. In dialogue with his fellow mathematicians, the cautious Dane was able to situate his desire to restructure the Euclidean method of presentation by couching his experience with the Greek author as unsettling and confusing. Blaming Euclid for perturbing him so, Fincke turned to praise Ramus for recovering the art that was there all along. Ramus's harmonisation of the luminescent qualities of reason with the dialectical qualities of mathematics was most clearly invoked in Fincke's *Prefatio ad Lectorem*:

> Therefore, thinking of another means of coming to know this divine knowledge than the one presented to me – for with this latter way, one would succeed little - I turned myself to Petrus Ramus's book of geometry, where I found immediately that which I had long desired in Euclid. Traces of the clearest methods presented themselves, and that particular art itself is seen to be taught more abundantly, and more brilliantly.[80]

This quest for clarity had led to the creation of *Geometriae rotundi*: Ramus, as the autodidactic Fincke's textual teacher, had shown the way through a humanist method marshalling copious authorities and definitions into a coherent, logical order. In doing so, the practically-minded French pedagogue had (in Fincke's eyes) made his methods and results congruent to reason, uncovering the building blocks of the art of geometry as he went—and revealing to his autodidactic charge the importance of logical structure to mathematics.[81] The visual, illuminating nature of such a description of method distinctly echoed Petrus Ramus's celebration of the discipline as coterminous with dialectic in *Dialecticae institutiones*, in which the author argued that the mathematical arts, understood through the correct use of dialectic, would both illuminate and purify all other disciplines and so elevate the understanding of all things.[82]

By way of this introductory dialogue, Thomas Fincke subtly deprecated his ambition to compare the astronomical calculations of Geber, Regiomontanus, and Copernicus with those of Ptolemy, Rheinhold, and Fincke himself, with a view to gauging which were most concise and efficacious. Instead, Thomas

[78] Brian Vickers, 'Epideictic and Epic in the Renaissance', *New Literary History*, 14.3 (1983), pp. 497–537, p. 498.

[79] See, for example, Fincke, 'Epistola Nuncupatoria', *Geometriae rotundi*, f. 2 r: 'Videre id cum aliis in rebus, tum literis humanioribus licet: maxime vero iis in artibus: quae ob certam suam, quam pariunt scientiam [...] mathematicae solae vocantur.' For Fincke's celebration of Frederick II's patronage, see Fincke, 'Epistola Nuncupatoria', *Geometriae rotundi*, f. 4 v: 'Quin et aequum esse arbitrates sum: ut grata Mathemata suos maxime Mecaenates et Patronos celebrarent. In quorum numero regiam T. M. consistere: vel insignis illa erga nobilissium et in Mathematis excellentissimum virum Dn. Tychonem Brahe magnificentia docere satis poterit: Academia vero Hafniensis nunquam tacebit.'

[80] Fincke, 'Praefatio ad Lectorem', *Geometriae rotundi*, f. 1 v: 'Alia itaque ad perscrutandam divinam hanc scientiam, via insistendum mihi putabam: cum hac successisset parum, itaque ad P. Rami me volumen Geometricum converti. inveni illico quod in Euclide desideraram diu, nam et methodi sese clarissima offerebant vestigia: et ars quoque ipsa copiosius aliquanto et luculentius instructa videbatur.'

[81] Fincke, *Geometriae rotundi*, f. b 1 v: 'Apuerit mihi vir hic mentis oculos: quod Logices usum in Mathematis egregiè monstrate est visus'.

[82] Ramus, *Dialecticae institutiones* f. 39 v: 'Itaque cum has disciplinas lumine suo dialectica lustraverit, quanto iam plenius naturalium principia rerum, et umbrarum illarum causae cernentur.' Goulding has argued that this section of Ramus's work outlines the author's view that mathematics is both improved by and identical with dialectic, with the discipline's lofty position owing to mathematics' ability to assess truth and certainty. Goulding, *Method and Mathematics*, p. 68.

Digges (or, indeed, any similarly expert reader) would be the judge.[83] Fincke could thereby be absolved of any accusations of arrogance, whilst still encouraging his audience to place him alongside the great theorists of his discipline. The author's employment of a respectable, epideictic rhetoric was crafted so as to encourage readers to witness him as an altruistic reformer, seeking to recover and advance mathematical thought in equal measure. To this end, *Geometriae rotundi*'s introductory epistles were in keeping with the style of literary rhetoric advocated by a range of humanists, amongst whom Sturm and Melanchthon were the latest inheritors. At the same time, Fincke's advocacy of Ramus's logical style and organisation of mathematical material over that of Euclid (and, indeed, Aristotle and the scholastic tradition) marked him as an active member of a new school of thought: one influenced by Melanchthon, Sturm, and Ramus, in which mathematics could be allied to dialectic as a method for discerning certainty.

Following its prefatory materials, *Geometriae rotundi*'s first five chapters took the form of an introduction to the geometry of the sphere, with particular focus granted to the radius: its relationship to methods of dividing the circle in right lines and triangles (Book I), and that same relationship to the creation of sines, tangents and secants of a semicircle (Book V). From Book VI onwards Fincke presented plane trigonometry, including methods to square the circle (Book VIII) and the trigonometric canons (Books IX and X), based on the calculations of Rheticus which dominated much of the volume. *Geometriae rotundi* then concluded with lessons on the construction and measurement of spherical triangles (Books XIII and XIV). Introducing the form of the circle in Book I, Fincke referenced Thales of Miletus, Ptolemy, Aristotle, and Euclid to demonstrate the proposition that the shape in either plane or solid form provides a maximum area compared to that of any other polygon with an equal perimeter.[84] Authorities were stacked higher and higher, with the author adding Theon of Alexandria's commentary on the *Almagest* as proof of the mathematical demonstration of this fact; but the bluntest proof is that of Ramus, expressed diagrammatically, in which the perimeters and areas of an equilateral triangle, a square, and a circle are presented.[85]

The organisation of this proposition followed Ramus's explanatory mode of mathematical presentation in its use of definition, enunciation, and construction. Aided by copious authority, the mental cognizance advocated by Fincke's presentational method chimes with the aforementioned identification of Ramism as an 'observational' methodology:[86] one wherein the reader was encouraged, thanks in no small part to the supposed clarity of this method, to mentally recognise the truth of these written propositions swiftly, and without significant difficulty.[87] As is frequently seen to be the case elsewhere in *Geometriae rotundi*, the composite parts of the argument were defined previously, with each conclusion built upon the foundations

[83] Fincke, 'Praefatio ad Lectorem', *Geometriae rotundi*, f. a 3 r: '(...) alis Mathematicis notum mihi primum factum est nomen tuum. Fac quaeso: ut sicuti scalas coelo compendiosas admovisti: sic brevissimos ad sydera in calculo accessus eligas. Quod utinam Copernici problemata praestarent. Ego enim quo pacto praestent videre nondum possem. Tu itaque, mi Thoma judicabis: et calculum Gebri, Regiomontani, itemque Copernici, cum eo, qui hisce in libris ex Ptolomaeo atque, Rheinholdo deducitur, conferes.'

[84] Fincke, *Geometriae rotundi*, p. 6: 'Nam ut in planis circulus quolibet rectilineo ordinato, sic in solidis sphaera ordinatis quibuslibet corporibus est ordinatior (...) Et vero axioma illud et principium Geometricum est: ex isoperimteris homogeneis ordinatius est majus, ex heterogeneis ordinatis terminatius.'

[85] For Ramus's treatment of isoperimetric figures, see Ramus, *Geometriae septem et viginti*, Book IV, pp. 19–33, and Book XIX, pp. 130–134. The diagram reproduced by Thomas Fincke is found in Fincke, *Geometriae rotundi*, p. 7, and is based on that of Ramus, *Geometriae septem et viginti*, p. 133.

[86] Menghini, 'Practical Geometry to Laboratory Method', p. 565.

[87] This chimes with Pierre Duhamel's assertion that 'one of the more or less explicit assumptions of the Ramist dialectic was the inevitability of the mind's assent to a true proposition, once it was presented to the mind'. Pierre Albert Duhamel, 'The Logic and Rhetoric of Peter Ramus', *Modern Philology*, 46.3 (1949), pp. 163–171, p. 169.

of the preceding one. Diagrams, used consistently (if sparingly, as in Ramus's own work), were secondary to the structure of the printed text.[88]

Thomas Fincke then bookended his mathematical material with additional philosophical excerpts from Plato, Aristotle, and Quintilian; each selected to portray the perfectability of circle and sphere and their value to the study of geometry, and, beyond that, to the study of natural order.[89] This is but one example of the way in which the arguments of *Geometriae rotundi* were consistently built from a cascade of sources. Their series of definitions were most often succeeded by blunt, explanatory sequiturs expressed in precisely the same format and diction as Fincke's mathematical proofs almost without fail: the implication being that many of these proofs were self-evident by observation of their preceding parts: a position that followed that adopted by Petrus Ramus. Fincke's commitment to the lessons of the French pedagogue might even at times be termed arch-Ramist, given the way the Dane abbreviated his authorities even more succinctly than might be thought necessary.

Such presentations shared two points of convergence, each crafted to secure the author's mathematical authority. Firstly, the author was careful to consistently reference authorities such as Euclid,[90] Regiomontanus,[91] and (most frequently) Ramus[92] by chapter and verse, indicating the precise points where readers might pick up his predecessors' work. This conscientious approach served in part to convince the reader that Fincke's conclusions might be verified by comparison with his voluminous sources; assuredly, this approach also worked to convince readers that his work was enough of a compendium to be trustworthy. Secondly, and more importantly, the author was just as careful to situate himself alongside classical mathematicians when subtly critiquing more recent theory, mounting stirring defences of the mathematical art as a product of rediscovery rather than novelty. In this manner, Fincke concluded his fourth book by marking how Hipparchus, Menelaus and Ptolemy improved the theory of chords only by abbreviation;[93] likewise, Copernicus, in his 'marvellous' work, had, along with Peurbach, succeeded by retaining much of the classical mathematicians' thought.[94]

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88 It should be noted that in this example the diagram which followed the proposition is visibly erroneous. Although the area of the circle is *numerically* significantly larger than the square or triangle, as the printed numbers indicate, the diagram itself was not drawn to scale.

89 Fincke, *Geometriae rotundi*, p. 7. This section is compiled from references cross-pollinating and supporting one another. The paraphrasing of Plato's *Timaeus*, for example ('Atque hinc Plato dixit, rotundam figuram omnium esse perfectissimam: ideoque Deum mundum Sphaericum figurasse: ut suo complex cuncta contineret') supports Ptolemy's idealisation of the sphericity of the heavens, in which the sphere is the figure with the 'freest motions', and therefore most suitable for the form of the heavens. *Ptolemy's Almages*t, trans. G.J. Toomer (Princeton, NJ: Princeton University Press, 1998; originally London: Duckworth Press, 1984) pp. 39–41.

90 Euclid's work is well represented throughout *Geometriae rotundi* in the italicised definitions and propositions strewn throughout each book. Mostly, material taken from the *Elements* is provided without naming the author, as, for example, in Fincke, *Geometria rotundi*, p. 8: '6. Circulus est rotundum planum. 15. d.1'. Euclid's fifteenth definition in the *Elements*'s Book I states: 'A circle is a plane figure contained by one line such that all the straight lines falling upon it from one point among those lying within the figure are equal to one another'. Thomas L. Heath, trans., *The Thirteen Books of Euclid's Elements, Volume I*, 2nd edn. (Cambridge: Cambridge University Press, 1926; reissued in paperback 2014), p. 153.

91 Similarly, Regiomontanus's work was used both within the text and to provide introductory definitions and guidance: see, for example, Fincke, *Geometriae rotundi*, p. 67: 'Radius aeque potest sinibus peripheriae et complementi. 1.p. Regio de sinib.'.

92 Fincke, *Geometriae rotundi*, p. 83: 'Nam quadratum semissis lateris de quadrato radii relinquit quadratum perpendicularis per 5. e. 12. R. (...)'.

93 Fincke, *Geometriae rotundi*, p. 63: 'Atque sic Ptolemaeo, quae debere ipsi volvi, exolvi: referens ipsius theoremata de subtensis: et ad suum locum ut puto referens. Hipparchus de subtensis scripsisse refertur libros 12. Menelaus (...) de iisdem libros sex consecerat. (...) Tantum in eo, hoc quidem in loco, brevitatis fuisse studium videmus. Brevitas hec tam grata accidit: ut a posteris Ptolemaica theoremata retenta fuerint.'

94 Fincke, *Geometriae rotundi*, p. 63: 'Retinuit Purbachius: retinuit Copernicus in opere suo mirando: retinuere alii'.

By this token, the effects of Johannes Sturm's influence on the young mathematician should not be underestimated, even if Sturm's professional interest in mathematics was limited. As a well-educated product of the Strasbourg model of education, Thomas Fincke presented his work within a context of rhetorical and mathematical appeals to key classical authorities, as taught by the humanist *gymnasia* and universities of his era. At the same time, the author utilised *Geometriae rotundi* as a theatre in which to rehearse and perform his extensive learning for reputational gain. Before presenting the sine canons which he hoped would help make his name, Fincke had to first of all guide his reader through the geometry of their construction. To do so, *Geometriae rotundi*'s Book VII defined how a circle circumscribed a series of regular polygons—including the equilateral triangle, pentagon, hexagon and decagon—and then used the sides of each polygon to compute the chords of the circle's circumference from a given radius.[95] Having obtained these chords, the mathematician could then follow Regiomontanus and Rheticus by decimalising the radius and its sines to ever greater degrees of accuracy.

Prior to his explication of the decimalisation of the radius, Fincke followed Ramus in situating his methodical presentation amongst a clutch of classical and contemporary authorities, buttressed by evidence from nature herself.[96] Citing Varro's conjecture on industrious bees, their busy feet, and their tiling of constructions of hexagons within the hive, both Ramus and Fincke spoke of nature and geometry as being in perfect accordance in the circumscription of the hexagon within the circle. Importantly, however, the Danish mathematician diverged from, and then returned to, Ramus's source material and its form of demonstration in his treatment of the triangle and hexagon.

As he had elsewhere, Petrus Ramus used *Geometriae septem et viginti*'s eighteenth book to edit and reorder several of the propositions of the *Elements*. Expanding on Euclid's work, the French pedagogue appended a summary of Varro's conjecture to Book XVIII's seventh proposition: a proposition which recapitulated the *Elements*' XIII:12 by stating that, if an equilateral triangle is inscribed in a circle, then the square is triple the square on the radius of the circle.[97] From there, the French pedagogue cited Pappus and Campanus of Novara to show that, if the side of a circumscribed hexagon is cut proportionally, then the larger segment would be the side of a decagon.[98] Ramus used this information to introduce the *Elements*' proposition XIII: 9, in which Euclid demonstrated that 'if the side of the hexagon and that of the decagon inscribed in the same circle are added together, then the whole straight line has been cut in extreme and mean ratio, and its greater segment is the side of the hexagon';[99] the pedagogue then considered (as Euclid had) the relationships between the figures of an equilateral pentagon, hexagon, and decagon inscribed within a circle.

In his adaptation of Ramus's geometry, Thomas Fincke retained much of his key source's order, content, and style. Yet rather than move exactly from the inscription of an equilateral triangle and its squares to the sides of a hexagon and decagon, as the French pedagogue had, Fincke in *Geometriae rotundi*'s Book VII instead inserted two additional propositions: proposition fifteen, detailing the construction and relationship

95 Fincke, *Geometriae rotundi*, pp. 89–104.

96 For Fincke's source material, see Ramus, *Geometriae septem et viginti*, p. 128.

97 Ramus, *Geometriae septem et viginti*, p. 128. Thomas L. Heath, trans., *The Thirteen Books of Euclid's Elements, Volume III*, 2nd edn. (Cambridge: Cambridge University Press, 1926; reissued in paperback 2014), pp. 466–467.

98 Ramus, *Geometriae septem et viginti*, p. 128: 'Si latus sexanguli secetur proportionaliter, majus segmentum erit latus decanguli.'

99 Heath, trans., *The Thirteen Books of Euclid's Elements, Volume III*, 2nd edn. (Cambridge: Cambridge University Press, 1926; reissued in paperback 2014), p 455. Ramus introduced this proposition with reference to a right line continued beyond the sides of the inscribed hexagon and decagon, cut proportionally. Ramus, *Geometriae septem et viginti*, p. 129: 'Si decangulum et sexangulum inscribantur eidem circulo, recta é latere utriusque continuata secabitur proportionaliter, et majus segmentum erit latus sexanguli: et si majus segmentum rectae proportionaliter sectae est latus sexanguli, reliquum erit latus decanguli. 9 p 13.'

of lines and triangles drawn from the end point of the side of an inscribed equilateral triangle,[100] and proposition 16, on the inscription of the heptagon from a point perpendicular to the centre of a circle and the side of an inscribed hexagon.[101] Proposition 15 was, in the author's argument, 'most noble for the construction of the canon of sines'.[102]

Displaying his own learning and the importance of continued inquiry, Fincke concluded his fifteenth proposition with Ramus's retelling of Varro's conjecture, before reporting how the great mathematician Pappus of Alexandria went beyond the hexagon in demonstrating that a series of other regular polygons were similarly circumscribed. Although the heptagon and nonagon could not be constructed in the same fashion, Pappus taught that they could be constructed from triangles, and their ratios calculated thereafter. This labour of the ancients had been passed down, with the authority of Pappus contrasted to Euclid, Proclus, Archimedes and Ramus; but as many had written, this art was difficult, with a variety of proposed solutions.[103] In the following proposition (proposition 16), the author went on to offer something of a resolution by outlining how 'mechanics' drew a heptagon by aid of the hexagon.[104]

As Ramus had, Fincke's seventeenth proposition of Book VII prefaced his treatment of proposition XIII:9 of the *Elements* by first treating the sides of a hexagon and decagon, following the order of operations previously set out in *Geometriae septem et viginti*. Aping the presentational method of the French pedagogue, Fincke began with a definitive statement detailing the problem, headed by italic type. Beneath this, brief statements of fact on the constituent parts of the topic were collated. Theory gave way to blunt, abbreviated definition, with copious authorities providing the necessary scaffolding. The author was often content to rely on his readership's ability to follow a step-by-step process: one whereby the factuality of the materials presented was recognised most frequently by brusque classification. However, his commitment to Ramist method did not mean that Fincke eschewed classical demonstration entirely. Despite his professed discomfort with the Euclidean presentation of geometry, the author incorporated a more demonstrative mode than that advised by Ramist method where necessary. Such an example can be seen in Book VII's seventeenth proposition. Whereas the first two paragraphs of this proposition followed Ramus's source text in content, in order and in presentation, the third paragraph broke from the pedagogue's method by clearly directing the reader's attention to the *apodeixis*, or proof, which followed.[105] Fincke's presentation in this section of the proposition thus belongs to a more classical style of mathematics common to works such as the *Elements*: one in which the proof is provided through a sustained exposition of the particular objects of a given diagram.[106]

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100 Fincke, *Geometriae rotundi*, p. 101: 'Si la termino lateris trianguli aequalteri inscripti duae rectae in puncta peripheria aequalitier a reliquo dicti lateris termino remota inscribantur: differentia inscriptarum aequatur inscriptae inter reliquum terminum et alterutram inscriptam.'

101 Fincke, *Geometriae rotundi*, p. 102: 'Perpendicularis a centro in latus inscripti sexanguli, est latus inscripti septanguli.'

102 Fincke, *Geometriae rotundi*, pp. 101–102: 'Differentia sinuum peripheriarum á sextante totius peripheriae aequali differentia majoris et minoris aequatur sinui differentiae. Consectarium certe pro constructione canonis sinuum nobilissimum'.

103 Fincke, *Geometriae rotundi*, p. 102: 'Apis enim, ait alicubi Varro, sexangulam cellam sibi architectatur, quot habet ipsa pedes; quod Geometrae ἑξάγωνομ fieri in orbe rotundo ostendunt: ut plurimum loci includatur. Hoc idem Pappo in Proemio libri quinti copiosius demonstratur. Et ita adscriptionem habemus trianguli, quadrati, quinquanguli, sexanguli, octanguli. Jam ad septangulum et nonangulum opus esset triangulo, cujus uter(que) angulus ad basin esset illic triplus hic quadruplus reliqui. In hujus inventione multum posuisse operae atq(ue) studii Geometras veteres accepimus. Quidam, ait Proclus, ab Archimedis, helicibus incitati in datam rationem datum angulum rectilineum secuerunt. Conatus illos Geometricos P. Ramus scholis suis Mathematicis inseruit lib. 12. in 4, Euclidis. Qui illic perlegi possunt: artificium est difficile, multiplex et varium.'

104 Fincke, *Geometriae rotundi*, p. 102: 'Mechanici tamen septangulum inscribunt opera sexanguli hoc modo.'

105 Fincke, *Geometriae rotundi*, p. 103: 'Hujus ἀπόδειξις haec est. A centro o in terminum e ducatur radius: fiet triangulum aequicrurum a o e cujus anguli ad a et e per 10. e. 6. aequantur: et uterque duplus est anguli a o e per 7. e. Nam a o ex thesi secatur proportionaliter et a e et majus segmentum.'

106 Reviel Netz, 'Proclus' Division of the Mathematical Proposition into Parts: How and Why Was It Formulated?', *The Classical Quarterly*, 49.1 (1999), pp. 282–303, p. 286.

Thomas Fincke's infrequent blending of Euclidean and Ramist forms of presentation is perhaps best understood in the context of his wider goals and the rhetorical strategies he applied in their pursuit. In defining and demonstrating how chords and their sines were found through the construction of polygons within a circle, Fincke demonstrated that his mathematical practice varied little from the methods of Ptolemy, Peurbach and Regiomontanus. He could nonetheless point to improvements in the precision of his calculations; by framing *Geometriae rotundi* in this way, Thomas Fincke cautiously plotted something of a precarious path for his textbook.[107] Making clear his knowledge and understanding of the works of Copernicus and Rheticus by comparison to their classical sources, the Dane strove to carve out for himself an uncontentious position as an expert humanist improver of spherical geometry, from which he might encourage his readers to see him as an altruistically-minded author whose comprehensive abbreviations and evaluations of his predecessors were for the benefit of teachers and students alike. In doing so, the author led his audience to see his work as part of a programme for the greater understanding and recovery of those sources, even as the discipline progressed.

At the same time, Fincke, an adherent and autodidactic reader of Ramus, sought to promote both the Ramist method as an ideal means to test the truth and validity of mathematical calculations, and to promote the use of this method to interpret the mathematical elements of Copernican theory. Through this methodical reading and application of geometry, Thomas Fincke presented his own canons as improvements upon those of Rheticus, and, even more ambitiously, sought to evaluate the trigonometric calculations of a wider range of authorities. Wrapped in the garb of Petrus Ramus's pedagogical method, however, *Geometriae rotundi* was itself part of another, much more cautious attempt at reform; one conceived in an attempt to keep the baby from being thrown out with the bathwater.

Although Thomas Fincke may have argued for the methodical presentation of mathematics to be altered according to the precepts of Ramus, he nonetheless wished to guarantee that classical works remained ennobled in the schools of early modern Europe.[108] The Dane admitted as much in the final lines of the tenth book of *Geometriae rotundi*, by which point he had largely abandoned one part of his original goal as put forward in his prefatory letter to Digges. If Ramus had charged Euclid with obscuring the natural reason of readers through abstruse constructions, Fincke could not afford the same accusation to be levelled at him. To avoid any possibility of *Geometriae rotundi* sowing confusion, students were strongly advised to remain in constant dialogue with their discipline's immortal masters, Regiomontanus and Ptolemy amongst them.[109]

In gathering up theoretical predecessors and saturating his text with reference to their works, Thomas Fincke had already expressed his intimacy with, and command of, the totality of their mathematical theory. The Dane's grafting of humanist learning on to the Ramist dialectical method was in one sense intended to provide pupils and their teachers with access to a style of presentation that he himself had found applicable, clear, and expeditious. As this was the method to which he attributed his own mathematical successes, the promotion of Ramist dialectic was thus intrinsically bound up with Fincke's astronomical calculations. In seeking to garner the disciplinary legitimacy required for his evaluation of astronomical authorities to be

[107] Considering the Dane's use of Copernican theory, Kristian Peder Moesgaard has described Thomas Fincke as a 'cautious' mathematician, and this term is suited to his work more generally. Kristian Peder Moesgaard, 'How Copernicanism Took Root in Denmark and Norway', in Jerzy Dobrzycki, ed., *The Reception of Copernicus' Heliocentric Theory: proceedings of a symposium organized by the Nicolas Copernicus Committee of the International Union of the History and Philosophy of Science, Toruń, Poland, 1973* (Dordrecht: Springer Science + Business Media BV, 1972), pp. 117–152, p. 119.

[108] Fincke, *Geometriae rotundi*, p. 63: 'Et nos retinere voluimus ob pleniorum textus Ptolemaici intellectum: qui miseré hodie é scholis, superioribus tamen annis a vide receptus, exclusus videtur'.

[109] Fincke, *Geometriae rotundi*, p. 295: 'Habui id quidem in animo ut calculum Regiomontani itemque Ptolomaei et Copernici cum hoc praesenti conferrem: Verum ego hinc tyrones turbari existimavi: nec ab horum artificum immortalitati consecratorum monumentis abducere studiosos volui. Ubi hic perceptus fuerit: facile collatio institui poterit'.

widely appreciated, Fincke had also used *Geometriae rotundi* to unseat one key authority in the teaching of early modern mathematics: Euclid, his supposed *bête noire*.

Seeing as it featured new approaches to spherical geometry in both terminology and, to a greater extent, methodological presentation, Thomas Fincke could hardly define his work as the epitome of a pre-existing and well-defined field of study. He could, however, present his text as a unique and forward-thinking synthesis of geometric knowledge, classical and modern: one that eschewed the dialogues, scholastic arguments, and other vestigial remnants of earlier texts, and advanced the discipline in the process. *Geometriae rotundi*'s paratextual materials, placed before and within the text itself, serve as appropriate frames through which to view the author's trigonometric and pedagogic constructions: through these frames, the author exercised his control over his reader's interpretation of his work. Crucially, an edifice built from copious reference to external authorities served to elevate Fincke himself to a position from where he could most appropriately present his mathematical textbook. Set against a growing number of classical and contemporary scholars, Euclid was often introduced as a lone voice: the engine which had moved the mathematician to the point of publication. On the one hand, Fincke could legitimately assail or correct the Greek author wherever necessary, destabilising where necessary Euclidean order, presentation and proof in the process;[110] on the other, the *Elements* functioned as the foundation of an art requiring intensive recovery.

As a result, Euclid's dual role as *Geometriae rotundi*'s progenitor and its pre-eminent antagonist was something of an authorial sleight of hand. Content to borrow the Euclidean definitions common to all who had studied mathematics in the period even as he decried what he perceived as the incorrect method of their presentation, Thomas Fincke nonetheless recognised the necessity of introducing readers to geometry through the terminology (and, occasionally, the mode of demonstration) used by the classical author. Only after the first five elementary books of *Geometriae rotundi* are completed did Fincke openly acknowledge that Euclid's teachings (along with those of Ramus, unsurprisingly) had functioned as a cornerstone on which to build sufficient understanding for a novel presentation of the geometric and trigonometric relationships between circles and rectilinear lines.[111] Without Euclid and Ramus, it would have been impossible (in Fincke's argument) to arrive at the point from which a more complex spherical geometry could be taught. How, then, did the users of such materials respond?

4. Using the Wittenberg *Sammelband*: Reading print, paratext and marginalia

If we take the Wittenberg *Sammelband* as indicative of an idiosyncratic crash-course in early modern mathematics in 1580s Germany, the purpose of binding texts from Petrus Ramus and Thomas Fincke together in a single, multi-faceted volume is clear. Where Ramus's works served as a suitable introduction to mathematical study (introductions that many would never graduate beyond), Thomas Fincke's textbook on spherical geometry was intended for a moderately more advanced audience, and one that might have at least some existing knowledge of geometry. The presence of John Peckham's *Perspectiva communis*, along with the manuscript notes on sexagesimal notation, suggests that the *Sammelband* was intended to function as a guide to a significant portion of the early modern *quadrivium*. As has previously been shown, introductions to more rudimentary arithmetic and geometry were common, if poorly taught, at secondary and tertiary institutions across Europe, and the study of the sphere was elementary to early modern astronomy. Writing to Conrad Dasypodius in March 1565 to outline the year's mathematical course, Johann Sturm

110 Schönbeck, 'Thomas Fincke und die *Geometriae rotundi*', pp. 89–90. Jürgen Schönbeck notes that Fincke offered an alternative and more elegant solution to Thales's inscribed angle theorem as postulated in Book III, Proposition 31 of Euclid's *Elements*. This solution found its way into mathematical teaching literature and was popularised in the eighteenth and nineteenth centuries.

111 Fincke, *Geometriae rotundi*, p. 80: 'Geometriam circularem nobis duplicé proposuimus: simplicem quidem solius circuli cum suis lineis et segmentis: conjunctam vero in adscriptione circuli et rectilinei ponimus. Cognitionem rectilineorum é Geometria vel Euclidis vel Rami huc adferri necesse est: nos ea saltem docebimus quae circulum attingunt'.

instructed that pupils be introduced to the study of the physical world through Proclus's *Sphere*, Aratus's *Phaenomena*, Euclid, Aristotle, and arithmetic;[112] reading Ramus's texts could aid with several of these works, with *Geometriae rotundi* something of a bridge between these introductory materials and the more demanding sub-disciplines of spherical geometry and the study of triangles that might follow. In these surroundings Peckham's text is the odd man out, belonging to both an older tradition of perspectival optics and to a more advanced study of mathematics than the volume as a whole appears to have witnessed.[113] Its inclusion may indicate an attempt by a book-seller or purchaser to include material that would be of use should a subsequent owner achieve high standards in their reading of mathematics. The almost total lack of annotation on its leaves suggests that this was not to be.

It is certain that the second batch of users after Hommer and Coppius, namely Joseph Lobhartzberger and David Klynaeus, were at least in attendance at an institution that taught mathematics alongside the mathematical precepts of astronomy and optics thereafter. That the users of the Wittenberg *Sammelband* were educated within the parameters of Melanchthonian doctrine is evident from the date and locations of their ownership inscriptions, and from a declaratory statement prior to the final, manuscript notes at the rear of the volume. Immediately preceding the volume's manuscript notes on sexagesimal astronomy is Plato's oft-paraphrased maxim 'God always geometrizes' ('ό ϑεός άεί γεωμετρεῖ'). The two latter works bound to the rear of the Wittenberg *Sammelband* were by-products of the study of mathematics as inspired by Melanchthon's influence. Students at Wittenberg and other Lutheran institutions would have studied 'lower' mathematics (arithmetic and geometry) and, depending on their progress, 'higher' mathematics (astronomy and astrology) as part of the arts course undertaken prior to the elevated disciplines of medicine, law and theology.[114] Although classical and medieval texts such as Euclid's *Elements* and Sacrobosco's *De Sphaera* remained predominant, the increased availability of print meant that students could supplement lecturers' notes (either distributed and copied, or taken by dictation in the classroom) with popular amended editions which included up-to-date commentaries—some, as in the case of those written by Conrad Dasypodius at the Strasbourg *Gymnasium*, authored by tutors themselves. In this vein, students at Wittenberg might have encountered works such as Sebastian Theodoricus Winshemius's *Novae questiones spherae, hoc est, de circulis coelestis, primo mobile* (Wittenberg, 1564), reprinted six times between 1567 and 1605.[115] When Fincke sat down to write his own textbook on the circle and sphere, he combined the Wittenberg interpretation of astronomy, a Ramist method insistent on clear and expeditious definitions, and a rhetoric which prioritised classical authority and noble use. Lessons from his recent past on dialectic, rhetoric, and the value of mathematics were surely fresh in his mind.

By consistently cross-referencing Ramist material, Fincke's text quite specifically suggested that his readers kept Ramus's *Geometriae septem et viginti libri* close to hand. While the Sammelband's contents suggest that this advice was adhered to, the volume's most frequent annotator (perhaps Johan Coppius, and likely a tutor of some sort) also appears to have used the volume as a springboard for the introduction of topics more appropriate for university study. The reading practices and scribal technologies found therein

[112] Letter from Johann Sturm to Conrad Dasypodius, March 1565, in Spitz and Tinsley, *Johann Sturm on Education*, p. 295.

[113] This may not preclude the text from taking its place in a wider conversation on Ramist-influenced mathematicians and their views, however. One noteworthy in this context is Ramus's former student Jean Pena (c. 1528–1558), whose Latin translation of *Euclidis optica et catroptica* (Paris: André Wechel, 1577) promoted the value of optics to the study of astronomy. See Peter Barker, 'Stoic Alternatives to Aristotelian Cosmology: Pena, Rothmann and Brahe', *Revue d'histoire des sciences* 61.2 (2008), pp. 265–286; on Pena, and Ramus's subsequent influence on French mathematics in the sixteenth century and beyond, see Isabelle Pantin, 'Teaching Mathematics and Astronomy in France: The Collège Royal (1550–1650)', *Science and Education*, 15 (2006), pp. 189–207.

[114] Nicholas Jardine, 'The Places of Astronomy in Early Modern Culture', *Journal for the History of Astronomy*, 29 (1998), pp. 49–62, p. 50.

[115] Owen Gingerich, 'From Copernicus to Kepler: Heliocentrism as Model and as Reality', *Proceedings of the American Philosophical Society*, *117.6, Symposium on Copernicus* (1973), pp. 513–522, p. 516.

are important examples of the types of public and private instruction delivered prior to, and, in many cases, alongside of, university tuition. At the same time, the annotator's efforts to paraphrase the texts of Ramus and Fincke's works in the style of these authors also provide evidence of the wider diffusion of this pedagogical methodology. Seen in this light, the Wittenberg Sammelband can be thought of as a scholarly 'instrument of knowing': one which, through its unique construction and use, helped to further systematise and standardise Ramist mathematical teaching through the transmission of both print and manuscript.

Defining the marginalia found in mathematical texts as indicative of evidence of readers' mathematical literacy is a complex task. In a recent article considering the mathematical literacy of annotators of Sacrobosco's *Sphere* in late sixteenth-century Paris and Cologne, Richard Oosterhoff proposed heuristically three broad categories of reading: the mining of mathematical or astronomical texts for material related to other, literary works; critical comparison of authorities on astronomical or mathematical knowledge; and, finally, calculations.[116] Whereas many of the annotations found in the Wittenberg *Sammelband* fall uncomfortably between each of these helpful categories, they can nonetheless be redefined to serve a similar analysis of the two key texts of this volume. Following Oosterhoff's template, three broad categories of reading are proposed to assist our interpretation of the reading strategies at work in the Wittenberg *Sammelband*. The first is a 'mining' of key sections, with their contents redacted and paraphrased in summaries in the text's margins. The second category takes the form of a visual restructuring of these printed materials and their figures, with textual content reduced to the dichotomous schema so familiar to historical studies of the printed Ramist method. The third strategy features occasional amendment to, and conversation with, the volume's printed content—at work both within the text and on the margins just beyond (Figure 1).

The volume's opening Ramist and proto-Ramist texts were annotated by a user displaying a keen eye for the reconstruction of mathematical practice. Commonplacing authorities, our predominant annotator studiously collected the names of the mathematicians identified in Thomas Fincke's *Praefatio ad Lectorem* prior to *Geometriae rotundi*. In contrast to the other annotated sections of the volume, these names were not scored through, but underlined and then listed in the margins.[117] Readers would have met with and understood such paratextual elements on their own terms: whereas the authoritative names listed in the *Praefatio ad lectorem* fulfil the role of dedicatees for the author, the contemporary reader would more likely have used these figures as points of reference by which to situate *Geometriae rotundi*'s mathematical theory. Ultimately, identification of authority served the early modern reader mainly for the purposes of collection and recall of information. Where reference is made to such theoreticians, the brief notes that accompany their reference perform the function of an index rather than a commentary, and one akin to existing reading practices by which the gathering and framing of authorities and their texts were central to the entire educational edifice of thinking, reading, teaching, speaking and writing.[118] Indeed, Thomas Fincke was as much a product of this practice as he was a proponent (Figure 2).

Where Fincke recognised Plato, Aristotle and Quintilian as being in agreement on the perfection of the form and of the sphere, his annotator followed suit, redacting key statements in the texts and citing their sources by name alone. In a section treating isoperimetric figures, Fincke showed that a circle (in plane) has a greater area than any other polygon with the same perimeter. Thales of Miletus, Ptolemy, Theon (via Zenodorus) and Ramus were all cited before the author railed against those wilfully ignorant

[116] Richard J. Oosterhoff, 'A Book, A Pen, and the *Sphere*: Reading Sacrobosco in the Renaissance', *History of Universities*, 28.2 (2015), pp. 1–54, particularly p. 17. Oosterhoff also provides an exemplary treatment of the transmission of mathematical pedagogy through text and manuscript in *Making Mathematical Culture*, pp. 56–85.

[117] Fincke, 'Praefatio ad Lectorem', *Geometriae Roundi*, f. a 2 v, Wittenberg *Sammelband*, O.B. RAM RAMUS, 30209019362777.

[118] Mary Thomas Crane, *Framing Authority: Sayings, Self, and Society in Sixteenth-Century England* (Princeton, NJ: Princeton University Press, 1993), p. 12.

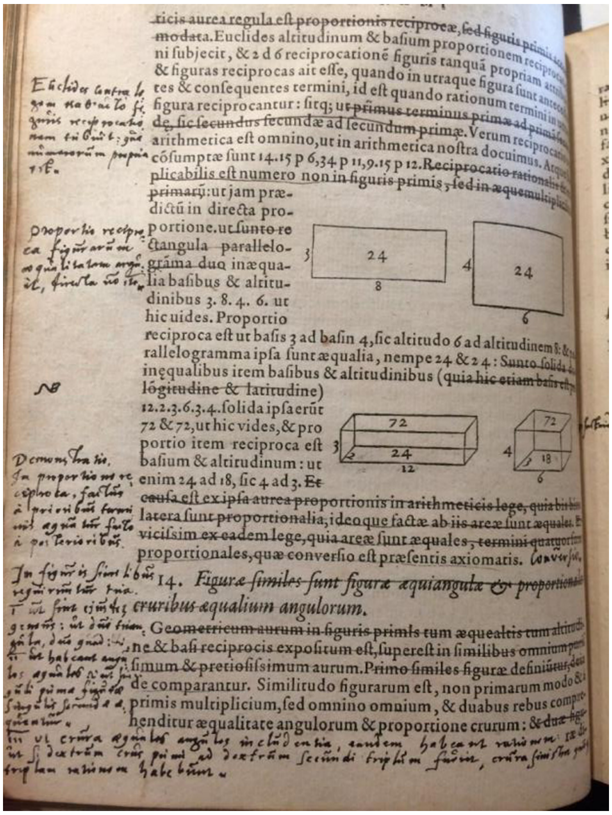


Figure 1. The annotating style most commonly found in the Wittenberg *Sammelband*. Sections are highlighted, struck through and paraphrased; occasional 'NB's specify important parts of the texts, and additional commentary (seen here under the section marked '*Demonstratio*') reframes the printed text of Ramus, *Geometriae septem et viginti*. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362784. By permission of Science Museum/Science & Society Picture Library.

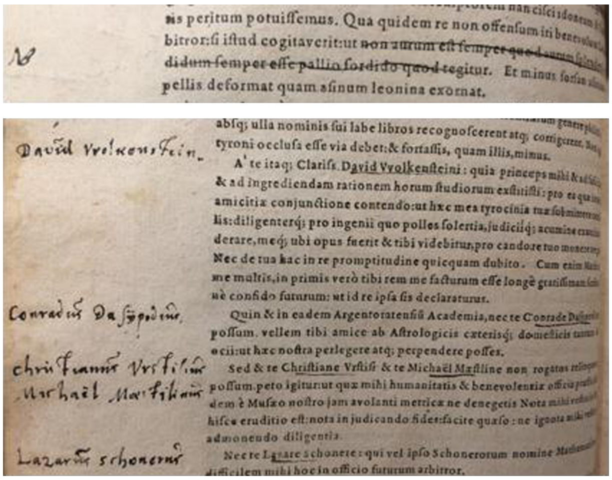
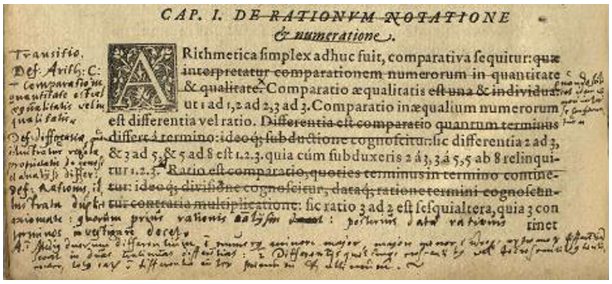


Figure 2. Composite image showing 'nota bene' ('NB') with text lined through, and, on the same page, a range of mathematical authors underlined in Fincke's *Praefatio ad Lectorem*. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362777. By permission of Science Museum/Science & Society Picture Library.


CAP. I. DE RATIONVM NOTATIONE & numeratione.

ARithmetica simplex adhuc fuit, comparativa sequitur: quæ interpretatur comparationem numerorum in quantitate & qualitate. Comparatio æqualitatis est una & individua: ut 1 ad 1, 2 ad 2, 3 ad 3. Comparatio inæqualium numerorum est differentia vel ratio. Differentia est comparatio quantum terminus differt à termino: ideoq; subductione cognoscitur: sic differentia 2 ad 3, & 3 ad 5, & 5 ad 8 est 1.2.3. quia cùm subduxeris 2 á 3, 3 á 5, 5 ab 8 relinquitur 1.2.3. Ratio est comparatio, quoties terminus in termino continetur: ideoq; divisione cognoscitur, dataq; ratione termini cognoscuntur contraria multiplicatione: sic ratio 3 ad 2 est sesquialtera, quia 3 continet

Figure 3. Excerpt of marginalia covering a third of a page of Petrus Ramus's *Arithmeticae Libri Duo*. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362784. By permission of Science Museum/Science & Society Picture Library.

of geometry;[119] despite the presence of these formidable thinkers, only the individual names of Plato and Aristotle recur in the margins, before a single repetition of the term 'ψευδογραφία' (pseudographia) marks the vulgar errors formed by ignorance of geometrical theory.[120] Our reader's interest in the force of these terms seems limited to summary, and not discussion. Mathematical material was considered on its own terms, with little evidence to suggest that this user was keen to read arithmetic or geometry in conversation with literary, philosophical, or spiritual authorities.

Collating mathematical authorities is merely one example of the way in which the Wittenberg *Sammelband* bears evidence of modes of mathematical study rather than any notable mathematical proficiency. For this individual, 'mining' the *Sammelband*'s works was a process characterised by extraction, collection, and reorganization, intended for their own pedagogical purposes. Redaction and repetition of the printed text dominate the marginalia of *Arithmeticae Libri Duo, Geometriae Septem et Viginti* and *Geometria rotundi* alike. Largely content to refrain from deconstructing literary and rhetorical allusions or arguments with the scholastic logic common to the universities, as might be expected, the main annotator of the Wittenberg *Sammelband* instead sought to replicate much of the volume's mathematical content in the abbreviated style of its authors. The form of this 'mining' is likely to have been the product of a master, with pupils copying dictation or written notes into their texts and notebooks: a clear example of this, rich in detail, occurs at the beginning of the first chapter of the second book of Ramus's *Arithmeticae Libri Duo* (Figure 3).

This particular example is typical of the annotating style brought to bear upon Ramus's text in particular, and several identifiable features common to the entire volume's marginalia can be seen. First of all, the title of the chapter—*De rationum notatione et numeratione*—has been struck through for emphasis: wherever annotation is present in the *Sammelband*, the majority of titles are found redacted in this manner. The volume's body text bears the same marking, with clauses lined through and then repeated in similar diction in the left-hand margin of the page. In this instance, the term 'transitio' serves as an important directive from the annotator, intended to aid future readers (or, perhaps, those listening to the annotator himself). Ciceronian rhetoric taught that through *transitio* the orator could recall what had been and introduce what would follow, its stylistic movement serving both to remind and to prepare.[121] Thus the marginal addition is a close analysis of the printed passage's stylistic movement, with the annotator's notes shepherding readers (or listeners) through three separate but interlinked definitions extracted from the text: the first, marking the definition of comparative arithmetic, whereby comparisons of quantity are defined either in terms of equality or inequality; the second, marking the rules by which differences in unequal numbers may be

[119] Fincke, *Geometriae rotundi*, pp. 6–7: 'Qui vel propterea cognoscendus est: ut constet quanto cum pudore philosophos sejactentii, qui elegantem hunc Geometriae usum ignorant: cum homini etiam rhetoric non sit ignotus'.

[120] Fincke, *Geometriae rotundi*, p. 7, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362777. The manuscript annotation reads 'Plato', 'Aristoteles', and 'ψευδογραφία ignoratione rationis in isoperimetris'.

[121] Cicero, *Rhetorica ad Herennium*, trans. Harry Caplan (Cambridge, MA: Loeb Classical Library and Harvard University Press, 1954), p. 319.

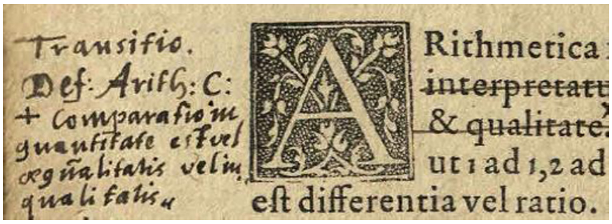


Figure 4. Expanded section of marginalia shown previously in Figure 2.12, detailing excavation and repetition of definitions from Petrus Ramus's printed text on arithmetic. Science Museum Library Shelfmark O.B. RAM RAMUS, 30209019362784. By permission of Science Museum/Science & Society Picture Library.

analysed; the third drawing attention to how numerical ratios may be used to illustrate the two previous axioms (Figure 4).[122]

The goal of this practice was twofold. First of all, in their process of extracting core definitions from the text and repeating them in the plain space of the margins, an authoritative reader has demonstrated their understanding of Ramus's key mathematical terms, and, more importantly, the importance of their order to building arithmetical understanding from more general parts to particulars. Secondly, whether for the purposes of lecturing, emphasis, or memorization, this annotator has further abbreviated the text through close reading, using the skills taught as part of the *trivium* to analyse the textual units of mathematics so that they might better present these units to younger and less capable learners.

The bulk of the marginal notes filling the blank spaces of the Wittenberg *Sammelband* were written in similarly abbreviated fashion. Although the lengthy notes surrounding the body text bear some resemblance to the commentary *scholia* written by Renaissance and early modern students in textbooks, their contents run exactly parallel to the text from which they are drawn with little in the way of critique or deviation and showcase more the contents, stylistic movement and presentation of the material to hand. In delineating how triangles of equal angles but different dimensions may be compared, Petrus Ramus had initially constructed his argument concerning equilaterals through a series of definitions aided by of simple geometrical figures. Following suit, our annotator used the margins of the printed work to unpack precisely the same argument verbally: although the form of the triangles was defined in terms similar to the Euclidean *ur*-text shared by both *Geometriae septem et viginti* and the later *Geometriae rotundi*, the logical structure of text and annotation is reorganised by a process of division and definition (Figure 5).

At times, these distillations highlight perfectly how the Ramist presentation of geometry could be ever more compacted by the 'active' techniques of Renaissance reading. Throughout Ramus's *Geometriae Septem et Viginti* this particular annotator was content to redact entire, introductory lines of propositions, occasionally indexing their contents only in the briefest possible terms. Headings such as Book II's 'fabrica peripheria'[123] (on the drawing of a circumference by extending a radius outwards from its central point) or Book VI's 'fabrica generalis omnis trianguli',[124] on the various ways of constructing triangles, serve as indexical bookmarks for the swift identification of various (and slightly more verbose) theoretical definitions. Rather than any meaningful commentary on Ramus's text, or, indeed, any significant efforts to draw its contents toward the other mathematical, philosophical or literary works these arguments might be com-

122 Ramus, *Arithmeticae Libri Duo*, p. 22, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362784. The annotation reads: 'Transitio. Def(initione): Arith(metica) C(omparativa): et comparatione quantitate est vel aequalitatis vel in[ae]qualitalis; Def(initione): differentiae quae illustrator regula proprietatis de genesi et analÿsi differ; Def(initione): Rationis illustrato duplici axiomate: quorum prius rationis analÿsin (docet): posterius datae rationis terminos investigare docet.'

123 Ramus, *Geometriae septem et viginti libri*, p. 12, Wittenberg *Sammelband*, O.B. RAM RAMUS, 30209019362784.

124 Ramus, *Geometriae septem et viginti libri*, p. 55, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362784.

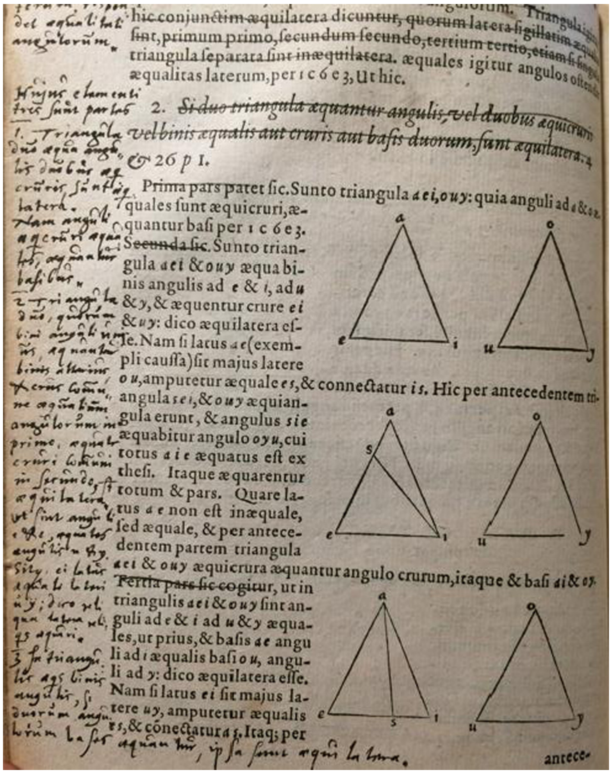


Figure 5. Annotations on equilateral triangles from Petrus Ramus's *Geometriae septem et viginti*. In the left-hand margin of the page, a tripartite system of division, definition and recast propositions can be seen. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362784.

pared with, such marginalia instead evince a user's systematic desire to parse the key elements of arithmetic and geometry to only the most essential units, most likely for swift identification and recovery.

This preponderance of summary is evidence of the repackaging of materials into ever more manageable packets of information best suited to more juvenile learners. Definitions were extracted from the text, their sources lined through, and their contents rewritten in the marginalia. However, as the volume's texts grew in complexity, a more interpretative style of annotation began to appear. As we have already seen, the first five chapters of *Geometriae rotundi* took the form of a Ramist-influenced reconstruction of introductory Euclidean geometry. Thomas Fincke presented these materials to inculcate in students a foundational understanding of the discipline, so that he might then move on to advanced treatments of spherical geometry and the importance of the radius to the calculation of trigonometric canons. Despite the author's best intentions, marginalia found in the Wittenberg *Sammelband* evince that students—somewhat understandably—required from their instructors a little more interpretation of geometrical theory than Fincke's text had initially provided. This user's interpretative additions to the volume's textual materials is witnessed only occasionally across the two key texts of the *Sammelband*, and most frequently in relation to Fincke's spherical geometry.

Where the printed text of *Geometriae rotundi* gave definitions of the diameter and radius as rectilinear lines inscribed within the figure of the circle, this was immediately followed by a manuscript reworking of the printed propositions: one which aimed to make clear more precisely what these propositions themselves had shown, and one in spite of Ramus's identification of apparent problems of logical method in Euclid's presentation. This tension is witnessed in the eleventh and twelfth propositions of *Geometriae rotundi*'s

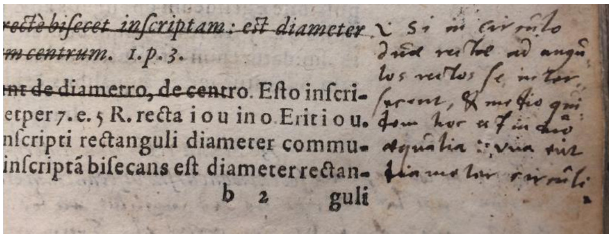

recte bisecet inscriptam: est diameter
in centrum. 1.p.3.
nt de diametro, de centro. Esto inscri-
etper 7.e.5 R. recta i o u in o. Erit i o u.
nscripti rectanguli diameter commu-
inscriptá bisecans est diameter rectan-
b 2 guli

Figure 6. The annotator's use of the Tironian symbol for 'est', followed by a brief note adding a Euclidean reference to Thomas Fincke's *Geometriae rotundi*. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362777. By permission of Science Museum/Science & Society Picture Library.

Book I.[125] The first of these propositions defined how a right line [eu] equal to a given line [a] drawn from the end point of the diameter [ei] of a larger circle radius could then be used to draw the circumference of a lesser circle, with the right line [eu] equal to both the given line [a] and the radius [eo]. The second demonstrated how, if the line [ui] bisected the line [ae] perpendicularly, then [y] would be the middle point of the diameter and the centre of its circle.[126]

Seeking to expand upon Fincke's eleventh proposition with reference to its source, the marginalia present benevolently reworded the text to instruct readers that 'this proposition demonstrates in what manner a right line equal to a given line is to be inscribed in a circle'—a definition quoted from Book XV of *Geometriae septem et viginti libri*.[127] Furthermore, the annotator has added a Tironian symbol to the following proposition,[128] expanding on the printed reference to Book III, Proposition 1 of Euclid's Elements. To more fully explain how this proposition is proven, the annotator expanded upon the printed text by adding the referenced Euclidean corollary: 'if, in a circle, two lines are cut one another at right angles, and in fact at the centre, that is into two equal parts, then in this way the diameter (of the circle) is found.'[129] Fincke, however, had directly followed Ramus by expressing the view that Euclid wished this definition impossible, and thus sought its antithesis through *reductio ad absurdum*.[130]

In some respects, this reflects a pragmatic approach to learning that today's teachers would no doubt recognise. Whilst every effort was made to situate mathematical learning within a Ramist structure, we can easily imagine why the annotator of these particular texts felt that it may have been necessary to succinctly adumbrate materials culled from external sources—even if these materials were, for the authors, a bone of some contention. Given that both Ramus and Fincke sought to educate expeditiously, the author of these marginal notes may not have had the time for his students to consult Euclid under their own steam. Emending the text with missing chunks of Euclidean theory and definition is likely to have been an educator's attempt at widening his pupils' understanding rather than any evidence of exasperation with the text itself, and the glosses which accompanied the text worked to supplement its original authors' deliberate elisions.

125 Fincke, *Geometriae rotundi*, p. 11: 'Si a termino diametri ex eaque radio aequante datam rectam peripheria describatur: recta a dicto termino in concursum peripheriarum inscribetur dato circulo, aequalis datae'.

126 Fincke, *Geometriae rotundi*, p. 11.

127 Fincke, *Geometriae rotundi*, p. 11, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362777. The annotation reads: 'Haec propositio docet quodmodo recta data sit inscribenda circulo aequalis datae'.

128 The Tironian symbol for 'est', shown in Figure 6, is listed by Michelle P. Brown in *A Guide to Western Historical Scripts from Antiquity to 1600* (London: The British Library, 1990), p. 136.

129 Fincke, *Geometriae rotundi*, p. 11, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362777. The annotation reads: 'si in circulo duae rectae ad angulus rectos se intersecant, et medio quidem hoc est in duo aequalia, (. . .) via erit diameter circuli.'

130 Fincke, *Geometriae rotundi*, p. 12: 'Euclides impossibile maluit, et ita cogit: quae est deductio ad sententiam absurdam'. See also Ramus, *Geometriae septem et viginti libri*, p. 115.

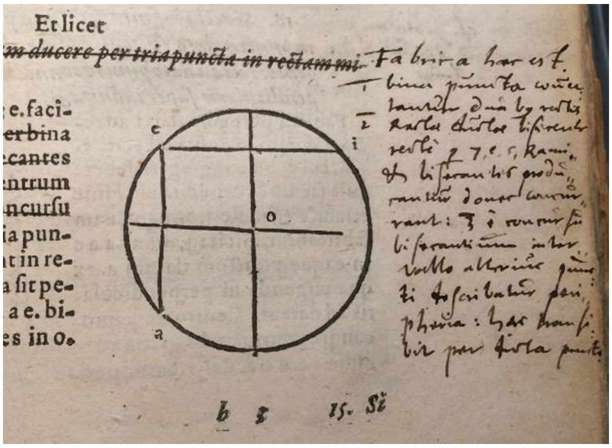


Figure 7. Annotations to Thomas Fincke's *Geometriae Rotundi* gloss the printed text with additional references, in this case from Petrus Ramus's *Geometriae septem et viginti*. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362777. By permission of Science Museum/Science & Society Picture Library.

'Mining' the text in the manner witnessed thus far was preparation for the second form of textual interaction most prominently witnessed in the *Sammelband*—the visual restructuring of theory. The body text of the volume was summarised and structured in such a way as to be more immediately accessible, and to retain the order of its source works. Textual material was then recast depending on the content of the information communicated: structured either by numbered points, or in the dichotomous trees of Ramus's organizational system. Both styles drew from the mode of presentation valued by Ramus and Fincke, even where the content of the texts being copied showed minor divergences from these authors. An example of this multi-level process of citation is seen in the first book of Fincke's *Geometriae rotundi*, on 'cutting' the circle with intersecting right lines.[131] Rather than replicate the diagram in the broad marginal space surrounding the text, the annotator has instead redacted the proposition's title and a key section of the text, before constructing a tripartite syllogism from the printed material (Figure 7).

The printed text of Fincke's fourteenth proposition concerns the construction of the right angle [aei] within the wider circumference of a circle.[132] The lines [ae] and [ei] are secants, each intersecting with two points on the circumference's curve. Each are themselves bisected at the central point, [o], with a vertical diameter bisecting [ei] and a misprinted, skewed line bisecting [ea].[133] The marginal annotation here recasts the printed text without disagreement, advising that, to construct the example, right lines would need to be drawn from two points.[134] Its author went on to repeat the postulation's connection of the points [a], [e], and [i] by two right lines.[135] Exceeding even Fincke's printed citations, the annotator has notably glossed this section with a cross-reference to the seventh proposition of Ramus's fifth book of *Geometriae septem*

131 The first book of Fincke's *Geometriae rotundi* is titled 'De circulis rectis secantibus'. Fincke, *Geometriae rotundi*, p. 5.

132 Fincke, *Geometriae rotundi*, p. 13. The proposition is entitled 'Peripheriam ducere per tria puncta in rectam minimè cadentia'.

133 Fincke, *Geometriae rotundi*, p. 13, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362777. The annotator has seen fit to redact (for emphasis) the lines 'Rectas enim interbina puncta inscriptas bisecantes duae in concursu suo centrum habent. Radius est a concursu in punctum.'

134 Fincke, *Geometriae rotundi*, p. 13, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362777. The annotation reads: 'Fabrica haec est: 1) linea puncta connectantur duobus rectis (. . .)'.

135 Fincke, *Geometriae rotundi*, p. 13, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362777. The annotation reads: '2) Recta ductae bisecentur recte p[er] 7.e.5. Rami, et bisecantes producantur donec concurrant'. For the relevant section in Ramus's work, see Ramus, *Geometriae Septem et Viginti*, p. 43. Ramus used the centre point of two separate yet coalescing circles to demonstrate how segments could be drawn between peripheral points using the equality of triangles.

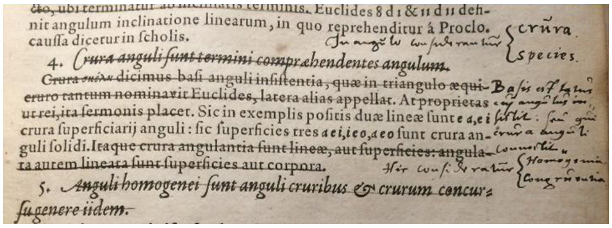


Figure 8. An example of branching, Ramist dichotomies drawn within Ramus's text. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362784. By permission of Science Museum/Science & Society Picture Library.

et viginti, where the bisection of right angle triangles within circumferences had previously been treated so as to further support *Geometriae rotundi*'s printed conclusions. Finally, the third description details (again, in keeping with the text) how the arc of the circle, held between these aforementioned points, is bisected by lines traversing its circumference.[136]

More eye-catching than the numbered summaries of logically ordered data are examples of the branching relationships drawn between thematically linked topics. There is little theoretical basis for the ways in which these flattened dichotomies have been presented: they are simply binaries, branching from a shared stem, presented visio-spatially for swift referral. By further subdividing the volume's texts into ever more abbreviated parts, the users of the Wittenberg *Sammelband* displayed their willingness to think within the dialectical method espoused by the very authors they were handling. To some extent, this can be seen as evidence of the successful inculcation of the Ramist method: users, be they teachers or students, adopted and standardised Petrus Ramus's ideas so entirely that they returned to pare down his texts according to the very methods of their instructor (Figure 8).

This reorganisation of Ramus's own text is present at a number of points. Dichotomous branches proliferated amongst geometrical propositions, separating objects by name and then by key properties. So it is that in *Geometriae septem et viginti*'s third book the annotator, following the author's terms and logical disposition almost to the letter, counsels that triangles must be considered first in terms of their sides and overall shape[137] and then afterwards by the homogeneity (or otherwise) of their angles within these lines.[138] Use of these graphic devices for mnemonic and organisational purposes is apparent in a number of additional examples from the Wittenberg *Sammelband*, and the practice is perhaps most apparent in the second book of Ramus's *Arithmeticae Libri Duo*. Books IV to VIII of the work move from the 'golden rules' of geometric proportion[139] to the explication of a method of inverse reciprocation, whereby the calculation of ratios is discussed with reference to the relationships of various figures and their common denominators.[140] Amidst the lined sections of text, an annotator selectively excerpted key phrases to keep in mind; thus a printed sentence comparing the structural importance of the golden rule and its analogical proportion to

136 Fincke, *Geometriae rotundi*, p. 13, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362777. The annotation to reads: '3. E(t) concursu bisecantium inter vallo alterius puncti describatur peripheria: hac transibit per dicta puncta'.

137 Ramus, *Geometriae Septem et Viginti*, p. 17, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362784. The annotation reads: 'In angulu(m) considerantur { crura / species'. Ramus's examples constructed the 'legs' (lines) or superficies of a triangle or triangular pyramid around a given angle (e or o) before using these terms to further define the formation of angles within plane and solid shapes. '*Crura'* in this annotation refers to the definition of the lines, and '*species*' to the more general shape thereafter.

138 Ramus, *Geometriae Septem et Viginti*, p. 17, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362784. The annotation reads: 'Basis est latus cujus angulus insistit: seu qui crura anguli connectit. Hic considerantur'.

139 Ramus, *Arithmeticae libri duo*, p. 26: 'Proportio arithmetica sic est, geometrica sequitur, in ratione(m) aequalitate (...) Si quatuor numeri sunt proportionales, factus a mediis, aequat factum ab extremis: et contra si equat, sunt proportionales. Haec proprietas propter admirabilem usum, vulgo regula aurea dicitur'.

140 Ramus, *Arithmeticae libri duo*, pp. 32–38.

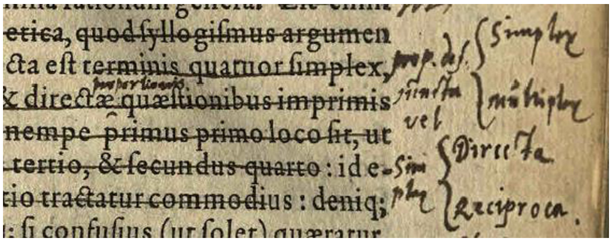


Figure 9. Further examples of the printed text of the Wittenberg *Sammelband* being dichotomised according to both Ramist method and the content of the text itself. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362784. By permission of Science Museum/Science & Society Picture Library.

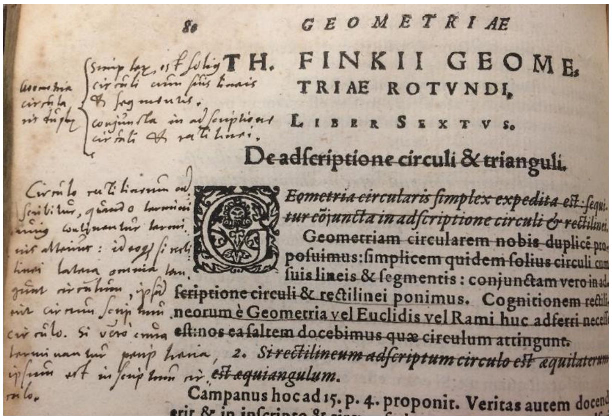


Figure 10. The system of dichotomy, division and definition continued throughout the Wittenberg *Sammelband*, as can be seen in the opening pages of Thomas Fincke's *Geometriae rotundi*, Book VI. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362777. By permission of Science Museum/Science & Society Picture Library.

arithmetic as akin to syllogistic reasoning in logic[141] is encapsulated as 'aurea regula: syllogismus' in the marginal space beside.[142]

Directly opposite this definition, appearing in the right-hand margin of the printed text, the quadripartite system of definitions that follows is distilled into a branching diagram illustrating proportion as composed of 'simplex' and 'multiplex' types: 'simple' proportion is then further constituted of direct and reciprocal proportions (Figure 9, above). The recursive circuit of Petrus Ramus's dialectical method from teacher to student was repeated logically in its transmission from Ramus to the autodidact Fincke to the *Sammelband*'s reader-annotator, and then beyond. In making Ramist dialectic instrumental to his presentation of spherical geometry, Thomas Fincke had explicitly praised the French pedagogue's method as the route by which he came to know mathematics. As the product of this knowledge, *Geometriae rotundi* was placed beside Ramus's works in the Wittenberg *Sammelband* to further inculcate and promote the value of Ramist pedagogy to the teaching and learning of mathematics. These lessons lasted long in the mind of the annotator of the Wittenberg *Sammelband*. As the text of *Geometriae rotundi* shed its more elementary garb, the annotator nevertheless retained a system of dichotomy and definition to organise his reading materials, marking the first page of Book VI with Ramist diagrams and their excerpted definitions (Figure 10).

141 Ramus, *Arithmeticae libri duo*, p. 27: 'Est enim analogismus proportionis in arithmetica, quod syllogismus argumentationis in logica'.
142 Ramus, *Arithmeticae Libri Duo*, p. 27, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362784.

Ramus's structure of logical organisation was proposed as but one part of the method by which the mind could be cleared of the obfuscatory logic and vestigial remnants of fabricated scholastic argument. The attentive reader at work in the *Sammelband* constructed familiar, branching diagrams in miniature to aid the recognition of these pathways; what these annotations serve to demonstrate is not so much an expressly critical or comparative form of mathematical reading, but rather one whereby the user-annotator accepted the materials on display as a viable means for the teaching and learning of mathematics, and practiced their teachings accordingly. This much is true even when authorities beyond those specified by the text are introduced in marginal commentaries. By gilding portions of Ramus's and Fincke's works with rare excerpts from other authors, the user identifiable throughout the *Sammelband* conveyed their acceptance of the printed mathematical models to their students or readers. As the third part of a reading practice which incorporated 'mining' the text for key definitions, representing those definitions and their relationships diagramatically or hierarchically, and glossing the structure with occasional notae from exterior authors, this final practice is of particular relevance when we consider how these annotations helped to organise and demonstrate mathematical theory for teaching purposes.

Though the manuscript notes surrounding the text focus most predominantly on applying the lessons of a logical, Ramist dialectic to the printed material at hand, their author was not so fixated on brevity as to leave his readers floundering (or himself stranded, should he return to the texts after any absence). On rare occasions, additional or indeed contrary information to aid the learner's development was provided, and in these moments the idea of a lecturer broadening students' horizons in conversation with the text is suggested. *Arithmeticae Libri Duo* Book 2's tenth chapter, on alligation, an arithmetical method used to calculate using mixed properties or denominations (for example, fluids commingled according to their ratios),[143] displays each of the two styles of annotation dealt with thus far: the text has been extensively redacted and repeated, although, somewhat curiously, the definition of alligation has escaped emphasis,[144] despite being dichotomised in the margin (Figs. 11 and 12, below).

Prior to these schematic marginalia, however, a point of contention with the main text was noted, even as the annotations themselves were dominated by a script which summarised (and tacitly bolstered) Ramist doctrine. An alternative voice was interjected to critique the veracity of the printed work. Marginal space was used to cite Lazarus Schöner's understanding of alligation as a method, and his concern over its value to the teaching of the doctrine of proportions (Figure 13).[145]

The introduction of Schöner illustrated a contemporaneous appreciation of the currents of mathematical theory in early modern Germany, and the citation of the respected mathematical educator again gives weight to the supposition that the content of these notes was ultimately the product of a lecturer. Excerpts drawn from additional alternative sources further support this view: in these notes are the hint of unasked questions, and of an educator constructing material so that it might be close to hand; whether to further

143 Petrus Ramus, *The Art of Arithmetike*, trans. William Kempe (London: Richard Field for Robert Dexter, 1592), p. 61. Kempe titled this section 'Mixture', and followed Ramus by explaining the practice as 'the mingling of divers sorts, whereof a meane is tempered: as in divers kinds of graine, liquid, mettall, pieces, weights, measures, and in all such things as may be mingled and tempered'. Similar terminology introduces the term in Robert Recorde, *The ground of artes teachyng the worke and practise of arithmetike* (London: Reynold Wolff, 1552), f. u 6 r. For a history of alligation for mercantile and medicinal calculation, see Alvan Bregman, 'Alligation Alternate and the Composition of Medicines: Arithmetic and Medicine in Early Modern England', *Medical History*, 49.3 (2005), pp. 299–320.

144 Ramus, *Arithmeticae libri duo*, p. 38: 'Alligatio est medii quaesiti vel dati'.

145 Ramus, *Arithmeticae libri duo*, p. 38, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362784. The annotation reads: 'Nota: Schonerus putat doctrina alligationis in differentis non in proportionibus locum habere, quia nec proportio est, nec proportionis necessario utitur, sed absque ea potuit intelligi'. I have to date been unable to precisely trace this remark. However, given Schöner's contemporary editing of Ramus's texts, and the presence of notes similar to Schöner's *De Logistice sexagenaria* in the Wittenberg *Sammelband* mentioned previously, I argue that this annotator was familiar with, and excerpting from, the works of Lazarus Schöner.

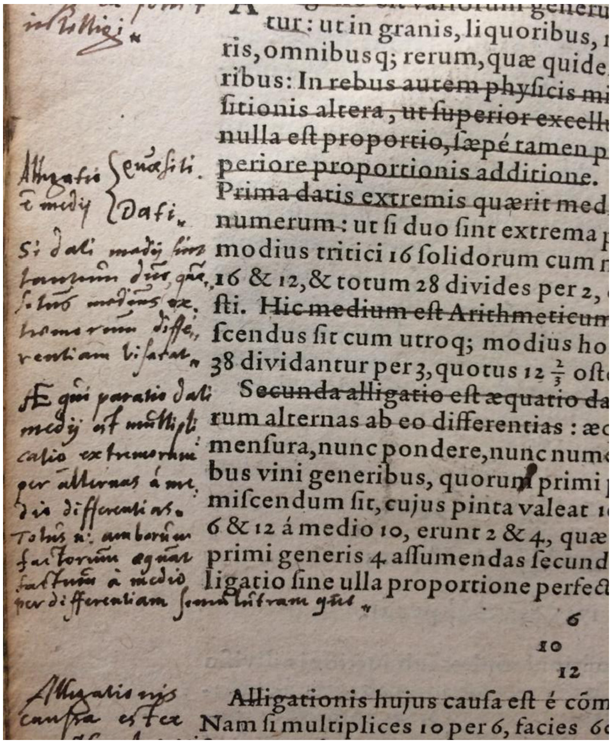

tur: ut in granis, liquoribus,
ris, omnibusq; rerum, quæ quide
ribus: In rebus autem phyſicis mi
ſitionis altera, ut ſuperior excell
nulla eſt proportio, ſæpé tamen p
periore proportionis additione.
Prima datis extremis quærit med
numerum: ut ſi duo ſint extrema
modius tritici 16 ſolidorum cum
16 & 12, & totum 28 divides per 2,
ſti. Hic medium eſt Arithmeticum
ſcendus ſit cum utroq; modius ho
38 dividantur per 3, quotus 12 $\frac{2}{3}$ oſt
Secunda alligatio eſt æquatio da
rum alternas ab eo differentias : æ
menſura, nunc pondere, nunc num
bus vini generibus, quorum primi
miſcendum ſit, cujus pinta valeat
6 & 12 á medio 10, erunt 2 & 4, quæ
primi generis 4 aſſumendas ſecund
ligatio ſine ulla proportione perfect
6
10
12
Alligationis hujus cauſa eſt é com
Nam ſi multiplices 10 per 6, facies 6

Figure 11. The text of Petrus Ramus' *Arithmeticae libri duo*, showing the styles of marginalia common to the *Sammelband*. Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362784. By permission of Science Museum/Science & Society Picture Library.

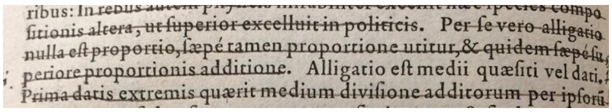

ſitionis altera, ut ſuperior excelluit in politicis. Per ſe vero alligatio
nulla eſt proportio, ſæpé tamen proportione utitur, & quidem ſæpé ſu
periore proportionis additione. Alligatio eſt medii quæſiti vel dati.
Prima datis extremis quærit medium diviſione additorum per ipſorū

Figure 12. The statement 'Alligatio est medii quaesiti vel dati' has escaped redaction, somewhat curiously given the standard practice in operation throughout the *Sammelband*. Wittenberg *Sammelband*, Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362784. By permission of Science Museum/Science & Society Picture Library.

elucidate a point or to head off a challenge. In this manner, the second page of Ramus's arithmetical text was inscribed with an alternative example on addition, adumbrating Euclid's first 'common notion', that things which are equal to the same thing are also equal to one another. The annotator chose to demonstrate this principle arithmetically rather than geometrically, constructing a monetary example of the ratios of *asses* to *aurei*, and *aurei* to *libella*, that was unmentioned in either the text or its Euclidean predecessor.[146]

Such minor amendments notwithstanding, the overall effect of the annotation found in the Wittenberg *Sammelband* was to construct and demonstrate a coherent practice of mathematical operations in agreement with the printed text. In this précis, occasional, minor emendations worked to assure readers of the suitability of method and outcome advocated by the two key texts of the volume. By drawing brief and occasional examples from the wider field of mathematics, the lecturer who annotated or delivered this in-

146 Ramus, *Arithmeticae libri duo*, p. 2, Wittenberg *Sammelband*, O.B. RAM RAMUS 30209019362784. The annotation reads: 'Euclides 1 axiomate 1. Numeri eidem aequales sunt inter se aequalibus: ut 2 et 2 sunt aequales tertie 2, s[un]t igitur inter se aequales. Hoc axiomate valoris aequalibus indicantur, quos numeros idem arguit: (. . . ?) totidem é partibus hoc et unitatibus (. . . ?) ut 260 asses faciunt 13 libellas: 5 aurei ft. 260 asses, ergo 5 aurei faciunt 13 libellas'.

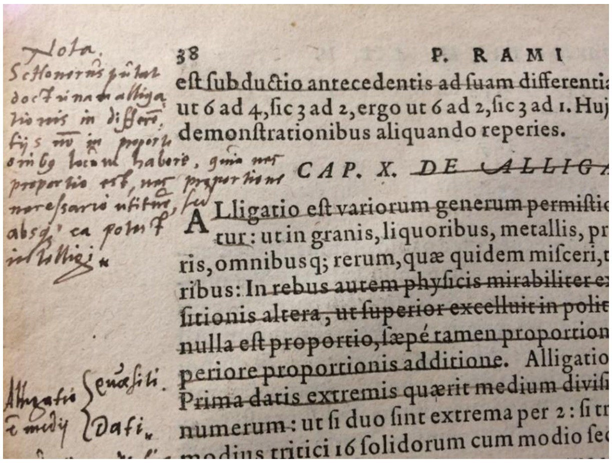

38 P. RAMI
eſt ſubductio antecedentis ad ſuam differenti
ut 6 ad 4, ſic 3 ad 2, ergo ut 6 ad 2, ſic 3 ad 1. Huj
demonſtrationibus aliquando reperies.
CAP. X. DE ALLIG
ALligatio eſt variorum generum permiſti
tur: ut in granis, liquoribus, metallis, pr
ris, omnibusq; rerum, quæ quidem miſceri, t
ribus: In rebus autem phyſicis mirabiliter e
ſitionis altera, ut ſuperior excelluit in polit
nulla eſt proportio, ſæpe tamen proportion
periore proportionis additione. Alligatio
Prima datis extremis quærit medium diviſi
numerum: ut ſi duo ſint extrema per 2: ſi tr

Figure 13. Additional information provided in contrast to that of the printed text: here the work of Schonerus (Lazarus Schöner) is referenced on alligation ('Nota…'). Science Museum Library Shelfmark O.B. RAM RAMUS 30209019362784. By permission of Science Museum/Science & Society Picture Library.

formation further assured their students of the suitability of these texts for study. Occasional, additional references to Euclid and Schöner in no way invalidated the conclusions of either Petrus Ramus or Thomas Fincke, as each theorist had shown themselves willing to utilise both classical and contemporary sources throughout their works. The markings found throughout the *Sammelband* do not betray disagreement with the authors, but rather evince a structured re-making of the text: one in keeping with the instructive syllabus of a teacher or lecturer. This annotator can be seen correcting his own notes on only three occasions throughout the volume, and there is no evidence to suggest that space has been left for text to be added later. The consistency and clarity presented in the vast majority of notes to the printed texts point toward a mature and knowledgeable author; a supposition further strengthened by the material qualities of the *Sammelband* itself.

5. Conclusion

The teaching and study of mathematics in the late sixteenth century was by no means independent of ongoing debates concerning dialectic, reason, and correct pedagogy. Such disputes wrought significant change in the teaching of the liberal arts curriculum in early modern Germany, where educational institutions from *scholae triviales* to gymnasia to the elite universities re-established the value of the mathematical disciplines. By the 1580s, the motivations of this educational structure were twofold. It combined the centuries-old teaching traditions and structure of the liberal arts—a tradition familiarising learners with the rudiments of rhetoric, grammar, dialectic, arithmetic, geometry, and astronomy, before ascending toward natural philosophy—to a more recent pedagogical approach tested over the preceding decades: one focussed on definition, division, and topical reorganisation. The *gymnasia* of early modern Germany, swept up in the vogue for Petrus Ramus's works, thereby produced learners taught according to the precepts of a dialectical model broadly in agreement with the earlier sixteenth-century reorganisation of the liberal arts as championed by Agricola and further emended by Melanchthon, Sturm, and Ramus.[147]

[147] Ong, *Ramus, Method and the Decay of Dialogue*, pp. 29–32. Ong divided Petrus Ramus's intellectual career into four main phases: rhetorical, methodical (focussed on dialectic and rhetoric), mathematical, and theological. Though a neophyte of Ramus's mathematical works, Thomas Fincke should be considered as influenced most strongly by the 'methodical' precepts of Ramus rather than his mathematical arguments.

An integral part of this shifting educational landscape was the output of Petrus Ramus: works marked by their rejection of existing scholastic principles and available to students in almost every branch of the liberal arts. By way of his controversial dialectical method, Petrus Ramus drew mathematics further into a wider programme of philosophical and pedagogical reform. Ramus's presentational style, regulated strictly by definition, rule, and diagrammatic constructions, was succinct and practically-minded. At the very least, his elementary works offered a coherent means by which to inculcate in students an ability to recognise and manipulate mathematical terms. Dispensing with Euclidean order and demonstrations, he applied his dialectical method as an 'instrument of knowing' to the study of arithmetic and geometry. In this way, the French pedagogue's sallies against Euclid in his *Scholae mathematicae* (1569) became a touchstone for the 'divergent philosophical and methodological stances' of the next generation of mathematicians.[148] Thomas Fincke was one such tyro, but, as the material form of the Wittenberg *Sammelband* shows, he was not alone in being guided by Ramist dialectic.

In a more cautious fashion, Fincke's *Geometriae rotundi* was crafted so as to reform mathematical teaching according to the lessons he had taken from the French philosopher and pedagogue. For Fincke, Ramist presentation would help to construct the platform from which he announced his skill in astronomical calculation. Though the latter was undoubtedly influenced by the former, it is important to note that the texts each author created shared a common well-spring in the educational movements of the mid-sixteenth century. Their impact had left humanist educators fighting a rear-guard defence of the scholastic philosophy, lest universities become infected by wave after wave of ill-prepared undergraduates reared on such insufficient methods. The intellectual coalescence of authors, consumers, and users on display inside the Wittenberg *Sammelband* is further evidence of the impact of this movement, and of its mode of transmission.

On the one hand we have an author—Thomas Fincke—who, under the tutelage of Johannes Sturm and Conrad Dasypodius at the Strasbourg academy, read the works of classical mathematical authors and, independently, learnt from Ramus's logical and mathematical works. Fincke appropriated Ramist method and terminology to promulgate a new model for mathematical pedagogy: one which challenged the importance of Euclid, just as it advanced the case for Regiomontanus as the ideal mathematical *praeceptor*. Much like Ramus, Fincke was himself a learner who found the mathematical discipline incomplete, obfuscatory, and unfit for his purposes. Much like Ramus, he drew upon the experiences of his background and education to resolve these matters.[149] On the other hand, the annotations populating the Wittenberg *Sammelband* embody the worst fears of the university lecturers of early modern Germany. The ownership inscriptions of Hommer, Coppius, Klynaeus, and Lobhartzeberger carry us from Leipzig to Wittenberg via Leisnig and Copenhagen. The bacillus of the Ramist educational method had infected these learners prior to their matriculation, and the transmission of this unique volume helps in part to demonstrate the hold such a methodology would exert on both students and educators. How this cultural phenomenon altered the minds and products of generations of learners remains under-explored, particularly in the history of mathematics.

Analysis of the network of influences present in the wider adoption of Ramist pedagogical principals encourages us to search for the rhetorical commonalities consistent in the presentation of innovative and novel mathematical ideas in the textual culture of the late sixteenth century. Existing research concerning the relevance of rhetoric to mathematical presentations has, understandably, focussed primarily upon the

[148] Stephen Johnston, 'John Dee on Geometry: Texts, Teaching and the Euclidean Tradition', *Studies in History and Philosophy of Science*, 43 (2012), pp. 470–479, p. 474.

[149] Ramus's socio-economic and meritocratic motivators are considered in Hotson, *Commonplace Learning*, p. 42; see also James Veazie Skalnik, *Ramus and Reform: University and Church at the End of the Renaissance* (Kirksville, MO: Truman State University Press, 2002). Given the nepotistic fiefdom Fincke later created for himself at the University of Copenhagen it is difficult to claim that he shared Ramus's zeal for social reform as based on absolute meritocracy. On Fincke's Copenhagen cabal, see Ole Peter Grell, 'Caspar Bartholin and the Education of the Pious Physician', in Ole Peter Grell and Peter Cunningham, eds., *Medicine and the Reformation* (London: Routledge, 1993), pp. 78–100, particularly pp. 89–91.

rhetoric of the mathematical 'author' rather than the possible response of the mathematical 'reader'.[150] But the importance of textual rhetoric must be considered a vital part of readers' responses, particularly when discrete texts are compiled and annotated as part of a single compendium.

The reshaping of the printed material of the Wittenberg *Sammelband* and the style and structure of such active manipulation is therefore indicative of the making of mathematical culture in the early modern period, and to the transmission and reception of this culture through social institutions. When seen in this light, a number of properties of the volume come into clearer focus. Its binding and composition suggest a salaried owner with an interest in mathematics, whilst its internal inscriptions speak of private and public instruction, as well as borrowing between contemporaries. The authoritative quality of its marginalia leaves little room for performativity or experimentation. Rather, key sections were only minimally refashioned outside of the boundaries of the printed text, their importance highlighted and further clarified for consumption by additional readers. Although these were literal rewritings of the printed works, contradictory or comparable treatments are missing, indicating that neither the methods nor the conclusions of the printed texts came into question. If anything, these texts are marked by users' unquestioning acceptance of their doctrines. As such they are not only Ramist materials, but literal evidence of attempts to think and reason within a newly systematised pedagogical model. The *Sammelband*'s annotations transformed these opening texts into a type of palimpsest: one which further enforced the value of Ramist method for the learning of mathematics, and which could, at times, be read almost independently of the printed works themselves. As much an instrument of knowledge as proof of the standardisation of a new mode of mathematical teaching and learning, there can be little doubt that the *Sammelband* is evidence of the multi-faceted ways in which mathematics were 'read' in the period.

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[150] Giovanna C. Cifoletti, 'Mathematics and Rhetoric: Introduction', *Early Science and Medicine*, 11 (2006b), pp. 369–389; and, in the same issue, Giovanna C. Cifoletti, 'From Valla to Viète: The Rhetorical Reform of Logic and its Use in Early Modern Algebra', *Early Science and Medicine*, 11 (2006a), pp. 390–423.

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