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Optimisation of heterogeneous wave energy converter arrays: A control co-design strategy

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ABSTRACT

The commercial development and deployment of wave energy converters (WECs) will require arranging these devices in groups known as 'arrays', similar to the deployment other large-scale renewable energy systems, such as wind farms, or tidal arrays. This study explores a novel control co-design (CCD) strategy for heterogeneous arrays of point absorber-type WECs, focusing on the simultaneous optimisation of buoy hull geometry and array layout to harness multi-directional wind and swell wave energy. The WEC array operates under a newly developed global centralised control algorithm, which supports displacement constraints, but allows for the assessment of array performance in the frequency domain. This approach has the potential to significantly speed up the numerical solution of the control co-design optimisation problem, compared to more traditional time-domain-based methods. The array optimisation problem is solved using a global optimisation method. The performance function aims to optimise the positive network effect of interactions between devices in the array, while simultaneously considering cost issues, quantified by device sizes. The investigation identifies optimal device geometry and array layouts for clusters of three, four, and five WECs, in two different wave climates: Irish and Portuguese coasts, allowing the sensitivity of optimal solutions to different wave climates to be studied.

1. Introduction

Innovative, cost-effective renewable energy solutions are urgently needed to address rising global energy demands and the challenges of climate change. The International Energy Outlook 2023 (IEO2023) predicts that global energy consumption will increase by 30% to 76% from 2022 levels by 2050, with renewable sources expected to meet 54% of total demand by then [1]. Among renewable options, wave energy is considered, as a currently unexploited resources, to have considerable potential [2,3].

Wave energy converters (WECs) are promising, but still underdeveloped technologies, for harnessing ocean energy [4]. The WECs discussed in this article are point absorbers [5,6], functioning as heaving buoys that oscillate with waves to generate electrical power via an attached power take-off (PTO) system (see Fig. 1). However, large-scale commercial deployment of WEC farms, or 'arrays' faces considerable challenges, particularly in optimising array configurations [7]. Studies suggest that heterogeneous arrays, where heaving buoys vary in size, can improve power generation compared to homogeneous arrays, primarily due to positive interactions where devices absorb the waves radiated by each other [8,9].

Designing effective heterogeneous WEC arrays requires advanced strategies to maximise energy capture while minimising cost of manufacturing. The conventional metric for evaluating renewable energy costs is the levelized cost of energy (LCoE), defined as the ratio of total expenditure (capital and operational costs, or CapEx and OpEx) to the energy generated over the device lifetime [10]:

$$LCoE = \frac{CapEx + OpEx}{Generated\,Power} \tag{1}$$

Achieving a competitive LCoE, compared to other renewable energy sources, is critical for advancing the Technology Performance Levels (TPL) of WEC technologies [11]. This requires novel strategies that integrate both engineering and operational factors to optimise system components. Research has demonstrated that simultaneous optimisation of control and layout in WEC arrays significantly enhances power generation compared to isolated devices [12]. Additionally, configuring

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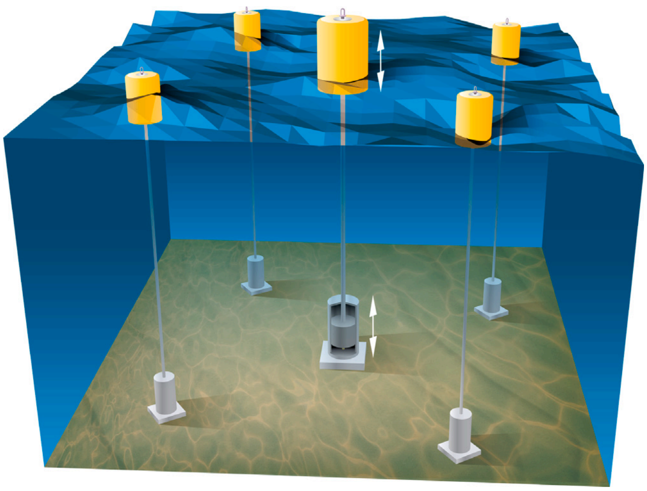

Fig. 1. Heterogeneous array of wave energy converters.

WECs into arrays not only boosts power capture potential but also reduces the capital costs (CapEx) of the devices, due to the economy of scale.

While initial studies on the interaction of point absorber WECs for enhancing power absorption started nearly five decades ago [13], determining the optimal layout of WEC arrays, configuring device shapes, and selecting control strategies remain significant challenges. Preliminary numerical studies have explored the benefits of integrating WECs of various sizes into heterogeneous arrays in the frequency domain in [14], although control optimisation is not addressed. Researchers have examined several aspects of WEC array optimisation, including the impact of different spatial configurations [15], wave directions [16,17], and parameters influencing array layout in varying sea states [18]. Studies indicate that optimal geometries and layouts can vary based on different control strategies [12,19]. Additionally, the effects of layouts, device spacing, and incoming wave directions on the power output of WEC arrays are assessed in [20]. The optimisation of array layout, considering the number of WECs as a variable, is explored in [21], while the independent optimisation of cylindrical buoy dimensions within arrays is examined in [22]. A new methodology for assessing the performance of WEC arrays in disturbed wave fields is proposed in [23].

At the same time, control co-design (CCD) for WEC arrays, which integrates control strategies to maximise power within the optimisation process from the outset, has garnered attention for addressing WEC array optimisation challenges in [12,24]. Studies demonstrate CCD influence on optimising various WEC parameters, such as the spectral control method combined with PTO constraint optimisation [25]. A comprehensive CCD approach, integrating advanced control algorithms with array layout and mooring design, is presented in [26]. The article [27] presents an open-source MATLAB toolbox for wave energy converter optimisation with adaptable co-design, power take-off modelling, and customisable constraints. A co-design optimisation approach, introduced in [28], incorporates the effects of all subsystems using an outer and an inner optimisation loop to achieve a fully optimised design of an oscillating surge wave energy converter (OSWEC). Several algorithms have been used to optimise WEC layouts, including using Differential Evolution (DE) algorithms for energy capture and interference reduction [29], and real-time control strategies to enhance energy output and manage loads [30]. However, optimising heterogeneous arrays involves numerous parameters (e.g., shape, size, draft, layout), making it computationally intensive, particularly when performance evaluations is conducted in the time domain, as most studies do [8,9,12,19].

This paper introduces a new methodology that integrates CCD for optimising the geometry and layout of heterogeneous WEC arrays, considering both physical constraints and economic considerations. Specifically, this study presents a novel CCD strategy focused on simultaneously optimising device dimensions and array configuration. The proposed algorithm evaluates WEC array performance in the frequency domain, utilising a new centralised constrained control method, which significantly speeds up computation compared to time-domain simulation. However, the inclusion of a nonlinear model, such as one accounting for slamming effects, is constrained by computational limitations, as frequency-domain analysis is challenging for nonlinear system models. Additionally, the analysis is restricted to system operation within the power production region, excluding extreme wave conditions, slamming phenomena, and similar effects. The presented model is a simplified framework designed to provide qualitative insights rather than a comprehensive simulation of real-world scenarios. It focuses on fundamental linear hydrodynamics and does not account for uncertainties in wave conditions, material properties, or device interactions. While the study assumes a specific wave climate, the application of Froude scaling allows for an understanding of the device operation at different scales. The methodology is applied to an optimisation problem inspired by the principles of LCoE, aiming to maximise positive device interactions while managing costs by penalising large WEC sizes. The employed differential evolution method for global optimisation accounts for multi-directional wave propagation with panchromatic spectra, for both swell and wind waves, reflecting realistic conditions off the Irish and Portuguese coasts. This allows for an assessment of the sensitivity of optimal arrays to regional differences. Optimal layouts and shapes for heterogeneous arrays with three, four and five devices are obtained.

The paper is structured as follows: Section 2 outlines the mathematical model for WEC array hydrodynamics, followed by performance metrics for array optimisation in Section 3. Section 4 details the centralised constrained control method, while the CCD problem is introduced in Section 5. Section 6 presents case studies based on realistic wave climates in Ireland and Portugal. Results of the optimisation algorithm are presented in Section 7, followed by conclusions in Section 8.

2. Mathematical model for an array of WECs

In this research, it is assumed that the displacement of each WEC within the array is limited to a single degree of freedom in the heave direction. The mathematical model is based on linear hydrodynamic theory, specifically Cummins' equation [31]. While the use of a linear model is a simplification, applicable only to relatively small displacements, it enables quick estimation of the motion of heaving buoy WECs in waves. Additionally, it allows for the use of frequency domain methods, making the computational problem more manageable. The basic equation for the WEC array model is given by:

$$\left(\boldsymbol{M} + \mathbf{M}_{\infty}\right)\ddot{\delta}(t) + \int_0^t \boldsymbol{B}_r(t-\tau)\dot{\delta}(\tau)d\tau + \mathbf{B_h}\dot{\delta}(t) + \mathbf{K_s}\delta(t) = \mathbf{f_{ex}}(t) + \mathbf{f_{pto}}(t) \quad (2)$$

where $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the WEC system mass matrix; $\mathbf{M}_{\infty} \in \mathbb{R}^{n\times n}$ is the added mass matrix at infinite frequency; $\delta(t), \dot{\delta}(t)$, and $\ddot{\delta}(t) \in \mathbb{R}^n$ are vectors representing positions, velocities, and accelerations for each of the n buoy hulls; $\mathbf{B_r}(t) \in \mathbb{R}^{n\times n}$ is the matrix of radiation damping impulse responses; $\mathbf{B_h} \in \mathbb{R}^{n\times n}$ is the linearised viscous damping matrix; $\mathbf{K_s} \in \mathbb{R}^{n\times n}$ is the hydrostatic stiffness matrix; $\mathbf{f_{ex}}(t) \in \mathbb{R}^n$ is the wave excitation force vector; and $\mathbf{f_{pto}}(t) \in \mathbb{R}^n$ is the power take-off (PTO) force vector.

The solution to Cummins' Eq. (2), which determines the frequency-domain response of an array of heaving buoys to various regular wave inputs, can be obtained using Ansys AQWA software [32], which utilises the boundary element method (BEM). It is assumed that there is no viscous force in the simulation $\mathbf{B_h}$=0. Thus, the solution for the displacement vector $\boldsymbol{\Delta}(\omega) \in \mathbb{R}^n$ of heaving buoy hulls, for a monochromatic wave with frequency ω, can be presented [33] as:

$$\boldsymbol{\Delta}(\omega) = \frac{1}{j\omega}\mathbf{Z}^{-1}(\omega)\cdot\left[\mathbf{F}_{\mathbf{ex}}(\omega) + \mathbf{F}_{\mathbf{pto}}(\omega)\right] \tag{3}$$

where

$$\mathbf{Z}(\omega) = \mathbf{B}_{\mathbf{r}}(\omega) + j\omega\left[\mathbf{M} + \mathbf{M}_{\mathbf{a}}(\omega) - \frac{\mathbf{K}_{\mathbf{s}}}{\omega^2}\right], \tag{4}$$

$\mathbf{B}_{\mathbf{r}}(\omega) \in \mathbb{R}^{n\times n}$ is a matrix of radiation resistance coefficients, $\mathbf{K}_{\mathbf{s}} \in \mathbb{R}^{n\times n}$ is the matrix of hydrostatic stiffness coefficients; $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the system mass matrix; $\mathbf{M}_{\mathbf{a}}(\omega) \in \mathbb{R}^{n\times n}$ is the frequency-dependent added mass matrix.

In the case of an array of n WECs, the elements of the intrinsic impedance matrix $\mathbf{Z} \in \mathbb{R}^{n\times n}$ describe the interactions among WECs in an array:

$$\mathbf{Z} = \begin{pmatrix} z_{1,1} & \dots & z_{1,n} \\ \vdots & \ddots & \vdots \\ z_{n,1} & \dots & z_{n,n} \end{pmatrix} = \begin{pmatrix} x_{1,1} + jy_{1,1} & \dots & x_{1,n} + jy_{1,n} \\ \vdots & \ddots & \vdots \\ x_{n,1} + jy_{n,1} & \dots & x_{n,n} + jy_{n,n} \end{pmatrix}, \tag{5}$$

For example, the element $z_{1,1}$ represents the intrinsic impedance of the first WEC, and the element $z_{n,1}$ represents impact of the WEC 1 on the WEC n. Each element in the matrix $\mathbf{Z}$ comprises a real part $x_{i,j}$ which represents radiation damping, and an imaginary part $y_{i,j}$, representing the integration of masses and hydro-static stiffness coefficients.

3. Performance metrics for heterogeneous array optimisation

In order to quickly obtain an evaluation of the performance of heterogeneous arrays, calculations are carried out in the frequency domain. An estimate of the average power generation P_{avr} for a specific sea state, in the frequency domain, can be computed by integrating the product of the average power generated at a given wave frequency $P(\omega)$ and the probability distribution function of wave frequencies for that sea state $p_{ss}(\omega)$ over all wave frequencies:

$$P_{avr} = \int_0^\infty P(\omega)\, p_{ss}(\omega)\, d\omega. \tag{6}$$

The probability distribution function for wave frequencies $p_{ss}(\omega)$ can be obtained from the wave spectral distribution $p_{bs}(\omega)$, by normalising it over all wave frequencies through integration:

$$p_{ss}(\omega) = p_{bs}(\omega) / \int_0^\infty p_{bs}(\omega)\, d\omega. \tag{7}$$

To accurately model the multi-directional propagation of water waves, the averaged power outputs, from various wave directions, are summed for each WEC in the array, providing a potentially more realistic representation of actual sea states:

$$P_{array} = \int_0^\infty \left\{ \sum_{j=1}^{k} \left[\sum_{i=1}^{n} P^{i,j}(\omega) p^{j}_{ss}(\omega) \right] \right\} d\omega, \tag{8}$$

where n represents the number of WECs, k is the number of chosen wave propagation directions, i is the WEC index, j is the wave type (swell or wind waves) and direction index, $p^{j}_{ss}(\omega)$ is the frequency distribution function of wave with direction j, and $P^{i,j}(\omega)$ is the power generated by WEC i, for waves with direction j.

The traditional performance evaluation metric, for homogeneous arrays of WECs, relies on the q-factor [22], which can be adapted to evaluate the performance of heterogeneous arrays of WECs. In heterogeneous arrays, q-factor is defined by the ratio between the total power generated by an array P_{array} and the energy absorption by the same number n of isolated devices $P^{i}_{isolated}$:

$$q_{factor} = \frac{P_{array}}{\sum_{i=1}^{n} P^{i}_{isolated}} \tag{9}$$

The interactions between the WECs in an array are constructive when $q_{factor} > 1$; otherwise, the interactions are considered destructive. Although a certain distance between devices may theoretically lead to positive hydrodynamic interactions, the q-factor is not expected to be the sole determinant of the economically ideal geometry and design of array layouts. This decision will primarily be influenced by significant cost factors, such as device geometry, cabling, and moorings.

In this study, the geometric parameters of the WECs in the array are also optimised, as well as the array layout. The focus is placed on cylindrical, semi-submerged devices characterised by two parameters: height H_0 and radius R_0. To address computational limitations in optimising the volume of each WEC, a single scaling factor $(a_1, a_2, \dots, a_n)$ is applied to both the height and radius of each device, such that $H_i = a_i H_0$ and $R_i = a_i R_0$. This approach reduces the number of parameters to be optimised while preserving proportionality in scaling. Thus, any changes in the volume of the WEC hull υ_i are proportional to a_i^3, since $\upsilon_i = \pi R_i^2 H_i = a_i^3(\pi R_0^2 H_0)$. Consequently, changes in the overall volume of the WECs in the array (and, as a result, the potential changes in associated capital costs due to variations in construction material requirements) can be determined as:

$$\upsilon_{overall} = \pi R_0^2 H_0 ({a_1}^3 + {a_2}^3 + \dots + {a_n}^3). \tag{10}$$

Thus, the quantity $\tilde{a}$ is selected as a measure of the 'average volume' scaling factor for the WECs in the array as

$$\tilde{a} = \sqrt[3]{{a_1}^3 + {a_2}^3 + \dots + {a_n}^3}. \tag{11}$$

The cubic root in Eq. (11) allows transitioning from a volume-based metric $[m^3]$ to a linear dimension metric $[m]$, since the radius and height are proportional to the same scaling parameter. This approach also mitigates the rapid growth inherent to cubic functions, ensuring a more balanced optimisation framework.

In this article, a new metric, termed the κ-factor, is proposed as the performance function for the optimisation problem discussed later. The definition of the κ-factor is inspired by the LCoE Eq. (1). Thus, the objective function is designed to capture the constructive interaction effects within the array, quantified by q_{factor}, while taking capital costs into account, which are quantified by the average volume scaling parameter $\tilde{a}$. In essence, the κ-factor represents the ratio of the geometric scaling rate of the devices, $\tilde{a}$, to the q-factor, which accounts for the increase in power absorption. By considering the ratio of $\tilde{a}$ to q_{factor}, the optimisation problem is formulated to maximise the positive array effects, while appropriately penalising capital costs:

$$\kappa = \frac{\tilde{a}}{q_{factor}} \rightarrow Min. \tag{12}$$

4. Optimal constrained control strategy for an array of WECs

This study assumes that the WECs in an array are operated using an extended version of the 'Simple and Effective' (SAE) controller [34–36]. The SAE controller frames the panchromatic control problem as an instantaneous monochromatic control problem, necessitating the use of an extended Kalman filter (EKF) for estimating the excitation force. Additionally, this control algorithm is designed to account for constraints on maximum buoy hull displacements. This section describes an extended version of the SAE controller specifically designed for heterogeneous arrays of WECs, which allows for the implementation of displacement constraints for each individual WEC within the array. Additionally, a method is derived here for evaluating the average power production of a WEC array under this centralised and constrained control algorithm. The validation of the presented frequency domain performance assessment method, against time-domain results [12], is conducted in [37]. Implementation of the SAE control in arrays of WECs, referred to as 'Array Simple and Effective' (ASAE) control for WEC arrays, requires modifications of the intrinsic impedance of the

PTO systems, denoted as $\mathbf{Z_{pto}}(\omega)$, which takes the following form:

$$\mathbf{Z_{pto}} = \begin{pmatrix} (2\gamma_1 - 1)x_{1,1} - jy_{1,1} & \dots & (2\gamma_n - 1)x_{1,n} - jy_{1,n} \\ \vdots & \ddots & \vdots \\ (2\gamma_1 - 1)x_{n,1} - jy_{n,1} & \dots & (2\gamma_n - 1)x_{n,n} - jy_{n,n} \end{pmatrix} \quad (13)$$

where γ_i represents the control variable for the PTO damping of the ith WEC, which can be utilised to impose constraints on the ith WEC motion. Note that the real part of each element in the ith column of the matrix (13) has the same tuning parameter $(2\gamma_i - 1)$. This approximation is based on the assumption of linear interactions among devices within the WEC array. Hence, adjustments to the radiation damping of a specific WEC will consequently impact its influence on all other devices in the array.

The total intrinsic impedance $\mathbf{H}$ of the WEC array, when combined with a centralised PTO, is given by the sum of $\mathbf{Z}$ and $\mathbf{Z_{pto}}$:

$$\mathbf{H} = \mathbf{Z} + \mathbf{Z_{pto}} = \begin{pmatrix} 2\gamma_1 x_{1,1} & \dots & 2\gamma_n x_{1,n} \\ \vdots & \ddots & \vdots \\ 2\gamma_1 x_{n,1} & \dots & 2\gamma_n x_{n,n} \end{pmatrix}. \quad (14)$$

For each WEC in the array, the corresponding velocities can be evaluated as $\mathbf{V} = \mathbf{H^{-1}F_{ex}}$ or

$$\mathbf{V} = \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} = \begin{pmatrix} 2\gamma_1 x_{1,1} & \dots & 2\gamma_n x_{1,n} \\ \vdots & \ddots & \vdots \\ 2\gamma_1 x_{n,1} & \dots & 2\gamma_n x_{n,n} \end{pmatrix}^{-1} \begin{pmatrix} F_1 \\ \vdots \\ F_n \end{pmatrix}. \quad (15)$$

The required inverse system matrix $\mathbf{H^{-1}}$ is given by the following equation:

$$\mathbf{H^{-1}} = \begin{pmatrix} \frac{C_{1,1}}{2\gamma_1|\mathbf{H}|} & \dots & \frac{C_{n,1}}{2\gamma_1|\mathbf{H}|} \\ \vdots & \ddots & \vdots \\ \frac{C_{1,n}}{2\gamma_n|\mathbf{H}|} & \dots & \frac{C_{n,n}}{2\gamma_n|\mathbf{H}|} \end{pmatrix}, \quad (16)$$

where $C_{i,j}$ are co-factors of the elements $x_{i,j}$, and $|\mathbf{H}|$ is the determinant of the original matrix $\mathbf{H}$.

Thus, the velocities for each WEC within the array can be evaluated as:

$$\mathbf{V} = \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} = \begin{pmatrix} \frac{1}{2\gamma_1}\left(\sum_{i=1}^{n} \frac{C_{i,1}F_i}{|\mathbf{H}|}\right) \\ \vdots \\ \frac{1}{2\gamma_n}\left(\sum_{i=1}^{n} \frac{C_{i,1}F_i}{|\mathbf{H}|}\right) \end{pmatrix}, \quad (17)$$

and the corresponding displacements $\boldsymbol{\Delta} = (\Delta_1, \Delta_2, \dots, \Delta_n)^T$ for the specified wave frequency ω for WECs can be evaluated as:

$$\boldsymbol{\Delta} = \frac{\mathbf{V}}{j\omega} = \frac{\left(V_1, V_2, \dots V_n\right)^T}{j\omega}. \quad (18)$$

It is clear that setting $\gamma_i = 1$ results in the classical complex conjugate control solution [38]. However, the solutions of complex conjugate control could require significant and unrealistic heaving buoy displacement. Therefore, in order to respect the necessarily finite stroke and force limitations of the PTO, the introduction of restrictions on maximum heave displacements is necessary [33].

Within the presented framework, these limitations can be achieved (while staying within a frequency-domain formulation) by adjusting the PTO damping parameters γ_i. If the displacement required for complex conjugate control exceeds realistic values (i.e., $|\Delta_i| > \Delta_{Max}$), the necessary adjustment for tuning PTO parameters γ_i can be derived from the following equation:

$$\text{If } |\Delta_i| > \Delta_{Max} \rightarrow \gamma_i = \frac{|\Delta_i|}{\Delta_{Max}}. \quad (19)$$

Then, the required PTO forces $\mathbf{F}_{pto} \in \mathbb{R}^n$ can be calculated as:

$$\mathbf{F}_{pto} = -\mathbf{Z_{pto}V}, \quad (20)$$

with Z_{PTO} calculated from (13), which includes the γ_i parameters.

The time-averaged power production vector $\mathbf{P} \in \mathbb{R}^n$, in the frequency domain, for each WEC in an array operating with an ASAE controller, can be computed as follows:

$$\begin{aligned} \mathbf{P} &= \begin{pmatrix} P_1 \\ \vdots \\ P_n \end{pmatrix} = \frac{1}{2} Re(\mathbf{Z_{pto}})|\mathbf{V}|^2 \\ &= \frac{1}{2} \begin{pmatrix} (2\gamma_1 - 1)x_{1,1} & \dots & (2\gamma_n - 1)x_{1,n} \\ \vdots & \ddots & \vdots \\ (2\gamma_1 - 1)x_{n,1} & \dots & (2\gamma_n - 1)x_{n,n} \end{pmatrix} \begin{pmatrix} |V_1|^2 \\ \vdots \\ |V_n|^2 \end{pmatrix}. \end{aligned} \quad (21)$$

The total power production P_{array} from a heterogeneous WEC array can be computed by substituting the evaluated range of power generation $P_i(\omega)$, assessed for each wave frequency ω, into (8), as detailed in Section 3.

The simplicity of the proposed ASAE method, for control calculation and performance evaluation, ensures rapid assessment, which is crucial for integration into system optimisation loops. Unlike time-domain performance methods, the proposed approach does not require lengthy virtual time simulation for control calculations at each time step, or convergence evaluation for power values.

5. Control co-design problem statement

This section is dedicated to presenting the CCD optimisation problem to which the methodology described in the previous sections will be applied. The research aims to find an optimal combination of device geometry and array layout for a heterogeneous WEC array. The overall goal is to maximise the beneficial, constructive effects of array interactions while maintaining the scale of the WECs within a realistic range, also subject to displacement and layout constraints. The optimisation problem also aims to harness multi-directional wind and swell wave power. The goal is to minimise the κ-factor (12). As the definition of this metric is inspired by the ideas underpinning LCoE, minimising the κ-factor should lead to array layouts which reduce the LCoE (1) of the generated energy. It is assumed that the WECs in the array operate under the new ASAE control, described in Section 4.

Consider a baseline point absorber WEC, with buoy hull given by a semi-submerged cylinder with radius $R_0 = 3$m and a height of $H_0 = 6$m. Using this baseline WEC, the geometry of each device in the array is adjusted in the optimisation algorithm using a scaling factor a_i, where the radius and height of the ith WEC are defined by $R_i = a_i R_0$ and $H_i = a_i H_0$ respectively. The scaling factor is constrained to the range $0.75 < a_i < 1.25$, reflecting practical considerations related to manufacturability, installation, and transportation of WEC hulls. The positions of the WECs in the array, denoted by (X_i, Y_i), are also optimised simultaneously with the hull scale a_i within a single optimisation problem, as depicted in Fig. 2. The optimisation algorithm is implemented in MATLAB, which calls ANSYS AQWA to estimate hydrodynamic parameters at each optimisation step. Parameters for WEC positions (X_i, Y_i) are subject to constraints that may vary, based on the number of devices. The water depth is considered to be 200 m.

The determination of hydrodynamic parameters and excitation forces for closely located multiple objects in waves is a challenging problem and has been studied by many researchers [39–41]. It is known that boundary element, or panel, methods may provide incorrect or overestimated results when the gap between multiple devices is small, and high-order asymptotic approximations are required for more accurate results in such a case [39,41]. These high-order effects become more significant with an increased number of devices [39]. Therefore, in this research, the following constraints are introduced on the minimum allowed distance between devices, to avoid overestimation of interaction for closely located devices. The minimum distance for three WECs is set at one third of the peak wavelength $L_{min} = \lambda/3$, while, for four or five WECs, it is three quarters of the wavelength $L_{min} = 3\lambda/4$.

The following constraints on the device spacing also ensure that devices do not collide and are not excessively spaced apart, which would incur excessive expenses for mooring lines and electrical cables [26]:

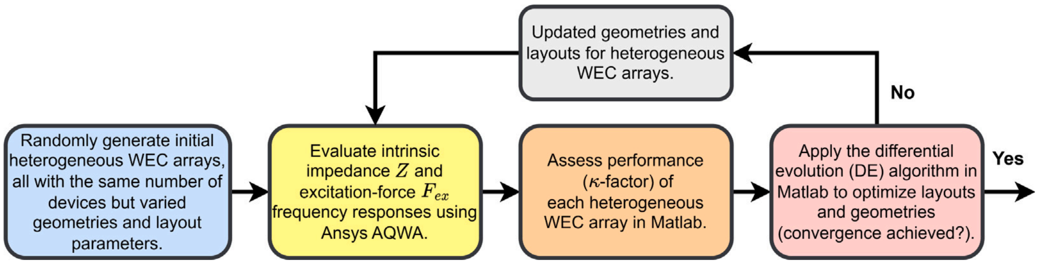

Fig. 2. Control co-design optimisation.

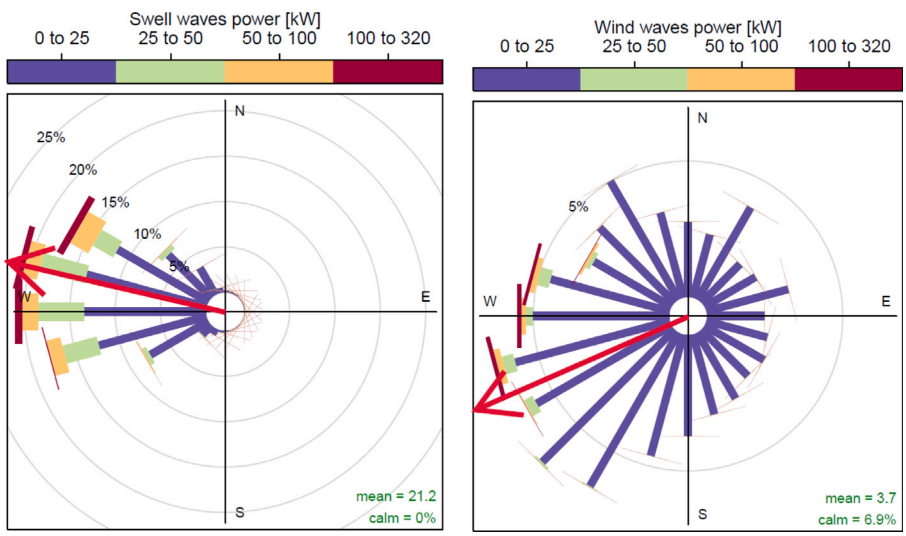

Fig. 3. Annual wave energy data for southwest coast of Ireland [longitude: −10°, latitude: 52.5°], the averaged properties of swell waves H_s=1.8 m T_p=9.3 s direction = 293°, and wind wave H_s=1 m T_p=4 s direction = 243°.

$$\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} > L_{min}$$
$$X_{min} < X_i < X_{max}, \qquad Y_{min} < Y_i < Y_{max} \tag{22}$$

In addition, the maximum possible displacement for WECs is constrained by $\Delta_i^{Max} = 0.4H_i$. This condition ensures that a WEC does not lose contact with the water and takes into account that a larger WEC may also experience a greater fluctuation magnitude in water waves.

The optimisation problem is tackled using a differential evolution (DE) algorithm, an adaptive meta-heuristic search method categorised within evolutionary computing algorithms [42]. DE demonstrates effectiveness in addressing the nonlinear challenges posed by WEC array optimisation. Engineered to uniformly and adaptively explore extensive search spaces [29,43], and [44], DE employs a differential mutation approach, inherently fostering diversity and mitigating premature convergence [45], a common issue in non-convex performance landscapes. In addition, the rotations of the obtained layouts, as well as attempts to approximate them with geometrically accurate shapes, are carried out to ensure that the obtained solution is a global minimum.

6. Selected wave climates

Traditionally, the optimisation of an array of WECs is conducted for a single wave direction, which corresponds to an averaged swell wave. In this study, the optimisation of the array is conducted for both swell and wind waves, which have different properties, and propagate from different directions. Wind waves usually have a smaller period and transfer much less wave energy; however, their inclusion could potentially lead to a 10%–20% increase in the wave power available for absorption as can be seen from Figs. 3 and 4. Unfortunately, in a real ocean state scenario, it would be difficult to predict the direction of wind waves and tune the PTO to align with their shorter periods. However, it is nonetheless interesting to assess how their inclusion would affect the optimal array layout and the scales of WECs within it.

To achieve a realistic representation of sea states, the ERA5 database is utilised to specify the wave climate in the two chosen locations [46]. ERA5 is the fifth generation of reanalysis produced by the European Centre for Medium-Range Weather Forecasts (ECMWF), covering global climate and weather data over the past eight decades, starting from 1940. Reanalysis integrates model data with worldwide observations to create a comprehensive and consistent global dataset, adhering to the laws of physics. ERA5 offers hourly estimates for numerous atmospheric, ocean-wave, and land-surface parameters.

Two wave climates are selected for heterogeneous wave array optimisation and assessment of the solution sensitivity to various waves types and directions. The first wave climate is based on data from southwest coast of Ireland at longitude −10° and latitude 52.5° (see Fig. 3), and the second wave climate is based on data from the Portuguese coast at longitude −9.5° and latitude 37.5° (see Fig. 4). These two regions are traditionally used as places for WEC prototype testing, and investigating their potential further deployment.

The wave roses, depicting the average wave power propagation for wind and swell waves recorded during 2023 for these two regions, are presented in Figs. 3 and 4. The dominant gradient directions, where maximum wave energy data is recorded, are highlighted with red arrows. The average wave properties corresponding to these directions are evaluated. It is evident that the *dominant* directions of wind and swell waves form approximately a 50^0 angle with each other for the Irish coast, while the angle is −55° for the Portuguese coast area. The irregular waves in the study are assumed to follow a Bretschneider

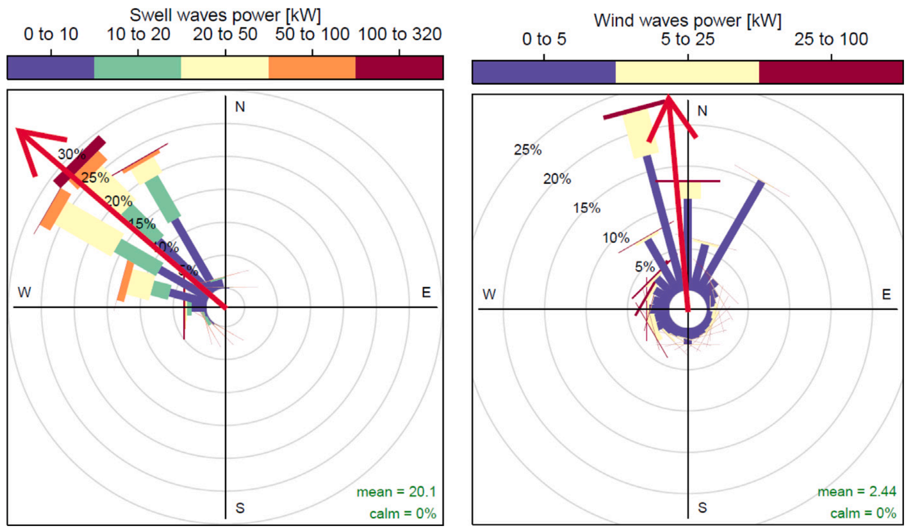

Fig. 4. Annual wave energy data for Portuguese Coast [longitude: −9.5°, latitude: 37.5°], the averaged properties of swell wave H_s=1.77 m T_p=9.6 s direction = 298.6°, and wind wave H_s=0.7 m T_p=3.6 s direction = 353.5°.

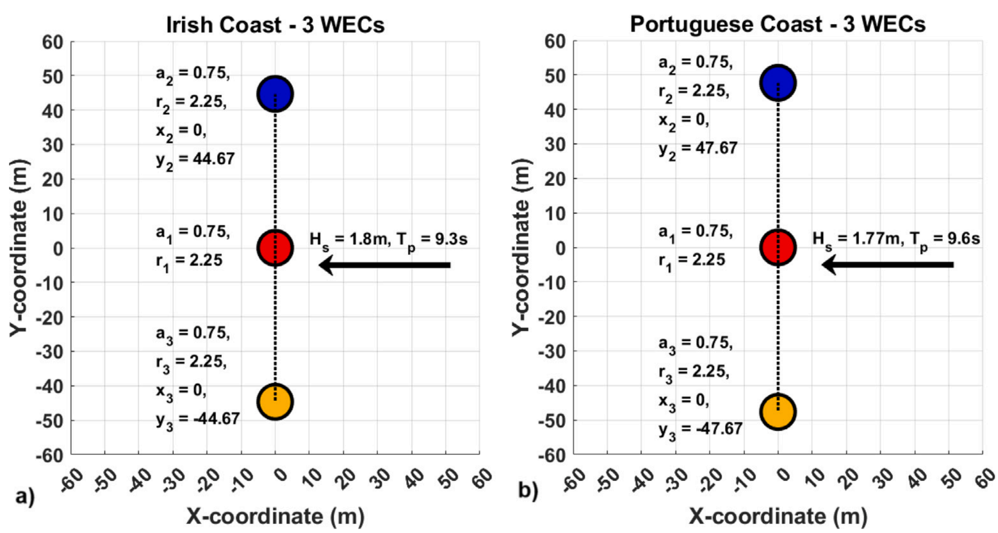

Fig. 5. Optimal arrays of 3 WECs for (a) Irish Coast and (b) Portuguese Coast climates (only for swell wave).

spectrum. The average swell and wind wave properties for the Irish coast are H_s=1.8 m T_p=9.3 s, and H_s=1 m T_p=4 s respectively. For the Portuguese coast, the swell and wind wave properties are H_s = 1.77 m, T_p = 9.6 s, and H_s = 0.7 m, T_p = 3.6 s, respectively. Thus, the values of the peak swell wavelengths for the Irish and Portuguese coast are λ_I=134 m and λ_P=143 m, which also impose different distance constraints for the two optimisation problems.

7. Results and discussions

This section presents the results of applying the proposed methodology for control co-design optimisation of heterogeneous WEC arrays consisting of 3, 4, and 5 devices. The DE optimisation algorithm is implemented in MATLAB and executed using resources provided by the Irish Centre for High-End Computing (ICHEC) [47]. Hydrodynamic parameters for the control and performance assessment algorithm are determined using ANSYS AQWA. The DE optimisation algorithm is initialised with 20 randomly generated WEC array configurations distributed within the allocated area, and constrained by (22). Approximately 200–500 iterations of the optimisation loop are conducted from each of the twenty starting positions until the solutions converge (see Fig. 7).

7.1. Case 1 - Optimisation results for 3 WECs

The first case study considers the optimisation of the shapes and positions of three WECs for the two selected wave climates: the Irish and Portuguese coasts. The area designated for WEC positioning is restricted to $[-2\lambda, 2\lambda]$ for both the X and Y axes, with a minimum distance of $\lambda/3$ m between any two WECs. It is assumed that the position of the first WEC is fixed at the origin, which reduces the number of optimisation variables to seven in total: four for the positions of the remaining two WECs and three for the size scaling parameters.

The first case scenario considers a traditional WEC optimisation problem with only unidirectional wave propagation of swell waves in the dominant direction, as detailed in Section 6. The optimal solutions for this optimisation problem are presented in Fig. 5. It is evident that the WECs form a straight line perpendicular to the incoming wave direction, which agrees with the traditional result for a small WEC array [22]. The obtained array is homogeneous, with all WECs having the minimum allowed size a_i = 0.75, and the distance between WECs is also the minimum allowed $\lambda/3$, for both wave climates. The κ-factors for the arrays are almost identical κ_I = 0.764 and κ_P = 0.766, as well as the q-factors, which are q_I = 1.416 and q_P = 1.412, respectively.

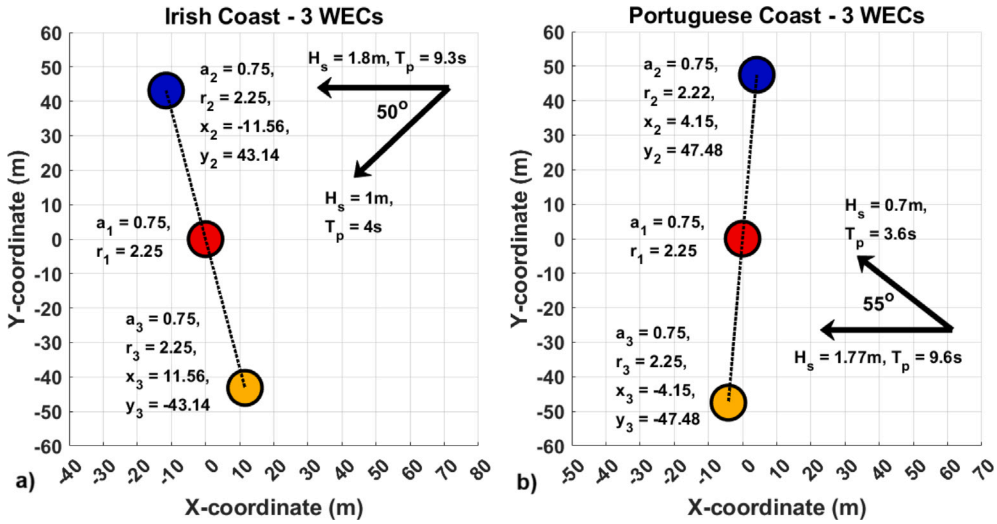

Fig. 6. Optimal arrays of 3 WECs for (a) Irish coast and (b) Portuguese coast climates.

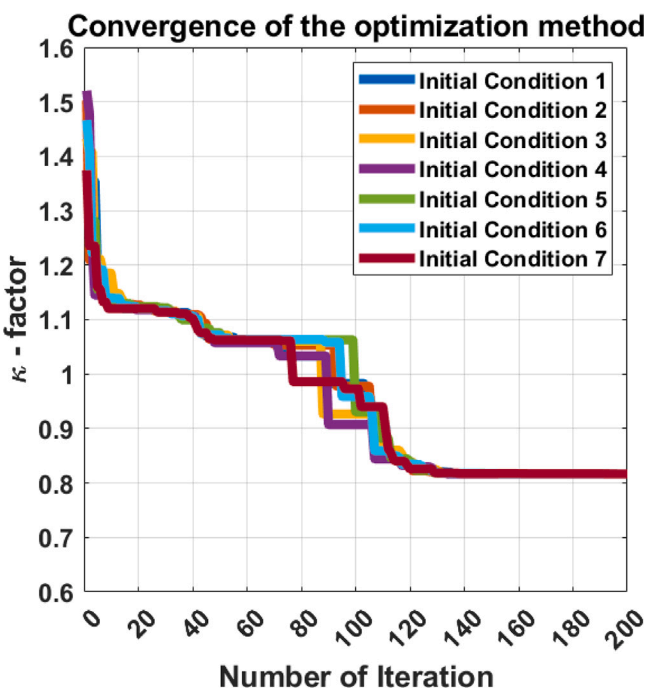

Fig. 7. Convergence of the differential evolution method for array optimisation for Irish coast.

However, the inclusion of wind waves in the optimisation problem changes the results, as shown in Fig. 6. It is clear that the optimisation leads to a homogeneous array, with the WEC sizes at their minimum boundary, with $a_i = 0.75$. The WECs are still arranged in a straight line, but the variations in the angle between wind and swell waves causes reorientation or rotation of the WEC layout. The shift in array orientation in the case of Irish coast is greater compared to that of the Portuguese coast, which can be explained by the larger peak period T_p and significant wave height H_s of the included wind waves. However, the κ-factor for the Irish coast, at $\kappa_I = 0.817$ is slightly larger (recall that a small k-factor is best) than the κ-factor for the Portuguese coast, which is $\kappa_P = 0.788$. It could be explained by a more favourable combination of swell and wind waves periods. The evaluated q-factors for the case of wind and swell waves $q_I = 1.32$, and $q_P = 1.37$, are small, compared to the case of only swell waves, which can be explained by lower positive interactions between WECs in the array in the case of the swell waves.

The convergence of the optimisation solution for the Irish coast is illustrated in Fig. 7 for seven initial configurations. It is evident that convergence of the seven randomly selected initial WEC positions is achieved after 140 iterations. It should also be noted that, as shown in [22], the optimal layout for a WEC array is a consequence of the selected control strategy, so changing the selected control strategy will change the optimal array layout. Thus, the layout of WECs presented in this article is optimal for the developed ASAE control strategy with the chosen constraints [48], and the particular objective function chosen.

7.2. Case 2 - Optimisation results for 4 WECs

The second case study involves optimising the shapes and positions of four WECs for the same selected wave climates of the Irish and Portuguese coasts. The selected area for WEC positions is limited to $[-2\lambda, 2\lambda]$ on both the X and Y axes, and it is required that WECs maintain a minimum distance of $L_{min} = 3\lambda/4$ from each other. The first WEC is again located at the origin of the coordinate system, which reduces the number of coordinate variables to 6, with the further addition of 4 scaling parameters: 10 in total.

The solution to the associated optimisation problem is presented in Fig. 8. It is evident that the arrangement of the four WECs is symmetrical and very close to a rectangular shape, with a single large WEC located behind (with respect to the wave direction) the three smaller ones. The smaller WECs are at the minimum allowed size a_i=0.75 or R=2.25 m, while the large WEC is at the maximum size, with a_i=1.25 or R=3.75 m. These results are in agreement with the results of the optimisation conducted in the time domain in [22], where the obtained layout also had larger WECs surrounded by smaller ones. The rectangle is oriented with respect to the angle between wind and swell waves. The κ-factor for the Irish coast is $\kappa_I = 0.98$, which is slightly higher than the κ-factor for the Portuguese coast, which is $\kappa_P = 0.956$, and the q-factors are $q_I = 1.506$ and $q_P = 1.595$, correspondingly. It is worth noting that the distances between the WECs in the array are, on average, slightly greater than the minimum allowed ($3\lambda/4$), and that the efforts to replace the obtained formation with a more exact square layout did not improved the κ-factor.

7.3. Case 3 - Optimisation results for 5 WECs

The third and final case study considers the optimisation of the shapes and positions of five WECs for the selected wave climates of

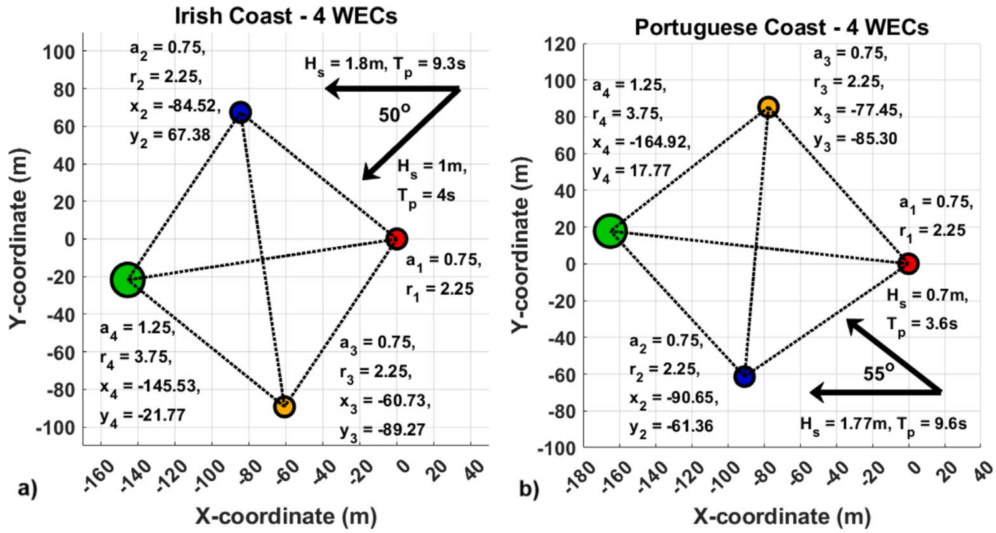

Fig. 8. Optimal Heterogeneous Arrays of 4 WECs for Irish and Portuguese Coasts Climates.

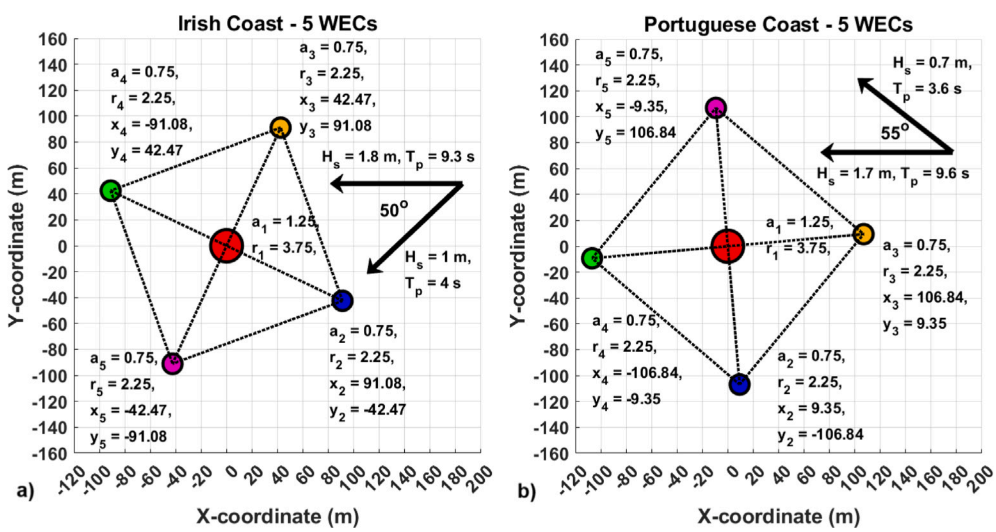

Fig. 9. Optimal Heterogeneous arrays of 5 WECs for Irish and Portuguese coasts climates.

the Irish coast and the Portuguese coast. The selected area for WEC positions is limited to $[-2\lambda, 2\lambda]$ for both the X and Y axes, while it is required for WECs to maintain a separation of at least $L_{min} = 3\lambda/4$ m. Since it is assumed that the first WEC is fixed at the origin, the number of variables is 8 for WEC positions and 5 for their scales, giving a total of 13 variables to be optimised.

The results of applying the proposed methodology to the optimisation problem are presented in Fig. 9. Once again, the layout has a high level of symmetry and closely approximates a square, on this occasion with the largest WEC at the centre (intersection of the main diagonals). The distances between the central WEC and the WECs located at the vertices are all equal to the applied constraint, which is $3\lambda/4$. Moreover, the square layouts are oriented towards the incoming wind and swell waves. The WECs located at the vertices of the square layout have their scaling parameters at the lower boundary of the constraint set, $a_i = 0.75$ while, for the central WEC, the scaling parameters are again at the maximum of the constraint set with $a_1 = 1.25$, for both wave climates. The obtained values for the κ-factor for the Irish and Portuguese Coasts are $\kappa_G = 1.014$ and $\kappa_P = 0.893$ correspondingly, and q-factors are $q_G = 1.517$ and $q_P = 1.722$.

The heterogeneous array layout obtained here, which represents a larger heaving buoy closely surrounded by smaller ones, closely resemble the results obtained in [22]. Interestingly, symmetry is not enforced in the optimisation formulation; however, the results of the optimisation algorithm exhibit geometrically regular shapes with a high degree of symmetry.

8. Conclusions

The presented methodology for the control co-design of heterogeneous arrays of WECs is both new and promising. By facilitating the enforcement of constraints in the frequency domain, this approach enables the discovery of new solutions while simultaneously optimising the shapes and layout of WECs in a heterogeneous array within an acceptable computational time-frame.

For the particular objective function studied here, it is shown that the optimal layout of a heterogeneous array is a geometrically regular shape, closely approximating a square layout in the case of 4 and 5 WECs, oriented towards the incoming wave, with the largest WEC located behind or surrounded by the smaller devices. The presented results align with those obtained in the time domain for the spectral control method in [22]. However, in that previous research, the authors did not vary positions for arrays with more than three WECs, adjusting only their scales.

Another feature of the presented methodology is its capability to optimise for the case of multi-directional wind and ocean waves. The proposed performance function, the new κ-factor metric, is inspired by the LCoE and allows for maximising the positive q-factor effect while maintaining the sizes of WECs within reasonable bounds.

However, the proposed methodology and the results obtained are subject to several limitations due to the use of linear models and frequency-domain assessments. The methodology cannot account for slamming effects, as frequency-domain analysis poses challenges for modelling nonlinear systems. Additionally, the analysis is confined to operations within the power production region, thereby excluding extreme wave conditions, slamming phenomena, and similar effects. The performance evaluation is based on a multi-monochromatic analysis, rather than a more comprehensive poly- or pan-chromatic approach. Additionally, all the considered devices are assumed to have the same shape, with proportional scaling in both height and breadth. Many of the solutions are constrained by the limitations of linear BEM analysis, which does not fully capture the potential of an optimally heterogeneous array. Finally, it should be noted that the manufacturing cost of a heterogeneous array is higher than that of a homogeneous array, and this factor is not considered in the 'economic' performance evaluation.

Nevertheless, the method to assess array performance in the frequency domain presented in this article opens new possibilities for further development of point absorber wave energy technology.

CRediT authorship contribution statement

Andrei M. Ermakov: Writing – original draft, Methodology, Investigation, Funding acquisition, Formal analysis, Conceptualization. **Zain Anwar Ali:** Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **Kumars Mahmoodi:** Methodology, Investigation, Formal analysis. **Oliver Mason:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization. **John V. Ringwood:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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