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Original Article

Estimating the proportion of modern contraceptives supplied by the public and private sectors using a Bayesian hierarchical penalized spline model

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Abstract

Quantifying the public/private-sector supply of contraceptive methods within countries is vital for effective and sustainable family-planning delivery. However, many low- and middle-income countries quantify contraceptive supply using out-of-date Demographic Health Surveys. As an alternative, we propose using a Bayesian, hierarchical, penalized-spline model, with survey input, to produce annual estimates and projections of contraceptive supply-share outcomes. Our approach shares information across countries, accounts for survey observational errors and produces probabilistic projections informed by past changes in supply shares, as well as correlations between supply-share changes across different contraceptive methods. Results may be used to evaluate family-planning program effectiveness and stability.

Keywords: Bayesian, correlation, family planning, hierarchical, splines, time-series

1 Introduction

The Family Planning 2020 Initiative (FP2020) sets the target of engaging 120 million new users of modern contraception in 69 of the world's poorest countries by the year 2020. This goal has now been extended to 2030 in line with the UN sustainable development goals (Bremner et al., 2021; FP2030, 2021). Consequently, there is a heightened need for annual, reliable estimates of family planning (FP) indicators for each country involved in the initiative. One such indicator is the most recent source of contraceptive commodities accessed by FP users. This indicator can be used to highlight where FP users obtain their contraceptives from, as well as to evaluate FP program effectiveness and forecast future commodity procurement (Data for Impact, n.d.). The family planning market is most successful when clients have a variety of methods and sources to choose from (Shelton & Finkle, 2016). Therefore, annual estimates and projections of the contraceptive method supply share are of particular use to countries who are trying to engage the private sector to rebalance the roles of the public and private sectors (Data for Impact, n.d.), meet the needs of all women within a country and enhance efforts to meet family planning goals (Sustaining Health Outcomes through the Private Sector (SHOPS) Plus, 2016).

In addition to their role as a stand-alone FP indicator, estimates of the contraceptive method supply share can be utilized with FP service statistics in a corrective step to reduce the bias of estimated modern contraceptive use (EMUs) rates (Track20, 2020b). EMUs represent the proportion

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of women in a particular country using modern contraception based on FP service statistic data (Cahill et al., 2021). Service statistics data are routinely collected in connection with family planning service delivery. These data provide high geographic detail, are relatively inexpensive to collect but often do not include contributions from the private sector (Magnani et al., 2018). The private sector is used by women across all socio-economic groups to access their family planning methods (Chakraborty & Sprockett, 2018), with 37% to 39% of all family planning users obtaining modern contraceptives through the private sector (Campbell et al., 2015). Therefore, the private sector makes up a significant portion of the family planning market. Service statistics data that are missing the private sector's contribution must be scaled-up to reflect the complete contraceptive market (Avenir Health, 2020). This adjustment to the service statistics data has spurred the need for annual estimates of the proportion of modern contraceptives supplied by the public, private commercial medical and private other sectors. Presently, annual estimates and projections with uncertainty for these proportions are not available when they are required for now-casting family planning indicators, such as EMUs.

Demographic and Health Surveys (DHS) provide information on the breakdown of public and private sector contributions to the contraceptive supply chain in the form of the percentage of contraceptive methods that are sourced from each sector. In this study, we consider 30 countries involved in the FP2020 initiative that have DHS data available after 2012. Notably, 54% of the countries considered do not have survey observations of the supply share indicator beyond 2015. In the absence of recent survey observations, we can rely on statistical model-based estimates and projections to bridge the knowledge gap and provide the recent private sector contributions to family planning service delivery with uncertainty.

Our proposed model falls into the class of temporal models for multiple populations (TMMP) (Susmann et al., 2022). Estimating demographic and health indicators using temporal statistical models with area-level variation is a well-established practice. The foundational 1979 Fay–Herriot model, which first considered estimates of income over time for small places, is an area-level model where the input data is made up of smaller sub-divisions, such as counties (Fay & Herriot, 1979). As such, it too would also fall under the class of TMMP. The Fay–Herriot model began a trend of using temporal models for small area estimation (SAE). As time and computing power has increased, so too have the complexity of the TMMP we use today. While our model is not considered a small-area estimation model, by merit of whole countries being too large to be considered small areas and the DHS data being adequately powered to capture observations at the national level, the proposed approach has some similarities to SAE models proposed in previous studies. In 2009, Ugarte et al. (2009) combined SAE modelling approaches with penalized splines to analyse trends and forecast dwelling prices in nine Spanish neighbourhoods. Our proposed model also uses penalized splines to capture the complex temporal nature of the data. While in 2013, Marhuenda et al. (2013) took advantage of the geographic nature of Spanish EU-SILC data within a spatio-temporal Fay–Herriot model to produce poverty indicators for Spanish provinces. Similarly, in the proposed model, we take advantage of the geographic nature of the dataset through Bayesian hierarchical modelling.

In this article, we describe a Bayesian hierarchical penalized spline model that produces annual, country, and method-specific estimates of the proportion of modern contraceptives coming from the public and private sectors. The utility of our model is in the ability to produce short-term projections (with uncertainty) beyond the year of the most recent survey. The model accounts for across method correlations within public and private sector contributions to family planning supply and relies on information sharing across methods and countries via a hierarchical modelling structure. The use of splines for demographic and health indicator estimation is growing in popularity. In recent years, many international health organizations use splines for the estimation of key demographic, health, and family planning indicators. These include the estimation and projection of under-5 mortality for United Nations Children's Fund (UNICEF) (Sharrow et al., 2022) and the estimation of excess morality due to Covid-19 for the World Health Organisation (WHO) (Knutson et al., 2023). We describe our model using two components—a process model and a data model. In the process model, we seek to model the unobserved latent trends over time using a hierarchical systematic element and stochastic temporal smoothing splines. In the data model, we account for the standard errors of the DHS observations.

2 Data

2.1 Definitions and data sources

According to Hubacher and Trussell, a modern contraceptive method is defined as 'a product or medical procedure that interferes with reproduction from acts of sexual intercourse' (Hubacher & Trussell, 2015). Modern methods of contraception considered in this study are female sterilization, oral contraceptive pills (OC pills), implants (including Implanon, Jadelle, and Sino-implant), intra-uterine devices (IUD, including Copper-T 380-A IUD and LNG-IUS), and injectables [including Depo Provera (DMPA), Noristerat (NET-En), Lunelle, Sayana Press, and other injectables].

The proportion of a modern contraceptive method provided by the public sector is defined as the percent of a modern contraceptive method that is supplied by any public sector outlet relative to all modern methods supplied in each country at a given time. Conversely, the proportion of modern contraceptives provided by the private sector come from the private sector outlets (Percent of contraceptives available from the private sector - MEASURE Evaluation, 2017). The public sectors include contraceptives supplied by government health facilities and home/community deliveries. Any supplies that come from sources outside the public sector can be defined as coming from the private sector. These include commercial, for-profit, and non-profit organizations (Jain et al., 2016). We consider three sector categories: public, private commercial medical and other private, where the private commercial medical and other private make up the total private sector. In this study, the outcome of interest for a given contraceptive method is the market share breakdown using the three sector categories.

A database of the public and private sector breakdown of modern contraceptive supply with their associated standard errors was created using data from the DHS (The DHS program, n.d.). The DHS use a two-stage sampling design, using census information as the sampling frame. First, the country of interest is stratified to have strata as homogenous as possible. This homogeneity minimizes the resulting sampling errors of the survey. Within each stratum, the census enumeration areas (EAs) form clusters. The households of each cluster are listed, and a fixed number of households within the selected cluster are chosen by systematic sampling. The sample selection process uses weights to address the any differences in probability of selection. This corrects any over- or under-sampling of different clusters during sample selection. The DHS standard model questionnaires are then utilized to collect data from the selected households (Croft et al., 2018). The cohort of interest for this study was women aged 15–49 years old who are taking a modern contraceptive method. The variable of interest was the current source of their modern contraceptive methods.

In this study, we consider countries involved in the FP2030 initiative. Table 1 lists the thirty countries used in this study. The total number of surveys carried out and the year of the most recent survey is listed for each country. There are five intermediate world regions considered in this study. Western Africa has the largest number of countries in the dataset with 12 countries, while South-Eastern Asia is the smallest with two countries in the dataset. Just under half of the countries included have survey data available after 2015, highlighting the need for annual up-to-date estimates of the contraceptive supply shares.

We calculated proportions and associated standard errors from DHS micro-level data. In line with STATcompiler (ICF, 2012) convention, we filtered the DHS survey microdata to only include observations for a given method m, at time t in country c where at least one sector (public, private commercial, or private other) has a sample size of at least 20 women. This removes sets of observations with large uncertainty due to small sample sizes. Sampling errors were calculated while accounting for the sampling design using a Taylor series linearization method to approximate the standard error of the calculated proportions (Binder, 1983; Lavrakas, 2013).

The final database included 1461 observations for 15 method-sector combinations. An example of the data is shown in Figure 1 where the proportion of modern contraceptives coming from each sector are plotted over time for Zimbabwe. Vertical bars indicate standard errors associated with each observation. Zimbabwe is considered a data-rich country as there are between three and five surveys available for each method. For all five methods, the public sector has the largest market share. However, in recent years, there is an increase in the proportion of OC pills and female sterilization supplied by the private sectors. In Figure 1b, the observed method mix for each of the five

Table 1. Summary of DHS microdata used during the study including the United Nation Statistics Division (UNSD) intermediate world region names, country names, the number of DHS surveys per country available and the year of the most recent DHS survey available. Just over 46% of countries have data available after 2015

UNSD intermediate world regions	Country	Total number of surveys	Recent Survey year
Southern Asia	Afghanistan	1	2015
Western Africa	Benin	5	2017
Western Africa	Burkina Faso	4	2010
Middle Africa	Cameroon	5	2018
Middle Africa	Congo	2	2011
Middle Africa	Congo Democratic Republic	2	2013
Western Africa	Cote d'Ivoire	3	2011
Eastern Africa	Ethiopia	5	2019
Western Africa	Ghana	5	2014
Western Africa	Guinea	4	2018
Southern Asia	India	4	2015
Eastern Africa	Kenya	5	2014
Western Africa	Liberia	3	2019
Eastern Africa	Madagascar	4	2008
Eastern Africa	Malawi	5	2015
Western Africa	Mali	5	2018
Eastern Africa	Mozambique	3	2011
South-Eastern Asia	Myanmar	1	2015
Southern Asia	Nepal	5	2016
Western Africa	Niger	4	2012
Western Africa	Nigeria	5	2018
Southern Asia	Pakistan	4	2017
South-Eastern Asia	Philippines	6	2017
Eastern Africa	Rwanda	6	2019
Western Africa	Senegal	11	2019
Western Africa	Sierra Leone	3	2019
Eastern Africa	Tanzania	6	2015
Western Africa	Togo	2	2013
Eastern Africa	Uganda	5	2016
Eastern Africa	Zimbabwe	5	2015

contraceptive methods is shown for each DHS survey year. We can see that as OC pills declines in the public sector supply share, so too does its popularity in the method mix for Zimbabwe. In the database, standard errors range from 0.015% to 18.2% points. For further details on how the standard errors were calculated and a summary of these errors, please see the online supplementary material, Appendix section 1.

3 Methods

The outcome of interest is the components of a compositional vector $\phi_{c,t,m} = (\phi_{c,t,m,1}, \phi_{c,t,m,2}, \phi_{c,t,m,3})$ denoting $\phi_{c,t,m,s}$ the proportion supplied by the public sector ($s = 1$), the private commercial

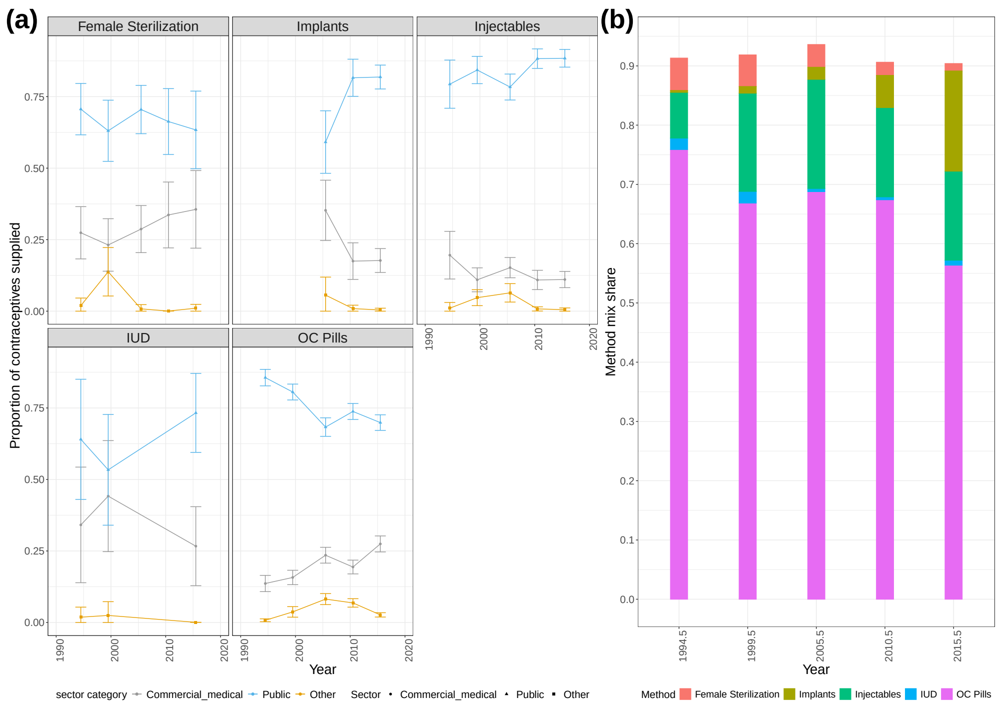

Figure 1. (a) Contraceptive supply in Zimbabwe. Observed proportions of the modern contraceptives supplied by each sector over time for Zimbabwe according to the DHS microdata. The sectors are shown by blue triangles for public, grey circles for commercial medical and gold squares for other private. The standard errors associated with each observation are used to create the vertical error bars. Zimbabwe is one of the 30 countries included in this study. (b) Method mix in Zimbabwe. The observed popularity for the five contraceptive methods of interest in Zimbabwe according to the DHS microdata. Each method is represented by a different colour. The year of the DHS survey is listed on the x-axis. On the y-axis, the height of each bar captures the proportion of modern contraceptive users utilizing each method in a given survey year.

medical sector ($s = 2$) and the other private sector ($s = 3$) of modern contraceptive method m, at time t, in country c.

We break the model specification into two parts, the process model that captures the underlying dynamics of the outcome of interest and the data model which links the observed data to the process model. In the process model, we model the logit-transformed proportion of the public-sector supply share and the ratio of private commercial medical to total private sector supply share using a Bayesian hierarchical penalized spline model. The logit-transformed data are linked to the process in the data model via a Normal distribution.

3.1 The process model

We begin by defining a regression model for $\phi_{c,t,m,1}$. Logit-transformed proportion $\psi_{c,t,m,1} = \text{logit}(\phi_{c,t,m,1})$ is modelled with a penalized basis-spline (P-spline) regression model:

$$\text{logit}(\phi_{c,t,m,1}) = \psi_{c,t,m,1} = \sum\nolimits_{k=1}^{K} \beta_{c,m,1,k} B_{c,k}(t), \tag{1}$$

where $B_{c,k}(t)$ refers to the kth basis function evaluated in country c, at time t and $\beta_{c,m,1,k}$ is the kth spline coefficient for the public sector supply ($s = 1$) of method m in country c. The basis functions B(t) are constructed using cubic splines. The basis is fitted over the years 1990 to 2025. We align the knot placement of the basis splines with the most recent survey in each country. As the most recent survey year varies by country, the basis splines $B_{c,k}(t)$ also vary by country. Between countries, the order of the splines and the number of knots in each set of basis functions remains the

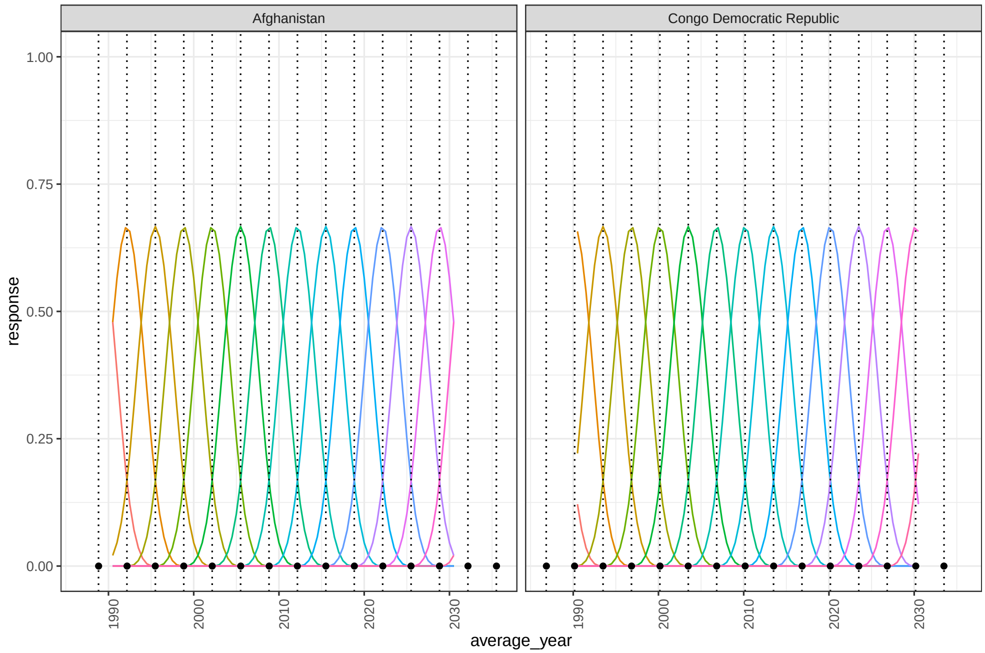

Figure 2. The set of basis functions plotted over time that are used to fit Afghanistan and Democratic Republic of Congo (DRC). The most recent survey for Afghanistan occurs in 2015 and the most recent survey in DRC occurs in 2013. Thus, the knot point locations, depicted as black dots with vertical dashed lines differ between the two countries.

same, but the location of these knot points may differ. An example of the country-specific basis functions are shown in Figure 2 where the basis are shown as a set of coloured curves plotted over time with the locations of the knots denoted in black dots with dash vertical lines along the x-axis.

Similarly, we model the latent variable, $\psi_{c,t,m,2}$, to capture the logit-transformed ratio of the private commercial medical supply share to the total private sector share. The model is specified as follows:

$$\text{logit}\left(\frac{\phi_{c,t,m,2}}{1-\phi_{c,t,m,1}}\right) = \psi_{c,t,m,2} = \sum\nolimits_{k=1}^{K} \beta_{c,m,2,k} B_{c,k}(t), \tag{2}$$

where $\beta_{c,m,2,k}$ is the kth spline coefficient for the private medical sector supply of method m in country c.

From the latent variable vector, $\boldsymbol{\psi}_{c,t,m}$, it is possible to infer the compositional vector $\boldsymbol{\phi}_{c,t,m}$.

$$\phi_{c,t,m,1} = \text{logit}^{-1}(\psi_{c,t,m,1}), \tag{3}$$

$$\phi_{c,t,m,2} = (1-\phi_{c,t,m,1})\text{logit}^{-1}(\psi_{c,t,m,2}), \tag{4}$$

$$\phi_{c,t,m,3} = 1-(\phi_{c,t,m,1}+\phi_{c,t,m,2}). \tag{5}$$

3.1.1 Parametrization of the spline regression coefficients

To estimate the spline coefficients, $\beta_{c,m,s,k}$, we use a random walk model of order 1 on spline coefficients such that the first-order differences, $\boldsymbol{\delta}_{c,m,s}$, are penalized. This model choice is motivated by prior work that used constant projections past the most recent data point (Track20, 2020a, 2020c).

The $\boldsymbol{\delta}_{c,m,s}$ vector is of length $K-1$, where K is the number of knots used in the set of basis functions $B(t)$. It is defined as

$$\boldsymbol{\delta}_{c,m,s} = (\beta_{c,m,s,2} - \beta_{c,m,s,1},\ \beta_{c,m,s,3} - \beta_{c,m,s,2},\ \ldots,\ \beta_{c,m,K} - \beta_{c,m,K-1}). \tag{6}$$

We assume that in country c, method m and sector s, the value of spline coefficient at knot point k^*, aligning with the year t^* where the most recent survey occurs, is $\alpha_{c,m,s}$. By doing this, we are assuming that the $\alpha_{c,m,s}$ parameter will act as the spline coefficient for the reference spline at k^*. We are then able to calculate the spline coefficients from the reference knot (k^*) using the estimated $\boldsymbol{\delta}_{c,m,s}$.

$$\beta_{c,m,s,k} = \begin{cases} \alpha_{c,m,s} & k = k^*, \\ \beta_{c,m,s,k+1} - \delta_{c,m,s,k} & k < k^*, \\ \beta_{c,m,s,k-1} + \delta_{c,m,s,k-1} & k > k^*. \end{cases} \tag{7}$$

3.1.2 Hierarchical estimation of the intercept

The parameter $\alpha_{c,m,s}$ acts similarly to an intercept term as it forms the baseline level from which forward and backward projections are based off. A change in $\alpha_{c,m,s}$ can lead to a systematic change on the estimated set of supply-share levels for a particular country. $\alpha_{c,m,s}$ is estimated hierarchically to allow parameter estimates to benefit from cross-method and then cross-country information sharing for each sector. The hierarchical distributions are given by:

$$\alpha_{c,m,s} \mid \theta_{r[c],m,s},\ \sigma^2_{\alpha,s} \sim N(\theta_{r[c],m,s},\ \sigma^2_{\alpha,s}),$$

$$\theta_{r,m,s} \mid \theta_{w,m,s},\ \sigma^2_{\theta,s} \sim N(\theta_{w,m,s},\ \sigma^2_{\theta,s}),$$

where, the subscript $r[c]$ denotes the region which country c belongs to. A parameter has the subscript w when it refers to the overall world-level parameter, where information from all regions is pooled for the parameter estimation.

Such that $\alpha_{c,m,s}$ is distributed around a sector, method, region-specific mean ($\theta_{r[c],m,s}$) allowing for a cross-method variance ($\sigma^2_{\alpha,s}$) for each sector. The geographic regions used to group countries together are the UNSD intermediate world regions (United Nations Statistics Division, 2022). The sector, method, region-specific means are distributed around a sector, method-specific world mean ($\theta_{w,m,s}$) allowing for a cross-regional variance ($\sigma^2_{\theta,s}$) within each sector. We chose this hierarchical setup as the private sector plays an important role in the supply of contraceptive products, such as OC pill and injectables, whereas the public sector tends to provide higher proportions of clinical contraceptive methods, including female sterilization, IUDs, and implants (Ayad et al., 1994; Ugaz et al., 2015). Thus, it made sense for the data to split the prior and hyper-prior mean parameters by method-type to respect these observed differences.

Finally, the overall sector-specific mean is given a vague Normal prior and the cross-method and cross-country standard deviation parameters are given half-Cauchy distributions (Gelman, 2006; Polson & Scott, 2012):

$$\theta_{w,m,s} \sim N(0, 100),$$

$$\sigma_{\alpha,s} \sim C^+(0, 1),$$

$$\sigma_{\theta,s} \sim C^+(0, 1).$$

3.1.3 Specification of the first-order differences between spline regression coefficients, δ

A multivariate normal prior centred on 0 was assigned to the vector of length M of first-order differences of the spline coefficients, $\boldsymbol{\delta}_{c,1:M,s,h}$, for all methods supplied by sector s in country c, at first-order difference h,

$$\boldsymbol{\delta}_{c,1:M,s,h} \mid \Sigma_{\delta,s} \sim MVN(\mathbf{0}, \Sigma_{\delta,s}),$$

where, $1 \le h \le K-1$ and

$$\Sigma_{\delta_s} = \begin{bmatrix} \sigma^2_{\delta_{1,s}} & \hat{\rho}_{1,2,s}\sigma_{\delta_{1,s}}\sigma_{\delta_{2,s}} & \cdots & \hat{\rho}_{1,M,s}\sigma_{\delta_{1,s}}\sigma_{\delta_{M,s}} \\ \hat{\rho}_{2,1,s}\sigma_{\delta_{2,s}}\sigma_{\delta_{1,s}} & \sigma^2_{\delta_{2,s}} & \cdots & \hat{\rho}_{2,M,s}\sigma_{\delta_{2,s}}\sigma_{\delta_{M,s}} \\ \cdot & \cdot & \cdots & \cdot \\ \hat{\rho}_{M,1,s}\sigma_{\delta_{M,s}}\sigma_{\delta_{1,s}} & \cdot & \cdots & \sigma^2_{\delta_{M,s}} \end{bmatrix}. \tag{8}$$

The variance terms $\sigma^2_{\delta_{m,s}}$ for $m = 1, .., M$ and $s = 1, 2$ are method-sector specific smoothness parameters that act as a penalization parameter on the first order differences of the spline coefficients. As $\sigma^2_{\delta_{m,s}}$ tends towards 0, deviations away from the sector, method, country-specific mean go to 0. Within each sector, we assume that the first-order differences are not independent across methods. This dependency is captured via the covariance $\hat{\rho}_{i,j,s}\sigma_{\delta_{i,s}}\sigma_{\delta_{j,s}}$ for $s = 1, 2, \quad i = 1, .., M$ and $j = 1, .., M$ and where $\hat{\rho}_{i,j,s}$ is the estimated correlation between first-order differences of spline coefficients for method i and method j supplied by sector s. Σ_δ plays a pivot role in the estimation process. It influences both the flexibility of the splines to move away from the most recently observed survey level, and the strength of the relationships that occur between the rates of change in contraceptive method supply share estimates. The deviation terms of the Σ_{δ_s} matrix are given vague uniform distributions,

$$\sigma_{\delta_{m,s}} \sim \text{Uniform}(0, 10).$$

The correlation terms of the covariance matrix, $\hat{\rho}_{i,j}$, were estimated using a maximum *a posteriori* estimator for the correlation matrix as described in Azose & Raftery (2018). This approach involves fitting a model where the covariance terms in $\Sigma_{\delta,s}$ are set equal to zero. The resulting estimates are then used to estimate the correlation between methods across time and all countries. Specifically, for sector s, the correlation between method i and method j is calculated as follows:

$$\tilde{\rho}_{i,j,s} = \frac{\sum_{c=1}^{C}\sum_{h=1}^{K-1}\tilde{\delta}_{c,i,s,h}\tilde{\delta}_{c,j,s,h}}{\sqrt{\sum_{c=1}^{C}\sum_{h=1}^{K-1}\tilde{\delta}_{c,i,s,h}{}^2}\sqrt{\sum_{c=1}^{C}\sum_{h=1}^{K-1}\tilde{\delta}_{c,j,s,h}{}^2}}, \tag{9}$$

where $\tilde{\rho}_{i,j,s}$ is the posterior sample of estimated correlations between method i and method j in sector s. $\tilde{\delta}_{c,m,s,h}$ are posterior samples of the estimated first order differences of the spline coefficients estimated in the zero-covariance model, C represents the total number of countries involved in the study, and h represents the number of differences ($h = K-1$) between the spline coefficients.

To ensure that the correlation matrix, ρ, is positive definite, we let:

$$\hat{\rho}_{i,j,s} = \bar{\rho}_{i,j,s},$$

where $\bar{\rho}_{i,j,s}$ is the posterior mean of $\tilde{\rho}_{i,j,s}$.

Figure 3 shows the heat map of the estimated correlations between the first-order differences in the spline coefficients for the five contraceptive methods studied for the public sector and ratio of commercial medical sector to total private sector. In general, the public sector has weak positive correlations across all method combinations (Figure 3). In both sectors, OC pills tend to have the strongest correlations with the other methods. The relationships between the other methods tend to be close to zero. In the public sector, injectables and OC pills have a weak-positive correlation at 0.17. This is the strongest average correlation calculated in the public sector. In the private sector, the OC pills-injectables relationship is again the strongest at 0.13 (Figure 3a; 0.17. Figure 3b; 0.13). In both instances, OC pills have a positive correlation with injectables. This implies that the rates of change in the supply of OC pills tend to increase jointly with the rates of change in the supply of injectables across the public and private sectors.

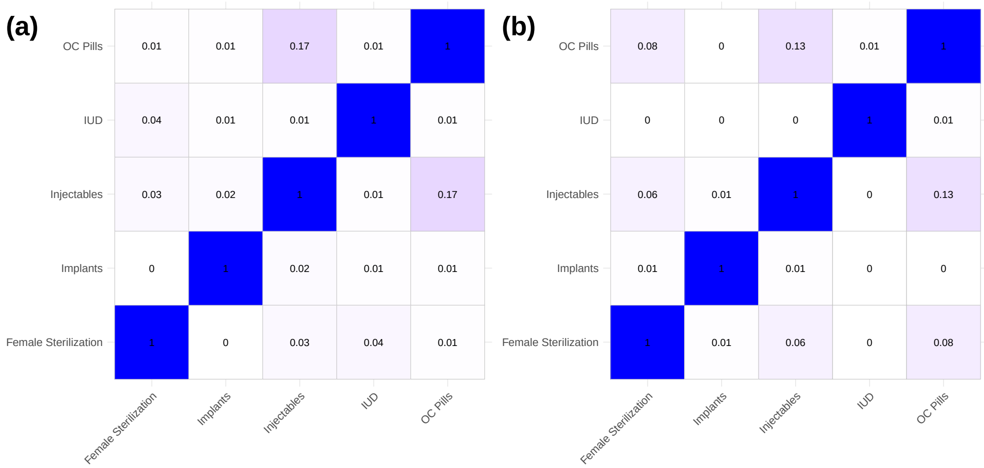

Figure 3. A heatmap of the estimated correlations between the changes in contraceptive supply of contraceptive methods (on the logit scale) present in the public (a) and private (b) sectors. The methods include female sterilization, oral contraceptive pills (OC pills) intrauterine devices (IUD), injectables, and implants. Correlation is measured on a scale from −1 (strong negative correlation) to +1 (strong positive correlation). The colours represent the strength of the correlation, blue indicates positive correlations, and red indicates negative correlations. The estimated correlation is shown in each entry of the correlation matrix.

3.2 The data model

The likelihood of the logit-transformed observed data, logit($\mathbf{Y}_{\mathrm{c,t,m,s=1:2}}$), the vector of observed logit-transformed proportions of modern conceptive method supplied by the public and commercial medical sectors ($s = 1$ and $s = 2$) for method m, at time t in country c, are modelled using Multivariate Normal distributions such that,

$$\mathrm{logit}(\boldsymbol{Y}_{c,t,m,s=1:2}) \mid \mathrm{logit}(\boldsymbol{\phi}_{c,t,m,s=1:2}) \;\sim\; \mathrm{MVN}(\mathrm{logit}(\boldsymbol{\phi}_{c,t,m,s=1:2}),\ \Sigma_{Y_{c,t,m}}),$$

where, $\mathrm{logit}(\boldsymbol{\phi}_{c,t,m,s=1:2})$ is a vector of the estimated logit-transformed public ($s = 1$) and private commercial medical ($s = 2$) supply proportions for the country c, time point t, method m, associated with vector of corresponding observations $\boldsymbol{Y}_{c,t,m,s=1:2}$. The variance–covariance matrix, $\Sigma_{Y_{c,t,m}}$, utilizes the standard errors (SE) and covariances calculated using the DHS survey microdata associated with the logit-transformed observations, logit $(\boldsymbol{Y}_{c,t,m,s=1:2})$. The calculated variance–covariance matrix is transformed onto the logit scale using the delta-method.

3.2.1 *The delta method for the transformation of variance terms on the logit scale*

To transform the variance–covariance matrix onto the logit scale, we use the multivariate delta method such that,

$$g(\Sigma_{Y_{c,t,m}}) = g'(Y_{c,t,m,s=1:2})\,\widehat{\mathrm{SE}}^2_{Y_{c,t,m,s=1:2}}\,g'(Y_{c,t,m,s=1:2}),$$

where, $g(\Sigma_{Y_{c,t,m}})$ is the variance–covariance matrix associated with the logit-transformed public ($s = 1$) and private commercial medical ($s = 2$) supply proportions for the country c, time point t, method m. $g'(Y_{c,t,m,s=1:2})$ is the derivative of the associated logit-transformed proportions, where $g'(Y_{c,t,m,s}) = \frac{1}{Y_{c,t,m,s}(1-Y_{c,t,m,s})}$. $\widehat{\mathrm{SE}}^2_{Y_{c,t,m,s=1:2}}$ are the estimated corresponding public ($s = 1$) and private commercial medical ($s = 2$) supply proportions for the country c, time point t, method m (Linzer & Lewis, n.d.).

3.3 Computation

We used R and JAGS (Just Another Gibbs Sampler) to fit the model. JAGS uses Markov Chain Monte Carlo (MCMC) algorithm to produce model estimates for Bayesian Hierarchical models

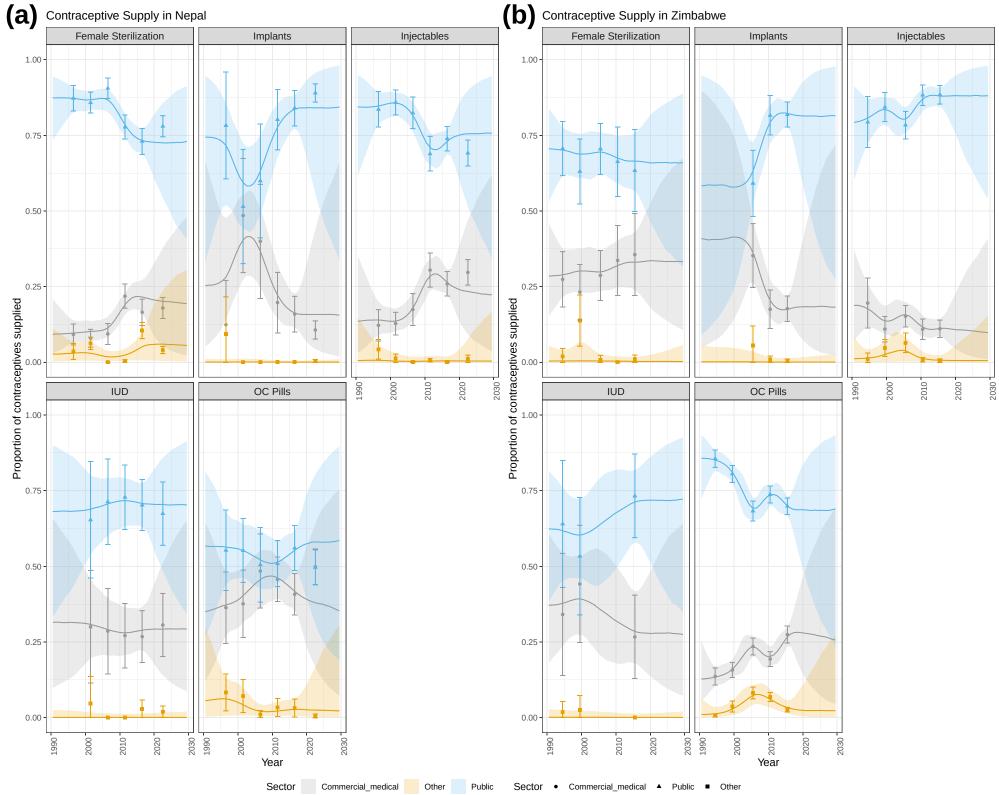

Figure 4. The projections for the proportion of modern contraceptives supplied by each sector in Nepal (a) and Zimbabwe (b). The median estimates are shown by the continuous line while the 95% credible interval is marked by shaded coloured areas. The DHS data point is signified by a point on the graph with error bars displaying the standard error associated with each observation. The sectors are shown by blue triangles for public, grey circles for commercial medical and gold squares for other private.

(Hornik et al., 2003). The number of iterations used was 200,000. The burn-in period was set to 20,000. The samples were thinned to every 90th sample. Consequently, the posterior distribution is made up of 2,000 samples. To assess convergence, we considered the convergence diagnostic $\hat{R}$ (Vehtari et al., 2021), as well as the individual parameter trace plots. Further details of the computation can be found in the online supplementary material Appendix section 3.

4 Country estimates

We produced estimates and projections of the public and private sector contraceptive supply share from 1990 to 2023 for five contraceptive methods in 30 countries involved in FP2030. Results are presented here for a subset of countries. Results for all countries are included in the online supplementary material, Appendix Figures A.1–A.25. We show results for five countries to illustrate the model's response to varying amounts of data available for each country in the dataset. These countries include Afghanistan, Democratic Republic of Congo (DRC), Mozambique, Nepal, and Zimbabwe. Afghanistan has at most a single survey data point for each method and sector (Table 1). The DRC and Mozambique have at most three survey data points for each method and sector (Table 1). Finally, Nepal and Zimbabwe have at least three survey data points for each method and sector (Table 1).

To begin, we will look at our model estimates in a data-rich setting. Historically, Nepal appears to distribute the pill supply share equally across the public and private sectors (Figure 4a). In contrast to this, Zimbabwe traditionally supplied most OC pills through the public sector (Figure 4b). However, in more recent years, Zimbabwe's private medical and private other sectors have been

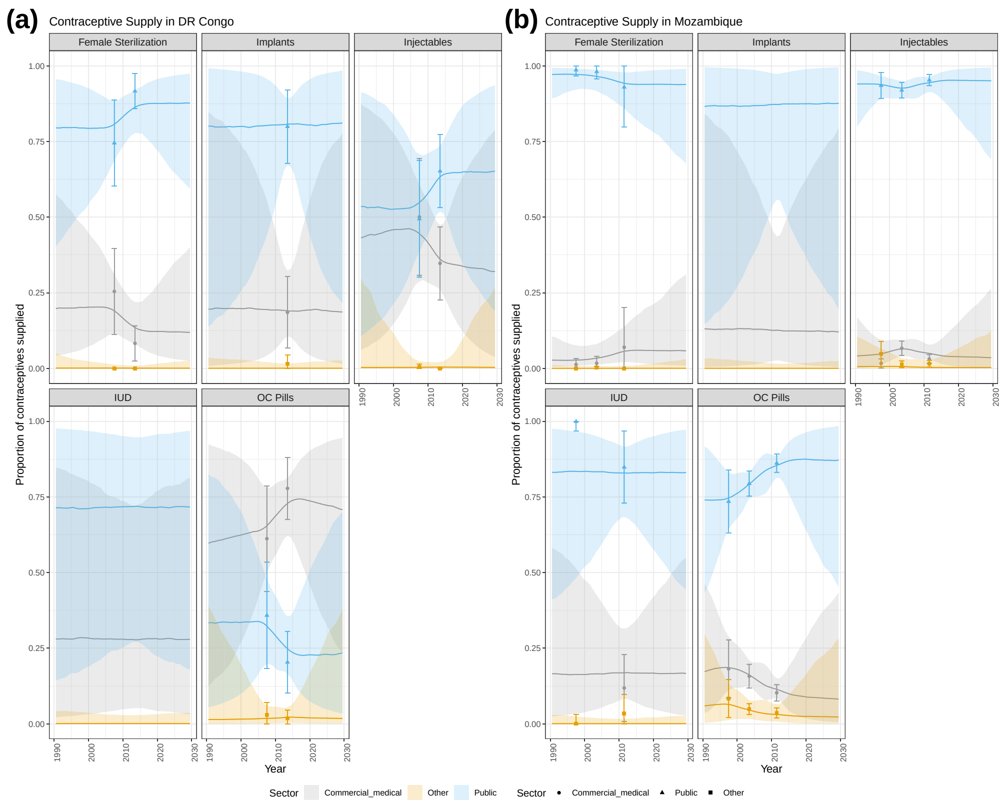

Figure 5. The projections for the proportion of modern contraceptives supplied by each sector in Democratic Republic of Congo (DR Congo) (a) and Mozambique (b). The median estimates are shown by the continuous line while the 95% credible interval is marked by shaded coloured areas. The DHS data point is signified by a point on the graph with error bars displaying the standard error associated with each observation. The sectors are shown by blue triangles for public, grey circles for commercial medical and gold squares for other private.

slowly growing in popularity. For other methods the public sector dominates the supply in both countries; however in Nepal, the public sector share of IUDs does appear to be decreasing slowly over time. In both Nepal and Zimbabwe, projections beyond the most recent survey are penalized to be as smooth as possible. The models are utilizing the most recently observed level ($\alpha_{c,m,s}$) as their baseline for extending estimates out into the future.

In the DRC, except for the OC pill, the public sector provides the largest share of all contraceptive methods (based on median estimates; Figure 5a). More recently, the proportion of OC pill supplied by the private sector seems to be declining (since ~2014). The public sector supply share is increasing over time for female sterilization. Injectables, which in the past projections appeared steady in the public sector supply share, shows an increase in supply from the public sector in more recent years (since ~2008). The private commercial medical sector injectables trend is being impacted by the across-method correlation structure imposed in the model. Changes over time in injectables supply from the private sector has a positive correlation with changes in OC pills (correlation = 0.13, Figure 3b). The behaviour of the OC pill supply share can be explained by the hierarchical structure of the fixed effects term in the model. In the absence of historical data, OC pill estimates begin at the sector, region, method-specific intercept, which is informed by the most recent supply-share level observed, and then the data begin to inform the estimates in more recent years (post-2014 approximately). The extrapolations into the future are steady as the spline coefficients used during estimation are based on the penalized first-order differences which are expected to result in a zero-unit rate of change between coefficients (equation 9).

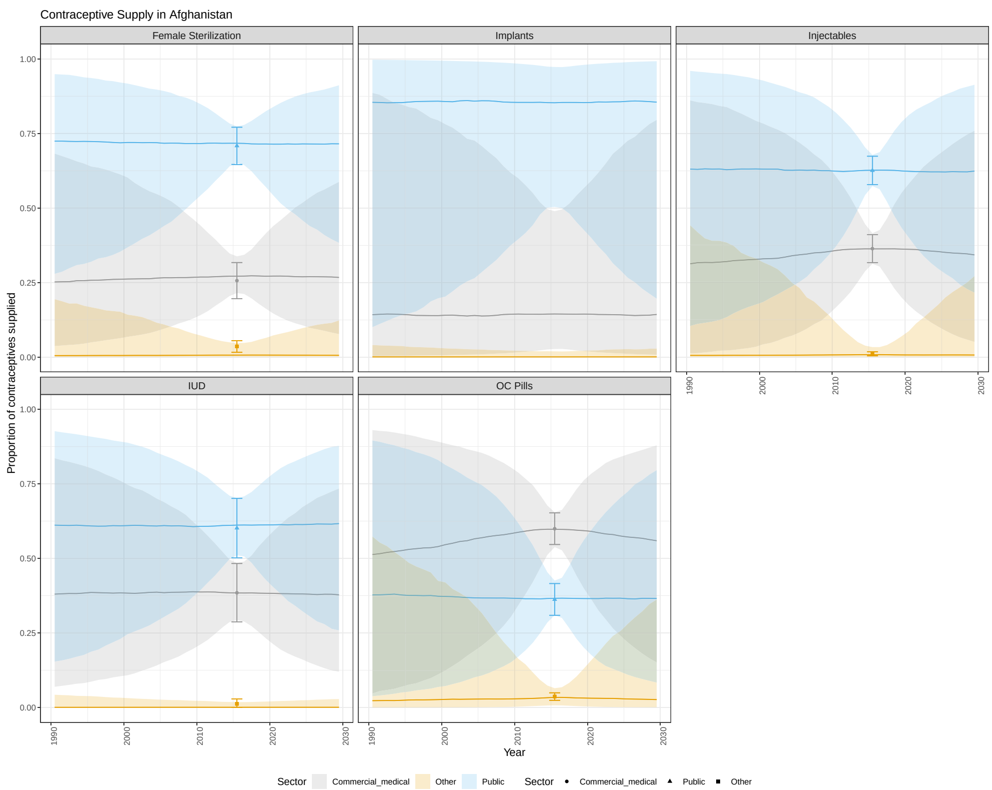

Figure 6. Contraceptive supply in Afghanistan. The projections for the proportion of modern contraceptives supplied by each sector in Afghanistan. The median estimates are shown by the continuous line while the 95% credible interval is marked by shaded coloured areas. The DHS data point is signified by a point on the graph with error bars displaying the standard error associated with each observation. The sectors are shown by blue triangles for public, grey circles for commercial medical and gold squares for other private.

In Mozambique, the public sector dominates the supply share for all methods (based on median estimates; Figure 5b). The public sector supply share for all methods is mostly steady over time. We see an increase in the public supply share of injectables and OC pills between 1995 and 2010 approximately. For implants, the national estimates are tending towards the sector-specific average of most recently observed supply share levels across the larger geographic region in the absence of data.

Lastly in Afghanistan (Figure 6), there is only one survey available. For all methods except for the OC pill, the public sector provides the largest supply share over the study period (based on median estimates). The supply share median estimates and uncertainty intervals for female sterilization, injectables, IUD and OC pills are influenced by single data points. In the absence of any data for implants, the estimates are centred on the sector, region, method-specific average of the most recently observed supply share levels.

Table 2 shows a summary of the contraceptive supply share for each method in 2023. Overall, the public sector supplies the highest proportion of each contraceptive method, and the private other sector supplies the smallest proportion. Implant supply is dominated by the public sector (approximately 83%). This may be since they require medical assistance for insertion and therefore require medically trained providers. This makes them more expensive to provide and less likely to be accessed through private means (Weinberger & Callahan, 2017). In contrast, the supply of OC pills is close to a 50:50% point split between the public and private sectors. OC pills can be supplied to users without medical assistance making them more popular in the private sector (Bradley & Shiras, 2020).

Table 2. Summary of contraceptive supply share proportions across all countries and in 2023 for each method and sector. The mean estimate for the percentage point supplied and the associated standard deviation (SD) are listed

	Private Commercial medical		Private other		Public	
	Mean (%)	SD (%)	Mean (%)	SD (%)	Mean (%)	SD (%)
Female sterilization	15.4	9.0	3.80	3.0	81.0	11.0
Implants	15.9	1.4	0.9	1.0	83.2	12.0
Injectables	21.9	14.0	2.6	2.0	75.4	15.0
IUD	25.7	12.0	0.7	0.01	73.7	14.0
OC pills	40.4	19.0	13.0	10.0	47.5	20.0

5 Discussion

In this paper, we proposed and validated a Bayesian hierarchical model to estimate the country and method specific changes in the contraceptive supply share over time with (sometimes) limited amounts of DHS survey data. The modelling framework uses penalized splines to capture the evolution of the contraceptive method supply from the public and private sectors while imposing a correlation structure to capture correlations between changes in the supply share. The penalized splines provide a flexible fit to the data without overfitting. The model structure imposes a hierarchy such that the expected sector, country, method-specific supply shares are informed based on the geographical relationships of the countries to promote information sharing across countries where smaller amounts of data are present. The model will produce estimates within the period of available survey data as well as projections beyond the most recent data point.

The model was used to estimate and project the contraceptive supply share from the public, private commercial medical and private other sectors for 30 focus countries of the FP2030 initiative, where relevant data were available. Case study examples illustrated the strength and flexibility of the modelling approach in its ability to capture the evolving nature of the contraceptive supply share over time. The hierarchical setup of the model and the imposed cross-method correlation structure allows the model to produce informed estimates (with uncertainty) even in cases where very limited data are available. Based on a series of validation measures, the proposed model is well calibrated and outperforms various model alternatives (see online supplementary material, Appendix for full details). To show the utility of the proposed approach for producing short-term projections, we use model validations where we leave out the most recent data points and compare the predictive performance of our proposed model to that of alternative simpler approaches, and check that forecasts are well calibrated. Through the validation exercises, we show that our model provides improved point estimates (as measured in terms of RMSE for predicting left-out observations) and is well calibrated as compared to alternative models.

An application of our model will be in the calculation of estimated modern use (EMU), a proxy for modern contraceptive prevalence derived from service statistics and frequently used in the Family Planning Estimation Model (Cahill et al., 2018, 2021). A description of how to calculate EMUs can be found in the online supplementary material, Appendix, section ‘Calculating Estimated Modern Use’. Service statistics data include the number of family planning commodities distributed to clients, commodities distributed to health facilities, family planning facility visits, or family planning facility users. Frequently, the raw data reported in a given country does not include the private sector contribution. As such, the raw data may not be fully representative of the country’s whole contraceptive market. Currently, to address this issue when calculating EMUs (Track20, 2020b), the most recent DHS survey estimate of the country’s contraceptive supply share is used to provide a breakdown of the contraceptive market and scale up the raw data accordingly. We instead propose to use our model-based estimates in this important supply-adjustment step of the EMU calculation and to propagate the posterior uncertainty for the contraceptive supply share of sector s, method m, at time t in country c into the EMU calculations. To maximize the engagement of Family Planning community with this model and its application to EMU calculation, we have carefully considered expert opinion and incorporated the current best practice approach to modern contraceptive supply share extrapolations (Track20, 2020a, 2020c). This is evident in the smooth projections of the model beyond the most recent data points (Figure 4). The smooth projections into the future promote the seamless fusion of the current EMU calculation process and our model-based estimates to produce improved EMUs with uncertainty.

Another potential application of these model estimates could be used in evaluating the security of contraceptive method supply chains. It is well-established that a total-market approach (TMA) is key to both the longevity and sustainability of the family planning market and in achieving equity among family planning users (Bradley & Shiras, 2020). Understanding and quantifying the supply shares of the private and public sectors in the contraceptive market contributes to a TMA approach to supply chain management (Moazzam, 2015). Presently, the latest DHS survey estimates are used to provide the proportion of modern contraceptives supplied by each sector. However, this relies on out-of-date information in relation the private sector supply in the absence recent surveys. The model estimates presented here can provide an essential measure of the private sector's role in modern contraceptive method supplies at a national level. Using the model estimates, it is possible to predict and reflect on overall contraceptive market supply trends. This feeds into indicator 1, 'Generating intelligence', and indicator 6, 'Accountability and transparency in health markets', of the TMA framework which will help to enable evidence-based strategies for resource management within the family planning market and improve the sustainability of future family planning initiatives (Cisek et al., 2019).

The estimates and projections presented here provide important and relevant information on the contraceptive supply share over time, at a national level. The data model allows for the incorporation of survey sampling errors to be included in the modelling process. The use of penalized-splines allows for data driven, flexible model-based estimates and the correlation structure imposed in the estimation of the splines allows the model to draw strength and information from the relationships between supply-shares of different contraceptive methods. Given that data are not necessarily available for all country-periods of interest, we use a smoothing model on the spline coefficients, such that differences between neighbouring spline coefficients are penalized. A similar approach to spline coefficient estimation can be found in Wang et al. (2022), where the authors also model the regression coefficients with a first-order random walk process (Wang et al., 2022). The Bayesian hierarchical framework allows for information sharing across methods and regions, which is of particular use for countries where data are limited. It allows us to capture differences between countries, and the similarities of countries within regions. This type of approach follows that used in other global estimation exercises, for example, for family planning use (Alkema et al., 2013) and estimating populations of women of reproductive age (Alexander & Alkema, 2022). Explorations carried out seem to suggest that there is no spatial autocorrelation in the model estimates. The hierarchical structure within the model, taking advantage of the geographic nature of the dataset, captures any potential spatial residuals within the data.

In future work, analyses that focus on using relevant covariates to try to explain or understand differences in supply shares across settings or within settings over time can be considered. Given data limitations and the variety of reasons why supply shares may vary, this falls outside the scope of our current study.

This consideration of the public/private sector supply of contraceptive methods within countries is vital for informing family planning programs that seek to improve access to and increase use of modern family planning methods (Ramakrishnan & Callahan, 2021). The private sector can play an important role in the sustainability of family planning markets and therefore the ability, via model-based estimates, to evaluate where countries currently stand in terms of private sector contributions to the contraceptive method supply can help to inform needs for private sector expansion and support the growth of contraceptive use (Ramakrishnan & Callahan, 2021).

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Conflicts of interest: none declared.

Data availability

The code and data used to create these results can be found at https://github.com/hannahcomiskey/mcmsupply and the corresponding R package, *mcmsupply* can be found on CRAN (Comiskey et al., 2023). The print for this manuscript is available on arXiv.

Supplementary material

Supplementary material is available online at *Journal of the Royal Statistical Society: Series A*.

References

Fay R. E., III, & Herriot R. A. (1979). Estimates of income for small places: An application of James-Stein procedures to census data. *Journal of the American Statistical Association*, *74*, 269. https://doi.org/10.2307/2286322

Alexander, M., & Alkema, L. (2022). A Bayesian cohort component projection model to estimate women of reproductive age at the subnational level in data-sparse settings. *Demography*, *59*(5), 1713–1737. https://doi.org/10.1215/00703370-10216406

Alkema, L., Kantorova, V., Menozzi, C., & Biddlecom, A. (2013). National, regional, and global rates and trends in contraceptive prevalence and unmet need for family planning between 1990 and 2015: A systematic and comprehensive analysis. *Lancet (London, England)*, *381*(9878), 1642–1652. https://doi.org/10.1016/S0140-6736(12)62204-1

Avenir Health. (2020). *SS to EMU Tool*. Avenir Health. http://www.track20.org/pages/our_work/innovative_tools/SS_to_EMU_tool.php. Date accessed March 9, 2022.

Ayad, M., Wilkinson, M., & McNiff, M. (1994). *Sources of contraceptive methods*. DHS Comparative Studies No. 11.

Azose, J. J., & Raftery, A. E. (2018). Estimating large correlation matrices for international migration. *Annals of Applied Statistics*, *12*(2), 940–970. https://doi.org/10.1214/18-AOAS1175

Binder, D. A. (1983). On the variances of asymptotically normal estimators from complex surveys. *International Statistical Review*, *51*(3), 279. https://doi.org/10.2307/1402588

Bradley, S. E. K., & Shiras, T. (2020). *Sources for family planning in 36 countries: where women go and why it matters*. Brief. Rockville, MD: Sustaining Health Outcomes through the Private Sector Plus Project, Abt Associates.

Bremner, J., Scoggins, S., Bai, Y., Farid, S., Sonneveldt, E., Emmart, J., & Stover, J. (2021). *FP2030 measurement report 2021*.

Cahill, N., Sonneveldt, E., Emmart, P., Williamson, J., Mbu, R., Fodjo Yetgang, A. B., Dambula, I., Azambuja, G., Mahumane Govo, A. A., Joshi, B., Felix, S., Makashaka, C., Ndaruhutse, V., Serucaca, J., Madzima, B., Muzavazi, B., & Alkema, L. (2021). Using family planning service statistics to inform model-based estimates of modern contraceptive prevalence. *PLoS One*, *16*(10), 1–14. https://doi.org/10.1371/journal.pone.0258304

Cahill, N., Sonneveldt, E., Stover, J., Weinberger, M., Williamson, J., Wei, C., Brown, W., & Alkema, L. (2018). Modern contraceptive use, unmet need, and demand satisfied among women of reproductive age who are married or in a union in the focus countries of the Family Planning 2020 initiative: A systematic analysis using the Family Planning Estimation Tool. *The Lancet*, *391*(10123), 870–882. https://doi.org/10.1016/S0140-6736(17)33104-5

Campbell, O. M. R., Benova, L., Macleod, D., Goodman, C., Footman, K., Pereira, A. L., & Lynch, C. A. (2015). Who, what, where: An analysis of private sector family planning provision in 57 low- and middle-income countries. *Tropical Medicine and International Health*, *20*(12), 1639–1656. https://doi.org/10.1111/tmi.12597

Chakraborty, N. M., & Sprockett, A. (2018). Use of family planning and child health services in the private sector: An equity analysis of 12 DHS surveys. *International Journal for Equity in Health*, *17*(1), 1–12. https://doi.org/10.1186/s12939-018-0763-7

Cisek, C. R., Klein, K., Koseki, S., & Wood, R. (2019). Strengthening family planning stewardship with a total market approach: Mali, Uganda, and Kenya experiences. *Public Administration and Development*, *39*(1), 47–56. https://doi.org/10.1002/pad.1840

Comiskey, H., Alkema, L., & Cahill, N. (2023). *mcmsupply: Estimating public and private sector contraceptive market supply shares. 0.1.1. Comprehensive R Archive Network (CRAN)*. https://CRAN.R-project.org/package=mcmsupply. Date accessed July 31, 2023.

Croft, T. N., Marshall, A. M. J., & Allen, C. K. (2018). *Guide to DHS statistics DHS-7*. https://dhsprogram.com/data/Guide-to-DHS-Statistics/index.cfm. Date accessed August 31, 2022.

Data for Impact (n.d.) *Source of supply (by method)*. https://www.data4impactproject.org/prh/family-planning/fp/source-of-supply-by-method/. Date accessed June 5, 2022.

FP2030. (2021). *Rights and empowerment principles for family planning. Technical report*. United Nations Foundation.

Gelman, A. (2006). Prior distributions for variance parameters in hierarchical models (Comment on Article by Browne and Draper). *Bayesian Analysis, 1*, 515–534. https://doi.org/10.1214/06-BA117A

Hornik, K., Leisch, F., Zeileis, A., & Plummer, M. (2003). *JAGS: A program for analysis of Bayesian graphical models using Gibbs sampling*. https://www.r-project.org/conferences/DSC-2003/Proceedings/Plummer.pdf

Hubacher, D., & Trussell, J. (2015). A definition of modern contraceptive methods. *Contraception*, *92*(5), 420–421. https://doi.org/10.1016/j.contraception.2015.08.008

ICF. (2012). *The DHS Program STATcompiler*. Funded by USAID. http://www.statcompiler.com/

Jain, M., Chauhan, M., & Talwar, B. (2016). Role of private sector in family planning programme in Rajasthan, India—A rapid assessment. *Int J Community Med Public Health*, *3(4)*, 869–874. https://doi.org/10.18203/2394-6040.ijcmph20160919.

Knutson, V., Aleshin-Guendel, S., Karlinsky, A., Msemburi, W., & Wakefield, J. (2023). Estimating global and country-specific excess mortality during the COVID-19 pandemic. *Annals of Applied Statistics*, *17*(2), 1353–1374. https://doi.org/10.1214/22-AOAS1673

Lavrakas, P. (2013). *Taylor Series Linearization (TSL). Encyclopedia of survey research methods*. SAGE.

Linzer, D. A., & Lewis, J. B. (n.d.). *Journal of Statistical Software poLCA: An R package for polytomous variable latent class analysis*. http://www.jstatsoft.org/. Date accessed December 10, 2023

Magnani, R. J., Ross, J., Williamson, J., & Weinberger, M. (2018). Can family planning service statistics be used to track population-level outcomes? *Global Health: Science and Practice*, *6*(1), 93–102. https://doi.org/10.9745/GHSP-D-17-00341

Marhuenda, Y., Molina, I., & Morales, D. (2013). Small area estimation with spatio-temporal Fay-Herriot models. *Computational Statistics & Data Analysis*, *58*, 308–325. https://doi.org/10.1016/j.csda.2012.09.002

Moazzam, A. (2015). *Ensuring contraceptive security through effective supply chains. Technical report*. World Health Organization, United Nations Population Fund.

Percent of contraceptives available from the private sector—MEASURE Evaluation. (2017). *Percent of contraceptives available from the private sector. Measure evaluation: Family planning and reproductive health indicators database*. https://www.measureevaluation.org/prh/rh_indicators/health-systems/ps/percent-of-target-population-using-fp-rh-or-hiv

Polson, N. G., & Scott, J. G. (2012). On the half-Cauchy prior for a global scale parameter. *International Society for Bayesian Analysis*, *7*, 887–902. https://doi.org/10.1214/12-BA730

Ramakrishnan, G., & Callahan, S. (2021). *Factors influencing the private sector's contributions to family planning market growth. Technical report*. Sustaining Health Outcomes through the Private Sector Plus Project.

Sharrow, D., Hug, L., You, D., Alkema, L., Black, R., Cousens, S., Croft, T., Gaigbe-Togbe, V., Gerland, P., Guillot, M., & Hill, K. (2022). Global, regional, and national trends in under-5 mortality between 1990 and 2019 with scenario-based projections until 2030: A systematic analysis by the UN Inter-agency Group for Child Mortality Estimation. *Lancet Global Health*, *10*(2), 195–206. https://doi.org/10.1016/S2214-109X(21)00515-5

Shelton, J. D., & Finkle, C. (2016). Leading with LARCs in Nigeria: The stars are aligned to expand effective family planning services decisively. *Global Health, Science and Practice*, *4*(2), 179–185. https://doi.org/10.9745/GHSP-D-16-00135

Susmann, H., Alexander, M., & Alkema, L. (2022). Temporal models for demographic and global health outcomes in multiple populations: Introducing a new framework to review and standardise documentation of model assumptions and facilitate model comparison. *International Statistical Review*, *90*(3), 437–467. https://doi.org/10.1111/INSR.12491

Sustaining Health Outcomes through the Private Sector (SHOPS) Plus. (2016). *Total market approach compendium*. https://shopsplusproject.org/tmacompendium. Date accessed August 31, 2022.

The DHS program. (n.d.) *The DHS program—Quality information to plan, monitor and improve population, health, and nutrition programs*. https://dhsprogram.com/. Date accessed December 9, 2021.

Track20. (2020a). *Monitoring progress in family planning estimated modern use (EMU): A new service statistics-based family planning indicator*. https://www.track20.org/pages/data_analysis/publications/methodological/family_planning_estimation_tool.php. Date accessed July 8, 2023.

Track20. (2020b). *SS to EMU tool: Process guide*. Avenir Health. http://www.track20.org/download/pdf/EMU. Date accessed March 9, 2022.

Track20. (2020c). *SS to EMU tool: Process guide*. Glastonbury, CT. http://www.track20.org/pages/track20_tools/SS_to_EMU_tool.php. Date accessed July 10, 2023.

Ugarte, M. D., Goicoa, T., Militino, A. F., & Durbán, M. (2009). Spline smoothing in small area trend estimation and forecasting. *Computational Statistics & Data Analysis*, *53*(10), 3616–3629. https://doi.org/10.1016/j.csda.2009.02.027

Ugaz, J. I., Chatterji, M., Gribble, J. N., & Mitchell, S. (2015). Regional trends in the use of short-acting and long-acting contraception accessed through the private and public sectors. *International Journal of Gynecology and Obstetrics*, *130*(S3), E3–E7. https://doi.org/10.1016/j.ijgo.2015.03.021

United Nations Statistics Division. (2022). *Methodology, standard country or area codes for statistical use (M49)*. https://unstats.un.org/unsd/methodology/m49/overview/. Date accessed May 12, 2022.

Vehtari, A., Gelman, A., Simpson, D., Carpenter, B., & Bürkner, P.-C. (2021). Rank-normalization, folding, and localization: An improved r^ for assessing convergence of MCMC (with discussion). *Bayesian Analysis (online)*, *16*, 667–718. https://doi.org/10.1214/20-BA1221

Wang, Z., Fix, M. J., Hug, L., Mishra, A., You, D., Blencowe, H., Wakefield, J., & Alkema, L. (2022). Estimating the stillbirth rate for 195 countries using a Bayesian sparse regression model with temporal smoothing. *Annals of Applied Statistics*, *16*, 2101–2121. https://doi.org/10.1214/21-AOAS1571

Weinberger, M., & Callahan, S. (2017). *The private sector: Key to achieving family planning 2020 goals*. Sustaining Health Outcomes through the Private Sector Project, Abt Associates.