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ORIGINAL RESEARCH

Exploring problem conceptualization and performance in STEM problem solving contexts

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Abstract

Problem solving abilities are critical components of contemporary Science, Technology, Engineering and Mathematics (STEM) education. Research in the area of problem solving has uncovered much about the representation, processes and heuristic approaches to problem solving. However, critics claim this overemphasis on the process of solving problems has led to a dearth in understanding of the earlier stages such as problem conceptualization. This paper aims to address some of these concerns by exploring the area of problem conceptualization and the underlying cognitive mechanisms that may play a supporting role in reasoning success. Participants (N = 12) were prescribed a series of convergent problem-solving tasks representative of those used for developmental purposes in STEM education. During the problem-solving episodes, cognitive data were gathered by means of an electroencephalographic headset and used to investigate students' cognitive approaches to conceptualizing the tasks. In addition, interpretive qualitative data in the form of post-task interviews and problem solutions were collected and analyzed. Overall findings indicated a significant reliance on memory during the conceptualization of the convergent problem-solving tasks. In addition, visuospatial cognitive processes were found to support the conceptualization of convergent problem-solving tasks. Visuospatial cognitive processes facilitated students during the conceptualization of convergent problems by allowing access to differential semantic content in long-term memory.

Keywords Problem conceptualization · STEM · Problem solving · Performance

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Introduction

An area that has garnered significant interest in the field of STEM education research is problem solving. While much is known about the processes and heuristics of problem solving, little is understood about the complexities surrounding problem conceptualization (Kirsh 2009). This concept relates to the manner in which an individual frames a problem or task prior to implementing a solution process (Adams 2001). Problem conceptualization is differentiated from the traditional notion of problem representation in subsequent sections. Much fruitful work has been conducted on the area of problem representation (Chi et al. 1981; Garderen and Montague 2003; Izsák et al. 2009; Koedinger and Nathan 2004) and this will be considered within the goals of the research presented in this study.

This research study focused on early stage aspect of problem solving within a STEM education context. In addition, this work explored the potential relationship between visuospatial cognitive processes and the conceptualization of convergent problem-solving tasks. This holds particular interest for the STEM education community as it has been widely established that spatial skills development has a positive effect on students' overall academic development in a variety of STEM disciplines (Sorby 2009; Uttal and Cohen 2012).

Of concern within this study is the nature of thinking indicative of convergent problem-solving activity, which is core to STEM educational practice. This requires an in-depth exploration of the process of problem solving.

Traditional problem-solving research

While problem solving is a specific and significant element of education curricula, the origins of research in the area stem from a general psychological context. The most significant body of work in the early era of problem solving research was from Newell and Simon (1972) who developed theories centring on a model of general problem solving. The primary contribution of this work identified the process of human problem solving as a heuristic search through a 'problem space' (Newell and Simon 1972; Novick and Bassok 2005). This conception of problem solving helped define the predominant direction that research in the field would take for the next few decades (Ohlsson 2012). Much is now known about the various heuristic search mechanisms that are employed during problem solving situations such as means-end analysis and hill climbing strategies (Novick and Bassok 2005). Heuristics limit the potential number of approaches an operator can take when searching a problem solving space thus reducing cognitive load and optimizing performance (Todd and Gigerenzer 2000). This research approach described the process of solving a problem as an interplay between the construction of a problem space (representation) and search (Pretz et al. 2003). The large body of research that has been conducted, based on this traditional information processing conception (Mayer 1996), has spawned several different process models of problem solving that are commonly found in various educational curricula. One of the most cited examples is that developed by Pólya (1957) where he described the process as encompassing four key stages: understand the problem, devise a plan, carry out the plan and review/extend.

While much research has been conducted on the process of solving problems, from an information processing perspective, there is a significant dearth in the field relating to the earlier stages of problem solving such as problem conceptualization (Ohlsson 2012). The traditional information processing paradigm has been criticized for treating problem solving as an abstracted process neglecting elements such as situated cognitive factors, personal traits and previous experience (Jonassen 2011; Kirsh 2009; Ohlsson 2012). The limitations of the assumptions underpinning the information processing perspective have, in recent years, come under scrutiny with educators' acceptance of constructivism. Advancing analogies of learning further, Jacobson et al. (2016) discuss a philosophical underpinning based on complex adaptive systems as an alternative to the information processing view for research in educational psychology. Complexity advocates a focus on characteristics such as sensitivity to initial conditions and emergence which positions a necessity to reconsider how we approach research in higher level cognitive activities such as problem solving (Jacobson et al. 2016). These factors are particularly important in the initial stages of interpreting a problem and especially when the individual is faced with an unfamiliar situation (Ohlsson 2012). While work has been conducted on the concept of problem spaces, this has tended to focus on problem representation at the expense of problem conceptualization.

Problem representation

Problem representation is typically described as a model that summarises an individual's understanding of a problem (Duit and Treagust 2012; Novick and Bassok 2005). This representation and the form that it takes has been demonstrated as a critical factor in determining an individual's success in solving a prescribed task (Boonen et al. 2014). Much work has been conducted on students external representation use in the field of mathematics (Boonen et al. 2014; Deliyianni et al. 2009) and science education (Bodner and Domin 2000; Wetzels et al. 2010). A representation can occur in external or internal domains and interact significantly with each other. When students engage with an externally presented representation, such as a problem statement, a prompt can occur which presses access to a long term store of concepts or schema (Wetzels et al. 2010). A representation can then be generated in working memory and used for the purposes of the task (Cox 1999). The model posited by Gick (1986) illustrates the interaction of representation and search processes in problem solving according to the information processing account (see Fig. 1).

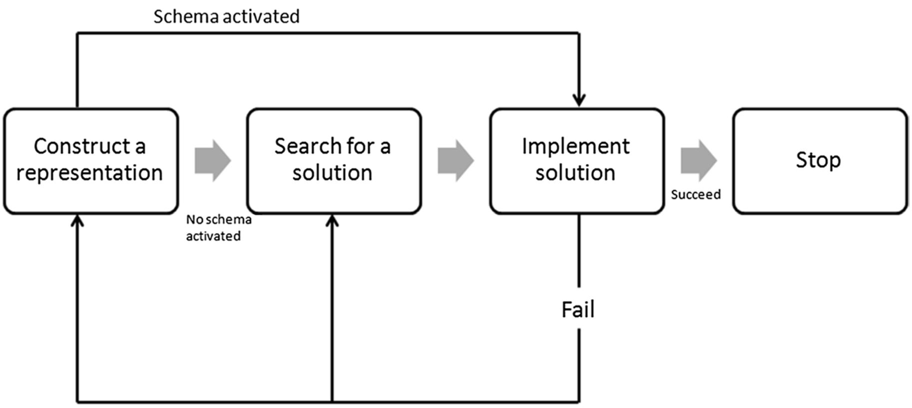


Fig. 1 Model of problem solving illustrating interaction of representation and search (Gick 1986)

In Gick's model, the most efficient case of attaining a solution is where a schema is activated by a representation leading to a direct solution. Although, arguably this does not constitute a problem which is typically defined lacking a direct route to the solution state (Dominowski and Bourne 1994). One of the key limitations of this model is the cursory treatment of representation construction and what is involved in this process (Ohlsson 2012). Illuminating this notion, Connell and Lynott (2014, p. 391) define a representation as "a specific, situated, contextual instantiation of one or more concepts necessary for the current task". A concept here being broadly defined as internal collection of data about an object stored in long-term memory. However, instantiation refers to the application of an instance which suggests a type of rigid input/output process from memory and harks back to the treatment of representation in the information processing tradition. This perspective may be overly simplistic in dealing with the notion of representation and has limitations when the style of problem reduces in the degree of a priori structure where access to stored representations are more complex. This paper therefore introduces a focus on the area of problem conceptualization.

Conceptualization

Accessing and instantiating stored representations is a very effective manner of solving problems, assuming the structure of the problem is clear, and the solver has the appropriate level of expertise. This becomes more difficult as the problem becomes more ill-structured and prompts are no longer as clear. Depending on the expertise hence quality of stored representations of the solver (Chi et al. 1981), a seemingly well-structured problem can also fail to provide appropriate prompts for a successful generation of a representation (Pretz et al. 2003). These are important considerations from an instructional design perspective given the developing nature of students' problem schema and mental models.

Critically, the process of representation involves both online and offline modes of cognitive processing where the human conceptual system takes advantage of the perceptual system for the construction of a representation (Connell and Lynott 2014). Moreover, this complex interactive process is partially captured by evidence of the correlational relationship between working memory and executive functioning in problem representation (Lee et al. 2009). This suggests an active and dynamic process of recollection, synthesis and construction preceding a problem representation. As discussed by Sherin (2001) there is more to this process than merely accessing memorized symbols and subsequently calling upon them in the current circumstance. This evidence presents a more complex and dynamic view of defining problem spaces than that advocated in the notion of representation from the information processing account.

Within the field of creativity research, Mumford et al. (1994) delineate a model of problem construction in dealing with ill-defined problems (see Fig. 2). Ill-defined problems may be referred to as 'representation-hungry' (Kiverstein and Rietveld 2018) and therefore involve active searching, combination, reorganisation and modification of elements of existing cognitive schema combined with perceptual input (Barsalou 1999; Mumford et al. 1994). Despite this idea being construed in the context of ill-defined domains, it is at least partly applicable to the current focus on convergent problems if it is assumed that these problems may be perceived as "ill-defined", mediated by the expertise of the solver. A clearer way of elucidating this is to consider the more novice problem solver as more

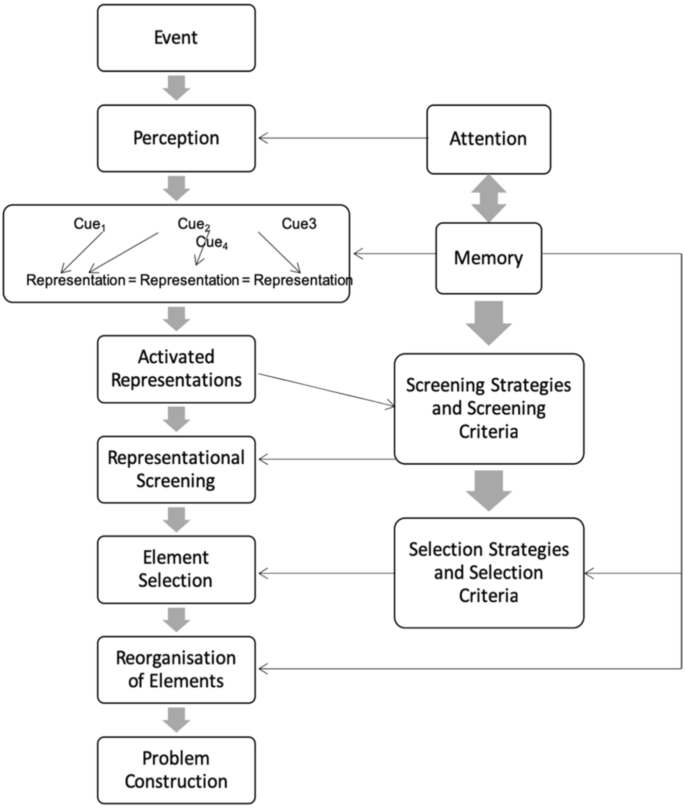


Fig. 2 Model of problem construction (Mumford et al. 1994)

'representation-hungry' necessitating a more active engagement with discrete cognitive schema and mental models as well as active combination and modification mechanisms.

To date problem representation has predominantly been dealt with within the information processing account of cognition and therefore the internal mechanisms of this active process of problem construction (Mumford et al. 1994) are relatively under researched (Jakel and Schreiber 2013). Based on the concept of problem construction (see Fig. 2), this paper introduces the notion of *problem conceptualization*. While the process of problem construction, as defined by Mumford and colleagues, is appealing in the sense of the more active and complex notion it affords problem representation, it is confined to the work on creativity and as such in areas such as design, where problems may not be presented but found (Jonassen 2011). To differentiate from this area and bring the focus back to more structured problem scenarios used at times in STEM education, *conceptualization* has been proposed as it connotes an action of forming an idea of something. This aligns with the core elements of active searching and combination of representational elements as in Fig. 2 as well as with the notion of modelling which is used by Goméz et al. (2000) to describe conceptualization. This paper therefore operationally defines problem conceptualization as the period of cognitive processing occurring before the externalization of any representation (e.g. a sketch, mathematical expression, verbal communication etc.).

There are a number of critical distinctions that need to be made here. Firstly, conceptualization is differentiated from classical problem representation by emphasis on the more active search, organization and combination of various elements of stored mental representations. This characteristic takes cognizance of the developments in memory research in which emphasizes that memory traces are not merely recalled from long-term memory but are actively constructed (Schacter and Addis 2007) and hence susceptible to influence from a variety of factors such as situational, emotional and attitudinal elements among others (Kensinger and Schacter 2005; Urbach et al. 2005). This problematizes the process of generating representations and places a focus on the very early stages of problem solving that have been neglected in the majority of research in the area. Secondly, this consideration also presents a differentiator from Mumford et al. (1994) notion of problem construction as it complexifies the role of memory and attention while considering more of the individual traits of the problem solver inter alia, attitudinal, emotional and cognitive propensities. These are all elements that have been claimed to be neglected in traditional accounts of problem solving research (Kirsh 2009) and hence conceptualization is differentiated from a traditional notion of problem representation through incorporation of aspects such as past experience, environmental constraints and epistemological orientation within an dynamic active process (Barsalou 2009).

The outcome of the conceptualization process may result in a representation for a problem but based on this theory it seems that these processes occur before any evidence of external representation is created by the individual. This presents limitations with studies that have used students' external representations as measures of a student's understanding and effectiveness in solving convergent tasks and presents a novel avenue to explore phenomena and potential issues within the early stages of students' problem-solving efforts.

Implications and research focus

Although problem solving ability is cited as a critical skill in contemporary STEM education and wider society, concerns have been raised about the lack of this ability in a variety of contexts. These include STEM graduates in the workplace (NAE 2004) and students in post-primary and higher education (Delahunty et al. 2012, 2018; McCormick and Davidson 2009; Seery and Delahunty 2013). One area that has been discussed is the lack of flexibility evident in students' approaches to solving problems leading to rigid and procedural tactics on the part of the problem solver (McCormick and Davidson 2009). Gaining an enhanced understanding of the processes of problem conceptualization could provide evidence to aid in addressing these concerns. Given the lack of empirical data evident in the literature (Jakel and Schreiber 2013), this paper will report on an exploratory investigation of problem conceptualization and potential relationships with problem solving performance. In particular, the research aimed to:

1. Investigate the relationship between the conceptualization of problems and associated performance.
2. Explore the underlying elements contributing to an individual's conceptualization in response to prescribed educational type problems.

Methods

Approach

A mixed methods approach was adopted for this study which included the use of electroencephalography as a key tool in capturing evidence of students' cognitive approach. The research approach allows a fit for purpose synthesis of research methods that are utilized to provide a "superior product" (Johnson and Onwuegbuzie 2004). The use of a mixed-methods approach was important to facilitate a pragmatic yet rigorous exploratory study, avoiding erroneously weighting one method as most important (Feilzer 2010) and providing a more complete picture of the hypothesized phenomenon of conceptualization. This is particularly important in a study that utilizes the potentially "seductive" details of neuroscience (Weisberg et al. 2008) to investigate a phenomenon that may be impacted by cognitive as well as sociocultural factors. The following sections will provide a brief overview of the methodological approach but for a more detailed description of this method see Delahunty et al. (2018). Based on the limited literature available on problem conceptualization, three key elements were chosen as necessary to explore this phenomenon effectively. These are the cognitive processes involved, problem solving performance and perception of the problem solver. These are summarized in Fig. 3 and elaborated on in subsequent sections.

Research instruments

Cognitive data

Considering the focus of the study, the research instruments needed to be capable of capturing different forms of data during reasoning episodes. As the conceptualization of problem situations is posited to be an extremely tacit phenomenon (Barsalou 2009), it was necessary to employ an objective measure of cognitive function to uncover evidence of the underlying cognitive processes. In order to achieve this, an electroencephalographic (EEG) headset was employed to collect this data. EEG offers a distinct advantage over other functional methods (e.g. fMRI) as it has superior temporal resolution and is more applicable (given wireless options) to an applied research context such as problem solving.

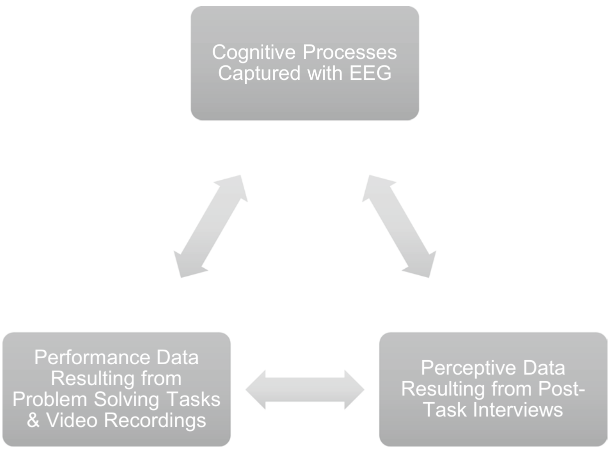


Fig. 3 Triangulated methodological approach

The EEG device utilized in this study was the Emotiv headset which incorporates 14 different sensors attached to the scalp which collect electrical data from the participants' scalp. This particular device has previous been used in several studies from different contexts including information presentation (Anderson et al. 2011) and spatial testing (Call et al. 2016).

Performance data

All participants (N = 12) were required to complete 15 tasks in total (detailed later in this section) and were given a maximum of 5 min to complete each task. The overall performance scores were determined using the participants' hard copy solutions. The solutions were assessed using a sliding scale of professional judgement, which is detailed in Table 1. Capturing performance data, while not allowing for direct evidence of the problem conceptualization, was necessary in order to address the first research question that sought to explore the link (if any) between conceptualization and problem-solving performance. Additionally, video recording was implemented to observe the process of problem solving as it unfolded in order to clarify assessment decisions using the sliding scale.

Perceptive evidence

In order to capture the perceptive evidence of the process, a series of post-task interviews were employed. The use of a post-task interview was necessary to uncover specific aspects of a participant's conception of the applied tasks. Bryman (2008) discusses the use of unstructured or semi-structured interviews which allow for a flexible investigation of underlying phenomena compared to that of structured or quantitative approaches. The co-authors generated a semi-structured interview scheme through careful deliberation and refinement to control for any 'leading' question items.

The triangulation of these different data sets was used to help explain the relationship between the conceptualization of problems and associated participant performance. All three data sets, including video recordings of the participants attempting to solve the assigned problems, were used to gain a rounded picture of participant's conceptualization of the prescribed educational problems. The electrical data from the EEG headset provided information on cognitive frequencies and cortical locations active during the early conceptualisation of the respective problems. The post-task interviews allowed participants the opportunity to further elucidate how they felt they had conceptualised the problem. Finally, these two data sets were cross referenced against the hard copy solutions submitted and the video recordings of participants attempting to solve the assigned educational problems to determine if their attempted solutions were reflective of a particular conceptualization of

Table 1 Breakdown of scores for scale of professional judgement

Sliding Assessment Scale	
Marks awarded	Assigned for
9–10	Effective approach and correct solution
6–8	Reasonable approach (likely to lead to a solution)
4–5	Ineffective approach (unlikely to yield solution)
1–3	Incorrect approach (marks for attempt)

each problems. These conceptualizations were then compared to participants' respective performance scores across the problems to explore the relationship between the conceptualization of problems and associated performance.

Design of tasks

The research was contextualized within STEM education, therefore the tasks were technical in nature and engaged problem-solving processes that are common in the STEM subjects. These include analytical and visuospatial reasoning skills among others (Ben Youssef and Berry 2012). To support the reliability and validity of the study, pre-existing tasks taken from the Improving Numerical Literacy Skills (INULIS) database were utilized, which were designed on the principles of the Program for International Student Assessment (PISA) (O'Donoghue and Kooij 2007). Full permission was sought and granted to the authors to employ tasks from the INULIS database as part of this study. These tasks were chosen firstly, to help control for task difficulty which can impinge on cognitive load and potentially impact the conceptualization process. PISA tasks are aimed at the 15-year-old, K-12 level and therefore difficulty should not have been a factor as the participant base was university students. Secondly, the tasks in the INULIS database are set for a general audience, which meant that high levels of domain specific knowledge were not necessary. Thirdly, the INULIS tasks are specifically modelled based on the mathematical and scientific competencies espoused by PISA (OECD 2010). Consequently, they not only include tasks of quantitative reasoning but also tasks that involve visuospatial approaches. This allowed for the selection of a database of tasks where different problem solving strategies were possible.

In total, 15 tasks were selected which encompassed three categories of mathematical, visuospatial and dual approaches. The three categories contained tasks, which were selected based on specific principles. The mathematical category encompasses a diverse range of tasks in relation to complexity. The tasks that are included in this category range from simple (e.g. application of formula) to the more complex (e.g. algebraic). The *visuospatial* category tasks were originally designed under the category of "space and shape" (OECD 2004). The tasks included in the *visuospatial* category ranged in complexity from simple (image holding and comparing) to complex (dynamic imagery) (Gaughran 1990). The *dual approach* category encompassed tasks that could be solved in two distinct manners, which were either mathematical or visuospatial. The focus of this category was on the flexibility of conceptualization approach a participant adopts so therefore it was critical that the tasks were of the same complexity to control, as much as possible, the influence of high levels of cognitive load on the process. The 15 tasks are presented in Table 2 with a sample task from each category shown in Fig. 4.

Participants

The participants for this study were recruited from a teacher education degree course where individuals specialize in technology education. Graduates from this program typically go on to teach technology and design subjects at the post-primary levels. The participants in the study ($N = 12$) were all third-year undergraduate students at the University of Limerick. Participants were recruited by email that included an information sheet outlining the details of the study and a consent form requesting participation on a voluntary basis. Informed consent was gained from 12 participants who voluntarily

Table 2 Task categories and types

Task	Category	Descriptor
A	Mathematical	Application of theorem
B		Simple arithmetic
C		Application of formula
D		Algebraic
E		Identification and application of theorem
F	Dual approach	Simple arithmetic/image holding and comparing
G		Simple arithmetic/image holding and comparing
H		Application of formula/image holding and comparing
I		Application of formula/image holding and comparing
J		Probability/image holding and comparing
K	Visuospatial	Dynamic imagery
L		Image holding and comparing
M		Planar rotation
N		Dynamic imagery
O		Kinetic imagery

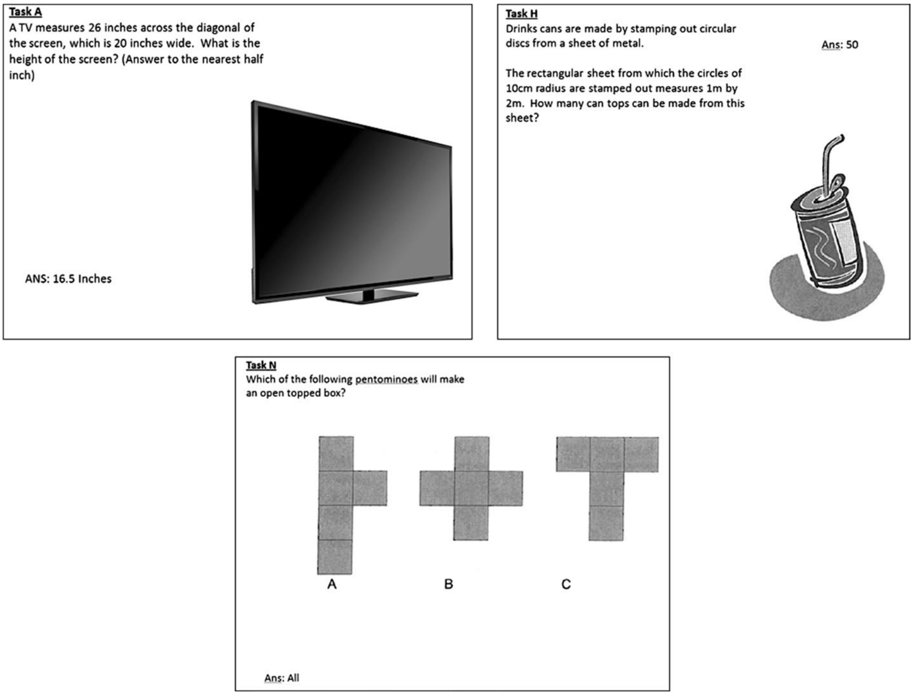


Fig. 4 Sample tasks from each approach category

agreed to take part in the study. It should be noted that all participants were male, as no females volunteered to participate. This study was approved by the Faculty of Science and Engineering research ethics committee at the University of Limerick (Ref: 2012_12_24_S&E). The authors declare that they have no conflict of interest.

Implementation

The study was conducted in a classroom setting (see Fig. 5) which students were familiar with from previous learning experiences. This setting was the same for all participants to reduce the potential influence of environmental affects during the gathering of data. The 15 tasks were presented on a PowerPoint presentation which was timed for five minutes per task as advocated by the INULIS guidelines (O'Donoghue and Kooij 2007). The visual and verbal data was gathered by means of a webcam and debut video capture software which was running in the background on the laptop as is shown in Fig. 5.

Shown in Fig. 5 is the EEG headset that the participant is wearing. The data for this were gathered on the researcher's laptop. The protocol for this data collection was based on that of Fink et al. (2009) and Delahunty et al. (2018) where a neutral stimulus was used between each task episode to record a baseline level of cognitive activity. This allowed for a comparison of each problem-solving episode with a hypothetical neutral period of cognitive activity (Pfurtscheller 1992). Pragmatically, this involved inserting a black cross in before each problem on the PowerPoint for a period of 20 s (Fink et al. 2009). During this period, the student was instructed to only observe the symbol until the slide progressed to the task.

Findings

This primary study generated a large amount of rich data for the problem-solving approaches adopted by participants. The method generated extensive amounts of cognitive data from all stages of participants' problem-solving episodes as well as the qualitative

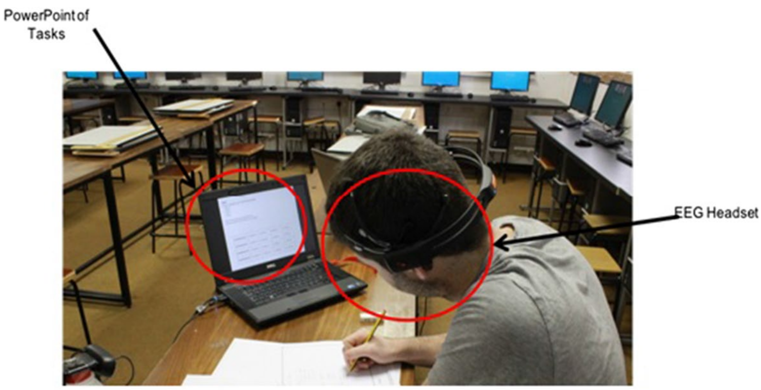


Fig. 5 Laboratory setting

data, which captured the behavioral and perceptive evidence. This section will first explore the performance data before moving onto an exploration of the cognitive and interview data related to problem conceptualization. In maintaining a systematic approach during this exploratory study, the first stage of analysis focused on the group level while later findings will focus on the individual cases level.

Participant group performance

Students displayed varying levels of performance in the tasks (Fig. 6). Here task performances are presented within each of their respective categories and re-ordered from high to low values for clarity. The *simple arithmetic/image holding and comparing* task (G) was completed successfully by all participants whereas the *probability/image holding and comparing* task (J) was not completed successfully by any of the twelve participants. Both of these tasks reside in the dual approach category. Overall, performance in the tasks was relatively poor given the experience level of the students and their subject backgrounds. One may expect third year undergraduates to be highly skilled in solving these tasks designed for 15-year-old high-school students.

To explore any overall differences in performances for each category a one-way ANOVA for repeated measures was conducted (data were first assessed for normality) with post-hoc comparisons using a Bonferroni adjustment to control for type 1 errors. A statistically significant difference in performance scores was found between categories, Wilk's Lambda = 0.231, $F(2,10) = 16.67$, $p < 0.005$. Post-hoc analyses indicated that the differences lay between the dual approach and mathematical ($p < 0.0001$) and the dual approach and visuospatial ($p < 0.05$) categories respectively. No differences were found in performance scores between the mathematical and visuospatial categories. Means and standard deviations are presented in Table 3.

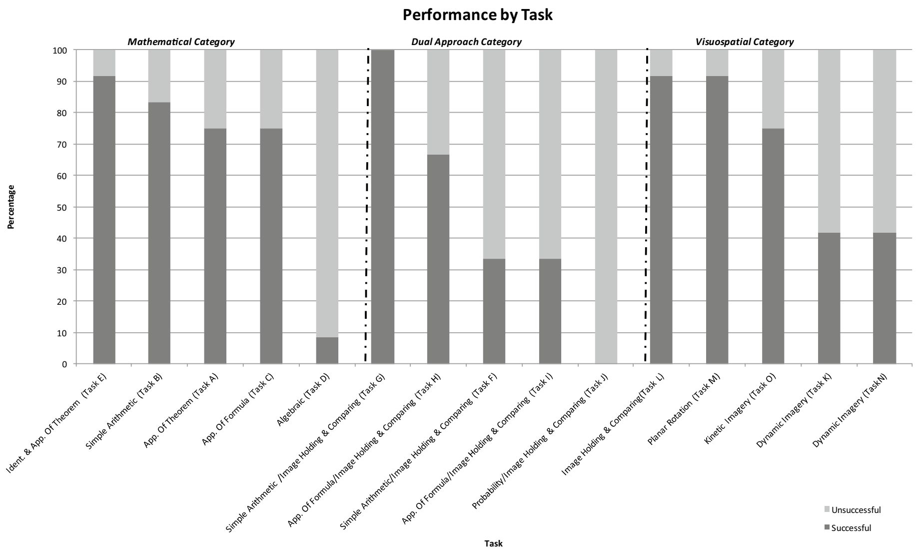


Fig. 6 Performance data

The data indicate that students performed significantly worse in the dual approach category where the approach needed to solve the problem was more ambiguous. This may suggest that students had difficulty in conceptualizing the tasks or in being flexible in terms of re-conceptualizing tasks when faced with uncertainty. The tasks in this category were controlled for in terms of difficulty in comparison to the other two.

Cognitive approaches in conceptualization and performance

This section of the paper will explore data relating to the EEG method employed for this study. It is not possible to present a lengthy account of the methodology and analysis for the EEG data due to space limitations and the need to devote care to unpacking the exploratory data as a basis for future research takes precedence. To overcome this potential issue, the authors produced an a priori publication of the methodology and data analysis procedure including detailed references and the reader is directed to Delahunty et al. (2018) for this information. Analysing EEG data is a complex undertaking and involves adopting principles and procedures from the field of signal processing. To determine if any cognitive patterns were evident in the EEG data, all participant readings were subjected to a Fourier transformation that decomposed the raw EEG readings into distinct frequency-power data. This is a common approach in EEG studies looking at cognitive function and often the two frequency bands that are selected are in the theta (4–8 Hz) and alpha (8–13 Hz) range. These frequencies are widely used in EEG studies in relation to problem solving and reasoning and are well established indicators of certain cognitive processes such as working memory, long-term memory and spatial cognition (Cabeza and Nyberg 2000; Fink et al. 2009; Razoumnikova 2000). The result of this process gives power values for both frequency bands across the 14 channels of the EEG headset. As the primary focus of this study was the investigation of the conceptualization period and its associated relationship with performance it was necessary to segment this theoretical period within the data. As the conceptualization and representation elements occur within the first stage of the problem solving cycle in most well established models such as that of Pólya (1957) or Sternberg (2003), it was prudent to focus specifically on the initial activity of each problem solving episode. This is a complex issue as there is no clear indication as to a temporality of this initial period and is likely person and situation specific. The decision of what time period to devote to conceptualization was further complicated by hardware and software limitations of the commercial EEG headset.

The headset used in this study collected data at a sampling rate of 128 Hz or 128 samples per second and the maximum number of data samples allowed in the analysis software available at one time was 4096 samples, equating to a period of 32 s. This processing limitation impeded the analysis in this study and is something that future research will need to consider. This coupled with the diverse range of times evidenced by all participants during each of the videoed solutions for the tasks (see Appendix 2) as well as the maximum time limit of 5 min per task informed the decision to focus on the first 32 s when exploring the conceptualization process. This provided a reasonable timeframe for analysis, considering data is removed in the

Table 3 Category performance

Category	Mean	SD	N
Maths	7.2667	1.32207	12
Dual	5.1833	0.95901	12
VisSp	6.8333	2.08341	12

process of screening and decontamination (Sanei and Chambers 2007), and in extreme cases (where the participant required the time limit of five minutes) this would provide evidence of at least 10% of the cognitive activity of the entire task period. This timeframe selection is in line with previous studies using FFT (e.g. Sun et al. 2013). It is important to note that problem conceptualization could be a much shorter time than the period of 32 s however, using this time frame allowed an adequate measure of activity for the purpose of spectral computation taking the entirety of the data collected into consideration.

Due to the dearth in research on the earlier stages of problem solving presented earlier, this study is concerned with an analysis of problem conceptualization specifically and this justified the focus on the initial period of activity of the problem-solving episode. Other research by Delahunty (2019) and Delahunty et al. (2018) have demonstrated the links between this initial activity epoch and remaining quartiles during the problem solving episode. In observing trends in cognition across a task epoch, Delahunty (2019) posited the effect that this initial cognitive period has on the remainder of the episode in the case of convergent problems. Therefore, it is appropriate here to just focus on this initial activity period.

In order to determine task related changes in cortical activity the data were further subjected to task related power (TRP) analyses using the formula devised by Pfurtscheller (1992) seen in Eq. 1.

$$TRP(logPowi) = \log(Powi\ activation) - \log(Powi\ reference) \tag{1}$$

The application of this formula results in power readings being obtained which are task relevant. A positive value indicates an increase in cortical activity, at each of the 14 sensors, during engagement with the specific task. A negative value indicates a decrease in cortical activity at each of the 14 sensor locations relevant to engagement with the task. This is a widely practiced data analysis technique in EEG studies focused on problem solving cognition and has been successful in demonstrating frequency related changes, indicative of specific cognitive processes, and location of cortical activations which further strengthens conclusions made about the type of cognition relevant to the task (Fink et al. 2009; Liu et al. 2018). Determining significant trends or patterns in EEG data is a difficult process due to the sheer complexity of the data and statistical approaches are typically used for data reduction and exploration purposes (Sanei and Chambers 2007). In order to determine any patterns of cortical activity at the group level, TRP values in each frequency were subjected to a repeated measures ANOVA using within-subject factors HEMISPHERE (left, right) and LOCATION (7 EEG channels per hemisphere), and the between-subject factors of GROUP (successful, unsuccessful). Within-subjects effects were interpreted with Greehhouse-Geisser values to correct for violations of sphericity. Values for these analyses in both frequency bands, for the first 5 tasks as an example, can be seen in Table 4.

The result of this process generated a large amount of data relating to the tasks. The data in general did not indicate a clear statistical pattern. This could be due to the small sample size of participants involved, or the idiosyncrasy of the conceptualization process as alluded to. Taking cognizance of the potential to draw conclusions from an argument of ignorance, it was decided to examine tasks and participants in further detail at the individual task level. For this purpose, it was decided to analyze a task where a high level of success was recorded and a task where a low level of success was exhibited. The tasks selected were Task A and Task D respectively (see Appendix 1).

Within the analysis of these two tasks, a focus on group and single cases was adopted to explore any potential activity of significance among participants. The use of single cases

Table 4 Values for repeated-measures ANOVA TRP analysis

Task	Group	HEMSPH	HEMSPH × Group	Location	Location × Group
Theta					
A	F = 0.26, p = 0.876	**F = 6.438, p < 0.05**	F = 0.05, p = 0.828	F = 0.769, p = 0.518	F = 0.47, p = 0.702
B	F = 0.15, p = 0.704	**F = 6.781, p < 0.05**	F = 2.534, p = 0.143	**F = 3.101, p < 0.05**	F = 1.076, p = 0.387
C	F = 0.401, p = 0.541	F = 1.503, p = 0.248	F = 0.033, p = 0.859	**F = 3.886, p < 0.005**	**F = 2.845, p < 0.05**
D	F = 0.501, p = 0.495	F = 0.112, p = 0.744	F = 0.005, p = 0.944	F = 1.443, p = 0.254	F = 1.019, p = 0.393
E	F = 0.333, p = 0.576	F = 0.379, p = 0.552	F = 0.061, p = 0.811	F = 0.505, p = 0.708	F = 1.089, p = 0.372
Alpha					
A	F = 0.001, p = 0.977	F = 0.529, p = 0.484	F = 0.629, p = 0.466	**F = 3.076, p < 0.05**	F = 1.203, p = 0.317
B	F = 0.887, p = 0.369	F = 0.231, p = 0.641	F = 0.009, p = 0.927	F = 2.349, p = 0.103	F = 0.962, p = 0.415
C	F = 0.006, p = 0.937	F = 0.007, p = 0.933	F = 0.062, p = 0.808	**F = 3.595, p < 0.05**	F = 0.773, p = 0.522
D	F = 3.443, p = 0.093	F = 0.630, p = 0.446	F = 0.953, p = 0.352	F = 1.442, p = 0.256	F = 0.985, p = 0.402
E	F = 0.43, p = 0.527	F = 0.728, p = 0.414	F = 2.724, p = 0.130	F = 1.479, p = 0.247	F = 0.463, p = 0.672

Bold values indicate the statistically significant results i.e. $p < 0.05$

can be a powerful technique in educational research and can often be used as a starting exploration in order to develop further theoretical research questions (Ledford and Gast 2018). Pragmatically, this involved taking a detailed look at the successful and unsuccessful participants in each of the two tasks followed by the analysis of a representative successful and unsuccessful participant from each category.

Task A (application of theorem) TRP analysis

Task A (Appendix 1) required students to calculate the height of a television screen when given the width and diagonal length. The most effective approach would involve the application of Pythagoras theorem to determine the missing variable. In both frequencies, TRP displayed no significant difference among performance groups (see Fig. 7), Theta ($F=0.026$, $p=0.876$), Alpha ($F=0.001$, $p=0.977$). Theta TRP for task A displayed a significant within-subject effect for hemisphere, $F=6.438$, $p<0.05$, partial $eta^2=0.392$, indicating increased left hemisphere cortical activation for the task in both performance conditions.

As shown in Fig. 7, successful individuals displayed higher activation in the left frontal and centrotemporal areas (F7, F3, FC5, T7) and left occipital regions (O1). Theta is a strong indicator of memory related cognitive processes (Klimesch 1999) and in particular working memory which is typically indicated over frontal regions. Alpha TRP displays some increased activity (indicated here by negative or decreased values compared to

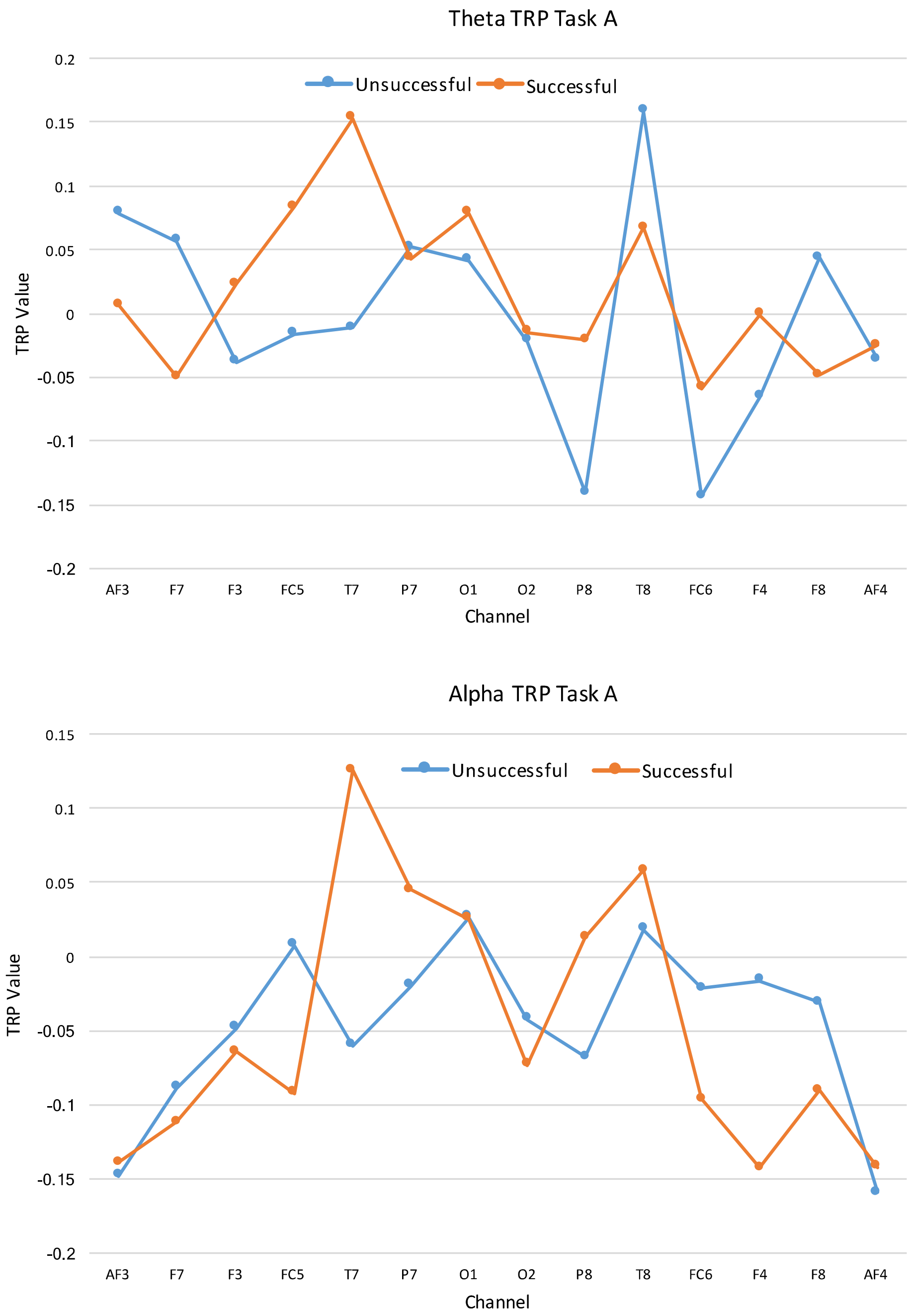


Fig. 7 Average task related power by performance for task A

unsuccessful students) for successful individuals over the occipital and frontal regions in the right hemisphere (Fig. 7) which are known to be involved in memory related and spatial cognitive processes (Razoumnikova 2000). The fact that these successful students indicate higher activation within Alpha in the right hemisphere could indicate a more holistic visual conceptualization (McGilchrist 2009). This indicates the use of visual approaches to conceptualizing the task that are predicated on forming some form of depictive mental model. This coupled with the prevalence of memory related spectral activity indicates an active interaction between the mental model and long-term memory in the form of schema.

Individual cases

Data for Participant 09 (successful) is presented in Fig. 8. The top left of the figure is the result of the task related power (TRP) calculations presented as percentage synchronization (increase in power) or desynchronization (decrease in power). Presented in the lower left corner of the figure is a graph, for illustrative purposes, indicating the extent to which certain cognitive processes are evident in the data. The graphic on the right of the image details the most active sensor sites on the headset during the conceptualization of the task. The participant's hard copy solution is also shown to the right of the image.

Within theta (Fig. 8), there is significant activity in the centrotemporal areas and the left parietooccipital area. The left parietooccipital area has been shown to incorporate various retrieval functions in long-term memory (Banich and Compton 2011). The function associated with synchronization of the theta frequency is known to be related to memory specifically (Klimesch 1999). Also of note is the de-synchronization, which can indicate cognitive activation (Fink et al. 2009), of alpha activity in the right parietooccipital areas. The right parietooccipital area is significantly involved in visuospatial processes, which have been known to be a key component of successful mathematical performance among others (Tversky 2005). The left frontal area is a known area for functions of working memory and in particular mathematical and analytical processes (Houdé and Tzourio-Mazoyer 2003). This would be expected in a task of this nature.

The activation of Theta in the left parietooccipital area indicates the recollection of Pythagoras' theorem as is shown in the hard copy solution (Fig. 8). Also noticeable is the activation of the right parietooccipital area within the alpha frequency which is known to be implicated in specific cognitive processes such as visualization (Rescher and Rapplesberger 1999). In Fig. 8 it is apparent that the hard copy reflects an emphasis

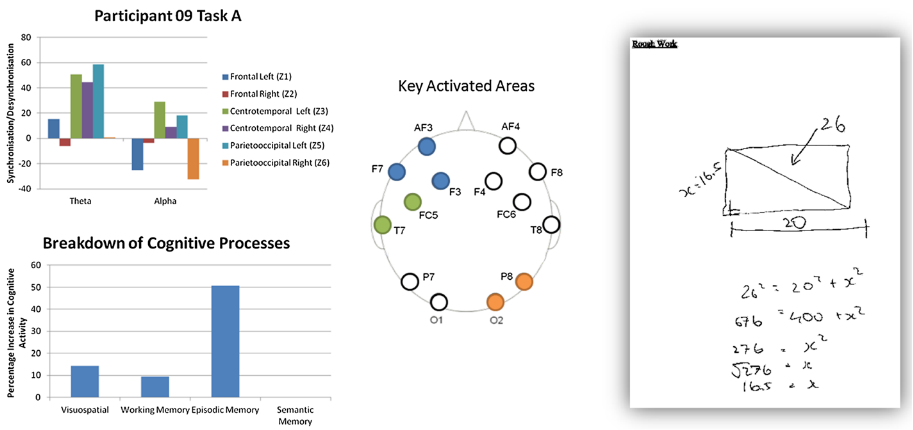


Fig. 8 Participant 09 application of theorem task (A) synchronization data and solution

on the use of a visual representation prior to application of a formula. These potential processes indicate a conceptualization process centered on the recall of information and its subsequent manipulation in working memory. The evidence of visuospatial cognition indicates that a visual representation or mental model was key to this students' approach. The conceptualization process for this student appears to involve the generation, maintenance and manipulation of a visual mental model that was then used as a means to access appropriate procedural schema in long-term memory.

Looking specifically at an unsuccessful approach, Fig. 9 illustrates the cognitive activity for participant 03 during task A. There is a different pattern of cognitive activity being displayed for this participant. The majority of theta is demonstrating a pattern of de-synchronization apart from the left centrotemporal area. This is indicative of a memory function as it is the sole synchronized activity for theta in this task. As it is not located in the frontal areas, which are the known seat of working memory (Stillings et al. 1995), it is likely an episodic style of memory (Tóth et al. 2012). The predominant activity in the alpha band is a de-synchronization in the frontal areas with increased activity in the left frontal area. This points to the expending of effort on an analytical function of working memory.

Alpha activity (Fig. 9) displays some activation in the right parietooccipital area which indicates a visual element of cognition. It could also indicate the blocking of visual strategies as alpha activity has been shown to block specific cognitive functions depending on task demands (Klimesch 1999). As already stated, this participant was unsuccessful. One of the core reasons for this lack of success is an inability to accurately recall the procedure. This is reflected in the participant's hard copy solution (Fig. 9) where it is clear that the formula was applied incorrectly. The cognitive evidence points to the role of a memory function during the conception and subsequently throughout the entire task. In contrast to the previous students' data, this conceptualization displays an attempt to access a memorized procedure rather than generating and utilizing a mental model to access the content of long-term schema that are more semantically rich in nature. It is possible this student relied on an overly episodic type of memory potentially attempting to recall a past instance of solving a similar problem.

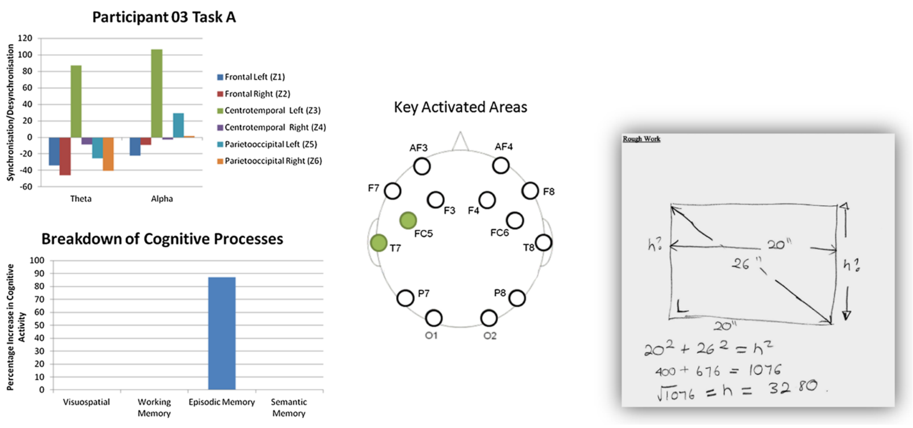


Fig. 9 Participant 03 application of theorem task (A) synchronisation data and solution

Task D (algebraic) TRP analysis

Task D was an algebraic type problem and involved the calculation of the area of a lawn after it had been extended under prescribed conditions. The most effective way of solving this task was the application of a binomial function using algebra to obtain the missing variables. Of the 12 participants in the study only one displayed an approach that could have yielded the correct solution for this task. Analyzing differences between the conditions in this case can be controversial given the sample sizes where the success group has a single participant. Mycroft et al. (2002) have analyzed this issue in the context of cognitive neuropsychological studies and have asserted that it is sufficiently valid and reliable to compare a single case with a group once caution is exercised in generalizing the results to the wider population. No significant effects between performance conditions were uncovered in either frequency TRP analyses (see Fig. 10), Theta ($F = 3.019$, $p = 0.113$), Alpha ($F = 3.178$, $p = 0.105$). Analyses of within-subject effects did not reveal any significant effects either.

Figure 10 illustrates some differences in cortical TRP for the successful participant when compared to the average for the unsuccessful students. In general, there appears to be a much higher level of TRP across the majority of locations in the theta frequency for the successful student. Increases in Theta TRP (synchronization) in conjunction with decreases in Alpha TRP (desynchronization) is indicative of cognitive engagement in a task (Fink et al. 2009). Figure 10 displays a pattern of higher cognitive engagement for the successful participant compared to the average for unsuccessful students. The centrotemporal and parietal regions seem to be key areas for this participant and may indicate a significant element of visuospatial cognition in conjunction with memory related processes. Again, this may point to the generation and manipulation of a visuospatial mental model in order to access appropriate task relevant data in long-term memory.

Individual cases

Participant 04 displayed the only successful approach to this task. Figure 11 illustrates the cognitive data captured during this participant's conceptualization episode.

There is significant activity within theta in the left parietooccipital areas indicative of long-term memory processes. This is coupled with further activity in alpha of the frontal areas in conjunction with theta of the frontal areas. This pattern is indicative of a higher cognitive load and possibly increased cognitive engagement. The nature of theta activation in the left parietooccipital area is likely in relation to a recollection process within long-term memory. Alpha displays further activity in the left hemisphere and this is tied to semantic processing (Cabeza and Nyberg 2000). There is significant activity in the alpha frequency located in the left occipital area throughout the task. This indicates a visuospatial process as the occipital areas cater for visual mental imagery in particular (Kosslyn et al. 2001). As is clear in this participant's solution, a visual representation was utilized followed by a more extensive mathematical reasoning process.

Participant 07, on the other hand, is an example of an unsuccessful approach to this task. As presented in Fig. 12, there is a significant amount of activity within the theta frequency located in the left parietooccipital area (O1 and P7). This area has been presented as a key region within the theta frequency in recalling past information (Banich and Compton 2011). There is also activity evident in the left centrotemporal (T7 and FC5) area, a key location for processes of long-term memory. There is a similar level

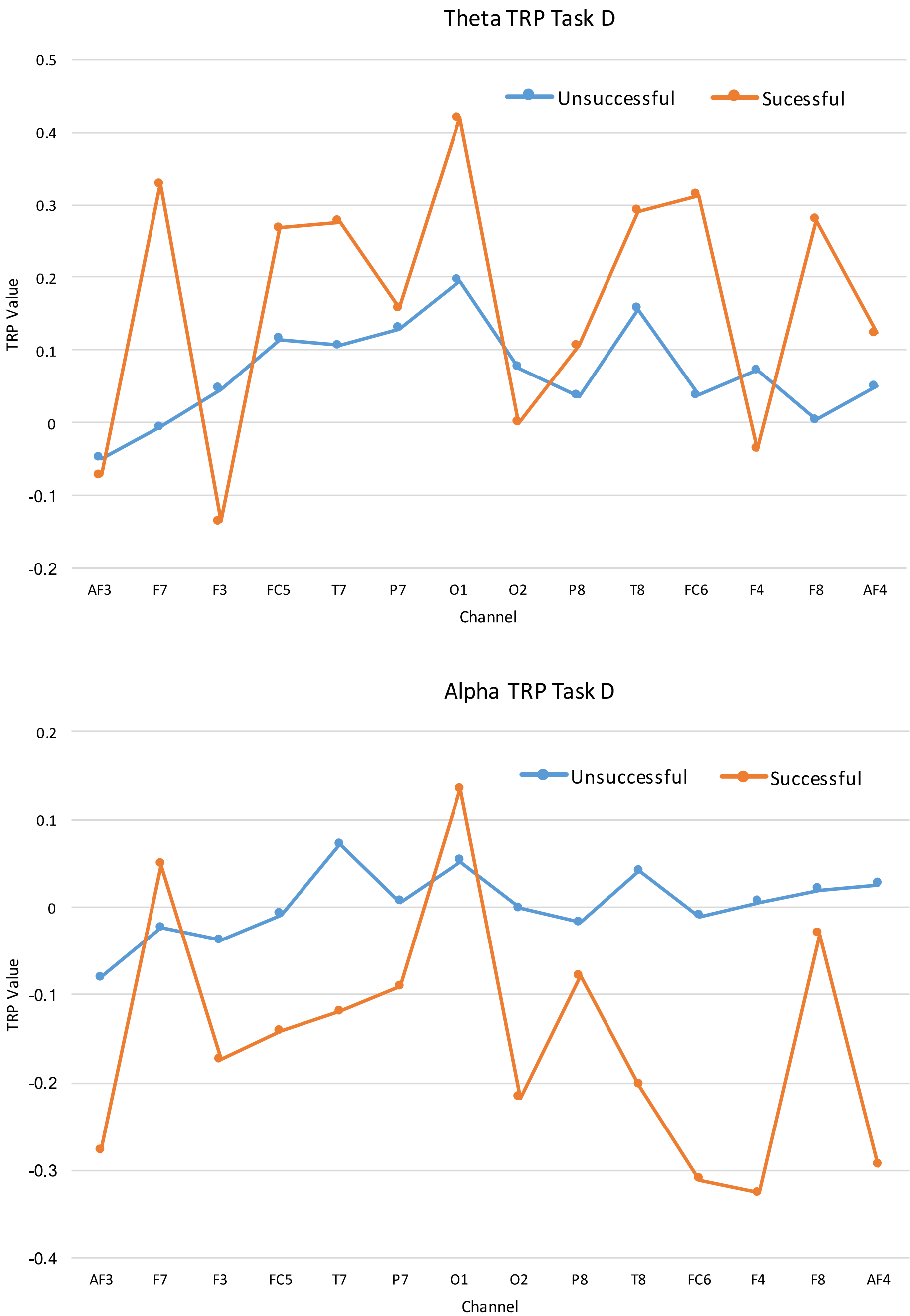


Fig. 10 Average task related power by performance for task D

of activity evident in the frontal areas within theta. These are key regions for working memory processes (Cabeza and Nyberg 2000).

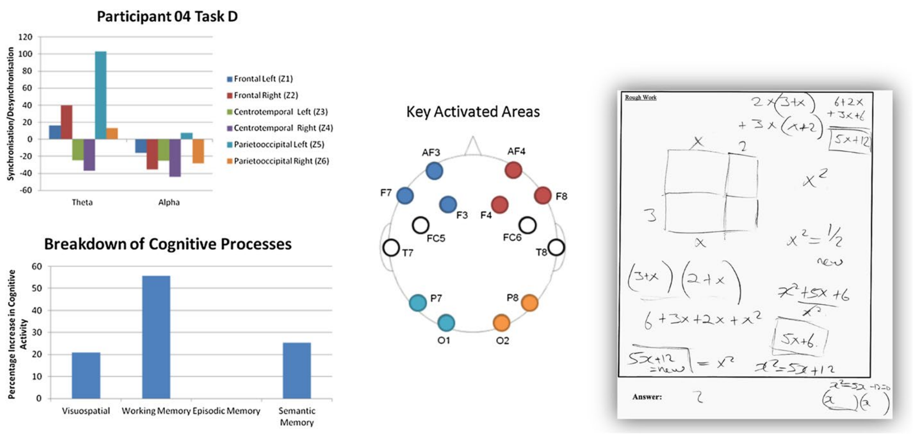


Fig. 11 Participant 04 algebraic task (D) cognitive data and solution

This pattern of activity indicates a focus on long-term recollection of information and given the high level of theta synchronization it is likely episodic in nature. The focus was potentially on trying to recall a past occurrence of a similar situation. There is also notable activity in the right parietooccipital area within the alpha frequency. This indicates the involvement of a visuospatial cognitive process during the reasoning episode (Cabeza and Nyberg 2000). Considering the solution produced and the nature of the theta synchronization it is probable that the visuospatial representation process used facilitated an episodic memory process. To clarify, the visual process may have activated an episodic recall for the participant where they recognized the situation but could not recall the exact process (semantically). This particular episode of cognitive activity displays a conceptualization that indicates a visuospatial approach. However, it appears that the mental model generated by the participant led to the access of inappropriate cognitive schema for the task at hand.

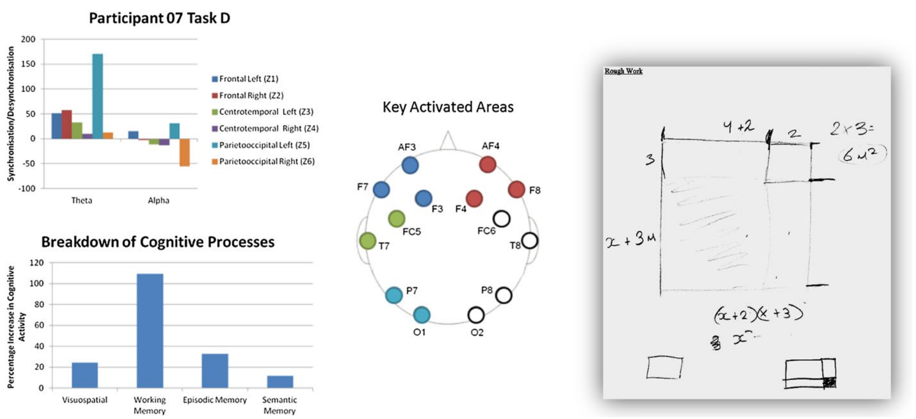


Fig. 12 Participant 07 algebraic task (D) cognitive data and solution

Perceptive evidence of conceptualization process

The previous section presented the objective evidence of the cognitive elements of the conceptualization processes utilized by students in this study. Another key element of the conceptualization process that must be considered is the perceptive stance of the individual and potential affective and experiential components of the conceptual frame. Representative individual cases were used for the focus of the cognitive analysis and qualitative data from the participant group will now be presented and discussed to triangulate these EEG findings with the perception of participants. It is important to note that subjectivity of interpretation was controlled for during these two stages of analysis as the EEG data was conducted by the principal author prior to any engagement with the interview data which was gathered, transcribed and initially coded by a research assistant (RA) that was involved in the study at the time.

All interview data were transcribed wholly and subjected to detailed analysis using NVivo. This met the consideration for ensuring a source of 'rich' data (Maxwell 2010) by including the entire verbal interaction between researcher and participant. Initial coding was undertaken by the RA and involved open coding (Bryman 2008) of the data which generated a significant amount of concepts. The lead author, following analysis of EEG data, contributed to this process with the RA. Following this stage, the authors refined these concepts into a series of larger categories (axial-coding) through a process of negotiated agreement (Campbell et al. 2013) where concepts were connected to create categories and any discrepancies were resolved through discussion. This ensured a trustworthy interpretation of themes within the data and limited potential bias from the lead author following analysis of the EEG data.

Overall, the predominant themes emerging from the data were the use of visuospatial approaches to conceptualization and a reliance on memorized processes, which support the EEG data presented in the previous section. In addition, affective components of the conceptualization process were uncovered in the form of prior educational experience. These themes will be presented and discussed in the proceeding sections where a sample of excerpts from the transcribed interview data will also be illustrated.

Visuospatial conceptualisation

Aligning with the EEG data, many of the participant's comments collected during the post task interviews suggest that a visuospatial approach was taken:

> I didn't even look at the question first I just looked at the pictures. And then I looked at the question, once I read the question I kind of had the answer in my head (Participant09)
> …my first port of call probably for all the problems was visualisation… (Participant 08)
> …. I think you need fairly good visualisation skills for them all…to understand what you been asked.… (Participant 04)

Participant 09 (Fig. 8) was asked to describe the overall approach adopted to the 15 tasks and there is again a strong indication of a visuospatial approach. The approach seems to take the form of a *"bottom-up"* style process where a visual representation is used to facilitate further reasoning (Tversky 2005).

…Just drawing them out basically, just putting it straight from what's in my head to paper and work from that… (Participant 09)

The intention to use a visual approach as an initial step in the problem solving process was also indicated by Participant 12:

(Researcher): So if you were to describe the strategy or approach you took how would you classify it?
Ah, visual. Am, I was just trying to directly, from what I could see, I was trying to draw out the amount of different ways it would work. Obviously I didn't draw it out too greatly here but … (Participant 12)

In this case, although the student adopted a visual conceptualisation, they lacked sufficient confidence in their ability to represent this. It is interesting to observe the number of visual conceptualisations participants alluded to using and the potential impact of attitudinal variables.

Visuospatial conceptualization facilitating memory and recall

Comments indicate a reliance on a visuospatial approach during the conceptualization stage of the tasks. This supported the student in progressing the solution for this particular task possibly through access to episodic memory as is alluded to in memory related data collected by EEG (Fig. 8). This bottom-up style of visual conceptualization is captured in other commentary as well.

Like, to answer you need both the question and the schematic if you like…I think you wouldn't have answered any of them without the diagram. (Participant 04)

Not only is the role of visuospatial cognition alluded to here, but a specific schematic type visual representation is referenced by Participant 04. This demonstrates a key role of spatial cognition in participants' conceptualization of the algebraic task.

This evidence of visuospatial conceptualization is also found in further commentary related to task D.

Well I got the garden that…increased it in size by three and two. And then I knew that the top right hand corner of the garden, it was going to be two meters by three meters… probably towards the end I was probably looking more into a quadratic equation… (Participant 07)
…I could picture that straight away but then….you kind of tend towards leaning towards doing it mathematically (Participant 08)
Again all the maths ones even the graphical ones I had to think about beforehand. Just in general there was a lot of them were you had to think before you, you could go way off" (Participant 11)

Here, Participant 11 alludes indirectly to the need to understand the task requirements before a solution is attainable. Again, this suggests the participant was trying to generate a mental model of the task to facilitate access to the most appropriate memory content for the task.

…I just drew the rectangle out and I knew it was extended by 3 m in one direction and 2 m in the other….I wasn't one hundred percent sure to work back from that… (Participant 07)

As can be seen in the commentary for this particular question there is an emphasis on utilizing a visual approach to build a representation of the problem. After this was built, Participant 07 was trying to recall a process from past experience (*"quadratic equation"*). This suggests that the visual approach used in building the conceptual model was utilized to aid in reconstruction from memory.

Episodic influence on recall

The content of the memory processes utilized by students during the conceptualization of the problems tended to focus on their past school experiences.

> Yeah maybe from school you kind of go into a bank of memory rather than maybe trying to work it out… (Participant 01)
> …I suppose it was…driven into me from that stage (school)… (Participant 02)
> Am what else is there, I can remember the lawn question I can remember doing questions like that in school for maths but I just can't remember how we done them (Participant 06)

In summary, the performance data indicates that these students in general displayed low levels of performance in the prescribed tasks. It must be remembered that these tasks were originally designed for 15 year old post-primary students so they should have been relatively straightforward to Year 3 undergraduates. Participants displayed the poorest levels of performance in the dual approach category of the tasks where there was more responsibility placed on the problem solver to conceptualize the task and approach. Cognitive data, gathered by means of EEG, revealed key roles for memory related (long-term and working memory) and visuospatial processes. These trends were also reflected in student commentary with experience from past schooling implicated as a key impacting variable on the memory related components of conceptualization. Overall, there are slight differences evident, in both selected tasks (A and D), between successful and unsuccessful individuals. Successful participants displayed higher activation in working memory sites in conjunction with enhanced use of visuospatial elements of cognition. Acknowledging the exploratory nature of this small scale study, triangulation of the resulting substantial data sets from the EEG headsets, the post-task interviews and the recorded solutions would suggest that there may be a relationship between the nature of the conceptualization of a problem and associated performance in the respective tasks that is worthy of further research. The triangulation of the different data sets would suggest that visuospatial cognition during the conceptualization of problems had a role to play in the performance of participants in these convergent educational problems.

Discussion

This paper presents an applied approach to exploring the mechanisms of problem conceptualization. The conceptualization procedure of an individual when engaging with a problem has a significant effect on the nature of proceeding performance. Therefore, enhancing understanding of this particular element of problem solving could have profound benefits to the design of instructional interventions in developing problem-solving skills within STEM education. The data presented in this paper outline a number of interesting elements relating to STEM students' approaches to conceptualizing convergent problems. Aligning with the theoretical definition of conceptualization introduced in this paper and the notion of problem construction posed by Mumford et al. (1994), this paper has provided empirical evidence of the complexity and variety of mental processes that occur in the early stages of problem solving, preceding the representation of the problem space. The precise elements of interaction and variables that are critical in this dynamic conceptualization process cannot be ascertained from this study due to its exploratory nature. However, the data do reveal a potential role for visuospatial cognition in this particular component of problem solving in convergent tasks.

Visuospatial cognitive processes were used by participants during the conceptualization and subsequent representation of the prescribed problems utilized in this study. The presence of spatial cognitive processes was not confined to the spatial style of tasks but were also utilized in mathematical type problems (as presented) and when dominant, led to increased levels of success for the student. Prior research by Wai et al. (2009), has demonstrated the significance of visuospatial ability in STEM and this paper has revealed a potential role for these capacities in the area of problem conceptualization. Given the dynamic nature of conceptualization where representations stored in cognitive schema and mental models are combined with perceptual input, situational/contextual constraints and affective individual variables, it is possible that these processes facilitate the active manipulation of cognitive representations and hence more effective access to appropriate memory systems. This active process of memory search and perceptual combination is the cornerstone of initial problem solving engagement (Todd and Gigerenzer 2000) and this itself is not a simple recollection of facts or data but an active constructional and relational process (Schacter and Addis 2007). This posited role of visuospatial cognition is supported by wider research from Hassabis and Maguire (2007) who posit a similar role indicating that visuospatial processes are key in reconstructing traces from episodic memory.

Building on this point, as memory is a constructive process rather than one of recollection, our ability to reconstruct cognitive representations stored in long-term schema is subject to potential interference (Barsalou 2009) and in particular can be influenced by attitudinal and emotional states (Kensinger and Schacter 2005). Evidence of this potential influence is provided in this study through participants reporting of the influence of school experience which often directed their problem-solving approaches. Beliefs generated by past experience in education have a lasting attitudinal and emotional effect on students and this demonstrates the need to consider these non-cognitive variables as key factors in problem conceptualization. This aligns with the critique of the information processing approaches' treatment of representation as abstracted and supports a more complex, situated view of problem conceptualization as key to convergent reasoning.

Conclusions and directions for future work

Overall trends between performance conditions were difficult to determine and this is likely due to the small sample size of the study. Some evidence in the data indicate increased load on working memory resources and with evidence of visuospatial and long-term memory related cognition (see Figs. 7 and 10). These data, although primarily statistically non-significant, were presented to demonstrate the thoroughness of data exploration and to safeguard against fallacious conclusions being drawn by narrowing the degrees of freedom afforded to the researcher (Wicherts et al. 2016). Given the exploratory nature of the research and acknowledging *argumentum ad ignorantiam* as described by Walton (1999), it was decided to adopt an approach inspired by single cases (Ledford and Gast 2018). Future work will build on this approach with a focus on larger samples.

Specific to STEM education, the data demonstrate a distinct role that visuospatial cognition has during the conceptualization of convergent problems. This study presents an initial exploration into the cognitive mechanisms underlying the process of problem conceptualization. In addition, it presents the specific role of visuospatial cognition in the conceptualization of problems by facilitating students' access to problem schema in long-term memory and subsequent manipulation in working memory. While spatial skills have been found to be a significant predictor of success in STEM education (Sorby 2009; Wai et al. 2009) there is no clear etiological understanding of this relationship (Delahunty et al. 2016). In order to effectively investigate this potential relationship further research is required.

In future work, it may be more prudent to consider alternative approaches including attitudinal variables such as self-efficacy or attitudes towards STEM. In addition, the inclusion of a psychometric measure of spatial ability would allow greater insight into the variance in this ability and conceptualization success. A key limitation of this paper that will need further work is the decision on the time period allocated to problem conceptualization in further neuropsychological approaches to the study of this phenomenon. The decision here was motivated by the exploratory nature of the study but in order to further refine this style of approach future work will need to consider more temporally accurate approaches such as ERP to isolate precise moments of cognitive change during the initial stages of the task.

In addition, future work should also look into the inclusion of a think aloud video protocol to complement the EEG dataset. While video recording was implemented here to aid in the assessment of performance, the participants did not concurrently verbalize their approaches to any great extent, and hence provided a limited lens on conceptualization. This study was predominantly concerned with the internal cognitive mechanisms of conceptualization which were hypothesized to be tacit to the individual and difficult to communicate, hence the researchers did not provide explicit instruction to do so during data collection so as to avoid any possible interference with the EEG recording. Future work could benefit from this inclusion and explicit instruction to verbalize thoughts during the problem-solving epoch providing the potential for real-time triangulation of tacit cognitive and overt verbal phenomena related to problem conceptualization.

Acknowledgements This study was approved by the Faculty of Science and Engineering research ethics committee at the University of Limerick (Ref: 2012_12_24_S&E).

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix 1

Task A

A TV measures 26 inches across the diagonal of the screen, which is 20 inches wide. What is the height of the screen? (Answer to the nearest half inch)

ANS: 16.5 Inches

Task B

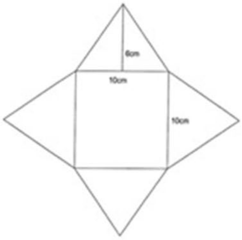
The recommended sizes for goals for mini-soccer are 3.66m wide by 1.83m high.

Approximately how much light weight tubing would be used to construct the front frame of the goal? (Answer to one decimal place)

ANS: 7.3m

Task C

The diagram shows the net of a pyramidal chocolate box. How many square centimetres of cardboard is needed to make the box?



ANS: 220cm squared

Task D

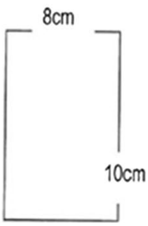
A square lawn was extended in width by 2m and in length by 3m. The area of the new lawn is twice as big as the area of the old lawn. What are the measurements of the old lawn?

ANS: 6m x 6m

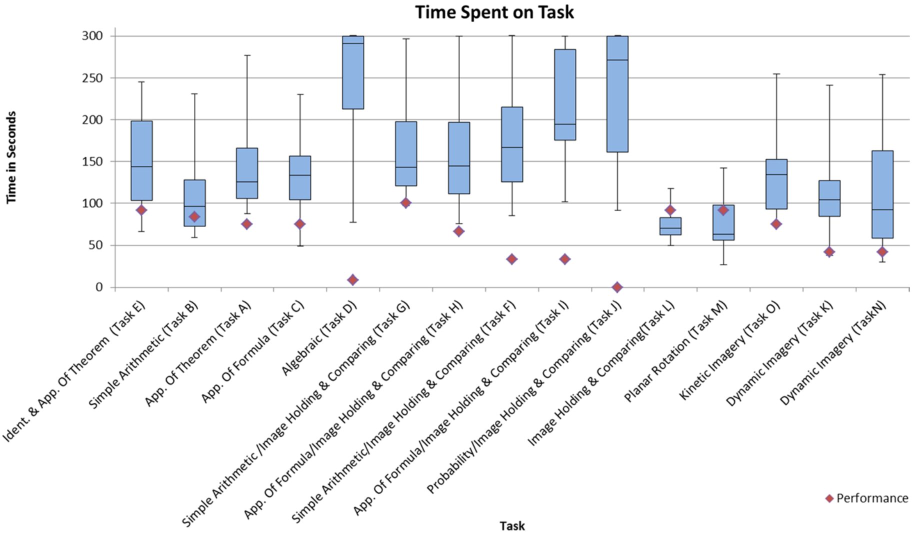
Task E

What is the length of the diagonal of the largest square-shaped piece of paper you can tear from the sheet of paper in the diagram below? (Answer to 1 decimal place)

ANS: 11.3cm



Appendix 2



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