DBmmWave: Chance-Constrained Joint AP Deployment and Beam Steering in mmWave Networks With Coverage Probability Constraints

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***Abstract*—At millimeter wave (mmWave) frequencies, high attenuation in propagation and severe blockage by obstacles lead to high uncertainty in the availability of links between access points (APs) and mobile devices. Considering this uncertainty in combination with the inherent user location uncertainty, we propose DBmmWave, as the first chance-constrained stochastic programming (CCSP) framework for joint AP *d*eployment and *b*eam steering in *mmWave* networks. Extensive results are generated to quantify the impact of channel conditions and user distribution on the network coverage and the required number of mmWave APs. Our results demonstrate the effectiveness of CCSP in handling the trade-off between the number of APs and the network coverage.**

***Index Terms*—MmWave networks, AP deployment, beam steering, coverage probability, chance-constrained stochastic optimization.**

I. Introduction

MILLIMETER wave (mmWave) communication is a promising solution for providing high-capacity wireless access to regions with high traffic demands [1], [2]. However, several technical challenges need to be overcome in order to exploit the benefits of mmWave network deployments. One prominent challenge is the high attenuation in mmWave propagation and severe blockage of mmWave links with obstacles [3], [4]. To overcome these challenges, mmWave systems typically use a large number of antenna elements both at the access points (APs) and mobile devices (MDs), which leads to highly directional communications [1]. The propagation features of mmWave frequencies, the directionality of mmWave links, and the stochastic blockage of mmWave signals, combined with the stochasticity in the user locations render *AP deployment* and *beam steering* critical challenges in designing mmWave networks to provide adequate coverage.

Recent thorough contributions on AP deployment in mmWave networks were reported in [5]–[8] and the references therein. However, they neither consider the beam steering problem nor account for the uncertainty in the user locations. In [5], the authors proposed an automated scheme for placing mmWave APs and gathering their line-of-sight coverage statistics to assist in modeling small-cell mmWave access networks. Considering the deafness and blockage problems in mmWave networks, the authors in [6], [7] proposed distributed schemes for association and relaying that improve the network throughput. Another AP deployment scheme was proposed in [8], where it was assumed that APs always direct their beams in one fixed direction and considered a fixed set of MDs with static locations.

Our Contributions: Considering the uncertainty in the availability of mmWave links between APs and MDs, combined with user location uncertainty, in this letter we propose a novel chance-constrained stochastic programming/optimization (CCSP) framework for joint AP **d**eployment and **b**eam steering in **mmWave** networks, called DBmmWave. The proposed DBmmWave aims at minimizing the required number of mmWave APs to achieve a minimum *network-wide* coverage probability of β, where β represents the requested QoS level. The *network-wide* coverage probability constraint formulated in this letter is in contrast to the *per-user* coverage probability constraint in [8]. Instead of formulating a constraint for each user to ensure that individual users are covered with a minimum probability of (say, β), we formulate a single constraint for the entire mmWave network that ensures that an arbitrarily selected user will be covered with a minimum probability of β. Formulating the stochastic coverage metric using the *per-user* coverage probability is the simplest approach (in terms of the mathematical formulation) to ensure that a network is probabilistically covered with a

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minimum probability of β. However, using the *network-wide* coverage probability significantly reduces both: (i) The computational complexity of solving the stochastic mathematical formulation and (ii) the required number of mmWave APs to achieve the requested stochastic coverage level. Using various reformulation techniques, we equivalently reformulate our stochastic program as a binary linear program (BLP). Finally, we numerically analyze the performance of DBmmWave under various system settings. To the best of our knowledge, DBmmWave is the first robust optimization framework for joint AP deployment and beam steering in mmWave networks that mitigates the negative impacts of channel availability uncertainty and user location uncertainty.

Stochastic Optimization: Stochastic optimization has been recently used to model several resource allocation problems in uncertain wireless networks [9]–[13]. In [9]–[11], several resource allocation problems in virtualized and software-defined wireless networks have been studied under different types of uncertainties. In [12], [13], the authors formulated stochastic resource allocation problems in LTE-A and LTE-U networks, respectively. CCSP is standard terminology in the stochastic optimization literature, used to describe a class of problems in which one (or more) of the constraints are probabilistic in nature.

Paper Organization: The rest of this letter is organized as follows. In Section II, we describe our system model and state our problem. The DBmmWave framework is formulated and analyzed in Section III. In Section IV, we discuss the corresponding numerical results. Finally, we conclude this letter in Section V and provide directions for future research.

II. System Model and Problem Statement

A. System Model

We consider a three-dimensional geographical area with a set $\mathcal{N} = \{1, 2, \ldots, N\}$ of candidate locations for deploying mmWave APs on the ceiling to cover the floor, as depicted in Fig. 1. In this context, r_d denotes the radius of the coverage area and r_b denotes the radius of the AP beam footprint.[1] The floor is divided into $K = \frac{r_d}{2\, r_b} + \frac{1}{2}$ annuli;[2] the *i*th annulus consists of M_i circles, where $M_1 = 1$ and $M_i, 2 \le i \le K$, is given by:

$$M_i = \left\lfloor \frac{2\pi}{2\sin^{-1}\left(\frac{r_b}{r_d - 2\, r_b(K-i) - r_b}\right)} \right\rfloor = \left\lfloor \frac{\pi}{\sin^{-1}\left(\frac{1}{2(i-1)}\right)} \right\rfloor. \quad (1)$$

We denote the set of circular areas by $\mathcal{K}$, where $|\mathcal{K}| = 1 + \sum_{i=2}^{K} M_i$. The *k*th circular area in $\mathcal{K}$, denoted by A_k, is represented by a pair (i_k, j_k), as illustrated in Fig. 1. MDs are distributed in the geographical area according to the distribution $f_Z(z)$. The link between a mmWave AP placed at location $n \in \mathcal{N}$ and the *k*th circular area, $k \in \mathcal{K}$, if one of the AP beams is steered to cover A_k, is only available with probability p_{nk}. The maximum number of beams that a mmWave AP can have is denoted by *B*.

[1]Note that we approximate the footprint of a mmWave AP by a circle, even if the AP covers a region that does not directly lie beneath it.

[2]Note that *K* needs to be an integer, and hence $r_d = (2K-1)r_b$ cannot be an even multiple of r_b.

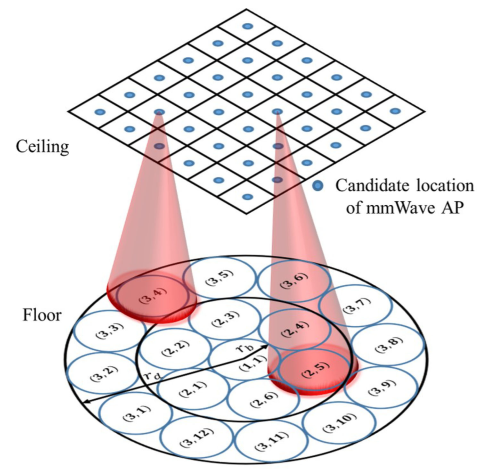

Fig. 1. Illustration of the system model considered in DBmmWave.

B. Problem Statement

Given $\mathcal{N}$, $\mathcal{K}$, B, β, $f_Z(z)$, and $p_{nk}, n \in \mathcal{N}, k \in \mathcal{K}$, we answer the following questions jointly:

1) What is the minimum number of required mmWave APs?
2) How to deploy them optimally?
3) How to steer their beams optimally?

while ensuring that an arbitrarily chosen user within the geographical area of interest will be covered with a probability $\ge \beta \in (0, 1)$.

III. DBmmWave Framework

A. Problem Formulation

Let $y_{nk}, n \in \mathcal{N}, k \in \mathcal{K}$, be binary decision variables; y_{nk} equals one if a mmWave AP is placed at location *n* and one of its beams is steered to cover region *k*, and it equals zero otherwise. The optimal values of $y_{nk}, n \in \mathcal{N}, k \in \mathcal{K}$ form the answer to the third question in Section II-B. Let $\mathbb{P}_{\text{cov}}$ be the network-wide coverage probability, i.e., the probability that an arbitrarily selected user in the network will be covered. Then, the joint AP deployment and beam steering problem can be formulated as follows.

Problem 1: Joint AP Deployment and Beam Steering

$$\underset{\{y_{nk}, n \in \mathcal{N}, k \in \mathcal{K}\}}{\text{minimize}} \quad \sum_{n \in \mathcal{N}} \mathbb{1}_{\{\sum_{k \in \mathcal{K}} y_{nk} \ge 1\}} \quad (2)$$

$$\text{subject to:} \quad \mathbb{P}_{\text{cov}} \ge \beta \quad (3)$$

$$\sum_{k \in \mathcal{K}} y_{nk} \le B, \forall n \in \mathcal{N} \quad (4)$$

$$y_{nk} \in \{0, 1\}, \forall n \in \mathcal{N}, \forall k \in \mathcal{K} \quad (5)$$

where $\mathbb{1}_{\{\cdot\}}$ is an indicator function; $\mathbb{1}_{\{\cdot\}}$ equals one if $\{\cdot\}$ is satisfied and zero otherwise, and $\beta \in (0, 1)$. The optimal objective function value in Problem 1 is the answer to the first question in Section II-B.

B. Coverage Probability Constraint

As stated earlier, the coverage probability is defined as the probability that an arbitrarily picked user lies within a circular area that is covered by at least one active beam. Hence, the

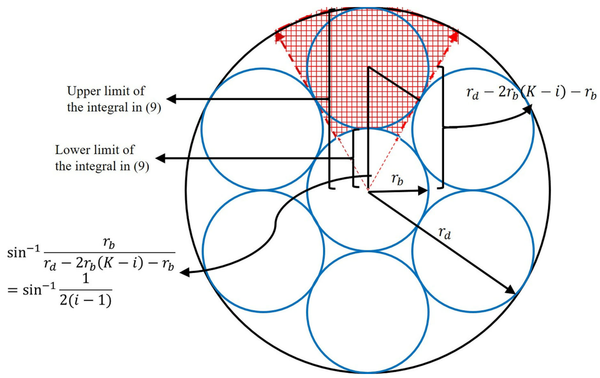

Fig. 2. Illustration of the coverage model considered in DBmmWave.

coverage probability when the arbitrarily picked user is located at z can be defined as:

$$\mathbb{P}_{\text{cov}}^{(z)} = \mathbb{E}\left[1 - \prod_{n \in \mathcal{N}} \left(1 - \delta_{nk^{(z)}} y_{nk}\right)\right] \tag{2}$$

where $k^{(z)}$ is the index of the circular area that contains location z. $\delta_{nk^{(z)}}$ equals one if there is no blockage between the AP candidate location n and the area $A_{k^{(z)}}$, and it equals zero otherwise. It is also noted that the expectation in (2) is over blockages, which are assumed to be independent across links. Therefore,

$$\mathbb{P}_{\text{cov}}^{(z)} = 1 - \prod_{n \in \mathcal{N}} \left(1 - p_{nk^{(z)}} y_{nk}\right) \tag{3}$$

where $p_{nk^{(z)}}$ is p_{nk} when $k = k^{(z)}$ and p_{nk} is defined in Section II-A. The unconditioned coverage probability is computed by taking the user distribution $f_Z(z)$ into consideration as follows:

$$\mathbb{P}_{\text{cov}} = 1 - \sum_{k \in \mathcal{K}} \left(\int_{A_k} f_Z(z)\, \mathrm{d}z \prod_{n \in \mathcal{N}} (1 - p_{nk}\ y_{nk}) \right). \tag{4}$$

The integration $\int_{A_k} f_Z(z)\ \mathrm{d}z$ over each circular area is upper-bounded by the integration over the sector enclosed in the i_k-th annulus between the two tangents of A_k, $2 \leq i_k \leq K$ (the red-shaded region in Fig. 2).[3] This upper bound is used in our analysis to enhance tractability. This enables us to use the probability distribution $f_{R_z}(r_z)$, where $R_z = \|z\|$, as explained in (9), as shown at the bottom of this page. The term $(2\sin^{-1}(\frac{1}{2(i_k-1)}))$ in (9) represents the angle of the sector enclosed in the i_k-th annulus between the two tangents of A_k, as shown in Fig. 2. The term a_{i_k} is added to ensure that $\sum_{k\in\mathcal{K}} \int_{A_k} f_Z(z) = 1$ (equivalently, to ensure gap-free coverage). To achieve this, a_{i_k} is computed as $(2\pi - 2M_{i_k}\sin^{-1}(\frac{1}{2(i_k-1)}))/M_{i_k}$. In Section IV, the following two user distributions are investigated.

- *Truncated Gaussian distribution*, where

$$f_{R_z}^{\text{gaus}}(r_z) = \frac{r_z \exp\left(-\frac{r_z^2}{2\sigma^2}\right)}{\sigma^2\left(1 - \exp\left(-\frac{r_d^2}{2\sigma^2}\right)\right)} \tag{5}$$

  and σ^2 represents the variance of the user distribution.
- *Uniform distribution*, where

$$f_{R_z}^{\text{unif}}(r_z) = 2r_z/r_d^2. \tag{6}$$

C. Equivalent Binary Linear Program (BLP)

First, note that the objective function of Problem 1 is nonlinear. Yet, it can be represented in a linear form by introducing new binary decision variables, $x_n \overset{\text{def}}{=} \mathbb{1}_{\{\sum_{k\in\mathcal{K}} y_{nk} \geq 1\}}, \forall n \in \mathcal{N}$, and reformulating the indicator function as follows [14][4]:

- The statement "if $\sum_{k\in\mathcal{K}} y_{nk} \geq 1$ then $x_n = 1$" can be equivalently and linearly reformulated as:

$$\sum_{k\in\mathcal{K}} y_{nk} - (M+\epsilon)x_n \leq 1 - \epsilon \tag{7}$$

  where M is an upper bound of $\sum_{k\in\mathcal{K}} y_{nk} - 1$ and $\epsilon > 0$ is a small tolerance beyond which we regard the constraint as having been broken. Selecting M and ϵ to be $B-1$ and 1, respectively, (7) reduces to $\sum_{k\in\mathcal{K}} y_{nk} \leq B\ x_n$.
- The statement "if $x_n = 1$ then $\sum_{k\in\mathcal{K}} y_{nk} \geq 1$" can be equivalently and linearly reformulated as[5]:

$$\sum_{k\in\mathcal{K}} y_{nk} + m\ x_n \geq m + 1 \tag{8}$$

  where m is a lower bound of $\sum_{k\in\mathcal{K}} y_{nk} - 1$. Selecting m to be -1, (8) reduces to $\sum_{k\in\mathcal{K}} y_{nk} \geq x_n$.

Based on the above, it follows that:

$$x_n = \mathbb{1}_{\{\sum_{k\in\mathcal{K}} y_{nk} \geq 1\}} \iff x_n \leq \sum_{k\in\mathcal{K}} y_{nk} \leq B\ x_n, \forall n \in \mathcal{N}.$$

Second, the coverage probability expression, $\mathbb{P}_{\text{cov}}$, includes the term $\mathbb{P} \triangleq \prod_{n\in\mathcal{N}}(1 - p_{nk}\ y_{nk})$, which is nonlinear in the decision variables $y_{nk}, n \in \mathcal{N}, k \in \mathcal{K}$. Expanding $\mathbb{P}$, we can see that the nonlinear terms in $\mathbb{P}$ are in the form of products of binary decision variables. For example, if $N = 3$, $\mathbb{P}$ can be expressed as:

$$\mathbb{P} = 1 - \sum_{i=1}^{3} p_{ik}\ y_{ik} + p_{1k}\ p_{2k}\ y_{1k}\ y_{2k} + p_{1k}\ p_{3k}\ y_{1k}\ y_{3k} + p_{2k}\ p_{3k}\ y_{2k}\ y_{3k} - \prod_{i=1}^{3} p_{ik}\ y_{ik}. \tag{10}$$

[3]We implicitly assume that the footprint of an AP beam covers a slightly greater area than a circular area on the floor. Hence, the red-shaded regions in Fig. 2 can be covered by the AP beams.

[4]The optimal values of x_n constitute the answer to the second question in Section II-B. They determine the subset of AP candidate locations on the ceiling that will be used to deploy mmWave APs.

[5]Note that this condition is equivalent to $\sum_{k\in\mathcal{K}} y_{nk} = 0 \Longrightarrow x_n = 0$, which is already enforced by the objective function, since it aims at minimizing the number of mmWave APs. Hence, this constraint is redundant.

$$\int_{A_k} f_Z(z)\,\mathrm{d}z \leq \left(a_{i_k} + 2\sin^{-1}\left(\frac{1}{2(i_k-1)}\right)\right) \int_{r_d - 2r_b(K-i_k+1)}^{r_d - 2r_b(K-i_k)} \frac{f_{R_z}(r_z)}{2\pi}\,\mathrm{d}r_z = \left(a_{i_k} + 2\sin^{-1}\left(\frac{1}{2(i_k-1)}\right)\right) \int_{2r_b(i_k-1.5)}^{2r_b(i_k-0.5)} \frac{f_{R_z}(r_z)}{2\pi}\,\mathrm{d}r_z \tag{9}$$

To linearize a product of N binary decision variables, e.g., $\prod_{i=1}^{N} y_{ik}$, we introduce a new auxiliary non-negative decision variable (let us denote it by y_k), replace $\prod_{i=1}^{N} y_{ik}$ by y_k, and add the following $(N + 1)$ constraints:

$$y_k \leq y_{ik}, \forall i \in \{1, \ldots, N\} \text{ and } y_k \geq \sum_{i=1}^{N} y_{ik} - (N - 1). \quad (11)$$

After reformulating the indicator function and $\mathbb{P}$, as explained above, Problem 1 becomes a BLP.

IV. Performance Evaluation

A. Evaluation Setup

Assuming an open indoor environment such as a sport center, r_d and r_b were selected to be 55 and 5 meters, respectively. Based on these values, we calculated the number of circular areas, as explained in Section II, and found that $|\mathcal{K}| = 92$. The maximum number of beams that a mmWave AP can have, B, is varied between one and four.[6] Two different user distribution were considered: (i) Gaussian distribution with mean $\mu = 0$ and variance $\sigma = 10$ and (ii) uniform distribution. The probabilities of link availabilities were calculated assuming that the channel between an AP and a user follows a Rician distributed small-scale fading, with fading parameter $K_{\text{Rice}} = 7$ dB. A link is considered unavailable if the SNR over this link is below 3 dB. The average SNR, denoted by $\bar{\gamma}$, varies from one link (AP-user) to another, following a uniform distribution [15]. It is also noted here that the proposed stochastic AP deployment and beam steering formulation can use any blockage model where blockages are independent across links such as [16] and can use other fading models as well.

While linearizing Problem 1 above, we assumed that, for each user, there are only three AP candidate locations that can cover it. These three AP locations form the best (most available) AP-user links, i.e., links with the highest $p_{n,k}$ values for a given k. We selected different values of N, the number of AP candidate locations, to better characterize the behavior of the system. The N AP candidate locations are distributed as a $\sqrt{N} \times \sqrt{N}$ grid on the ceiling. DBmmWave is evaluated in terms of the required number of APs for different coverage probabilities β. The optimization problem is solved using CPLEX.

B. Numerical Results

Fig. 3 shows the number of required APs as a function of the minimum required coverage probability (β). In this figure, the number of AP candidate locations was chosen to be $N = 46$, the users were assumed to be distributed according to a Gaussian distribution, and $\bar{\gamma}$ was sampled from a uniform distribution on [0, 30] dB. It can be seen that as β increases, the number of APs needed to satisfy the coverage demand increases almost exponentially. Hence, reducing β by a small value reduces the required number of APs significantly. This is the power of stochastic resource allocation compared to deterministic resource allocation. Furthermore, increasing the number of beams at each AP reduces the required number of APs. This is expected, as having more beams at an AP allows it to cover more users.

[6]Note that the proposed framework and the trends in the numerical results remain the same for any range of values of B.

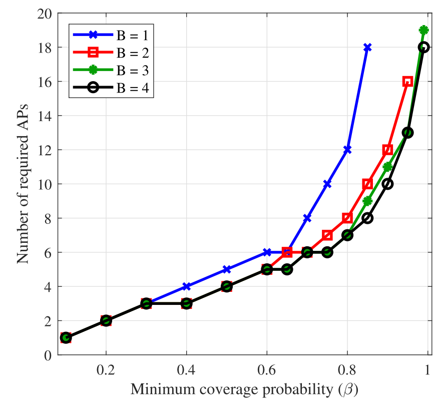

Fig. 3. Number of required APs vs. minimum coverage probability for Gaussian distributed users and different number of beams ($N = 46$ and $\bar{\gamma} \in$ [0, 30] dB).

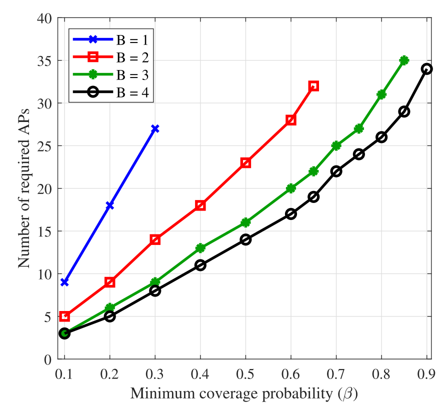

Fig. 4. Number of required APs vs. minimum coverage probability for uniformly distributed users and different number of beams ($N = 46$ and $\bar{\gamma} \in$ [0, 30] dB).

Fig. 4 is similar to Fig. 3, but assumes that the users are uniformly distributed. Both figures show similar trends. However, the number of APs required to satisfy a certain coverage probability is higher when the users are uniformly distributed. In the case of Gaussian distribution, users are clustered in the geographical area (in contrast to the case of uniform distribution). This clustering results in reducing the number of required APs.

In Fig. 5, we study the effect of $\bar{\gamma}$ on the performance of DBmmWave. For each type of user distributions, a single value of B is considered with two ranges of $\bar{\gamma}$ (namely, [0, 10] dB and [0, 30] dB). It can be seen that the lower the average SNR ($\bar{\gamma}$) the more APs are needed to meet the required coverage probability. Note that $\bar{\gamma}$ affects the required number of APs much more for larger values of β. For example, the number of required APs is almost doubled when $\bar{\gamma} \in$ [0, 10] dB compared to the case when $\bar{\gamma} \in$ [0, 30] dB for $B = 1$ and $\beta = 0.65$.

Finally, Fig. 6 illustrates the effect of the number of AP candidate locations on the number of required APs to meet a

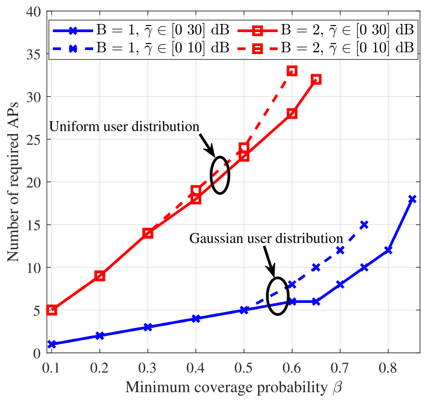

Fig. 5. Number of required APs vs. minimum coverage probability for different number of beams and $\bar{\gamma}$ ($N = 46$).

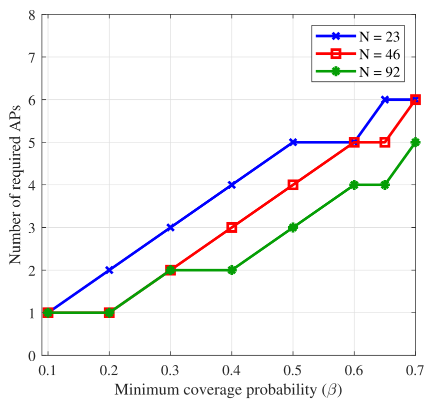

Fig. 6. Number of required APs vs. minimum coverage probability for Gaussian distributed users and different values of N ($B = 2$ and $\bar{\gamma} \in$ [0, 10] dB).

certain coverage probability. It can be seen that as N increases the number of required APs decreases. This is due to the fact that increasing N expands the feasibility region of the allocation problem, which opens the room for better solutions (i.e., with lower objective function value). Hence, if we afford increasing the complexity of running our optimization problem, the actual required number of APs might be reduced.

V. Conclusion and Future Research

To remedy the uncertainty in mmWave links availability and user locations, in this letter we developed DBmmWave, a novel chance-constrained stochastic optimization framework for joint AP **d**eployment and **b**eam steering in **mmWave** networks operating under coverage probability constraints. To the best of our knowledge, DBmmWave is the first robust optimization framework for joint AP deployment and beam steering in mmWave networks that mitigates the negative impacts of channel availability uncertainty and user location uncertainty.

Numerical results were generated to study the impact of channel conditions and user distribution on the network coverage and the required number of mmWave APs in DBmmWave. Corroborating our analytical derivations, our numerical results demonstrated the optimal behavior of DBmmWave under various system settings. They showed the effectiveness of DBmmWave in handling the trade-off between the number of APs and the network coverage. They also illustrated the advantages of stochastic network deployment and resource allocation compared to the deterministic approaches.

To achieve a better network deployment, in terms of reducing the cost and increasing the coverage probability, we aim to modify DBmmWave to support adaptive beam steering, where beam steering adapts to the distribution of MDs.

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