On data-based control-oriented modelling applications in wave energy systems

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Abstract—The development of effective energy-maximising control strategies has a crucial role in the empowerment of wave energy technology, and in its improvement towards economic viability. Within the state-of-the-art, most of the strategies adopted to maximise the absorbed energy exploit a model of the wave energy converter (WEC) to be controlled, *i.e.* they are model-based. These models attempt to replicate the WEC dynamics with a sufficient degree of fidelity, trying, at the same time, to minimise their associated computational burden. However, due to the presence of the hydrodynamic effects, which inherently characterise wave energy systems, simultaneously achieving high-fidelity and computational efficiency is not trivial. Oversimplification of the problem through, for example, linearity assumptions, could lead to non-representative models and/or large uncertainty levels. To overcome these issues, in the last decade, several approaches based on data have been proposed in the wave energy field. These approaches, falling under the umbrella of system identification techniques, exploit data coming from experimental tests or high fidelity simulations, and build control-oriented models with a pre-defined level of complexity. In this paper, we analyse the different strategies that have been adopted in the literature to build data-based control-oriented models for WECs, highlighting the characteristics of each approach, together with their opportunities and inherent drawbacks. Conclusions are drawn regarding the capabilities that this type of approach has in (at least partially) solving the modelling issues that affect WEC control system design, and the pitfalls that pure adoption of these strategies has when applied on larger scales, or in the operational stage.

Index Terms—Data-based modelling, System identification, Control-oriented modelling

I. INTRODUCTION

IN the context of a growing effort in the attempt to push wave energy technology forward, one of the main challenges to be overcome is the development of effective and efficient control strategies [1, 2]. These are responsible for the process of energy maximisation and for the reduction of the stresses associated to the operations in harsh environments like ocean and marine ones. A successful fulfilment of these purposes can actively contribute to the reduction of the levelised cost of energy (LCoE), by increasing the absorbed energy and reducing the operational costs.

The control problem of wave energy converters (WECs), due to its energy-maximising nature, falls under the *optimal control* category, where the performance function to be maximised is a measure of the absorbed energy [3]. Moreover, the control strategy is responsible of maintaining the wave energy systems inside the inherent motion and force constraints characterising these conversion devices [4]. With the only exception of *model-free* control strategies, like *extremum-seeking* [5, 6], *reinforcement learning* [7, 8], and *surrogate-optimisation-like* control [9, 10], most of the strategies employed to solve the wave energy optimal control problem (OCP) rely on a model of the controlled wave energy system (*i.e.* are *model-based*). These models are the result of a trade-off between accuracy and complexity, which is motivated by the need of guaranteeing the feasibility of the OCP numerical solution in real-time. The reduction of complexity of WEC models is usually obtained through simplifications of the Navier-Stokes equations (which precisely describe the motion of wave energy systems inside the water). These simplifications are often based upon linearity or small oscillations around equilibria assumptions [11, 12]. However, these hypotheses are usually not representative of the system dynamics in controlled conditions, due to the tendency of optimal control actions to emphasise the WECs motion. These latter considerations are in constrast with the hypotheses made in the modelling stage, leading to a controller that inherently invalidates the model upon which it is synthesised, and to the so-called 'modelling paradox' [11]. Moreover, the process of computation of the hydrodynamics contributions is usually affected by uncertainties which are not easily identifiable [13]. With the aim of solving part of these issues, in the last decade several attempts of directly exploiting measured data coming from (real or numerical) WECs to model these latter were made. Motivated by the above considerations, this paper attempts to precisely analyse the studies related to the data-based modelling approaches that have been applied to obtain control-oriented models of wave energy systems. Considerations are made on the type of approach adopted in the modelling stage and on the type of process of identification itself. Moreover, a further distinction is made on the basis of the origin of the the employed data.

The remainder of the paper is organized as follows. In Section II, a brief introduction to WEC modelling is presented. Section III describes the WEC OCP, highlighting the role of models in the control synthesis
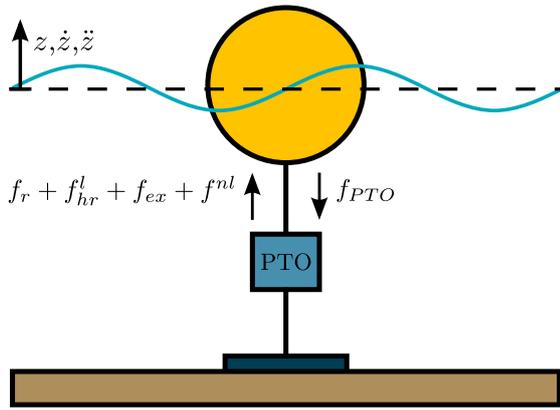
Fig. 1. Heaving point absorber WEC.

process. In this context, the WEC modelling paradox is introduced together with its consequences in Section IV. The main data-based modelling approaches are described in Section V, where time domain and frequency domain techniques are also faced (Section V-A and V-B respectively). Finally, Section VI aims at critically compare the data-based modelling applications in wave energy, while Section VII is devoted to some conclusions and considerations are drawn regarding the opportunities that this type of approaches have with respect to the problems characterising WECs modelling.

II. WAVE ENERGY SYSTEMS MODELLING

WEC dynamics is mainly the result of the interaction between a floating body (the hull), with the surrounding fluid and a controlled actuation system (the so-called *power take-off*, PTO). Potentially, some inner conversion mechanism could be present [14, 15, 16]. In general, wave energy systems can be classified on the basis of their geometries and working principles [17] into four main classes: *Point absorbers* (PAs) [18], *oscillating water columns* (OWCs), [19], *terminators*, [20, 21], and *attenuators* [22]. For the sake of simplicity, we consider a single[1] degree-of-freedom (DoF) WEC device throughout this paper, based on the schematic shown in Fig. II. The equation of motion for such class of devices can be given by[2]:

$$m\ddot{z} = f_r + f_{hr}^l + f_{ex} + f^{nl} - f_{PTO}, \qquad (1)$$

where z is the device heave displacement, f_r is the radiation force, f_{hr}^l is the linear component of the hydrostatic restoring force, f_{ex} is the wave excitation force, f^{nl} represents a potential source of nonlinearity that depends on displacement $z(t)$ and velocity $\dot{z}(t)$ (*e.g.* nonlinear hydrostatic effects or viscous drag forces), and $f_{PTO}(t)$ is the controllable force exerted by the PTO. Apart from f^{nl}, the terms in Equation (1) are normally modelled on the basis of linear potential flow theory, which make assumptions of frictionless and irrotational flow, linear wave theory, and amplitude of

[1]It must be highlighted that similar considerations can be formulated for multi-DoF devices (see, for instance, [23]).
[2]From now on, the dependence on t is dropped when clear from the context.

motion significantly smaller than the dimension of the floating body. Under these assumptions, excitation and radiation effects are numerically computed through *Boundary Element Method* (BEM) solvers. Nonetheless, these simplifying assumptions add a certain degree of uncertainty to the employed model, which can consequently breakdown the energy absorption performances, and increase the risk of unsafe operations [13]. Moreover, as deeper described in Section IV, the application of energy-maximising control strategies naturally lead the system to operating conditions which are distant from those assumed by the modelling hypotheses.

III. WEC OPTIMAL CONTROL PROBLEM

As briefly introduced in Section I, the control strategies employed in wave energy field are of the energy-maximising type. As a consequence, the synthesis of such controllers is the result of the solution of an OCP, whose performance function $\mathcal{J}$ to be maximised is a measure of the energy absorbed by the device over a certain time interval $\mathcal{T} = [a, b] \subset \mathbb{R}^+$. A standard formulation of such performance function is given by the absorbed mechanical energy:

$$\mathcal{J}(f_{PTO}) = \frac{1}{T} \int_a^b f_{PTO}(\tau)\dot{z}(\tau)d\tau, \qquad (2)$$

where $T = b - a$. Other performance metrics are also possible, trying to take into account of the net power, or of the energy at different levels of the conversion chain (*e.g.* by exploiting *wave-to-wire* [24, 25] or *wave-to-grid* [26, 27] models). Apart from maximising the energy extraction, WEC control is responsible of avoiding exceeding physical system specifications to enable safe operations [4]. With this purpose, *soft constraints* can be implemented via additional terms in $\mathcal{J}$, or *hard constraints* can be formulated along with Equation (2), as:

$$\begin{cases} |z| \le z_{max}, \\ |\dot{z}| \le \dot{z}_{max}, \\ |f_{PTO}| \le f_{PTO,max}, \end{cases} \qquad (3)$$

with $\{z_{max}, \dot{z}_{max}, f_{PTO,max}\} \subset \mathbb{R}^+$, leading to a constrained optimisation problem. As a consequence, the resulting OCP to be solved in the WEC controllers synthesis can be written as:

$$\begin{aligned} f_{PTO}^{\text{opt}} = \arg\max_{f_{PTO}} \ \mathcal{J}(f_{PTO}) \\ \text{s.t.:} \\ \text{WEC dynamics (1),} \\ \text{Motion and input constraints (3).} \end{aligned} \qquad (4)$$

Several control strategies have been employed in the WEC control literature [28] in the attempt of solving the OCP presented in Equation (4). Within the different attempts, two main approaches can be identified: *optimisation-based* and *non-optimisation-based* controllers [29]. Optimisation-based control includes all controllers that require the online numerical solution of the OCP in Equation 4 for the control action to be calculated. These include *e.g. model predictive control* (MPC) [3,
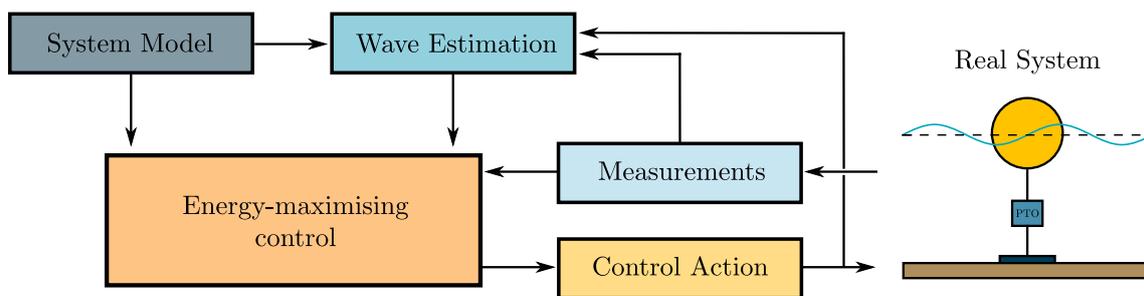
Fig. 2. Typical WEC control system: Schematic representation of the control loop.

30], *moment-based control* [31], and *spectral* and *pseudo-spectral control* [32]. Non-optimisation-based control instead includes all controllers that try to emulate the so-called *impedance-matching* condition [33, 34] trying to maximise the energy outcome. Relevant examples of this type of controllers are given by the *Linear Time Invariant Controller* (LiTe-Con) [35, 36], and by *Linear Quadratic Gaussian* (LQG) control [37, 38].

A. What role do models play in WEC control?

Most of the control solutions presented above are *model-based*, *i.e.* they rely on a model of the controlled WEC. This means that a control-oriented model of the system is required to synthesise the control strategy itself. The role of models in WEC control is related to several aspects of the control loop, different from the pure control synthesis. The availability of a model enables, for example, the adoption of estimators, employed to estimate the (unmeasurable) force f_{ex} acting on the WEC [39]. As a consequence, such information of this disturbance (which is also the origin of the absorbed energy) can be considered 'available' at each time instant during the optimisation stage characterising the control synthesis (Equation (4)). Moreover, such signal can also be forecasted [40], and the knowledge of future samples employed in the optimisation. In addition, models allow the system dynamics to be propagated into the future, enabling the constraint handling in hard manner in the computation of the control action. The role of WEC models in the synthesis process of typical controllers for wave energy systems is shown in Figure 2. It must be highlighted that the performances of model-based control are strictly dependent on the fidelity of the model in the operational conditions. As a consequence, if the errors injected during the modelling stage (which could be parametric or consequence of unmodelled dynamics) are relevant, not only the performances are degraded, but the behaviour could act in an unpredictable manner, causing problems or even jeopardising safe operations.

IV. THE WEC MODELLING PARADOX AND ITS CONSEQUENCES

As outlined in Section I, one of the characteristics of WEC control problem is the presence of the so-called WEC modelling paradox. The nature of this paradox lays in the contrast between the linear assumptions (associated with small oscillations) adopted in the models used to synthesise the control, and the objective of the control itself. In fact, maximising energy indirectly amplifies the amplitude of the WEC motion, especially when reactive controls are applied [1, 11]. As highlighted by Figure 3, adapted from the deep analysis related to WEC modelling paradox in [11], wave energy systems tend to operate with bigger amplitudes whenever an energy maximising strategy is applied. In [11] different control strategies (with growing levels of aggressiveness in terms of control action) have been applied, in the attempt to show how an optimal control action forces the WEC to work outside the range of motion assumed in the modelling stage. As a consequence, the power production estimation can be significantly over-estimated whenever reactive control is applied. Also, in the same study [11], one of the conclusions that have been highlighted is that models obtained through data-based methodologies considering an average system dynamics over a range of operation, are usually more representative than the linear ones developed on BEM-based potential flow solutions [41]. Motivated by this consideration, in the following Sections V and VI the principles of data-based modelling and its applications on wave energy systems are analysed and compared.
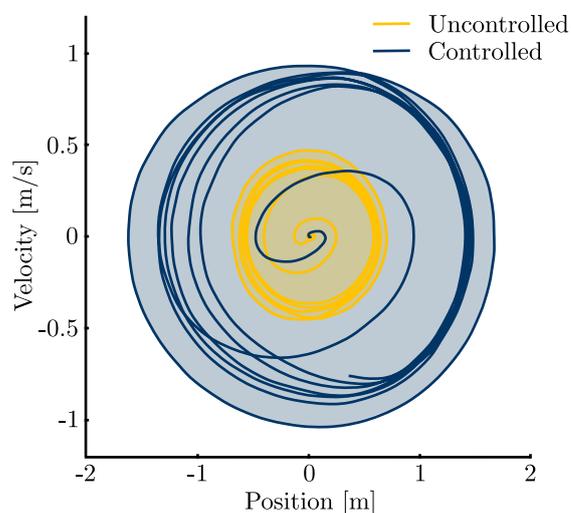

Fig. 3. Phase space of a heaving WEC device in controlled and uncontrolled conditions, starting from zero initial conditions and excited by regular waves (adapted from [11]).

V. Data-based WEC modelling

In recent years, data-based modelling of wave energy systems has became more common [42], mainly due to the growing popularity of *system identification* [43] applications in the wave energy field. Motivated by the issues related to WEC modelling mentioned in the Sections II and IV above, several studies have been presented, trying to exploit techniques from the system identification field in the attempt of obtaining sufficiently descriptive models, with reduced computational complexity.

In this context, the control-oriented models that can be identified are usually used to describe the relation that occurs between the external inputs of the system (the wave force f_{ex}, and the control input f_{PTO}) with the defined output (the velocity $\dot{z}$ or the displacement z), in time or frequency domain. Different model structures could be employed. For linear models, most popular are the transfer functions (in continous or discrete time domain), and linear state space representations. To identify nonlinear models instead, polynomials, and neural networks are the most employed structures. Both the linear and nonlinear models are usually parametrised on θ, and the modelling process related to the identification consists in the minimisation of a cost function, $\mathcal{J}_{ID}$ which depends upon θ, and usually is formulated as a function of the square of the L^2 norm of the error between the expected model output and the training set measurements [44]:

$$\theta_{ID} = \arg\min_{\theta} \ \mathcal{J}_{ID}\left(\theta, e(\theta)\right), \tag{5}$$

where θ_{ID} represents the model parameters identified at the end of the identification process, while $e(\theta)$ is the vector containing the errors between the measured output and the one estimated by the model with θ parameterisation. The adopted approaches can be classified in terms of the domain over which the optimisation process in Equation (5) is carried on (*i.e.* the error e is computed). The resulting methodologies can be divided in *time-domain* and *frequency-domain* approaches [45, 46]. These two differ mainly for the form of $\mathcal{J}_{ID}$ to be minimised during the identification process, and each one has its own advantages and disadvantages, as deepened in Sections V-A and V-B.

A. Time-domain modelling approach

Whenever the modelling process is made considering data in time domain (*i.e.* the data considered to compute the error in $\mathcal{J}_{ID}$ are measurements at different time instants) the identification process is a time-domain one. Usually, the cost function to be minimised during the identification process in Equation (5) for this type of approaches is formulated as a 'least-squares' identification[3]:

$$\mathcal{J}_{ID}\left(\theta\right) = \|e(\theta)\|_2^2 = \sum_{n=1}^{N_t} e(\theta, nT_s)^2, \tag{6}$$

where N_t represents the amount of data samples considered in the identification (each one sampled every T_s sampling time), and $e(\theta, nT_s)$ represents the error between the output of the model parametrised by θ and the measured output at $t = nT_s$. This is the consequence of the definition of the error in time-domain as:

$$e(\theta, t) = y(t) - \hat{y}(\theta, t), \tag{7}$$

where $y(t)$ is the measured output at time t, and $\hat{y}(\theta, t)$ is the estimated output of the identified model (parameterised by θ) at the same time instant. It must be highlighted that this definition of the error stresses the dependence on the domain t over which the time-domain techniques compute the error. Since the formulation of the cost function to be minimised directly takes into account discrete time measurements (the output is sampled), the majority of the identified time-domain models are discrete time models. Among them, structures like *autoregressive exogenous* (ARX), *autoregressive moving average exogenous* (ARMAX) for linear models, *nonlinear autoregressive exogenous* (NARX), and *nonlinear autoregressive moving average exogenous* (NARMAX) for nonlinear models, are the most employed. The main advantage of time-domain models is their variety, that easily enables the management of the level of complexity, which can be straightforwardly increased during the identification routine. This allows (especially for the nonlinear identification) the inclusion of possible known elements (*e.g.* whenever a well-known nonlinearity is present) in the model. However, the choice of the right model is a trade-off between complexity and accuracy. This is not only related to the goal of 'obtaining the best model which is the less computational demanding', but also to avoid the overfitting phenomenon. High complexity models tend to attempt to model also the noise/disturbance dynamics whenever not suitably limited.

B. Frequency-domain modelling approach

If the modelling process employs data in frequency domain (*i.e.* the data considered to compute the error in $\mathcal{J}_{ID}$ are considered in frequency domain, by means for example of their Fourier transforms). In this type of approaches, the cost function is usually parameterised in terms of θ and of the frequency ω, as[4]:

$$\mathcal{J}_{ID}\left(\theta, \omega\right) = \left\|E(\theta, \omega)\right\|_2^2 = \left\|\frac{Y(\omega)}{U(\omega)} - G(\theta, \omega)\right\|_2^2 \left\|U(\omega)\right\|_2^2, \tag{8}$$

where $E(\theta, \omega)$, $Y(\omega)$, and $U(\omega)$ are the Fourier transforms of the output error, the output, and the input respectively, while $G(\theta, \omega)$ represents the frequency response of the identified model. With this kind of formulation of the cost function, the identification process attempts to 'fit' the identified model to the empirical transfer function $Y(\theta, \omega)/U(\theta, \omega)$. Among the main advantages of this kind of approach is the possibility of restricting the frequency range over which

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[3]For the sake of simplicity, the system to be identified here is supposed to be SISO. Similar formulations can be made for MIMO systems.

[4]For the sake of simplicity, the system to be identifies is supposed to be a SISO, with a transfer function formulation. Similar considerations can be made for MIMO system of for other model structures.

the identification is performed to a suitably designed range of interest. This can be of particular interest in application like the wave energy one, where the system is continuously excited by the wave disturbance which has a spectrum limited in frequency, and thus has only a certain range of frequency over which the modelling must be performed precisely. Moreover, this characteristic of frequency-domain approaches, coupled with the design of frequency bounded identification signal, enables the possibility of modelling focused tests design. Additionally, it is possible to include in a straightforward manner (differently from time-domain approaches) different identification tests with different excitation amplitudes by averaging their resulting frequency responses, allowing in this way an identification that takes into account (in an average sense) of a broader set of operating conditions (different from a linearisation around equilibria that hypotheses small oscillations). However, since the minimisation that is performed during the identification process of this kind of approaches is not considering the error signal but its Fourier transform, standard validation indexes are in general slightly poorer than the one obtained with time-domain identification (which directly take into account the error, assuming the risk of overfitting) [47]. Finally, given the nature of the frequency-domain, such type of modelling enables the possibility of also continous time domain models, which are instead not easily obtainable with time-domain modelling techniques and sampled identification signals.

VI. DATA-BASED MODELLING TECHNIQUES OF WAVE ENERGY SYSTEMS

In the wave energy field, a comprehensive study on the application of system identification, is presented

TABLE I
SUMMARY REVIEW TABLE. SIM: DATA FROM SIMULATED ENVIRONMENT, EX.: EXPERIMENTAL DATA, TD: TIME-DOMAIN IDENTIFICATION, FD: FREQUENCY-DOMAIN IDENTIFICATION, ▲: LINEAR STATE-SPACE, ▵: TRANSFER FUNCTION, ■: NONLINEAR MODEL.

Ref.	Type of data		ID domain		Model type	WEC type
	Sim.	Ex.	TD	FD		
[48]	●		●		▵ ■	PA
[49]	●		●		▲ ■	PA
[50]	●		●		▵	PA
[51]	●		●		▲	PA
[52]		●		●	▵	PA
[53]		●		●	▵	PA
[54]	●		●		■	PA
[55]	●			●	▲	PA
[56]		●	●		▵ ■	PA
[57]		●	●		▵ ■	OWC
[58]		●	●		▵	OWC
[59]		●	●		▵	OWC
[60]		●		●	▲	PA
[61]	●			●	▵	PA
[41]	●			●	▲	PA
[62]	●			●	▵	PA
[11]	●			●	▲	PA
[63]	●			●	▵	PA
[64]	●		●		■	PA
[65]		●		●	▲	Ter.

in [49] and [48], including the design of identification-oriented tests to be performed on a point absorber in a numerical wave tank [49], together with the consequent development of control-oriented models, using different black-box identification methods [48]. Similar tests are employed to perform grey-box identification in [50]. In [51], system identification is performed to find a state-space model of a point absorber, subsequently employed by a predictive fuzzy logic controller [66]. In [52], identification-oriented multisine signals are designed to perform experimental tests on a scaled WEC, with the resulting responses used to identify a black-box model of the system. A scaled multi-DoF WEC model is also identified from experimental tests in [53], and the obtained model is adopted to synthesise different energy-maximising control strategies, subsequently experimentally assessed. In [54], a nonlinear reduced-complexity model is determined following a data-based *moment-matching* [67] approach, on the basis of simulated data. Data obtained in a simulated environment are also adopted in [55], where a control-oriented WEC model, including nonlinear Froude–Krylov effects [68], is identified and subsequently employed to synthesise a moment-based control solution. In [56], recorded tank test data are used to identify nonlinear Kolmogorov-Gabor models. Applications of system identification techniques to experimental data from tests on scaled OWC devices can be found in [57] and [58], where the authors identify the dynamical relationship between free-surface elevation and water column displacement inside the OWC chamber. In [59], the same dataset is used to model the OWC dynamic system describing the turbine pressure drop between the chamber and atmosphere, driven by the variation in free-surface elevation. In [60], a parametric model of the dynamics of a point absorber is obtained from wave tank tests, and used to synthesise a LiTe-Con strategy, which is then experimentally assessed. The studies in [61] investigate the process of identifying a model from data obtained in a numerical wave tank, exciting the system with different classes of signals, evaluating the uncertainty on the obtained models, and assessing the performance of a corresponding robust control strategy. In [41], the influence of the excitation amplitude in the process of system identification is highlighted, with similar wave tank experiments. In [62, 63], identification-oriented signals are applied to a WEC coupled with a mooring system (usually neglected in control-oriented models) in a simulated environment, and the obtained linear description of the system is used to synthesise a reactive controller. In [11] identification focused signals are applied on a point absorber in a numerical wave tank, and the linear model identified from these tests is validated in uncontrolled and controlled conditions in the same high-fidelity numerical wave tank. In the study [64], the authors propose a novel approach to system identification of a point absorber (the so-called 'green-box' identification), which attempts to take into account of the carbon footprint of the model identification and simulation in the process of model choice. Finally, in [65] system identification is employed to

obtain a representative linear model of a flap-like device to synthetise an unknown-input estimator from experimental data coming from an hardware-in-the-loop system.

A. Techniques comparison

In Table I a comparison between the different data-based modelling applications in wave energy is reported. Several considerations can be made. The first one is related to the data source. As it can be observed, data-based techniques are employed with both simulated and experimental data. Data coming from a high-fidelity environment can be useful to be employed (even if they are not sampled from the real device) because, coupled with system identification techniques, can be used to get less complex (and control-oriented) models which are more representative than the ones based on simple linearisation around equilibria. Such data, in this way, are employed to partially attempt to solve the uncertainty related to standard BEM models. Other considerations can be made instead regarding the relation between the identification domain and the type of model. Regarding this, it can be noticed that whenever the identification is made in time domain, and the target model is a linear one, the employed structure is a transfer function. This can be explained by the popularity of ARX models in discrete time domain. This linear combination of present and past values of output and inputs is equivalent to a discrete transfer function. Moreover, all the nonlinear data-based models are identified in time domain, since, apart from [54] that exploits the moment-based representation, all of them can be traced back to forms of NARX structures. The linear state-space representation is instead employed only with frequency domain techniques. Finally some consideration can be drawn regarding the choices of input and output of the developed models. The most common choices for the input are the wave elevation $\eta(t)$ and the force exerted at the conversion axis [33], while usually the output is defined as the WEC motion (displacement and velocity). However, in the cases of OWC devices, the input/output choice is different. In [57, 58] the developed models define the output as the undisturbed wave elevation $\eta(t)$ (*i.e.* the wave elevation experimentally measured whenever the OWC is not installed in the facility), while the output is the water displacement in the chamber in the same testing condition. In [59] the authors defined the output as the chamber pressure.

VII. CONCLUSIONS

In this paper we analyse the different data-based modelling techniques that have been applied in the wave energy field. In the last years, several applications have been proposed, and all fall under the categories of time- and frequency-domain identification approaches. We provide a comparison of the wave energy applications, highlighting the domain of identification and the model structure definition. These two kind of approaches offer several opportunities (variety of parameterisation for the time-domain

model structures, choice of the identification frequency range for frequency-domain methods), but also have some kind of limitations (risk of overfitting for the time-domain approach, worse validation indexes for frequency-domain methods). These approaches must be seen as complementary approaches, not competitive. The availability of previous knowledge on the model structure or on the operational frequency range can define which of the two approaches could fit better than the other. In general, at the state-of-the-art, data-based techniques have been already employed to find linear and nonlinear control-oriented models, and these model have been already successfully employed to synthetise controllers and estimators. However, better understanding of the model performances in both uncontrolled and controlled (with exaggerated motion) conditions must be investigated in experimental setups [69], to understand the real capabilities of data-based approaches. Moreover, to really unlock the true potential of these techniques, it is important to investigate a 'non-blind' identification approach, which exploits the most from the knowledge offered by physics-based analysis, in the attempt to include in a conscious manner the nonlinearities that characterise wave energy systems.

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