



## Sequential city growth: Empirical evidence

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### ABSTRACT

Using two comprehensive datasets on populations of cities and metropolitan areas for a large set of countries, I present three new empirical facts about the evolution of city growth. First, the distribution of cities' growth rates is skewed to the right in most countries and decades. Second, within a country, the average rank of each decade's fastest-growing cities tends to rise over time. Finally, this rank increases faster in periods of rapid growth in urban population. These facts can be interpreted as evidence in favor of the hypothesis that historically, urban agglomerations have followed a sequential growth pattern: Within a country, the initially largest city is the first to grow rapidly for some years. At some point, the growth rate of this city slows down and the second-largest city then becomes the fastest-growing one. Eventually, the third-largest city starts growing fast as the two largest cities slow down, and so on.

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### 1. Introduction

The study of how cities develop and grow has attracted the attention of economists for a long time. However, most of the existing studies use very recent data (Glaeser et al., 1995; Henderson and Wang, 2007) or focus on one or two countries only (Eaton and Eckstein, 1997; Ioannides and Overman, 2003). This paper undertakes a thorough analysis of urban growth by taking into account a longer time period and a greater number of countries. I first show that, in most decades and countries, the distribution of cities' growth rates is skewed to the right, indicating that a few cities grow much faster than the rest. Second, I study the behavior of the cities that grow the fastest in each decade. I found that these cities tend to maintain their lead above other cities. Cities which were initially largest were also the first to grow and they do so at a rate that is faster than the rest up until a critical size. Only when such cities reach their critical size does the second-largest city start growing at a significant pace until it too reaches a critical size, and so on. This trend of sequential growth is markedly pronounced in episodes of intense urban population growth.

These novel empirical facts indicate that city growth processes vary a great deal over time and across cities but follow a remarkably similar pattern across countries. The specifics of this pattern have interesting implications both for policy makers and academics.

First, they are useful to formulate effective policies in countries whose population is changing rapidly, mainly located in Asia and Sub-Saharan Africa.<sup>1</sup> For example, an extensive literature in economic development emphasizes the importance of infrastructure investment on the economic performance of less-developed countries, particularly at early stages of urbanization (Bennathan and Canning, 2000). My study contributes to the design of strategies on infrastructure investment by presenting data on how urban development evolves over time. As such, policy makers can make informed decisions on where and when to invest in urban infrastructure, taking into account the country's geographical structure.

This new evidence can also contribute to the debate on the effectiveness of foreign aid in developing countries. One particular puzzling aspect that has been discussed in the literature (see Calderón et al., 2009) is that foreign aid disbursements are not conducive to an improvement in the distribution of income in recipient countries. My findings provide a possible explanation for this. Even if aid flows were equally spread among cities or regions, basic urban economic forces would lead to a concentration of resources in the initially largest cities, at least for some period of time, hence

<sup>1</sup> Recent studies estimate that, if current trends continue, the urban population should increase in East Asia by about 450 million people over the next two decades. The increase is predicted to be almost 350 million in South and Central Asia, and 250 million in Sub-Saharan countries. See the World Bank's *World Development Report* (2009).

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limiting the spread of wealth to other geographical areas of the country.

While the previous examples suggest how these new facts on city growth can be useful for less-developed countries, one can easily think of several applications that may be of interest for policy makers in other regions as well. For instance, these findings can be used to predict the geographical evolution of regions that experience natural disasters or wars that fundamentally alter their urban structure (see Davis and Weinstein, 2002) or to analyze how labor and capital flows evolve in regions that are part of a process of economic and political integration, like the European Union.

From an academic point of view, my findings are important to enhance our understanding of city growth and in particular the effect that a country's urbanization process has on its urban structure. Additionally, the three new stylized facts presented in the paper would presumably be valuable inputs to develop new theories of urban growth or extend the existing ones to improve their goodness of fit.

This paper is based on the empirical analysis described in Cuberes (2009). However, there are a number of important differences between the two papers. First, the analysis of the current paper uses data on both administratively defined cities and metropolitan areas, whereas Cuberes (2009) only discusses the former unit of analysis. Second, my second result – the positive trend of the rank of the fastest-growing cities – is obtained here by estimating a regression of the logarithm of this rank on time, the number of cities/MAs, and their square. This is a more suitable specification than the one used in Cuberes (2009) for reasons stated below. Third, the current paper includes a detailed example that facilitates the interpretation of my estimates. This example is then followed by a graphical analysis for all the countries in my sample, showing how cities and metropolitan areas grow sequentially and that they do so faster when their urban population increases rapidly. Fourth, I distinguish my findings from the well-known empirical regularity that urban primacy ratios – defined as the ratio of the population of the largest cities of a country over that country's total or urban population – follows an inverse U-shape pattern as countries develop. Fifth, I report the existence of significantly different city growth patterns in different regions of the world. Finally, I discuss here a number of robustness checks that show that the results are not sensitive to any of the assumptions imposed in the analysis.

The paper is organized as follows. In Section 2, I review the literature that most closely relates to my analysis. Section 3 provides a brief summary of an existing theory of sequential city growth, based on Cuberes (2009). In Section 4, I describe the dataset used throughout the paper and discuss the method of sample selection. The three new empirical findings are presented in Section 5. Section 6 shows some robustness checks, and Section 7 concludes.

## 2. Related literature

Several papers have used historical data on the population of cities and metropolitan areas to study the properties of their growth process. One prominent example is Eaton and Eckstein (1997), who analyze the evolution of the transition matrices of France's and Japan's largest metropolitan areas and conclude that they remained constant during the time intervals of 1876–1990 for France and 1925–1985 for Japan. Another important paper along these lines is Ioannides and Overman (2003). Using data on for the largest US metropolitan areas in the 1900–1990 period they estimate city growth non-parametrically and show that deviations from Gibrat's law<sup>2</sup> are not statistically significant. Other papers that

<sup>2</sup> This law states that the growth rate of a city's population is independent of its size. See Gibrat (1931) for a general statement of the law and Gabaix and Ioannides (2004) for an excellent review of studies that apply it to cities.

analyze the evolution of the US population using long time series are González-Val (forthcoming), Beeson et al. (2001), Beeson and DeJong (2002), Ehrlich and Gyourko (2000), and Kim (2007).

My paper differs from Eaton and Eckstein (1997) and Ioannides and Overman (2003) on several dimensions. First, I provide results for both administratively defined cities and metropolitan areas, while they only analyze the latter. Second, the number and identity of urban agglomerations in the aforementioned studies are constant over time, while I allow for the entrance of new cities and metropolitan areas into my sample as countries urbanize. Finally, from a methodological point of view, I focus on a simple statistic that summarizes the process of city growth in the fastest-growing cities instead of analyzing properties of the entire distribution of cities growth rates.

In terms of theories, Henderson and Venables (2009) and Cuberes (2009) have recently developed models of city formation in which urban agglomerations grow sequentially. In these models, the initially largest cities are the first to grow and they do so until they reach a critical size, at which point they are followed by the second-largest cities, then the third-largest ones, and so on.<sup>3</sup> The empirical facts reported below are consistent with the main predictions of these models, which I review in the next section.

## 3. Theoretical background

In this section, I sketch a theoretical framework that rationalizes the empirical exercises carried out in Sections 5 and 6. To my knowledge, Henderson and Venables (2009) and Cuberes (2009) are the only two papers that explicitly model sequential city growth. Although they are substantially different, the two models assume irreversible investment and predict that cities grow sequentially, with the initially largest ones being the first to develop and grow. They also predict that this process is more pronounced the faster the growth rate of a country's urban population. Henderson and Venables (2009) develop a rich model that offers predictions on the role of various institutions in driving different equilibria and on housing price cycles, among others. The model proposed in Cuberes (2009) is more stylized, but it captures the process of sequential city growth in a straightforward way. For simplicity, I summarize here the main setup of the latter model, although both theories predict the three empirical facts described in Section 5.

The benchmark model consists of two cities that are modeled as Cobb–Douglas production functions.<sup>4</sup> Each city uses labor and capital to produce a homogenous good. Firm  $i$  located in city  $j$  produces output  $Y^{ij}$  according to

$$Y^{ij} = (N^{ij})^\alpha (K^{ij})^{1-\alpha} (K^j)^\psi$$

where  $N^{ij}$ , and  $K^{ij}$ , respectively, represent the firm's labor and capital inputs.  $K^j$  is the total stock of capital installed in city  $j$  (i.e.,  $K^j \equiv \sum_{i=1}^I K^{ij}$ ), and  $I$  is the number of firms operating in that city. The parameter  $\alpha$  is between zero and one and  $\psi > 0$  captures the positive external effect of aggregate city capital on any firm that

<sup>3</sup> Duranton (2007) presents a model of city and industry growth that links the growth rate of cities' employment with changes in their industry composition. I do not model a city's firm or industry composition here, but it would be interesting to test to what extent the introduction of this source of heterogeneity in my model affects the prediction of sequential city growth. See also Findeisen and Südekum (2008) for an empirical application of Duranton's model.

<sup>4</sup> Cuberes (2009) presents the optimal and the decentralized solutions to the model. Here I focus on the latter, because I am only interested in the theory's positive predictions. Another difference between the two papers is that in this summary I discuss neither the existence of a unique equilibrium nor the extension of the model in the case of an arbitrarily large number of cities.

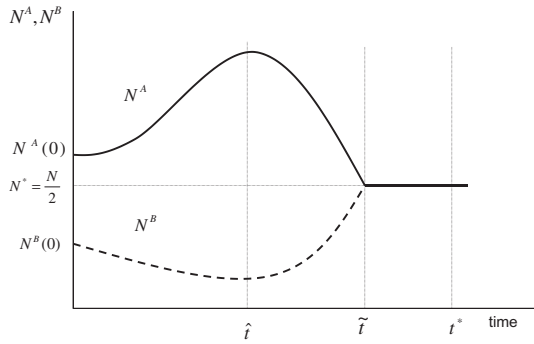


Fig. 1. The evolution of population in the model.

operates in the city.<sup>5</sup> Moreover, firms pay a fraction  $\frac{1}{l}$  of the congestion cost  $g(K^j)$  generated by the total stock of capital installed in the city where they operate, where  $g(\cdot)$  is an increasing and convex function. Therefore, normalizing the price of the good to one, profits for a given firm can be expressed as:

$$\pi^{ij} = (N^{ij})^\alpha (K^{ij})^{1-\alpha} (K^j)^\psi - (r^j + \delta)g(K^{ij}) - \frac{1}{l}g(K^j) - \omega N^{ij}$$

where  $\delta \in (0, 1)$  is the rate at which capital depreciates, and  $r^j$  and  $\omega$  denote the rental price of capital and the wage rate, respectively. Free labor mobility then implies that the population ratio between the two cities satisfies

$$\frac{N^A}{N^B} = \left( \frac{K^A}{K^B} \right)^{\frac{1+\psi-\alpha}{1-\alpha}} \quad (1)$$

From the firms' first-order conditions, one also has

$$r^j = f_j - \delta - g'(K^j) \quad (2)$$

where  $f_j \equiv (1 - \alpha)(N^j)^\alpha (K^j)^{\psi-\alpha}$  is the gross marginal product of capital in city  $j$ . Households invest in capital and supply labor inelastically. They solve the following problem:

$$\begin{aligned} & \max \int_0^\infty e^{-\rho t} \ln(c) dt \\ & \sum_{j=A,B} \dot{i}^j + c = \omega + \sum_{j=A,B} r^j z^j \\ & \dot{i}^j \geq 0, \quad \forall j = A, B \\ & z_0^j \text{ given}, \quad \forall j = A, B \end{aligned}$$

where  $c$  is per-capita consumption, and  $\rho \in (0, 1)$  is the household's discount rate.  $z^j$  represents the amount of assets invested in city  $j$ . As mentioned above, an important assumption of the model is that households face the irreversibility constraints  $\dot{i}^j \geq 0$ ,  $j = A, B$  reflecting the fact that, once installed in a city, physical capital cannot be relocated to the other city or destroyed. Finally,  $z_0^j$  is the initial stock of assets in city  $j$ , which is taken as given.

The model next assumes that at the initial date, city  $A$  has a slightly larger stock of capital than city  $B$  and that congestion costs in city  $A$  are relatively small compared to the productivity gains associated with its large size. With these assumptions, the evolution of the population in each city follows the pattern displayed in Fig. 1.

Initially, city  $A$  has more population than city  $B$  because the former also has a larger initial stock of physical capital and, from Eq. (1), population moves together with capital in the model. From the initial period to period  $\hat{t}$ , the population moves from city  $B$  – the

smallest one – to city  $A$  since in this time interval all new investment goes to the latter city.<sup>6</sup> At period  $\hat{t}$ , the rise in congestion costs in city  $A$  makes capital equally productive in the two cities. Therefore, investment becomes positive again in city  $B$  until both cities have the same stock of capital (at period  $\tilde{t}$ ). After this period, the two cities are identical, and thus the population is equally split between them until the economy reaches its steady state (at period  $t^*$ ). This model has the following three testable implications that are explored in Section 5. Within a country the model predicts.

### 3.1. City growth rates are skewed to the right

In the model, the population of one of the cities grows much faster than the rest at each point in time during the transition to the steady state. If one thinks of an extension of the model with more than two cities (see Cuberes, 2009), this implies that the coefficient of skewness of cities' growth rates must be positive along this transition.

### 3.2. The rank of the fastest-growing cities increases over time

The model also predicts that, at each point in time, the fastest-growing city is the biggest one, conditional on the fact that congestion costs are not too large in that city. This means that cities grow in a precise sequential order: after the largest city grows alone for a number of periods, the second-largest city takes the lead, then the third one, and so on. Therefore the rank – with the largest city having rank 1- of the cities that grow the fastest is predicted to increase over time.

### 3.3. The increase in rank is faster the faster urban population grows

It is shown in detail in Cuberes (2009) that, in this model, exogenous increases in population  $N$  are associated with faster sequential city growth, because as the country's urban population grows, there is more pressure on the existing largest cities. These cities then reach their congestion costs earlier on, and so the population moves to the second-largest city earlier. This implies that the average rank of the fastest-growing cities should raise faster the higher the urban population growth.

## 4. The data

There exist three datasets for international comparisons of the populations of urban agglomerations over long time intervals. The first one, from Vernon Henderson, contains data on metropolitan areas (henceforth, MAs) in different countries during the 1960–2000 period. The second one, by Thomas Brinkhoff, presents information on the populations of various administratively defined cities (henceforth, cities) in 79 countries during the 1970–2000 period. Finally, the most comprehensive dataset, by Jan Lahmeyer, includes the size of the largest cities for all countries up to the year 2000 and going as far back as 1790 in some cases. In the three datasets city population is available on a decade frequency.

Cities and MAs are in most cases very different units of analysis.<sup>7</sup> Since the theory sketched above can, in principle, be applied to both definitions I proceed to test its main implications using the two types of data.<sup>8</sup> The paper combines city data from

<sup>6</sup> Capital is most productive in city  $A$  since the difference between its gross  $MPK$  and its marginal congestion costs (see Eq. (2)), is much larger there than in city  $B$ .

<sup>7</sup> For instance, using the definition of an administratively defined city, New York had a population of 8,008,278 in the year 2000. Its MA, however, includes a much larger geographical area, and so the figure becomes 21,199,865.

<sup>8</sup> This has been a common practice in empirical studies of city growth like for instance Glaeser et al. (1995) and Eeckhout (2004).

<sup>5</sup> If one interprets  $K$  in a broad sense, this positive effect may be generated for instance from the existence of knowledge spillovers between firms.

54 countries from the Lahmeyer's and Brinkhoff's datasets and data on the MAs of 115 countries from Henderson. A list of the countries and decades used is displayed in Table A1 of the appendix.

#### 4.1. Sample selection of cities

Heterogeneity in data availability and time span across countries makes it difficult to conduct appropriate cross-country comparisons. In this paper, I follow the methodology used in Henderson and Wang (2007) to address this issue. They order cities by size and select the first  $s$  cities such that the  $s + 1$  city would be below a relative cut-off. This cut-off is defined as the ratio of the minimum (100,000) to mean (495,101) city size in their sample of countries in 1960. Henderson and Wang argue that this sample selection method has the advantage of allowing one to analyze a portion of the city size distribution that is comparable across countries and over time. I use their same cut-off in the exercises that involve MAs, but I choose a different one for cities for two reasons. First, cities tend to be considerably smaller units than MAs. Second, my sample of cities expands back to 1790 in some cases, when most cities were much smaller than in 1960. I consider the distribution of city sizes in the United States in 1790 and select the cities that have a relative population above 0.6.<sup>9</sup> This threshold comes from dividing the US median city size in 1790 (5077) by its average (8402).

### 5. New empirical facts on city growth

#### 5.1. Right skewness of cities' growth rates

In this section, I show that the distribution of cities' growth rates is skewed to the right in most countries and decades.<sup>10</sup> Let  $x$  denote the variable on which one wants to calculate the coefficient of skewness, and let  $x_i$ ,  $i = 1, \dots, n$  be an individual observation on  $x$ . The coefficient is then defined as  $m_3 m_2^{-3/2}$ , where  $m_r$  is the  $r$ th moment about the mean  $\bar{x}$ , i.e.,  $m_r = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^r$ . A positive (negative) skewness indicates a distribution with an asymmetric tail extending toward more positive (negative) values.

In 88% of my sample of cities, the coefficient of skewness is positive. I next run a normality test that reveals that in 78% of the cases these coefficients are statistically different than zero.<sup>11</sup> For MAs, 78% of the cross-sections are skewed to the right. When I run the test described above, the 60% of the observations reject that MAs growth rates distributed normally. These percentages are remarkably high, given the fact that the number of observations is quite small in many periods and countries.

#### 5.2. Average rank of the fastest-growing cities

In this section, I expand the previous finding and investigate from what part of the city size distribution the fastest-growing cities come in each decade. I begin by ranking each country's cities by size – in terms of population – at every decade, with the largest city having rank 1, the second largest having rank 2, and so on. Next, for each country-decade, I calculate the 75th percentile of cities' growth rates and consider the cities whose growth rate is larger or equal to this threshold.<sup>12</sup> I refer to these cities as

"fast-growers" in that decade, and I calculate their average rank. It is crucial to understand that the logic of this exercise is *not* to follow specific cities over time, but to determine which cities grow the fastest *at each point in time*. In particular, I attempt to answer two questions. First, for a given decade, do the large cities (low rank) or the small ones (high rank) grow the fastest? Second, does this pattern change from decade to decade?

##### 5.2.1. An example using historical data on French cities

I illustrate here my procedure using as an example the growth of French cities in the mid 19th century. The second and third columns of Table 1 display the population of the largest French cities in the years 1851 and 1861 respectively. Cities are ordered in decreasing order by their size in 1861. The growth rates of cities' population between these two years are reported in column 5. The sixth column illustrates the method of sample selection employed throughout the paper. Following the discussion from Section 4, only cities with a relative population (relative to the country's average in 1861) above 0.6 are selected. In this example, Toulouse is the last city that satisfies this constraint.

Next, I consider the subsample of cities whose growth rate is strictly larger than the 75th percentile of the growth rates of Table 1. In this example, this percentile corresponds to a growth rate equal to 0.675, and so only the cities of Lyon and Lille are classified as "fast-growers." The ranks of these two cities are 2 and 5, respectively, so that the average rank in this case is 3.5. If one carries on these calculations a decade later, the 75th percentile of growth rates of the selected cities corresponds to a growth rate equal to 0.2 and so the fast-growers in this decade are Marseille, Lille, Saint-Etienne, and Reims, and their average rank is 7.25. Therefore in 1861 the 75th percent fastest-growing cities comes from a group of relatively large cities (with an average rank of 3.5) and in 1871, the fastest-growing cities are relatively smaller (a rank equal to 7.25). I then repeat this routine for every decade and thus end up with a time series of the rank of the fastest-growing cities which is plotted in Fig. 2. The average rank goes from 3.5 in 1861 to 62.6 in 1936, and up to 164.5 by the end of the period.

In this example, early on in the process of urbanization, the largest cities grow fastest. As time passes, population growth in the larger cities declines and the fastest population growth can be found in smaller cities farther down in the urban hierarchy. This is consistent with the theory of sequential city growth summarized in Section 3.

One important feature of the two datasets used throughout the paper is that the number of cities with available information on population significantly increases over time.<sup>13</sup> This is also the case in France and therefore a possible concern with the example described above is that the rank of the fastest-growing cities may grow in part because there are more cities in the sample as time goes by. By construction, a larger number of cities imply a higher probability that, in a random draw, one of them has a high rank. To check that the positive trend of Fig. 2 is not an artifact of the data one should then take into account the growing number of French cities in the sample. In results not shown here I find that the slope of the average rank is indeed positive even after controlling for this margin. In the next two subsections I explain in more detail how I deal with this issue when using data for the entire sample of countries.

##### 5.2.2. A systematic analysis using all countries

The previous section has shown that in France, the average rank of the fastest-growing cities exhibits a clear, positive trend in the 1851–1999 time interval. Here I examine whether this is also the

<sup>9</sup> As a robustness check, I have chosen different cut-offs to select my sample of cities; although the composition and size of the resulting samples change, the qualitative results do not vary much. See Section 6.1.

<sup>10</sup> Glaeser and Gyourko (2005) document this fact but they only use US data during the 1920–2000 period.

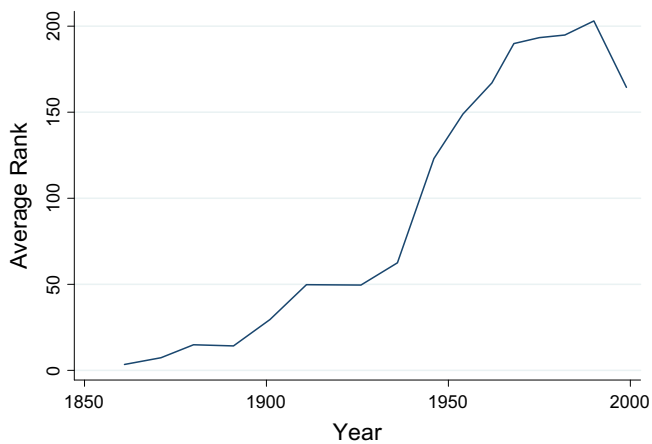
<sup>11</sup> This normality test requires a minimum of eight observations, and its null hypothesis is that the distribution of the data is normal. More details can be found in D'Agostino et al. (1990) and Royston (1991).

<sup>12</sup> This exercise has been carried out using different percentiles and the results are very similar. See Section 6.2.

<sup>13</sup> In the Lahmeyer–Brinkhoff dataset this number grows on average 14% during the time interval considered. The corresponding figure for the Henderson dataset is 23%.

**Table 1**  
Rank and growth rate of the largest French cities in 1861.

City	Pop in 1851	Pop in 1861	Rank in 1861	Growth rate	Ratio pop/avg. in 1861
Paris	1053,300	1696,100	1	0.61	9.25
Lyon	177,200	318,800	2	0.8	1.74
Marseille	193,300	260,900	3	0.35	1.42
Bordeaux	130,900	162,800	4	0.24	0.89
Lille	75,800	131,800	5	0.74	0.72
Nantes	96,400	113,600	6	0.18	0.62
Toulouse	94,200	113,200	7	0.2	0.62



**Fig. 2.** The evolution of the average rank of the fastest-growing cities in France.

case in my panel of countries. As mentioned above, an important feature of both datasets (cities and MAs) is that the number of observations in the sample grows over time. To account for this, I include the number of cities (or MAs) without missing data on population for each decade and country as a control variable. I hence estimate the following panel-data regression:

$$\log RANK_{25jt} = \eta_j + \beta_1 t + \beta_2 N_{jt} + \beta_3 N_{jt}^2 + \varepsilon_{jt} \quad (3)$$

where  $RANK_{25jt}$  and  $N_{jt}$  are the average rank of the 25% fastest-growing cities (or MAs) and the number of cities (or MAs) in country  $j$  and period  $t$ , respectively. In some specifications I also include the square of the number of cities as a control in order to better capture the relation between the number of cities in the sample and the dependent variable.  $\eta_j$  is a country-fixed effect that is meant to control for unobservable country time-invariant factors that could affect the evolution of  $RANK_{25}$  over time. Examples of such unobservable variables are aspects of geography and culture that may have an impact on a country's city growth process. The variable  $t$  measures time in decades, and  $\varepsilon_{jt}$  is a standard error term.

Following the Zipf's law literature (see Gabaix, 1999) I use the logarithm of  $RANK_{25}$  as the dependent variable. There are two main reasons for doing so. First, it ensures that the predicted values will be positive, a desirable property given the nature of my dependent variable. Second, and more specific to my exercise, the variable  $RANK_{25}$  may potentially be influenced by large outliers – especially in countries with few decades of data – and so taking logarithms reduces the impact of these observations on the estimation.<sup>14</sup> Table 2 shows the estimates of Eq. (3) for both cities and MAs.

Specification [1] of the table shows that  $RANK_{25}$  clearly increases as time goes by. Although including the number of cities as a regressor (specification [2]) has a large effect – the size of

the time coefficient drops by a factor of two – the positive sign of the trend coefficient remains statistically significant. This is also the case when the square of the number of cities is included as an additional control (specification [3]). The coefficient associated with the squared term is negative indicating a concave relation between  $RANK_{25}$  and the number of cities in the sample. The results for MAs also support the hypothesis that  $RANK_{25}$  exhibits a positive time trend (specification [4]). As is the case with cities, including the number of MAs as a regressor has a big impact on the magnitude of the time-trend coefficient, although it remains statistically significant (specification [5]). Finally, the inclusion of the squared term does not change much the size of the time estimate and again suggests some degree of concavity between the dependent variable and  $N$  as in the regression for cities.

These estimates are consistent with the well-known fact that urban primacy ratios – defined as the fraction of the population in the largest  $N$ th cities of a country relative to its total or urban population – follows an inverse U-shaped pattern when plotted against time. Fig. 3 from Cuberes (2010) reproduces these patterns for four countries.<sup>15</sup>

Notice that sequential city growth is sufficient to generate the inverse U-shaped pattern, because, if cities grow sequentially, the initially largest ones must represent a large share of the total (or urban) population of the country in the initial years and a relatively smaller one later on. This pattern could also be consistent with non-sequential growth, however. For instance, suppose that the initially largest city grows alone for a few years, and, after that, all cities grow at a rate equal to or higher than the first city. In this situation, one would have a bell-shaped pattern because the largest city will represent an increasing share of total population in the initial years and this share will decline as the rest of the cities grow faster. Yet, growth would not be sequential in the sense that one would not see the second city grow faster than the third one for a few decades, and so on. Therefore, although it is interesting to map my results with the urban primacy literature, the two approaches are different and complementary.

### 5.2.3. The relation between changes in $RANK_{25}$ and changes in urban population

Theories of sequential city growth imply that faster growth of the urban population should cause existing cities to reach their critical size faster. In this subsection I test this prediction by exploring whether changes in  $RANK_{25}$  and changes in urban population are positively correlated.

Urban population is defined here as the sum of the population of the cities (or MAs) that are above the 0.6 (0.202) cut-offs for cities (MAs) defined in Section 4.<sup>16</sup> For each country, I first calculate the average growth rate of  $RANK_{25}$  in periods of unusually rapid growth – defined as decades with a growth rate of urban

<sup>14</sup> The results are qualitatively similar if I estimate this regression in levels or using a log-log specification. This suggests that the presence of outliers is not an important concern.

<sup>15</sup> The figures represent the cumulative share of the (initially) largest cities on total population. This inverse U-shaped pattern has also been reported in Wheaton and Shishido (1981) and Junius (1999) among others.

<sup>16</sup> Defining urban population as the sum of the population of all cities with 5000 inhabitants or more does not alter the results in any significant way.

**Table 2**  
A regression of  $RANK_{25}$  on time, the number of cities/MAs, and its square.

	Cities			MAs		
	[1]	[2]	[3]	[4]	[5]	[6]
Time	0.143*** (0.02)	0.07*** (0.02)	0.034*** (0.01)	0.105*** (0.02)	0.077*** (0.02)	0.055*** (0.02)
Number cities/MAs		0.011*** (0.002)	0.033*** (0.003)		0.013** (0.006)	0.044*** (0.008)
Square number of cities/MAs			-0.000*** (0.000)			-0.000*** (0.000)
Constant	1.469*** (0.1)	1.5*** (0.08)	1.258*** (0.08)	1.14*** (0.05)	0.973*** (0.1)	0.667*** (0.11)
R <sup>2</sup>	0.372	0.558	0.698	0.09	0.146	0.24
Number of observations	536	536	536	448	448	448

Note: Robust standard errors in parentheses.  
\*\* Denote significance at the 5% level.  
\*\*\* Denote significance at the 1% level.

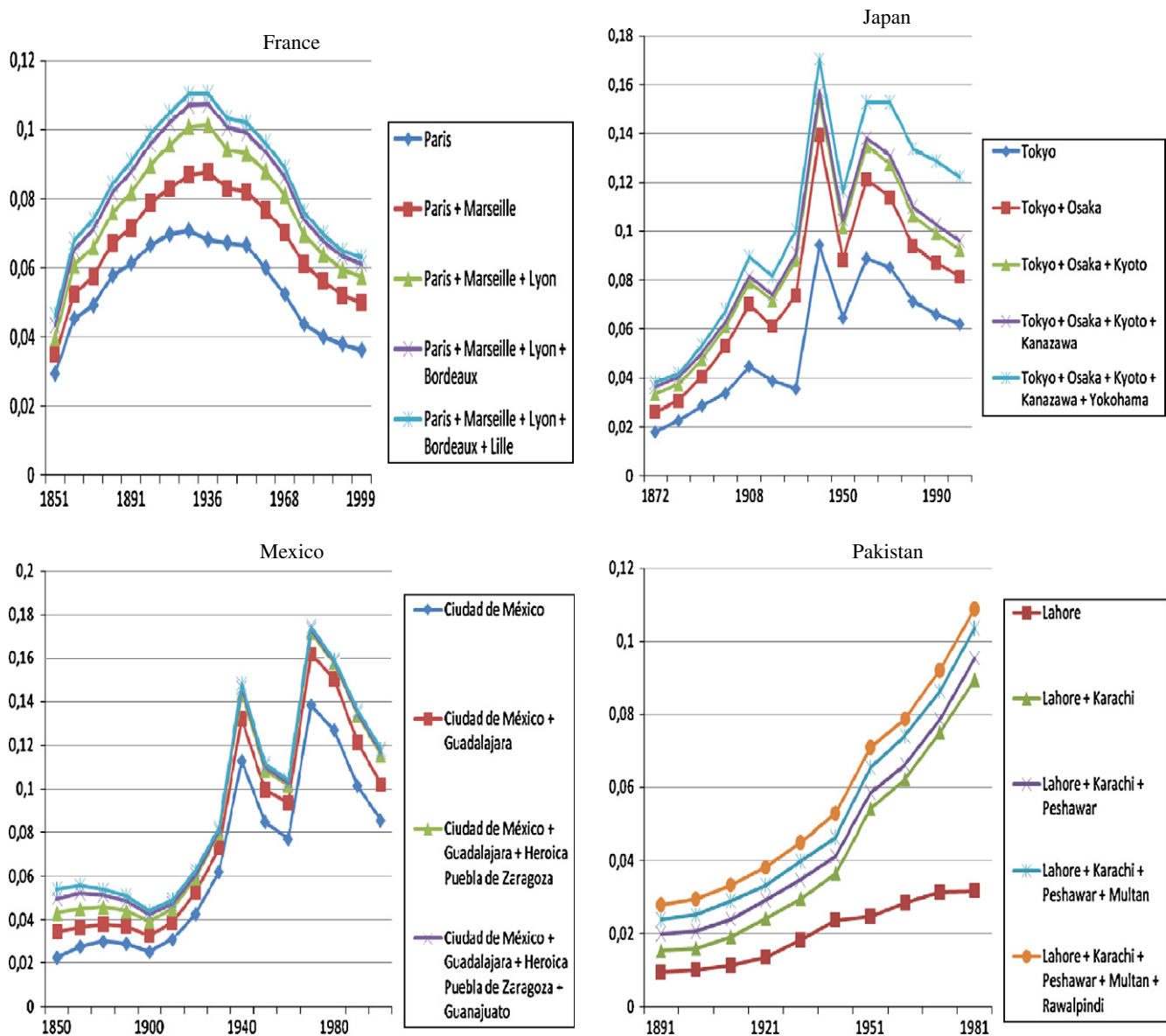


Fig. 3. Historical urban primacy ratios in different countries.

population above the country's average – and compare it with the corresponding figure for the rest of the periods. For cities, this aver-

age is much larger in the 214 periods of rapid increases in urban population than in the rest of periods (1.03 vs. 0.15), suggesting

**Table 3**A regression of the growth rate of  $RANK_{25}$  on the growth rate of urban population and the growth rate of the number of cities/MAs.

	Cities		MAs	
	[1]	[2]	[3]	[4]
Growth rate of urban pop	1.303*** (0.18)	0.581*** (0.21)	0.16** (0.08)	0.238*** (0.06)
Growth rate of number of cities/MAs		0.8*** (0.04)		0.728*** (0.2)
Constant	−0.006 (0.07)	−0.045 (0.09)	0.253*** (0.04)	0.072 (0.05)
$R^2$	0.231	0.624	0.007	0.193
Number of observations	479	479	332	332

Note: Robust standard errors in parentheses.

\*\* Denote significance at the 5% level.

\*\*\* Denote significance at the 1% level.

that sequential growth is indeed more pronounced during the rapidly urbanizing decades. For MAs, there are 198 episodes of rapid urbanization and 245 of slow urbanization, and, on average,  $RANK_{25}$  also grows faster in the former than in the latter (0.47 vs. 0.23).

Another strategy to analyze the relation between the growth rate of  $RANK_{25}$  and the growth rates of the urban population is to regress one on the other. As in the estimation of (3), the relation between these two variables may be in part driven by the fact that the number of observations increases over time in our sample. To take this into account, I include the growth rate in the number of cities (or MAs) as an additional regressor.<sup>17</sup> The specification I estimate is then

$$g_{RANK_{25jt}} = \delta_j + \beta_1 g_{U_{jt}} + \beta_2 g_{N_{jt}} + u_{jt} \quad (4)$$

where  $g_{RANK_{25jt}}$ ,  $g_{N_{jt}}$ , and  $g_{U_{jt}}$  denote the growth rate of  $RANK_{25}$ , the growth rate in the number of cities (or MAs), and the growth rate of urban population in country  $j$  and period  $t$ , respectively;  $\delta_j$  is a country-fixed effect; and  $u_{jt}$  denotes a standard error term.

The results of estimating (4) are shown in Table 3. For both cities and MAs, the coefficient on urban growth is significantly positive, indicating that rapid growth in a country's urban population is associated with a larger slope of  $RANK_{25}$  (specifications [1] and [3]). Controlling for the growth rate in the number of available cities in the sample (specifications [2] and [4]) has the effect of lowering the magnitude of the coefficient on urban growth, although its statistical significance is preserved.

To facilitate the interpretation of this finding, Fig. 4 plots the evolution of the urban population (in millions) and  $RANK_{25}$  using city data on four different countries. Consistent with the previous regression results, it is apparent that the two lines display a strong positive correlation in these examples.<sup>18</sup>

## 6. Robustness checks

In this section, I provide several robustness checks that confirm the validity of the empirical results presented in Section 5.

### 6.1. Different cut-offs to select cities

The choice of the cut-off used to select the relevant sample of cities in Section 4 is somewhat arbitrary. Nevertheless, I show next

<sup>17</sup> Including the square of the growth of the number of cities/MAs does not alter the rest of the estimates. These coefficients are statistically insignificant for both units of analysis.

<sup>18</sup> The figures for all the countries are available at [http://merlin.fae.ua.es/cuberes/Appendix\\_C\\_new.pdf](http://merlin.fae.ua.es/cuberes/Appendix_C_new.pdf) (cities) and [http://merlin.fae.ua.es/cuberes/Appendix\\_D\\_new.pdf](http://merlin.fae.ua.es/cuberes/Appendix_D_new.pdf) (MAs).

that none of the empirical results presented above hinges on its specific value. Tables 4 and 5 reproduce the main results using a cut-off equal to zero (i.e., all available cities are selected).<sup>19</sup>

The estimates of these tables are qualitatively similar to the corresponding ones for cities using the 0.6 cut-off (Tables 2 and 3).  $RANK_{25}$  significantly increases over time and its growth rate is faster in periods of rapid growth in urban population. Moreover, in results not shown here, it is still the case that the vast majority of the country-decade cities' growth rates exhibit significant right skewness.

### 6.2. Different percentiles to define the fastest-growing cities

Here I use different percentiles in my definition of what constitutes a "fast-growing" city. In Section 5, a city is a "fast-grower" in a given decade if its growth rate is above the 75th percentile of the growth rates of cities in that country and decade. Tables 6 and 7 show the results that correspond to choosing the 70th percentile of the cities' growth rates (i.e., the 30% fastest-growing cities).<sup>20</sup> The estimates are again similar in sign and significance to those of Tables 2 and 3. The same is true when I choose the 80th, 90th, or 95th percentile of growth rates to define a fast-growing city or MA.

### 6.3. Regional analysis

In this subsection, I analyze how accurately the three facts predicted by the theories of sequential city growth describe the behavior of cities and MAs in different world regions.<sup>21</sup> To do this, I use dummy variables for eight world regions. The first six are defined in the World Bank Classification<sup>22</sup>: East Asia and Pacific (EAP), Europe and Central Asia (EUCA), Latin American and the Caribbean (LAC), Middle East and North Africa (MENA), South Asia (SA), and Sub-Saharan Africa (SSA). In order to include most of the countries in my sample, I add a dummy variable for Europe (EU) and another one for North America (NAM).

Table 8 shows the percentage of observations with a positive coefficient of skewness (columns 2 and 4) and the fraction of these observations for which the normality test described in Section 5.1 is rejected (columns 3 and 5). The percentage of cross-sections with right skewness ranges between 67% and 100% for cities and

<sup>19</sup> Note that this robustness check cannot be performed in the sample of MAs because the Henderson dataset only has data available for cities whose relative population is above 0.202.

<sup>20</sup> The dependent variable for this robustness exercises is accordingly relabeled  $RANK_{30}$ .

<sup>21</sup> One difficulty with this exercise is that the number of observations is quite low for some of these regions, hence reducing the accuracy of the estimates.

<sup>22</sup> See <http://web.worldbank.org/WBSITE/EXTERNAL/COUNTRIES/0,pagePK:180619~theSitePK:136917,00.html>.

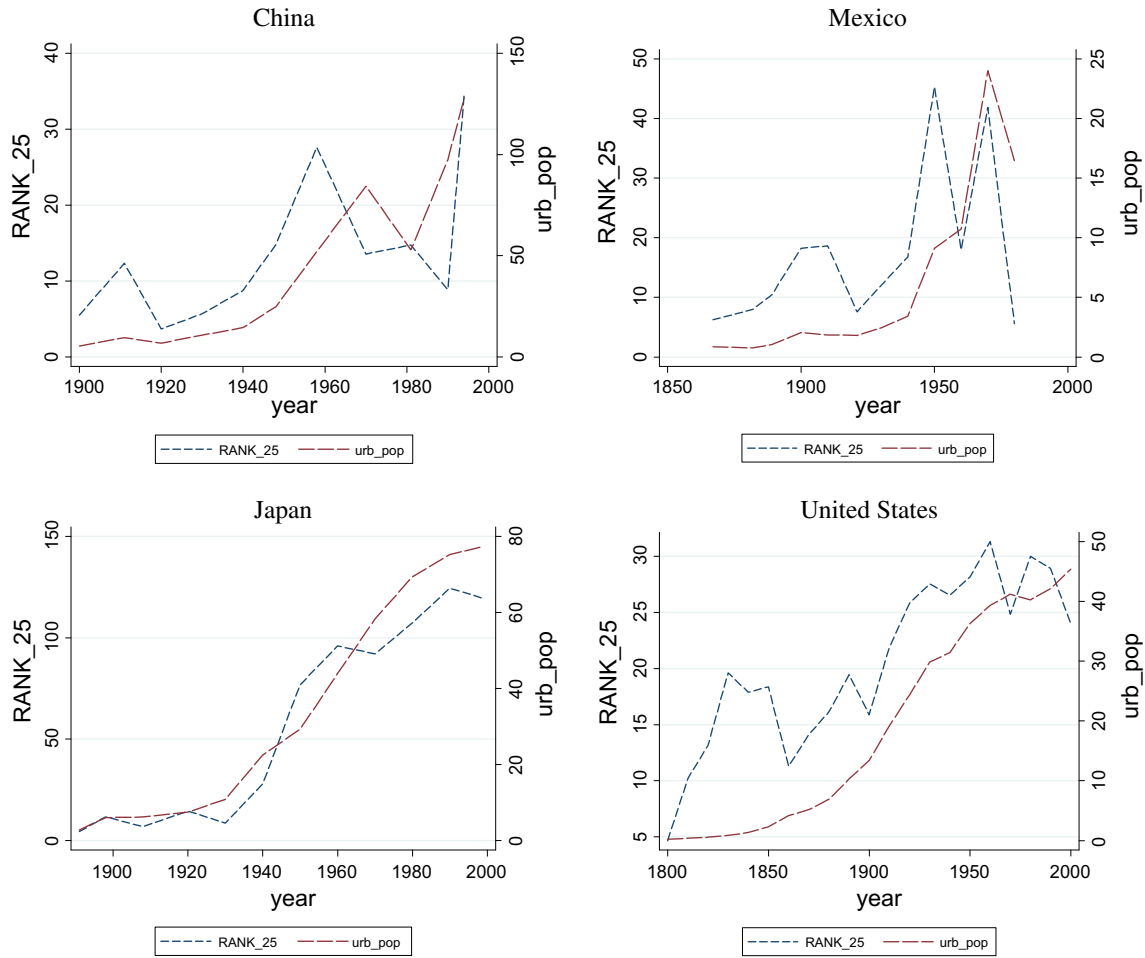


Fig. 4.  $RANK_{25}$  and urban population in different countries.

Table 4

A regression of  $RANK_{25}$  on time, the number of cities/MAs, and its square. Zero Henderson–Wang cut-off.

	[1]	[2]	[3]
Time	0.16*** (0.02)	0.091*** (0.02)	0.053*** (0.01)
Number cities/MAs		0.004*** (0.001)	0.013*** (0.001)
Square number of cities/MAs			-0.000*** (0.000)
Constant	1.972*** (0.1)	2.037*** (0.07)	1.862*** (0.08)
$R^2$	0.408	0.562	0.686
Number of observations	536	536	536

Note: Robust standard errors in parentheses.  
\*\*\* Denotes significance at the 1% level.

Table 5

A regression of the growth rate of  $RANK_{25}$  on the growth rate of urban population and the growth rate of the number of cities/MAs. Zero Henderson–Wang cut-off.

	[1]	[2]
Growth rate of urban pop	1.34** (0.59)	1.279** (0.57)
Growth rate of number of cities/MAs		0.581*** (0.03)
Constant	0.119 (0.23)	-0.188 (0.22)
$R^2$	0.07	0.451
Number of observations	479	479

Note: Robust standard errors in parentheses.  
\*\* Denote significance at the 5% level.  
\*\*\* Denote significance at the 1% level.

from 61% and 100% for MAs and in all but one regions – for both cities and MAs – the normality test is rejected by at least half of the observations. One can conclude from this exercise that the first empirical fact described in the paper is quite ubiquitous across different world regions.

In Table 9, I show that the coefficient associated with time is significantly positive in three regions: East Asia and Pacific, Europe, and South Asia. The estimates of Table 10 indicate that the third empirical fact – the positive relation between the growth rate of  $RANK_{25}$  and the growth rate of urban population – is verified in

Latin America and the Caribbean, Europe, Middle East and North Africa, and North America.<sup>23</sup>

In results not reported here, I estimate that, using the Henderson dataset on MAs, Latin America and the Caribbean, East Asia and Pacific, and Europe and Central Asia exhibit strong sequential city growth, whereas Latin America and the Caribbean, Europe, and North America match the rank-urban growth prediction.

<sup>23</sup> East Asia and Pacific has a surprising negative and significant estimate.



**Table 6**A regression of  $RANK_{30}$  on time, the number of cities/MAs, and its square.

	Cities			MAs		
	[1]	[2]	[3]	[4]	[5]	[6]
Time	0.143*** (0.02)	0.07*** (0.02)	0.035*** (0.01)	0.105*** (0.02)	0.076*** (0.02)	0.055*** (0.02)
Number cities/MAs		0.011*** (0.002)	0.032*** (0.003)		0.014** (0.006)	0.044*** (0.008)
Square number of cities/MAs			−0.000*** (0.000)			−0.000*** (0.000)
Constant	1.47*** (0.11)	1.5*** (0.08)	1.263*** (0.07)	1.158*** (0.05)	0.988*** (0.09)	0.686*** (0.1)
$R^2$	0.387	0.58	0.72	0.103	0.171	0.278
Number of observations	536	536	536	448	448	448

Note: Robust standard errors in parentheses.

\*\* Denote significance at the 5% level.

\*\*\* Denote significance at the 1% level.

**Table 7**A regression of the growth rate of  $RANK_{30}$  on the growth rate of urban population.

	Cities		MAs	
	[1]	[2]	[3]	[4]
Growth rate of urban pop	1.332*** (0.21)	0.69** (0.26)	0.165** (0.08)	0.238*** (0.06)
Growth rate of number of cities/MAs		0.702*** (0.04)		0.687*** (0.18)
Constant	−0.049 (0.08)	−0.084 (0.09)	0.204*** (0.03)	0.033 (0.04)
$R^2$	0.278	0.637	0.01	0.24
Number of observations	479	479	332	332

Note: Robust standard errors in parentheses.

\*\* Denote significance at the 5% level.

\*\*\* Denote significance at the 1% level.

**Table 8**

Percentages of observations with positive coefficients of skewness and percentage of observations for which the normality test is rejected. Different regions.

Region	Cities		MAs	
	Positive	Reject	Positive	Reject
EAP	0.89	0.68	0.95	0.81
EU	0.91	0.8	0.65	0.73
EUCA	0.89	0.75	0.61	0.48
LAC	0.85	0.78	0.81	0.59
MENA	0.78	0.8	0.62	0.79
NAM	1	0.87	1	0.62
SA	0.69	0.77	0.92	0.67
SSA	0.67	0.5	0.63	0.52

**Table 9**A regression of  $RANK_{25}$  on time and the number of cities. Different regions.

	LAC	EAP	EU	SSA	SA	MENA	EUCA	NAM
Time	0.017 (0.02)	0.166** (0.05)	0.074** (0.03)	−0.037 (0.02)	0.092* (0.04)	0.000 (0.05)	0.062 (0.03)	0.007 (0.02)
Number of cities	0.023*** (0.004)	0.008** (0.002)	0.01*** (0.003)	0.111* (0.04)	0.007*** (0.001)	0.057** (0.01)	0.03*** (0.005)	0.03* (0.003)
Constant	1.464*** (0.09)	1.2*** (0.21)	1.553*** (0.14)	0.54 (0.3)	1.47*** (0.15)	1.04*** (0.05)	1.229*** (0.14)	1.659** (0.03)
$R^2$	0.565	0.688	0.575	0.497	0.613	0.612	0.753	0.883
Number of observations	80	44	211	28	40	37	62	34

Note: Robust standard errors in parentheses.

\* Denote significance at the 10% level.

\*\* Denote significance at the 5% level.

\*\*\* Denote significance at the 1% level.

## 7. Conclusions

In this paper, I study the evolution of city growth in different countries over long periods of time using data on administratively defined cities and metropolitan areas. I document three novel empirical facts. The first is that the cross-section of cities' growth rates is clearly skewed to the right in most countries and decades. This indicates that, within a country, at each decade, a few cities grow much faster than the rest. Second, the rank of these fast-growing cities rises as time goes by, implying that early on in the process of urbanization, the largest cities grow fastest. As time passes, population growth in the larger cities declines and the fastest growth can be found in smaller cities farther down in the urban hierarchy. In other words, cities grow in sequential order, with the

**Table 10**A regression of the growth rate of  $RANK_{25}$  on the growth rate of urban population. Different regions.

	LAC	EAP	EU	SSA	SA	MENA	EUCA	NAM
Growth of urb pop	0.41 <sup>*</sup> (0.18)	-0.412 <sup>**</sup> (0.11)	1.03 <sup>***</sup> (0.14)	0.626 (0.85)	0.231 (0.12)	0.652 <sup>*</sup> (0.25)	-0.102 (0.61)	0.212 <sup>**</sup> (0.005)
Growth of num of cities	0.904 <sup>***</sup> (0.2)	1.229 <sup>***</sup> (0.05)	0.772 <sup>***</sup> (0.04)	0.461 (0.31)	0.42 (0.22)	0.518 <sup>*</sup> (0.21)	1.05 <sup>**</sup> (0.33)	1.25 <sup>*</sup> (0.11)
Constant	-0.065 (0.05)	0.33 <sup>**</sup> (0.08)	-0.032 (0.07)	-0.3 (0.53)	0.058 (0.05)	-0.123 (0.07)	0.225 (0.15)	-0.081 (0.02)
R <sup>2</sup>	0.589	0.673	0.732	0.422	0.248	0.571	0.515	0.849
Number of observations	71	39	191	24	35	32	55	32

Note: Robust standard errors in parentheses.

<sup>\*</sup> Denote significance at the 10% level.<sup>\*\*</sup> Denote significance at the 5% level.<sup>\*\*\*</sup> Denote significance at the 1% level.

initially largest ones being the first to develop. Finally, I show that this sequential growth process is more pronounced in decades where urban population grows rapidly, and that there are important differences in this city growth pattern across world regions. These results are shown to be robust to the cut-off that determines the sample selection, and to the definition of what constitutes a “fast-growing” city.

These findings can be interpreted as providing strong support for the recently proposed theories of sequential city growth and are valuable inputs for policy makers, especially in countries that are urbanizing rapidly.

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### Appendix A

See Table A1.

**Table A1**Countries and years used.<sup>a</sup>

Country	Years	Country	Years
<i>Lahmeyer-Brinkhoff dataset</i>			
Afghanistan	1950–1988	Japan	1881–1999
Albania	1923–1989	Kenya	1931–1999
Algeria	1882–1987	Luxembourg	1901–2001
Argentina	1947–1999	Lybia	1929–1988
Austria	1870–2001	Malaysia	1921–1991
Bangladesh	1891–1991	Mexico	1850–1980
Belgium	1894–1999	Morocco	1931–1982
Bolivia	1881–2001	Nepal	1961–2001
Brazil	1890–2000	Netherlands	1795–1999
Bulgaria	1888–1990	Nigeria	1909–1991

**Table A1** (continued)

Country	Years	Country	Years
Canada	1861–1996	Norway	1801–1980
China	1890–1994	Pakistan	1891–1981
Colombia	1902–1999	Poland	1851–2000
Czech Rep.	1880–1991	Portugal	1864–2001
Ecuador	1930–2001	Romania	1890–2000
Egypt	1897–1996	Russia	1897–1991
Finland	1881–2000	South Africa	1911–1991
France	1851–1999	South Korea	1920–2000
Greece	1920–2001	Spain	1860–2000
Honduras	1901–2000	Sudan	1937–1993
Hungary	1858–1999	Sweden	1910–1994
India	1865–1991	Switzerland	1910–1990
Indonesia	1920–1990	Turkey	1927–2000
Iran	1910–1996	United Kingdom	1851–1981
Ireland	1891–1991	Uruguay	1919–1996
Israel	1931–2000	United States	1790–2000
Italy	1800–2001	Venezuela	1921–1990
Henderson dataset <sup>b</sup>			

Afghanistan, Albania, Angola, Argentina, Australia, Austria, Azerbaijan, Bangladesh, Belarus, Belgium, Benin, Bolivia, Bosnia and Herzegovina, Botswana, Brazil, Bulgaria, Burkina Faso, Cambodia, Cameroon, Canada, Chad, Chile, China, Colombia, Congo Dem. Rep., Cote d'Ivoire, Croatia, Cuba, Czech Republic, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, Finland, France, Germany, Ghana, Greece, Honduras, Hungary, India, Indonesia, Iran, Iraq, Ireland, Israel, Italy, Japan, Jordan, Kazakhstan, Kenya, Kuwait, Kyrgyzstan, Laos, Latvia, Lebanon, Lithuania, Madagascar, Malawi, Malaysia, Mali, Mexico, Morocco, Mozambique, Myanmar, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Paraguay, Peru, Philippines, Poland, Portugal, Puerto Rico, Reunion, Romania, Russia, Saudi Arabia, Senegal, Sierra Leone, Slovak Republic, Slovenia, Somalia, South Africa, South Korea, Spain, Sri Lanka, Sudan, Sweden, Switzerland, Syria, Tajikistan, Tanzania, Thailand, Tunisia, Turkey, Turkmenistan, Ukraine, United Arab Emirates, United Kingdom, United States, Uzbekistan, Venezuela, Vietnam, Yemen, Zambia.

<sup>a</sup> Details on the sources of data for each country can be found in the web page of their authors: <http://www.library.uu.nl/wesp/jalahome.htm> (Lahmeyer), and <http://www.citypopulation.de> (Brinkhoff), and <http://www.econ.brown.edu/faculty/henderson/worldcities.html> (Henderson).

<sup>b</sup> In all cases, the time interval covered is 1960–2000. I have dropped 33 countries that lack comprehensive data.

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