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Students' Views on Transition to University: The Role of Mathematical Tasks

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Abstract In this article, we use a case study to explore the views of first-year university students on the differences between mathematics at school and at university, and on the changes to their study methods as they make the transition to university mathematics. We also consider their views on the differences and affordances of tasks that they encounter on either side of the transition. The students in this study were registered on differential calculus modules where non-routine tasks were employed. We find that students are aware of the increased emphasis on conceptual understanding and reasoning at university and of the need to be an independent learner. We also see that this awareness is raised through engagement with mathematical tasks and that working on tasks is an integral part of students' study methods. We conclude that mathematical tasks have a role in making lecturers' expectations clear to students and also in giving students' opportunities to develop mathematical thinking skills and work independently.

Résumé Dans cette étude, nous utilisons une étude decas pour sonder les opinions d'étudiants en première année à l'université quantaux différences qui existent entre les mathématiques du niveau scolaire etcelles du niveau universitaire ainsi que les changements que la transition versles mathématiques universitaires apporte aux méthodes d'étude des étudiants.Aussi, nous tenons compte de leurs perspectives sur les différences etcapacités suggestives d'action des tâches rencontrées avant et après latransition. Les étudiants sondés étaient inscrits à des modules de calculdifférentiel dans lesquels les tâches employées ne leur sont pas familières.Nos résultats montrent que les étudiants sont conscients qu'à l'université, onmet davantage l'accent sur la compréhension conceptuelle ainsi que sur leraisonnement et qu'à ce niveau, on doit être un apprenant autonome. Nousconstatons de plus que cette prise de conscience est facilitée par laréalisation de tâches mathématiques et que leur exécution fait partieintégrante des méthodes d'étude des étudiants. Enfin, nous concluons que lestâches mathématiques jouent un rôle dans la capacité des chargés de cours àfaire en sorte que leurs attentes sont clairement comprisespar

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lesétudiants et leur donner des occasions de développer des compétences enréflexion mathématique ainsi que travailler de façon autonome.

Keywords Non-routine tasks · Secondary-tertiary transition · Mathematical thinking skills · Differential calculus

Introduction

The transition from school-level mathematics to university-level mathematics has been a topic of research in mathematics education for many years. However, this transition is still seen to be problematic for both students and instructors (Gueudet et al., 2016, p. 19–20). Indeed, the European Mathematical Society has recently called for further consideration of the challenges involved in the secondary-tertiary transition (STT) as a step to making progress in this area (Koichu & Pinto, 2019). In this article, we will consider a specific aspect of STT, that is the views of students enrolled in a first-year university mathematics module on the differences between mathematics teaching and learning at school and at university, and in particular, on the impact of the mathematics tasks that they encountered on both sides of the STT. The data on which this work is based was gathered as part of an evaluation of a task-design project initiated by the authors (Breen & O'Shea, 2019). We have previously published a study of students' views on some of the tasks involved (Breen et al., 2016) but here we will analyse interview data in which the general notion of STT was explored. We build on the data analysis that we reported in Breen and O'Shea (2015).

Literature Review

Thomas et al. (2015) conducted an international survey of undergraduate mathematics educators to investigate their views on STT. They found that 91% of respondents felt that students experience problems in the transition from school to university mathematics, and attributed these difficulties to a lack of preparation, weak algebraic skills, the emphasis on concepts rather than procedures at university, large class sizes at university, and personal issues. When asked what could be done to alleviate the problem, the majority of respondents referred to changes that could be made at school level and very few made suggestions for innovations at university level. Gueudet et al. (2016) also reported that when addressing difficulties in STT, suggestions are often made on how to change mathematics at second level but that mathematics at university level seems "unquestionable and thus untouchable" (p. 20). However, Thomas et al. (2015) recommended that mathematics departments at university should also make changes and suggested adjusting the syllabi of first year modules, providing guidance on proof and proving, and introducing bridging courses.

Some bridging courses concentrate most of their efforts on strengthening students' algebraic skills but Leviatan (2008) described a course which aimed to reinforce students' school level mathematical knowledge while also bridging the "cultural gap" between school and university. To achieve this aim, students were carefully introduced to mathematical culture such as language and rules of logic, and also mathematical activities like generalisation, deduction and definitions. There is evidence that such activities are relatively rare in school classrooms (Jäder et al., 2019). In Ireland, O'Sullivan (2017) analysed tasks from three secondary school textbook series; he found that the vast majority of tasks required students to practice previously seen algorithms and could be completed without providing justifications. However, there is evidence at school level that students value challenging tasks (Ingram et al., 2019) and opportunities to collaborate (Wilkie & Sullivan, 2018), and research has shown that the types of tasks experienced by students can have a large impact on their learning (Jonsson et al., 2014). Furthermore, the types of activities with which students engage can influence their view of the subject of mathematics itself (Lithner, 2011), and for these reasons, Gueudet (2008) advocated that care should be given to the selection and design of mathematical tasks in the transition to university. De Guzman et al. (1998) advised university instructors to design problems that target the type of thinking they themselves value, and to resist assigning purely technical exercises, in order to help students appreciate the type of mathematical thinking required at university. They believed this would alleviate transition problems.

However, tasks that aim to develop mathematical thinking skills and conceptual understanding can be rare in first-year university modules aimed at non-specialist students. In a study of three different introductory calculus modules in two universities in Ireland, Mac an Bhaird et al. (2017) found that the proportion of summative assessment tasks which required creative reasoning as opposed to imitative reasoning (Lithner, 2008) was very low. Maciejewski and Merchant (2016) found that the types and levels of tasks assigned in first-year undergraduate courses were different to those used in later years. First year courses had tasks that focused more on lower level skills, while in later years, the tasks required more high-level cognitive processes (along with memory and recall). Given this situation, it may be difficult for students in STT to recognise the new expectations placed on their mathematical work and this may hamper their transition (Jablonka et al., 2017).

This is all the more pertinent given that the transition to university mathematics is an opportunity for personal growth for first-year undergraduates (Clark & Lovric, 2008); indeed, Hernandez-Martinez et al. (2011) found that students reported overcoming their struggles and felt that this was important in their growing-up process. Hernandez-Martinez et al. (2011) and Lithner (2011) emphasised the importance of providing students with opportunities to engage meaningfully with their mathematical difficulties and allowing them space to overcome these problems themselves.

This is one of the aims of our task design project (Breen & O'Shea, 2019), which we hope to achieve by creating tasks which give students opportunities to develop their conceptual understanding and mathematical skills, while making clear the new expectations at university.

In particular, we hope that the experience of working on non-routine tasks would help students to think more flexibly and overcome their problems independently. According to Maciejewski and Merchant (2016), different types of tasks can affect students' approach to study which in turn affects student outcomes. Thus, we are interested in finding out more about students' (self-reported) approaches to study and study methods and their perspectives on the tasks they were assigned on both sides of the STT. Hernandez-Martin et al. (2011) and Di Martino and Gregorio (2019) explored students' feelings on the subject of STT but relatively few other studies have done this, and in particular students' views on the differences between school and university mathematics are seldom sought. There is evidence that mathematics at second level is often seen as a series of unconnected tasks and solution methods (Bosch et al., 2004), but on the other hand, mathematics at university level can appear to be a series of theoretical results with little effort made to build intuition or to motivate the introduction of concepts (Gueudet et al., 2016). There is evidence that students in Ireland have few opportunities to work on non-routine tasks both at school (O'Sullivan, 2017) and at the beginning of their university studies (Mac an Bhaird et al., 2017). In fact, over the last 20 years, there have been many studies in Ireland which have highlighted the emphasis on procedural mathematics in second level classrooms (Lyons et al., 2003; Lubienski, 2011; NCCA, 2012a; Davis, 2013).

Here, we will investigate how the participants in our study see the differences between mathematics (and mathematics tasks) at school and university level and in particular how their study methods change during the transition. We have seen above that the tasks with which students engage can have a powerful effect on their learning and so we will consider the students' view of the role that tasks could play in the STT.

Our research questions are:

1. What are the views of students in this case study on the difference between mathematics at school and at university?
2. What are the views of students in this case study on the difference between study methods in mathematics at school and at university?
3. What are the views of students in this case study on the difference between mathematics tasks at school and at university?

The third research question is obviously narrower than the first; in order to study the first question, we will only consider students' answers to general questions about the differences between their mathematical experience at school and university (without any mention of tasks). Given the objective of our task design project, we particularly wanted to study students' views of tasks in STT and so we later asked specific questions about this.

Methodology

This project uses case study methodology. Stake (2000) defined three types of case study: intrinsic—where the sole focus is to understand a single case; instrumental—where a particular instance of a phenomenon (the case) is explored with the aim of learning more about the phenomenon in general; and collective—where a representative sample of cases is chosen in order to test a hypothesis. Our study is an instrumental case study; the case explored is the experience of STT of students in two calculus modules in which non-routine tasks were utilised. The case study approach allowed us to explore this complex phenomenon from the students' point of view (Moore et al., 2012).

The Modules

The authors are faculty members at mathematics departments in two universities in Ireland. At the time of this study, both were teaching courses on differential calculus to first-year undergraduate groups. These two modules followed a standard syllabus similar to typical Calculus 1 courses offered in North American universities (Rasmussen et al., 2014); that is, they covered functions, limits, and rules and applications of differentiation. Some proofs were given, but it should be noted that these modules were calculus and not analysis modules. The Maynooth University course used Larson's Essential Calculus textbook (Larson et al., 2008), while the DCU course did not follow a textbook. Both modules were the first university level mathematics modules that the students encountered; the Maynooth University module was delivered in the first semester, whereas the Dublin City University (DCU) module was year-long.

The modules were delivered using traditional lectures, with both lecturers frequently asking students to work on tasks in class either individually or in groups. Students also took part in small group tutorials facilitated either by the lecturer (in DCU) or a tutor (Maynooth University). In addition, the students at Maynooth University were able to avail of the services of the Mathematics Support Centre, where they could work with a tutor either individually or in a small group. Each week, the lecturers presented students with problem sets containing tasks for them to work on either independently (as homework) or in small groups (during tutorials).

The Students

There were 130 students registered to the module at Maynooth University and 35 in the module in DCU. Both modules were part of BA programs in their individual institutions; these programs are usually 3-year degree courses in which students take three or four subjects in their first year and then two in their second and third years. The DCU module was also taken by students who chose to specialise in Mathematics as part of a 4-year BEd (primary) program. The nature of the "mathematics" streams chosen by both BA and BEd students were more akin to "mathematical studies" in nature than "pure mathematics" and the BA students involved were usually aiming for a career in Teaching or Business.

The students' in the study had all completed the Leaving Certificate course in Mathematics. The Leaving Certificate program (NCCA, 2012b) takes place during the last two years of second-level education in Ireland, and the Leaving Certificate Examinations are high-stakes state-run tests which take place in June each year. Entry to university is decided using a point system which is based solely on the results of these examinations. Mathematics is offered at three levels: foundation, ordinary, and higher levels. Students who have taken mathematics at Foundation level are not eligible to study at university. Students in our modules had taken either ordinary level or higher level mathematics and so had studied some differential calculus at school level.

The Task Design Project

Both authors designed a series of unfamiliar non-procedural tasks for use in these modules, in an effort to give students opportunities to develop their thinking skills. An "unfamiliar task' is one for which students have no algorithm, well-rehearsed procedure or previously demonstrated process to follow. We aimed, by presenting the students with unfamiliar tasks, to discourage a tendency to rely on past experience when working on problems (Lithner, 2000) and to help students to develop the flexibility in their thinking and reasoning characteristic of mathematicians. The tasks designed in this project required students to make use of definitions, generate examples, generalise, make conjectures, analyse reasoning, evaluate statements, or use visualisation. To explain the motivation behind this choice of task types, we mention only Watson and Mason's (2002) study on the use of student-generated examples to construct mathematical meaning, and promote consolidation and reorganisation of knowledge structures. With a focus on the development of conceptual understanding, they identified five reasons for asking learners to generate examples for themselves: experiencing structure, experiencing and extending the range of variation, experiencing generality, experiencing the constraints and meanings of conventions, and extending example-spaces and exploring boundaries. For further rationale behind the choice of task types, please see Breen and O'Shea (2019). In addition, the types of tasks selected were ones that students had most probably seldom encountered in Irish secondary schools (O'Sullivan, 2017). Samples of the tasks designed are shown in the following.

Example Generation Task

a) Give three examples of functions that are continuous everywhere.
b) Give an example of a function, f, that is not continuous at 0 because $f(0)$ does not exist.
c) Give an example of a function, f, that is not continuous at 0 because $\lim_{x\to 0} f(x)$ does not exist.
d) Give an example of a function, f, which is not continuous at 0 because $\lim_{x\to 0} f(x)$ does not exist, although $f(0)$ exists.
e) Give an example of a function, f, which is not continuous at 0 because $\lim_{x\to 0} f(x) \neq \mathrm{f}(0)$, although both exist.

Conjecturing Task

Suppose f is a continuous function on $[a, b]$ and suppose that $f(a) > 0$ and that $f(b) < 0$.
What can you say about the number of times that the graph of f crosses the x-axis?
What about the number of times the graph of f touches the x-axis?
Explain your answer.

Visualising Task

Draw a rough sketch of the graphs of the following functions:

a) A function f which is continuous everywhere except at $x = -3$ and $x = 4$.
b) A function g which is continuous everywhere except for removable discontinuities at $x = -3$ and $x = 4$.
c) A function h which is continuous everywhere except for a removable discontinuity at $x = -3$ and a non-removable discontinuity at $x = 4$.

In contrast, a routine or procedural task for these students would be one such as "find the limit (if it exists) of $\lim_{x \to \infty} \frac{x^2-x-6}{x^2-5x+6}$."

Between 20 and 40% of the tasks on the weekly problem sets distributed during the two differential calculus modules were "unfamiliar." Some of these non-routine homework tasks were graded, and the scores contributed to the final module mark. The terminal examination for the modules also included some non-routine tasks.

The Interviews

Volunteers to participate in interviews were sought toward the end of the modules; at Maynooth University, all 130 students were e-mailed, 8 volunteered and because of timetable constraints five students were interviewed. Three of these were interviewed individually (students labelled F, I, J), and the remaining two were interviewed together (G and H)—we will refer to this as the joint interview. Fourteen DCU students (those who had attended two particular tutorials) were invited to participate in interviews. Six students volunteered but because of timetable constraints only five were interviewed individually (students with pseudonyms A–E). The interviews were semi-structured and of 16-min duration on average, ranging from 13 to 22 min. They were audio-recorded and were anonymised and fully transcribed by a research assistant.

The interview schedule produced in advance of the semi-structured interviews included the following questions:

A What do you think of your mathematics course (at university)? How is it different from school mathematics?
B Describe your study habits at school. Are they different at university?
C Different types of questions were used on the problem sheets in the calculus course, were you aware of the difference? Can you give examples? What did the tasks help you learn? [Note that the interviewer (an independent researcher) had a selection of problem sheets available for the students to consult during the interview in order to help them choose examples of tasks.]

Data Analysis

The two authors coded transcripts of the interviews independently using a general inductive approach (Thomas, 2006). We first read the transcripts multiple times in order to become familiar with the data. We then constructed a table for each interview question which contained the answers of each student. We coded the data question by question by highlighting phrases from the participants' answers. We each gathered these phrases into themes or categories and then met on various occasions to discuss and compare our coding. This was an iterative process and when we were satisfied with the coding, we agreed on the final summary categories. For example, the phrases that contributed to the Emphasis on Understanding at University category were: more about thinking, more in-depth, understanding the principles/how it works, explain the principles. In this way, the categories emerged from our analysis of the data. In particular, we did not consciously use predefined themes from our reading of the relevant literature to construct the categories.

Findings

When students were asked what they thought of their university mathematics course, they were positive on the whole. Seven of the ten students said they liked the course, one (a student in the joint interview) did not comment, and two said they found it difficult because of having to work independently or because of the speed of the course. The students who expressed a positive view of the course also mentioned that the course was challenging but rewarding, entailed a lot of work, and involved going into more depth than they were used to.

What Are Students' Views on the Differences Between Second-Level Mathematics and University-Level Mathematics in General?

Eight of the students (all but the two in the joint interview—students G and H) were asked about the differences between mathematics at second level and at university. The main categories of differences that emerged from the analysis of the responses to this question were as follows: the emphasis on understanding at university, and the need to be an independent learner at university (see Table 1 below). We might have expected students to focus on the large class sizes at university, but nobody did. In addition, only one student (student J) mentioned new terminology and concepts.

Emphasis on Understanding at University

Seven of the eight participants spoke about the procedural nature of mathematics at school as opposed to the emphasis on understanding at university. These students drew distinctions between mathematics at school and university by describing second-level mathematics as involving memorisation and computation but lacking in explanation and connections, while university-level mathematics appeared to them to be more about theory and understanding. For example, student A said:

Table 1 Students' views on the differences between mathematics at school and university

Theme	Number of students	Names of students
Emphasis on understanding at university	7	A, B, C, D, E, F, I
Independent learners	5	A, B, E, F, I
Were not asked this question	2	G, H

> ... it's a fair bit different. Because it's a lot more about the theory behind it, whereas in school you were just told well you do this and you just do it. You don't ask why, you just do it.

The students were aware of the emphasis on proof and explanations at university and this awareness seemed to come from two main sources: the lectures, and the assigned tasks. For example, student F explains the different approaches at school and in university lectures to the rules of differentiation:

> Like in 6th year [at school] there was a lot of things I just took for granted.. (…), the Product or Quotient Rule for example, we'd just accept the formula as it is. But in first year college we would explain how that formula develops. …That's the big difference, I think.

Student B spoke about the change in emphasis in the context of tasks:

> Like in school you do an awful lot of learning off and it's just procedural, like you're going through the procedure for each question. Whereas now like — just you have to — think about more what we are doing.

In fact, when describing the differences between mathematics at school and at university, five of the students spoke about doing questions or exercises. In particular, some of the students described the school syllabus in terms of Leaving Certificate examination questions (the Leaving Certificate is a high-stakes state examination held at the end of secondary education in Ireland). They saw the secondary syllabus not as a coherent set of topics and concepts but as an unconnected series of questions:

> It [university mathematics] kind of ties together really, everything from the Leaving Cert. It kind of explains why you were doing it before. And it kind of interlinks some more. For the Leaving Cert there were separate questions and they didn't really tie together at all. (Student D)

Independent Learners

Five of the eight students spoke about the necessity to be an independent learner at university by mentioning working on their own or self-motivation. This is an issue for any student moving from school to university, as student I said:

> I find that ahm, a lot – it's up to you basically how to learn. Nobody is there to actually push you or to give out to you [ie scold] if you don't have your homework or assignments done.

However, there seems to be some features of this element of transition that are particularly important in mathematics. Two students called attention to the fact that they were used to being guided step by step through tasks at school but at university they have to think for themselves (see Student B quote above). This was echoed by student E:

> It's just more, you have to think for yourself, like filling in formulas, things like that. I think it is much harder than the Leaving Cert. (…) It's just more the work you do yourself. Like in Leaving Cert you're kind of shown the formula and you fill into this…

Table 2 Students' views on the differences between study methods at school and university

Theme	Number of students	Names of students
Emphasis on understanding at university	6	A, B, C, D, I, J
Independent learners	2	B, E
Learning through tasks	8	A, B. C, D, E, F, G, H

The other independence-related issue that arose in the interviews concerned the fact that the students had previously studied calculus at school and were now studying the same material (but at a deeper level) at university. One of the students (student F) highlighted the tension that this creates for learners, since they may be confused whether they may use their previous knowledge or not:

> It takes a while to get used to, ya. Because you don't know where to accept it as it is or use the new kind of concept.

It seems that this student may be moving from being happy to accept facts to recognising the need to justify them.

What are Students' Views on the Differences in Study Methods Between School and University?

All ten students were asked this question. We categorised their responses into three categories as shown in Table 2: the contrast between a procedural focus at school and an emphasis on understanding at university, being an independent learner, and the view that study is primarily accomplished through working on tasks.

The first two categories here also appear in the previous section and the participants' responses were similar to those outlined above. For that reason, we will concentrate on the third category here.

Learning Through Tasks

All but two of the students (all but I and J) spoke about doing questions, exercises, or problems when describing their study methods at school and at university. In fact, working on tasks was the first thing that the majority of the participants mentioned when asked this question even though the interviewer had not yet mentioned tasks at that point in the interview. For example, student B's response when asked what is different about the way they study at university was

> I find now with the problem sheets the questions were very different to just say what school would have been like.

The students spoke about the differences in study methods between school and university by focusing on the experience of working on tasks; the main difference they identified was that at second level they needed to know how to apply a formula or method, while at university level they needed to understand what they were doing and why. For instance, student A commented that although she could apply the Quotient Rule and Chain Rule at school, she could not derive them. While she had seen some proofs at school, she did not really understand why things were done in a certain way.

Some of the students referred to the fact that their lecturer asked unfamiliar or unseen questions and that because of this they needed to understand the underlying concepts in order to be able to solve the problem (students A and D). Other students seemed to draw a distinction between the emphasis in school on "doing" and that in university on "understanding":

> Last year I would have put a lot of emphasis on doing the exam papers and then just doing problems and stuff, but now this year it's like read over the lecture notes … it's not all about actually doing the work, doing the actual maths problems themselves, but understand the concept (Student C).

Of the students who spoke about the second-level syllabus, all mentioned it in the context of questions on the examination paper and revision using old examination papers. For example, student D was asked about study methods at school and said:

> I'd basically get one of those revision guides and just learn - just basically what you need to pass pretty much. (…) it was like past exam papers the whole time. You're just going through the

motions really. You're not really - you don't think much about it at this stage. Because like even in the past exam paper it's for the Leaving Cert, they ask the same questions every couple of years.

Two students mentioned "practicing" questions as a study method: student G used examination papers as a source of questions at university, while student D used questions from a textbook.

The students also mentioned other study methods in the context of attempting tasks. For instance, most of the interviewees (all except students G and J) spoke about looking for explanations in their notes or in books, and some (A, B, H) mentioned looking for examples in their notes or textbooks. Some students spoke about memorisation at second level (A, B, D). Student A elaborated on this to explain how in school she memorised proofs without explanations but at university she memorises definitions and uses them as a starting point of her work on problems. Students from DCU often mentioned going to the library (B, C, D) to seek help from books, while students from MU mentioned looking for help in the Mathematics Support Centre (F, G, H, I) when working on assignment tasks.

What Are Students' Views on the Differences Between Tasks at School and University?

The students were asked if they were aware that their lecturers were assigning different types of tasks on the module problem sheets. Eight of the ten participants immediately said that they were aware of the differences, one student (student J) initially misunderstood the question but then said that she was aware that the lecturer assigned unfamiliar tasks, and one student (student H—a mature entrant) said that he was not aware of this. The participants were then given a selection of problem sheets that had been used during their modules and asked to pick out question types that were unfamiliar to them from school. All but one student (student J) chose tasks designed by the authors in response. Thus, on the whole, the participants seem to concur that the designed task types were not familiar to them from their school days. Furthermore, some students indicated that they would have been unlikely to encounter novel tasks in school:

We hadn't done that in class [referring to a designed task], so we had to try to figure it out for ourselves. Whereas in school the teacher would have done that with you. (Student B)

All students chose procedural tasks when asked to pick task types that were familiar to them from school. In fact, all but two of the students (H and J) spoke about the algorithmic nature of tasks that they had encountered at second level; for example, they talked about applying a method (student C), following the steps (student F), and working out an answer (student E). Student I picked a procedural task and said:

That's the sort of question that I would have expected in a mathematics book of a secondary school … It's just a matter of you know given a problem and solve it. You aren't to explain why or why is this not the right answer.

The students were asked what they gained from the tasks assigned to them during their calculus modules (Table 3).

The comments of a number of the interviewees indicate that to perform the tasks designed for this study they were forced to apply and find relationships between previously learned concepts. The quote

Table 3 Students' views on the benefits of unfamiliar tasks

Theme	Number of students	Names of students
Making connections	5	A, B, D, H, I
Assessing understanding	2	A, E
Developing new thinking practices	8	A, C, D, E, F, G, H, I

below from student A arose from her response when she was asked to compare two particular tasks, the first one familiar (Q1—evaluating limits) and the second one unfamiliar to the student (Q2—the example generation task given earlier in this article). Her answer suggests that she appreciated the benefit of working on a novel problem and that it necessitated the making of connections between concepts.

> So you're kind of bringing together what you know from other things whereas in Q1 you kind of — you're told what you have to do. So you're literally just kind of following a procedure really... whereas for the second one you kind of actually are more thinking yourself...it is more difficult, ya, but it kind of helps you understand it better. You see the relationship between them.

The interviewer then asked this student how she approached the two tasks. Student A's reply gives us some insight into the thinking process sparked by these questions:

> Q1 I just launched straight into that because I know what I'm doing, whereas Q2 I'd have to take more time. And I'd have to go through the different – I'd say well, if it's not continuous because f(0) doesn't exist, that means it's undefined, so how do I find functions undefined? And like even where the limits don't exist I'd have to go through - well that implies that as x approaches 0 from below is not equal to as x approaches 0 from above. […] Like even if I get something like that I won't know the answer straight away but I'd know if I'd break it down I'd get to it eventually. […] If this is wrong you're still learning something from it.

Other participants also explained that the tasks designed by the authors helped them to develop new thinking practices and ways of working. Some interviewees described certain tasks, which they had identified as unfamiliar or non-procedural, as encouraging habits such as thinking, analyzing, questioning, or exploring patterns. Eight of the ten students interviewed (A, C, D, E, F, G, H, I) asserted that the unfamiliar tasks made them think more or think for themselves. For instance, student C said about a conjecturing task where students were encouraged to predict formulas for higher derivatives of certain functions:

> You have to think about it and then, it's not actually a procedure, it's about you analysing the pattern and stuff.

One of the analyzing reasoning tasks designed for this project asked students to determine whether or not an argument was a satisfactory proof of a statement, and if not to indicate all errors or omissions in the reasoning. Student B described how working on this task prompted her to ask herself questions:

> Ahm, because it makes you analyze the proof and ask yourself questions like why you do things like that. Whereas if you were just given the proof, the correct one, you just take it for granted that that was correct.

Student H spoke about exploring his example space when working on an example generation question which involved finding rational functions with specific limiting behaviors (question A of Breen et al. (2016)):

> But you really have to think more about these and understand the concepts and the different – ahm - possible solutions that may be there and why one solution isn't going to work.

Some of the students (A, B, C) also mentioned that, when approaching an unfamiliar task, they refer to the basic definitions or theorems on the course and think about what they know (in relation to the task) and how they can apply it. We have seen above that Student A described undertaking an example generation exercise by breaking it down or taking it step-by-step but also "drawing on other things" that she knew and bringing them together. Student C described using the following techniques (sometimes in combination) when confronted with various unfamiliar tasks: generating examples, examining different cases, generalising, working backwards, sketching a graph. In addition, two students also spoke of the benefit of unfamiliar tasks for assessing their own understanding of concepts.

Discussion

In the case study reported on here, we have explored students' views on the differences they have noticed between school and university mathematics, including the differences in study methods and in mathematics tasks themselves. We found that students concentrated on three areas; the emphasis on understanding at university, the need to be an independent learner, and learning through tasks. Previously, there has been little written on the subject of student views on STT, but we have seen that our interviewees are aware of the new expectations placed on them at university and in particular of the fact that they must now go beyond procedural fluency and pay attention to justification and the development of understanding. They themselves spoke about the need for more relational and conceptual understanding at university as well as more flexibility in thinking and in approaching mathematical problems. While it would be remiss to attempt to extrapolate the findings from a case study involving only 10 students, it is interesting to note that this awareness has not always been seen in other studies (Jablonka et al., 2017). Our interviewees often indicated their awareness of the change in expectations when speaking about tasks and only sometimes in the context of lectures. Indeed, when asked about differences in mathematics or in study methods, they almost always responded by referring to their experience with mathematical tasks. The move from an emphasis on procedural fluency at school to conceptual understanding at university seemed to be flagged to the students by the contrast in the types of tasks that they encountered. The students in our study indicated this by expressing the view that they had to think for themselves when working on tasks at university and understand underlying concepts rather than simply following procedures. This seems to accentuate the role played by tasks and suggests that careful attention should be paid to the selection and/or design of tasks for students.

For the students interviewed here the connection between study methods and working on problems seems very strong; recall that 8 of the 10 students spoke about tasks when asked about their study methods even before tasks were mentioned in the interview. If this is the way that students are likely to study, then mathematics instructors at university could influence their undergraduates' study, and thus their learning (Jonsson et al., 2014), by the types of tasks that they use. Departments might well be reluctant to make large-scale changes to their syllabus or teaching methods (and this is possibly why the survey carried out by Thomas et al. (2015) did not yield many suggestions for changes at university level); however, adapting the types of tasks that are used in first-year courses is feasible. Furthermore, if students engage primarily with procedural tasks only (as seen in Mac an Bhaird et al., 2017), then this might lead to a more difficult STT. Kajander and Lovric (2005) reported finding that most of their first-year undergraduates are fearful of new types of mathematical experience and often adopt a surface-learning approach. However, our data appears to suggest that, with the aid of the "unfamiliar" tasks assigned to them, our students were no longer adopting a surface-learning approach. For example, student H described how he constructed his own example space, and how he needed to draw on and develop his understanding of concepts to do this. Also, although they reported having struggled with the unfamiliar tasks initially, the students adapted to the new types of tasks and found them beneficial. Thus, the tasks appear to have allowed students to engage meaningfully with their mathematical difficulties and provided them with the space to overcome these difficulties as advocated by Hernandez-Martinez et al. (2011).

De Guzman et al. (1998) recommended a list of actions that university instructors could take to help alleviate students' transition problems. They warned mathematicians that students are unlikely to develop insight by working on computations alone and thus advised lecturers to design problems aimed at engaging students in the type of thinking valued. We aspired to follow that advice through the design of our tasks, and in doing so, to help students to bridge the "cultural gap" between expectations of mathematics at school and university. Gueudet (2008) agreed that first-year mathematics students are often unaccustomed to using sophisticated mathematical thinking skills and the type of reasoning required at university, while other authors have raised the question as to whether the teaching in first year of university permits progress in this direction (Maciejewski & Merchant, 2016). The first-year students we interviewed, however, seemed to be developing mathematical thinking skills such as making connections, exploring patterns, using definitions, etc. and spoke about using these new skills in the context of engagement with the tasks we had designed. For example, we have seen here that students have reported using pattern recognition in the context of a conjecturing task, making connections between different concepts in an example generation task, and evaluating statements in an analyzing reasoning task. Thus, while being mindful of the difficulties in generalising the findings from our case study, we propose that the tasks assigned to these students have not only afforded them the opportunity to develop conceptual understanding, but also raised their awareness of these requirements, thereby easing their transition to university practices. We also note that these tasks were incorporated within a first-year module, reducing the need for a separate bridging course as in Leviatan (2008).

Stein, Grover and Henningsen (1996) asserted that "students should not view mathematics as a static, bounded system of facts, concepts, and procedures to be absorbed but, rather, as a dynamic process" (p.456) involving them in purposeful activities to not only gather but also discover and create knowledge. As instructors wishing to help students navigate the STT, we are concerned not only with students' learning of mathematics but how they view the subject. We suggest that our case study points to a threefold role for tasks in the transition from school to university. Firstly, we have seen that tasks can be used to convey a particular view of mathematics and mathematical thinking to students: rather than focusing exclusively on procedural fluency, tasks can highlight different facets of mathematical proficiency (Kilpatrick et al., 2001) and foreground the importance of conceptual understanding if desired. Secondly, tasks can make new expectations clear to students through the mathematical habits of mind they invoke or invite students to engage in. In addition to that, the opportunities for students to develop these mathematical habits of mind are inherent in the tasks themselves. Towers et al. (2017) suggest that students' autobiographical accounts of their experiences learning mathematics have the potential to occasion advances in our knowledge of students' experiences learning mathematics and we have found this to be the case here in terms of the modules we were teaching.

Gueudet et al. (2016, p.27) call for more work on STT which focuses on opportunities for learning rather than studies of knowledge gaps or obstacles in the transition to university. The data presented here contends that the choice and design of tasks in introductory calculus courses are opportunities to help students make a successful transition to university-level mathematics. Moreover, although further data has not yet been systematically collected, the authors' experiences of using the same types of non-routine tasks for other mathematical topics (e.g., linear algebra, number theory, discrete mathematics) suggest that these opportunities for learning are not limited to the study of calculus. In fact, we recommend that unfamiliar tasks be incorporated in Mathematics modules taken by students throughout their programs to afford them opportunities to develop the mathematical thinking skills desired at university level. It has often been suggested that changes should be made at secondary school level (Thomas et al., 2015; Gueudet, 2016) leaving the approach to mathematics at university untouched. However, we, as university lecturers, have not only the opportunity but also the responsibility to help our students with the secondary-tertiary transition. Our contribution may be to inform ourselves of the past mathematical experiences and backgrounds of our students and then select or design tasks to best meet their needs.

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