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Trajectory Tracking of Tractor-Trailer Wheeled Mobile Robots via Dynamic Feedback Linearization in Forward and Backward Motion

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***Abstract*— One type of wheeled mobile robot widely used in public transportation and for carrying high payloads is the tractor-trailer wheeled robots (TTWRs). This study considers a differentially-driven tractor under pure rolling conditions, which is subject to nonholonomic constraints. Controlling a TTWR in both backward and forward motion is challenging due to inherent instability. To address this issue and achieve trajectory tracking control for these systems, this paper employs dynamic feedback linearization (DFL) to overcome the limitations of static feedback linearization (SFL). The system's response will be examined under various trajectories and initial conditions for both forward and backward motion.**

Keywords*— *Backward Motion – Dynamic Feedback Linearization – Tractor-Trailer – Trajectory Tracking – Wheeled Mobile Robots

I. Introduction

Wheeled mobile robots are increasingly popular in industry due to their versatility. To enhance their payload capacity, a trailer is often attached to the primary motorized robot, known as the tractor [1]. Most research on TTWRs focuses on forward motion and motion planning, while studies on control, especially during backward motion, are limited. There is a need to explore and develop simple, effective, and practical methods for controlling these systems in backward movement.

In academic discussions, three categories of N-trailers with fixed axles are identified. The first category, known as Standard N-Trailers (SNT), includes those with joints positioned directly on the previous wheel-axles. The second category, called non-Standard N-Trailers (nSNT), comprises those with joints located off the previous wheel-axles. Lastly, the third category, General N-Trailers (GNT), features a mix of on-axle and off-axle hitches [2].

Most research on controlling TTWRs relies on the kinematic model of these systems. In [3], a full-state trajectory tracking controller is proposed to control a tractor-trailer system for the trajectory tracking problem. This controller, based on Lyapunov methods, is capable of following desired paths in both forward and backward motion. The study assumes non-slip conditions for the wheels and uses a kinematic model for the control algorithm design. In [4], a robust adaptive controller is designed for a TTWR, using the kinematic model and non-slip conditions to handle uncertainties and disturbances. In [5], a car-like robot is controlled with both SFL and DFL, though there is no mention of backward motion.

In [6], a TTWR is considered with non-slip and non-holonomic constraints, and it is able to follow a path despite uncertainties using a Lyapunov-based controller combined with a neural network. In[7], a TTWR is controlled using the backstepping algorithm while considering wheel slip conditions. In [8], a

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robust Lyapunov-based controller is designed to estimate wheel slip and can control TTWRs in both backward and forward motion. Although this controller performs commendably, it is not based on the dynamic model of the TTWR.

In [9], the dynamics modeling of a TTWR takes wheel slip into account, and a controller using the SFL control algorithm is developed to follow a desired path in forward motion. However, this study is unable to control the robot in backward motion and is unstable.

The DFL method was applied to a wheeled mobile robot in [10], , where the researchers demonstrated that DFL eliminates internal dynamics, allowing for full-state linearization.

The identified research gap reveals that DFL has yet to be applied to a TTWR. Previous studies highlight the challenges of controlling a TTWR in reverse motion due to its internal dynamics [11]. In this paper, we first provide a detailed system description of the TTWR, followed by its kinematic modeling. By selecting an appropriate output, we apply the DFL method. In the simulation section, we compare our proposed control method with two pivotal studies on trajectory tracking for a TTWR.

II. System Description

A. System Description

The wheeled mobile robot in this study consists of a differentially-driven wheeled platform, called the tractor, and a passive wheeled platform, named the trailer. The system considered in this study features off-axle hitching. The tractor and trailer are connected with an inactive pin at point C_0, which is at the midpoint of the two active wheels of the tractor. Additionally, point C_1 is at the midpoint of the two passive wheels of the trailer.

In Fig. 1, the configuration of the tractor trailer system is described by $\mathbf{q} = [x, y, \alpha_1, \alpha_0]^T$ in which (x, y) is the coordinate vector of point C_1. The orientation of the tractor with respect to the inertial frame is shown by α_0 and α_1, respectively [12].

B. Kinematics Modeling

The tractor-trailer system is subject to m nonholonomic constraints, which in Pfaffian form are written as

$$\mathbf{A}(\mathbf{q})\dot{\mathbf{q}} = \begin{bmatrix} \sin\alpha_0 & -\cos\alpha_0 & -l\cos(\alpha_0-\alpha_1) & 0 \\ \sin\alpha_1 & -\cos\alpha_1 & 0 & 0 \end{bmatrix}\dot{\mathbf{q}} = 0 \tag{1}$$

Kinematic equations of the mobile robot can be written as

$$\dot{\boldsymbol{q}}(\boldsymbol{t}) = \mathbf{S}(\boldsymbol{q})u \tag{2}$$

where $u = (u_1, u_2)^T$ with u_1 being the linear velocity of trailer at point C_1 and u_2 being the angular velocity of the tractor. The matrix $S(q)$ can be written as follows

$$\mathbf{S}(\mathbf{q})^T = \begin{bmatrix} \cos\alpha_0 & \sin\alpha_0 & \frac{1}{l}\tan(\alpha_0-\alpha_1) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

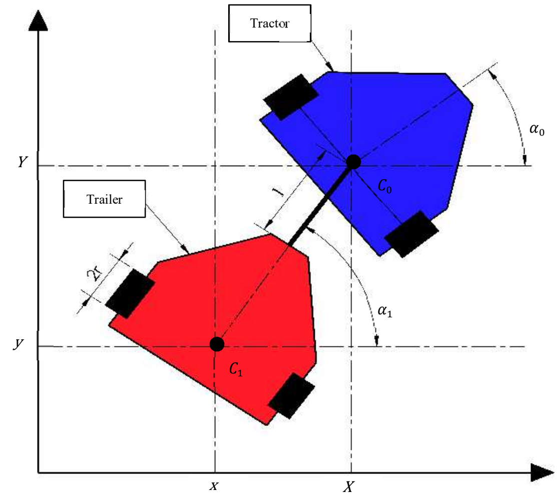

Figure 1. Schematic illustration of a tractor trailer wheeled mobile robot

The state equations can be derived as

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\alpha}_1 \\ \dot{\alpha}_0 \end{bmatrix} = \begin{bmatrix} \cos\alpha_1 & 0 \\ \sin\alpha_1 & 0 \\ \frac{1}{l}\tan(\alpha_0-\alpha_1) & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \tag{4}$$

It can be seen from (4) that there is a model singularity when $\alpha_0 - \alpha_1 = \frac{\pi}{2}$, which indicated that the tractor and trailer would become jammed. This situation should be avoided.

III. Dynamic Feedback Linearization

In static feedback linearization, the outputs should be chosen such that their derivatives reveal all the inputs and the decoupling matrix is full rank. To ensure that all inputs appear for a tractor-trailer system, a look-ahead point is used for the outputs. In contrast, with dynamic feedback linearization, the outputs can be selected more flexibly by redefining the inputs. In this method, the first derivative of the outputs is computed. If all inputs are not present, the inputs that do appear are redefined as new states. The second derivative is then calculated, and this process continues until the decoupling matrix achieves full rank. This approach is also referred to as dynamic extension, as it extends the states by incorporating the inputs. Additionally, it is known as input redefinition [13].

For the tractor trailer system, DFL is applied by defining the outputs as follows

$$\eta = \begin{bmatrix} x \\ y \end{bmatrix} \tag{5}$$

Taking the first derivative of the outputs gives

$$\dot{\eta} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ s_1 & 0 \end{bmatrix}\begin{bmatrix} v \\ \omega \end{bmatrix} \tag{6}$$

c_1 denotes $\cos\alpha_1$ and s_1 denotes $\sin\alpha_1$.

It is evident that the decoupling matrix is rank-deficient and ω can not be derived from the equations. By considering input v as a new state and adding it to the set of states, the second derivative of the outputs is then taken.

$$\xi_1 = v,\ \dot{\xi}_1 = v' \tag{7}$$

$$\ddot{\eta} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = A + B \begin{bmatrix} v' \\ \omega \end{bmatrix} = \begin{bmatrix} -s_1 v \dot{\alpha}_1 \\ c_1 v \dot{\alpha}_1 \end{bmatrix} + \begin{bmatrix} c_1 & 0 \\ s_1 & 0 \end{bmatrix} \begin{bmatrix} v' \\ \omega \end{bmatrix} \tag{8}$$

Clearly, the decoupling matrix B is rank-deficient, so v' is added to states.

$$\xi_2 = v',\ \dot{\xi}_2 = v'' \tag{9}$$

Taking third derivative of η gives

$$\dddot{\eta} = \begin{bmatrix} \dddot{x} \\ \dddot{y} \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} + D \begin{bmatrix} v'' \\ \omega \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{10}$$

In which

$$\sigma_1 = -\frac{1}{l}\left((-\dot{\alpha}_1)\sec^2(\alpha_0 - \alpha_1)\, v + \tan(\alpha_0 - \alpha_1)\, v'\right) s_1 v - \dot{\alpha}_1{}^2 c_1 v - 2\dot{\alpha}_1 s_1 v' \tag{11}$$

$$\sigma_2 = -\frac{1}{l}\left((-\dot{\alpha}_1)\sec^2(\alpha_0 - \alpha_1)\, v + \tan(\alpha_0 - \alpha_1)\, v'\right) c_1 v - \dot{\alpha}_1{}^2 s_1 v + 2\dot{\alpha}_1 c_1 v' \tag{12}$$

The decoupling matrix is written as

$$\boldsymbol{D} = \begin{bmatrix} c_1 & -\frac{1}{l} s_1 v^2 \sec^2(\alpha_0 - \alpha_1) \\ s_1 & \frac{1}{l} c_1 v^2 \sec^2(\alpha_0 - \alpha_1) \end{bmatrix}, \tag{13}$$

And the determinant of decoupling matrix **D** is calculated as

$$\det(\boldsymbol{D}) = \frac{v^2}{l\cos(\alpha_0 - \alpha_1)} \tag{14}$$

The matrix D is singular when $v = 0$ or $\alpha_0 - \alpha_1 = \frac{\pi}{2}$. By avoiding these conditions, the new inputs can be calculated as

$$\begin{bmatrix} v'' \\ \omega \end{bmatrix} = \begin{bmatrix} c_1(r_1 - \sigma_1) + s_1(r_2 - \sigma_2) \\ -\frac{1}{v^2} l^2 c_{01}^2 \left(s_1(\sigma_1 - u_1) - c_1(\sigma_2 - u_2)\right) \end{bmatrix} \tag{15}$$

In which r_1 and r_2 are the auxiliary inputs and are calculated as

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} k_1(x_r - x) + k_2(\dot{x}_r - \dot{x}) + k_3(\ddot{x}_r - \ddot{x}) + \dddot{x}_r \\ k_4(y_r - y) + k_5(\dot{y}_r - \dot{y}) + k_6(\ddot{y}_r - \ddot{y}) + \dddot{y}_r \end{bmatrix} \tag{16}$$

It should be noticed, as in (14), $v \neq 0$. By choosing a proper initial condition for v, the robot will move either backward or forward.

$$\begin{cases} v(0) > 0, & Forward\ Motion \\ v(0) < 0, & Backward\ Motion \end{cases} \tag{17}$$

The stability analysis of the proposed method is presented below.

In the dynamic feedback linearization method, the original system has four states, and the dynamic controller introduces two additional states. Consequently, the sum of the relative degrees of the outputs matches the state space dimension, eliminating any internal dynamics. This results in a control law that achieves full-state linearization.

The diffeomorphism $T(x)$ is considered as below.

$$z = T(x) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{bmatrix} = \begin{bmatrix} h_1 \\ \dot{h}_1 \\ \ddot{h}_1 \\ h \\ \dot{h} \\ \ddot{h} \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ y \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} x \\ c_1 v \\ -\frac{1}{l} tan(\alpha_0 - \alpha_1)\, s_1 v^2 + c_1 v' \\ y \\ s_1 v \\ \frac{1}{l} tan(\alpha_0 - \alpha_1)\, c_1 v^2 + s_1 v' \end{bmatrix} \tag{18}$$

To verify that $T(x)$ is a diffeomorphism we calculate its Jacobian and observe that it has full rank. This confirms that $T(x)$ is indeed a diffeomorphism. Additionally, since no extra states need to be added, there are no internal dynamics, ensuring the system's dynamics remain intact.

For the stability proof, the third derivative of the outputs are calculated as follows.

$$\dddot{x} = k_1(x_r - x) + k_2(\dot{x}_r - \dot{x}) + k_3(\ddot{x}_r - \ddot{x}) + \dddot{x}_r$$
$$\dddot{y} = k_4(y_r - y) + k_5(\dot{y}_r - \dot{y}) + k_6(\ddot{y}_r - \ddot{y}) + \dddot{y}_r$$

Obviously

$$\dddot{e}_1 + k_3\ddot{e}_1 + k_2\dot{e}_1 + k_1 e_1 = 0 \tag{19}$$

$$\dddot{e}_2 + k_6\ddot{e}_2 + k_5\dot{e}_2 + k_4 e_2 = 0 \tag{20}$$

By choosing appropriate $k_i > 0\, , i = 1, \ldots, 6$ such that (19) and (20) are Hurwitz, the system can achieve exponential stability for any initial state. This topic is discussed extensively in [13] within the feedback linearization section.

IV. Simulation And Results

In this section, the proposed controller is applied to the tractor-trailer system, and the results will be discussed.

The configuration of the tractor-trailer system considered in this paper is given in table 1.

Table 1- Robot Configuration

Parameters	Nominal Values
l	$0.41\ m$
r	$0.05\ m$
ω	$0.135\ m$

A sinusoidal reference is defined as

$$\begin{cases} x_r(t) = 0.1t\ (m) \\ y_r(t) = 2\sin(0.05t)\ (m) \end{cases} \tag{21}$$

Backward motion is desired. The initial conditions are as

$$q(0) = \begin{bmatrix} -0.2 & 0.1 & {}^{\pi}/_{2} & 1.2217 & -0.5 & 0 \end{bmatrix} \tag{22}$$

The chosen k_i values are

$$[k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_5 \quad k_6] = [100 \quad 40 \quad 32 \quad 100 \quad 80 \quad 40]$$

In Fig. 2, the robot successfully follows the reference trajectory. The magnitude of the inputs is limited, with the maximum linear velocity being 0.5 m/s and the maximum rotational velocity of the tractor being 5.76 rad/s. To achieve these limits, a saturated velocity is considered.

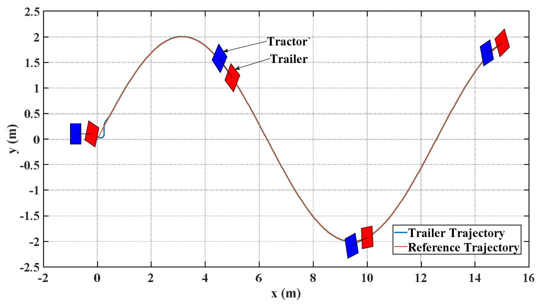

Figure 2. Trajectory tracking of the proposed controller for a sinusoidal trajectory in backward motion

The mean of error e_x is 2.2509×10^{-4} and the mean of error e_y is -2.7266×10^{-4} for this trajectory.

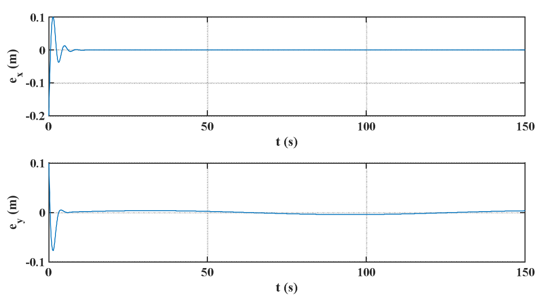

Figure 3. Errors for the sinusoidal trajectory in backward motion

Simulations for two trajectories are presented and compared with the controllers from [3] and [4]. The results demonstrate that the proposed controller outperforms both the Coordinate-Change Lyapunov-based Controller (CCLC) controller from [3] and the robust adaptive feedback linearizing dynamic controller (RAFLD) controller from [4]. The proposed controller in this study is named dynamic feedback linearizing controller (DFLC).

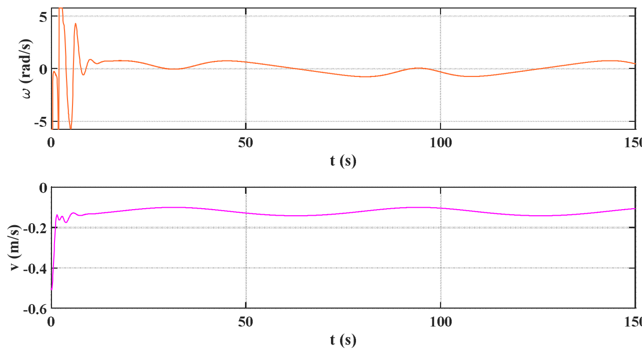

Figure 4. Control input signals for the sinusoidal trajectory in backward motion

The reference trajectory in [3] is defined as

$$\begin{cases} x_r(t) = 0.02\left(30 + \cos\left(\frac{\pi t}{20}\right)\right)\cos\left(\frac{\pi t}{120}\right)\ (m) \\ y_r(t) = -0.02\left(30 + \sin\left(\frac{\pi t}{20}\right)\right)\sin\left(\frac{\pi t}{120}\right)\ (m) \end{cases} \tag{23}$$

The proposed controller performs excellently in both backward and forward motion, showing better results than the two previous studies.

The initial conditions for forward motion are

$$q(0) = [-0.8901 \quad 0.6243 \quad 0.0042 \quad 0.6612 \quad 0.2 \quad 0] \tag{24}$$

These initial conditions are described as challenging in [3]. The controller in [4] cannot successfully track the trajectory under these conditions. However, the proposed controller in this study successfully follows the trajectory, with the error decreasing to zero more effectively and quickly than the controller in [3]. Although the DFLC relies on exact cancellation, its ability to stabilize the system exponentially for any initial condition allows it to outperform the two Lyapunov-based controllers, which are not globally stable or effective under all conditions.

The chosen k_i values are

$$[k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_5 \quad k_6] = [250 \quad 800 \quad 700 \quad 600 \quad 500 \quad 750]$$

In Fig. 5, it is evident that the DFLC successfully follows trajectory B, outperforming the CCLC. In contrast, the RAFLD controller fails to follow the trajectory. The DFLC demonstrates a quicker response and better adherence to the trajectory compared to the CCLC.

The initial conditions for backward motion are:

$$q(0) = [-0.8901 \quad 0.6243 \quad 0.0042 \quad 0.6612 \quad -0.2 \quad 0] \tag{25}$$

The chosen k_i values are the same as for the forward motion.

In Fig. 8, trajectory B in backward motion is successfully followed. Although backward motion is not typically addressed in the literature, the comparison with the CCLC and RAFLD controllers in forward motion highlights the DFLC's effectiveness in this context. Despite the inherent challenges of backward motion, the DFLC demonstrates superior performance compared to the other two controllers.

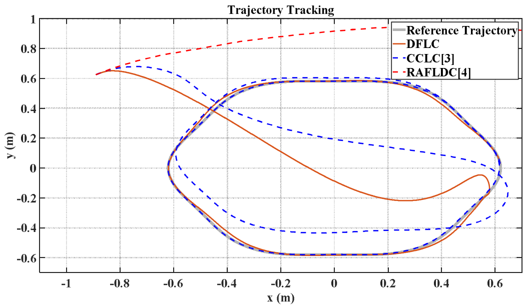

Figure 5 Performance of three controllers for trajectory tracking in forward motion

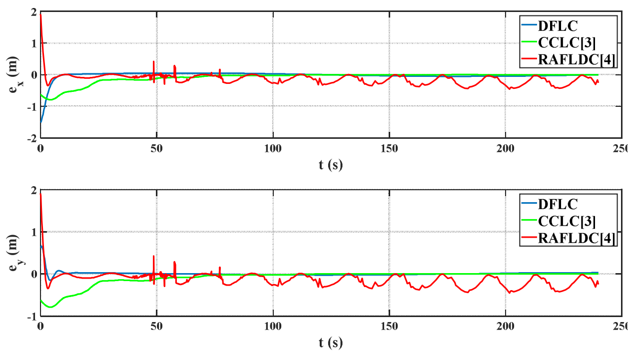

Figure 6. Error plots in forward motion

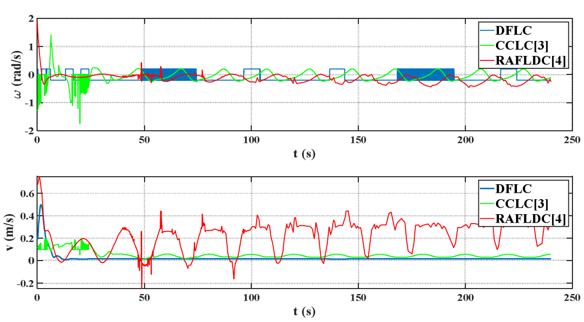

Figure 7. Control input signals in forward motion

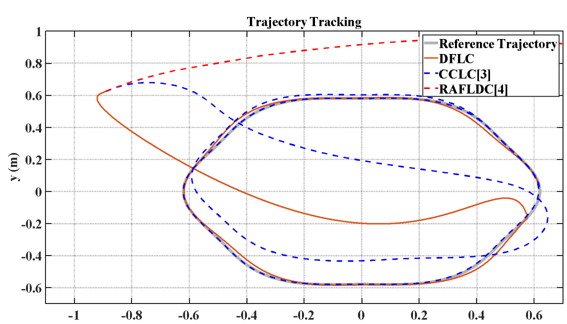

Figure 8. Trajectory tracking in backward motion

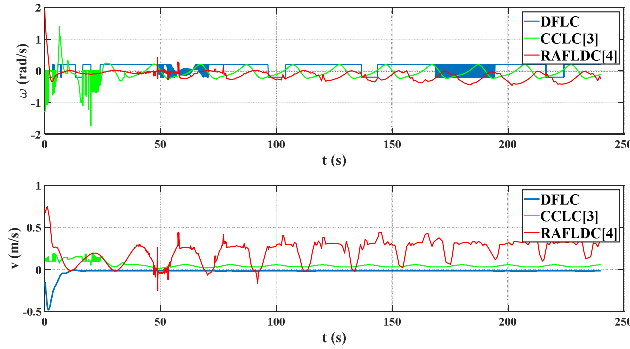

Figure 9. Error plots in backward motion

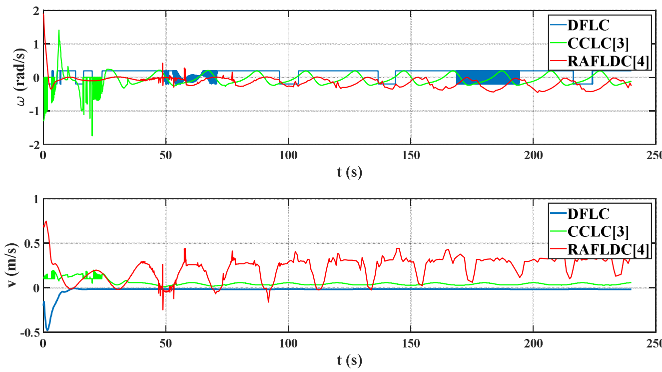

Figure 10. Control input signals in backward motion

The proposed controller performs excellently in both backward and forward motion, showing better results than the two previous studies. In table 2 and 3, the mean errors for the three controllers are written for forward and backward motion.

Table 2- The mean erros of the controllers in backward motion

Controller	$mean(e_x)$	$mean(e_y)$
DFLC	-0.0188	0.0032
CCLC [3]	0.0383	-0.0844
RAFL [4]	0.2156	-0.0668

Table 3- The mean erros of the controllers in forward motion

Controller	$mean(e_x)$	$mean(e_y)$
DFLC	-0.026	0.0019
CCLC [3]	0.0383	-0.0844
RAFL [4]	0.2156	-0.0668

The proposed controller remains effective even with modeling errors in the system dynamics. When parameters such as l and r are perturbed by 10%, the mean error e_x in forward motion changes slightly from -0.0188 to -0.0189, while the mean error e_y stays at 0.0032. This indicates that the proposed controller is robust to perturbations in the system modeling.

V. CONCLUSION

In this study, we propose a controller for the trajectory tracking problem of a tractor-trailer system using the Dynamic Feedback Linearization (DFL) method. The kinematic model of the system is derived under the assumption of pure rolling conditions, allowing the DFL method to be applied effectively. It is shown that this controller can follow any reference trajectory in both backward and forward motion, as DFL, unlike Static Feedback Linearization (SFL), lacks internal dynamics and stabilizes the system in both directions. Simulations for two trajectories are presented, demonstrating that the proposed controller outperforms existing controllers from previous studies. However, there are drawbacks: determining appropriate gain values can be challenging and must be done for each trajectory. Additionally, this method cannot prevent jack-knife incidents, and it operates offline, requiring enhancements for online situations. The current study considers only pure rolling conditions based on kinematics, leaving room for future research to explore dynamics, including wheel slip in backward motion. Integrating neural networks or reinforcement learning could also provide solutions to mitigate jack-knife risks.

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