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A Frequency Domain-Based Control Methodology for Performance Assessment and Optimisation of Heterogeneous Arrays of Wave Energy Converters

Andrei Ermakov[1], Zain Anwar Ali[1], Kumars Mahmoodi[2], Oliver Mason[1] and John V. Ringwood[1]

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Abstract— **The development of wave energy absorption technology has progressed to a stage where researchers and industry companies are assessing the potential performance of wave energy converter (WEC) farms. Besides the evident financial benefits of deploying WECs in farms, numerous analytical and experimental studies have shown that the average power production of WEC arrays can surpass that of isolated devices when an appropriate layout and control strategy are implemented. However, optimising the layout, shapes of devices, and control strategy for WEC arrays requires extensive investigation, with computational costs being a major challenge.**

This paper introduces an innovative approach for the rapid assessment of heterogeneous WEC array performance in the frequency domain, operating under a new optimal centralized constrained control strategy. This frequency-domain method holds promise for faster simulation/optimisation compared to traditional time-domain approaches. The framework integrates hydrodynamic modeling with optimal constrained control, addressing optimisation problems for both array layout and individual WEC design, in order to maximise wave power absorption. The proposed performance function combines time-averaged power production with wave frequency distribution functions and wave propagation direction roses, providing a comprehensive representation of sea states. The paper includes examples illustrating the approach and validates it against previous time-domain simulations for homogeneous and heterogeneous arrays.

I. INTRODUCTION

Wave energy converters (WECs) harness the renewable energy potential of ocean waves. The types of WECs that are under consideration in this study are point absorbers (such as the commercial CorPower Ocean device [1]). These devices can be described as heaving buoys that fluctuate in ocean waves and are connected to ocean-bed-referenced power take-off (PTO) systems. Regrettably, further development of WEC Technology Performance Levels (TPLs) [2], and widespread adoption of WEC technology, has been impeded by an associated high Levelized Cost of Energy (LCoE) [3].

While integrating WECs into arrays or farms offers clear financial benefits and allows for a further decrease in the Capital Cost (CapEx) of the devices, numerous analytical and experimental studies have demonstrated that simultaneously optimising the layout and control methods of WEC arrays can significantly enhance their collective power generation compared to individual devices [4]. Although initial studies on the interaction of point absorber WECs in an array for power absorption enhancement were conducted almost five decades ago [5], determining the optimal layout for WEC arrays, configuring the shapes of the devices within the array, and re-optimising essential control strategies present a challenging computational problem.

Preliminary numerical studies of the advantages of integrating WECs of various sizes into heterogeneous arrays were conducted in the frequency domain [6], though control optimisation was not considered. Subsequently published researches have explored various aspects of heterogeneous arrays optimisation, such as the influence of spatial configuration [7], effects of wave direction [8], [9], factors affecting array layout in irregular waves [10], and strategies to minimise power fluctuations through different WEC sizes and park geometries [11]. The research conducted in [12], [13] shows that the diverse optimal geometries of devices and their positions can be attained based on the nature of the energy-maximising control employed.

The impact of different layouts, WEC separation distances, and incident wave directions on array power production for different Italian locations was analysed in [14]. Array layout optimisation, where the number of WECs in the array is considered a variable, was conducted in [15], while individually optimised dimensions for cylindrical buoys within arrays was investigated in [16]. In [17], researchers assessed a possibility of protecting coastlines by offshore wave farms. Some new insights into position optimisation of WECs were obtained in [18] using hybrid local search, while recently published research also highlights the potentially higher efficiency of heterogeneous arrays of WECs, comparing them with homogeneous arrays [19], [20].

One of the tools for the WECs design optimisation (WecOptTool) was developed by Sandia National Laboratories [21]. WecOptTool facilitates a co-design (WEC & control) approach where, for each WEC design considered in the optimisation, the optimal control strategy for that design can be found.

However, further analysis and optimisation of heterogeneous arrays of WECs are associated with significant computational costs. The mathematical model of a heterogeneous array may contain too many parameters for optimisation (such as positions, shapes, and submergence drafts), which

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[1] Centre for Ocean Energy Research, Maynooth University, W23 A3HY, Co. Kildare, Ireland (andrei.ermakov@mu.ie)

[2]Kumars Mahmoodi with Process Control Laboratory, Faculty of Natural Sciences and Engineering, Abo Akademi University, Finland

are considered variables in the global optimisation problem. It should be noted that in the referenced studies [12], [13], [19], [20], the evaluation of array performance was conducted in the time domain, and such an approach would be close to computational intractability.

This paper introduces a novel methodology for swiftly evaluating the performance of diverse arrays of WECs in the frequency-domain, employing a new optimal centralised constrained control strategy. Thus, the method is loosely based on the approach proposed in [6]. The suggested frequency domain approach holds the promise of being notably swifter, compared to conventional time-domain simulations.

The proposed control method is based on an extended version of the 'Simple and Effective' controller [22], which was developed and successfully tested for single WECs. The control strategy extension develops a centralised control algorithm that implements constraints on the buoy hull displacement and/or PTO force response magnitudes by adjusting the global intrinsic impedance matrix Z_{pto} of the PTO system. The corresponding performance is evaluated in the frequency domain by combining time-averaged power production with the wave frequency distribution function and wave propagation direction roses, providing a representative depiction of sea states.

The examples presented in this article, demonstrating the performance enhancements for heterogeneous arrays of WECs conducted within the frequency domain-based method, are in agreement with the fundamental findings of previous research conducted in the time domain. This provides validation for the proposed methodology.

The article is organized as follows: Section II defines the proposed performance metrics for the optimization of arrays of WECs in the frequency domain, followed by the mathematical model for the array of WECs defined in Section III. Section IV presents the developed optimal centralised constrained control for arrays of wave energy converters. The illustrative examples and validation results of applying the proposed frequency domain-based method are in Section V. Finally, conclusions are drawn in Section VI.

II. Performance Metrics

To swiftly assess the efficiency and performance of heterogeneous arrays, their performance is evaluated in the frequency domain. However, to accommodate the panchromatic nature of ocean waves and their multi-directional propagation, the proposed performance function combines time-averaged power production with the wave frequency distribution function $p_{ss}(\omega)$, and wave propagation direction roses, offering a representative depiction of real sea states.

$$P_{array} = \int_0^\infty \left\{ \sum_{j=1}^{k} \left[\sum_{i=1}^{n} P_{i,j}(\omega) \right] d_j \right\} p_{ss}(\omega) d\omega, \quad (1)$$

where n is the number of WECs, k is the number of selected waves propagation directions, i is the WEC index, j is the wave direction index, $p_{ss}(\omega)$ is the wave frequency distribution function, d_j is the discrete probability of waves propagating from direction j, and $P_{i,j}(\omega)$ is the power generated by WEC i, for waves with direction j.

The traditional performance assessment metric for a homogeneous array of WECs is based on the q-factor [16], which can be adapted for performance evaluation of heterogeneous arrays of WECs. For heterogeneous arrays, Q_{Het} represents the ratio of the total output power of an array P_{array} to the sum of potential power generated by *each* of the n devices working in isolation $P^i_{isolated}$, as

$$Q_{Het} = \frac{P_{array}}{\sum_{i=1}^{n} P^i_{isolated}}. \quad (2)$$

For the case of a homogeneous array, the power generated by each of n isolated devices is identical, so (2) reduces to

$$Q_{Hom} = \frac{P_{array}}{n\, P_{isolated}}. \quad (3)$$

The positive interaction between WECs in an array occurs when $Q > 1$; otherwise, the interaction is considered destructive.

III. Heterogeneous WEC array mathematical model

The present study is based on linear hydrodynamic theory and limits the displacements of the WECs in an array to heave direction only. The fluctuations of heaving buoy WEC positions in waves are traditionally modeled by Cummins' equation [23]:

$$(\mathbf{M} + \mathbf{M}_\infty)\, \ddot{\mathbf{a}}(t) + \int_0^t \mathbf{a}(\tau)\mathbf{B_r}(t-\tau)d\tau + \mathbf{B_h}\dot{\mathbf{a}}(t) + \mathbf{K_s}\mathbf{a}(t) = \mathbf{f_{ex}}(t) + \mathbf{f_{pto}}(t), \quad (4)$$

where $\mathbf{a}(t) \in R^n$ is the vector of the vertical displacement for each of the n buoys hulls, $\mathbf{M} \in R^{n\times n}$ is the mass matrix of the system, and $\mathbf{M}_\infty \in R^{n\times n}$ denotes the added-mass matrix at infinite frequency, $\mathbf{B_r}(t) \in R^{n\times n}$ is the radiation damping impulse response matrix, $\mathbf{K_s} \in R^{n\times n}$ is the hydrostatic stiffness matrix, $\mathbf{B_h} \in R^{n\times n}$ is the viscous damping matrix, $\mathbf{f_{ex}}(t) \in R^n$ is the vector of wave excitation force, and $\mathbf{f_{pto}}(t) \in R^n$ describes the vector of PTO forces.

The solution of Cummins' equation (4) for an array of heaving buoys experiencing waves is obtained in the frequency domain utilising the boundary element method (BEM) based software Ansys AQWA [24]. The resulting solution for the heaving buoy hull displacement vector $\mathbf{A}(\omega) \in R^n$, at each specific frequency of a regular wave ω, can be represented as a product of the sum of excitation $\mathbf{F_{ex}}(\omega) \in R^n$ and $\mathbf{F_{pto}} \in R^n(\omega)$ force vectors, and the inverse of the intrinsic impedance matrix $\mathbf{Z}(\omega) \in R^{n\times n}$ of the WECs system [25], divided by $(j\omega)$:

$$\mathbf{A}(\omega) = \frac{1}{j\omega}\mathbf{Z^{-1}}(\omega) \cdot \left[\mathbf{F_{ex}}(\omega) + \mathbf{F_{pto}}(\omega)\right], \quad (5)$$

where

$$\mathbf{Z}(\omega) = \mathbf{B}(\omega) + j\omega\left[\mathbf{M} + \mathbf{M_a}(\omega) + \mathbf{M}_\infty - \frac{\mathbf{K_s}}{\omega^2}\right], \quad (6)$$

with $\mathbf{B}(\omega) \in R^{n\times n}$ the radiation resistance matrix, $\mathbf{K_s} \in R^{n\times n}$ is the hydro-static stiffness matrix, $\mathbf{M} \in R^{n\times n}$ is the mass matrix of the system, and $\mathbf{M_a}(\omega) \in R^{n\times n}$ is the added mass matrix after the singularity at infinite frequency $\mathbf{M}_\infty \in R^{n\times n}$ is removed.

In the case of and array of n WECs, the intrinsic impedance, $\mathbf{Z} \in R^{n\times n}$ represents the matrix that accounts for interactions between WECs in an array:

$$\mathbf{Z} = \begin{pmatrix} z_{1,1} & \dots & z_{1,n} \\ \vdots & \ddots & \vdots \\ z_{n,1} & \dots & z_{n,n} \end{pmatrix} = \\ = \begin{pmatrix} x_{1,1}+jy_{1,1} & \dots & x_{1,n}+jy_{1,n} \\ \vdots & \ddots & \vdots \\ x_{n,1}+jy_{n,1} & \dots & x_{n,n}+jy_{n,n} \end{pmatrix}, \quad (7)$$

where, for example, $z_{1,1}$ is the intrinsic impedance of the WEC 1, and $z_{n,1}$ is the influence of the WEC number 1 on the WEC number n. Each element of the $\mathbf{Z}$ matrix consists of a real part $x_{i,j}$ which represents the radiation damping, and an imaginary part $y_{i,j}$, which is a combination of the masses and hydro-static stiffness coefficients. The excitation forces $\mathbf{F}_{ex}$ in the heave direction, for each array WEC, form the following vector:

$$\mathbf{F}_{ex} = (F_1, \dots, F_n)^T. \quad (8)$$

IV. Optimal Centralised Constrained Control for Heterogeneous Arrays of WECs

The study assumes that the WECs in an array operate under a newly developed array version of the 'Simple and Effective' controller (SAE) [22]. This control represents a constrained modification of the traditional complex conjugate control method [25]. While the original SAE controller was developed and successfully tested for isolated devices, in this section, a new method that allows an extension to the case of heterogeneous arrays of WECs is described. An assessment method, for the time average power production of the array of WECs operating under this new control strategy, is also articulated.

The application of SAE control to arrays of WECs (now dubbed ASEA, for 'arrays') requires the adjustment of the overall intrinsic impedance of the PTO systems, $\mathbf{Z_{pto}}(\omega)$, determined by the following equation:

$$\mathbf{Z_{pto}} = \\ \begin{pmatrix} (2\alpha_1-1)x_{1,1}-jy_{1,1} & \dots & (2\alpha_n-1)x_{1,n}-jy_{1,n} \\ \vdots & \ddots & \vdots \\ (2\alpha_1-1)x_{n,1}-jy_{n,1} & \dots & (2\alpha_n-1)x_{n,n}-jy_{n,n} \end{pmatrix} \quad (9)$$

where α_i represents a tuning parameter for the PTO damping of WEC i, which effectively imposes constraints on the system motion.

Notably, each element in the column of the matrix (9) is proportional to $(2\alpha_i - 1)$, this approximation is based on the assumption of linear interaction between WECs in the array. Thus, the changes in the radiation damping of one WEC will also change the influence of this WEC on all the others in the array.

Then, the overall intrinsic impedance $\mathbf{H}$ of the WECs in the array in combination with their PTO, can be found as a sum of $\mathbf{Z}$ and $\mathbf{Z_{pto}}$:

$$\mathbf{H} = \mathbf{Z} + \mathbf{Z_{pto}} = \begin{pmatrix} 2\alpha_1 x_{1,1} & \dots & 2\alpha_n x_{1,n} \\ \vdots & \ddots & \vdots \\ 2\alpha_1 x_{n,1} & \dots & 2\alpha_n x_{n,n} \end{pmatrix}. \quad (10)$$

The corresponding velocities for each WEC in the array can be found as $\mathbf{V} = \mathbf{H^{-1}F_{ex}}$ or

$$\mathbf{V} = \\ \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} = \begin{pmatrix} 2\alpha_1 x_{1,1} & \dots & 2\alpha_n x_{1,n} \\ \vdots & \ddots & \vdots \\ 2\alpha_1 x_{n,1} & \dots & 2\alpha_n x_{n,n} \end{pmatrix}^{-1} \begin{pmatrix} F_1 \\ \vdots \\ F_n \end{pmatrix}. \quad (11)$$

The required inverse system matrix $\mathbf{H^{-1}}$ can be calculated in the following form:

$$\mathbf{H^{-1}} = \begin{pmatrix} \frac{C_{1,1}}{2\alpha_1\Delta} & \dots & \frac{C_{n,1}}{2\alpha_1\Delta} \\ \vdots & \ddots & \vdots \\ \frac{C_{1,n}}{2\alpha_n\Delta} & \dots & \frac{C_{n,n}}{2\alpha_n\Delta} \end{pmatrix}, \quad (12)$$

where Δ is the determinant and $C_{i,j}$ are the co-factors of the matrix whose elements are $x_{i,j}$. Then, the velocities, for each WEC in the array, can be evaluated as:

$$\mathbf{V} = \begin{pmatrix} V_1 \\ \vdots \\ V_n \end{pmatrix} = \begin{pmatrix} \frac{1}{2\alpha_1}\left(\sum_{i=1}^n \frac{C_{i,1}F_i}{\Delta}\right) \\ \vdots \\ \frac{1}{2\alpha_n}\left(\sum_{i=1}^n \frac{C_{i,1}F_i}{\Delta}\right) \end{pmatrix}, \quad (13)$$

and corresponding displacements A_i for WECs for the selected wave frequency ω can be evaluated as:

$$(A_1, A_2, \dots A_n)^T = \frac{(V_1, V_2, \dots V_n)^T}{j\omega}. \quad (14)$$

Consequently, in the case when all $\alpha_i = 1$, the traditional complex conjugate control solution is obtained [26]. However, it is a well-known fact that solutions involving complex conjugate control often require unrealistic heaving buoy displacement magnitudes. Therefore, limitations on the maximum heave displacements are necessary to remain within physical PTO constraints, while also satisfying the assumptions of the linear theory [25]. Such limitations can be achieved by adjusting the PTO damping parameters α_i. In cases where the displacements, required by complex conjugate control, exceed realistic values, i.e. $|A_i| > A_{Max}$, the corresponding correcting tuning PTO parameter α_i can be found using the following equation:

$$\text{If } \ |A_i| > A_{Max} \ \rightarrow \ \alpha_i = \frac{|A_i|}{A_{Max}} \tag{15}$$

The required PTO forces $\mathbf{F}_{pto} \in R^n$ can be evaluated as:

$$\mathbf{F}_{pto} = -\mathbf{Z_{pto}}\mathbf{V} \tag{16}$$

The limitation on the maximum available $\mathbf{F_{pto}}$ could also potentially be integrated into the system, imposing additional requirements on the PTO damping parameters α_i.

The time-averaged power production $\mathbf{P} \in R^n$ vector, in the frequency domain, for each WEC operating under the presented ASAE controller, can be evaluated as:

$$\mathbf{P} = \begin{pmatrix} P_1 \\ \vdots \\ P_n \end{pmatrix} = \frac{1}{2} Re(\mathbf{Z_{pto}})|\mathbf{V}|^2 =$$

$$= \frac{1}{2} \begin{pmatrix} (2\alpha_1 - 1)x_{1,1} & \dots & (2\alpha_n - 1)x_{1,n} \\ \vdots & \ddots & \vdots \\ (2\alpha_1 - 1)x_{n,1} & \dots & (2\alpha_n - 1)x_{n,n} \end{pmatrix} \begin{pmatrix} |V_1|^2 \\ \vdots \\ |V_n|^2 \end{pmatrix} \tag{17}$$

The overall power production P_{array} by the array can be evaluated by substituting the range of obtained $P_i(\omega)$ values, evaluated for each wave frequency ω, into (eq. 1), as presented in Section II.

The simplicity of the presented ASAE method for control calculation and performance assessment promises computationally fast numerical performance evaluation, which is particularly important for incorporation within system optimisation loops. In comparison with time-domain-based performance assessment methods, this method does not require long virtual time simulation, conducting control calculations at each time step, or assessing convergence for power values.

V. Results

This section is dedicated to the validation of the proposed frequency domain-based ASAE method against results obtained in the time domain by previous researchers [12]. The computational process involves establishing the overall layout of the array, followed by specifying the geometric characteristics of WECs. Subsequently, hydrodynamic parameters ($\mathbf{Z}$ and $\mathbf{F_{ex}}$) are computed utilising Ansys AQWA [24]. After that, the optimal constrained array control and its time-averaged performance are assessed, employing the techniques described in Section IV. The overall array performance is evaluated in terms of Q_{Het}-factor, as elaborated in Section II.

Case 1: Validation for Regular waves

In order to validate the proposed frequency domain control and performance assessment methodology against previous research conducted in the time domain, we present a comparison of q-factors obtained using the proposed frequency

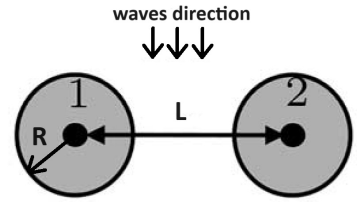

Fig. 1: Layout of the WEC array which is studied in [12].

domain method ASAE with results from [12], which utilised global control (GC) in the time domain.

The WEC array studied in [12] consists of two fully submerged devices with R=6.25m and draft 4m, and has unidirectional wave propagation as illustrated in Fig. 1. The study in [12] involved assessing the optimal distances between devices in an array in terms of L/R. For this performance assessment, a regular wave with H=1m and T=9s was considered. In order to prevent unrealistic solutions from the proposed frequency domain-based control (ASAE), the maximum allowable magnitude for the buoy hull displacements is alternatively set to A_{max}=2m and A_{max}=4m, as defined in eq. (15). The non-constrained solution is also considered ($A_{max} = \infty$).

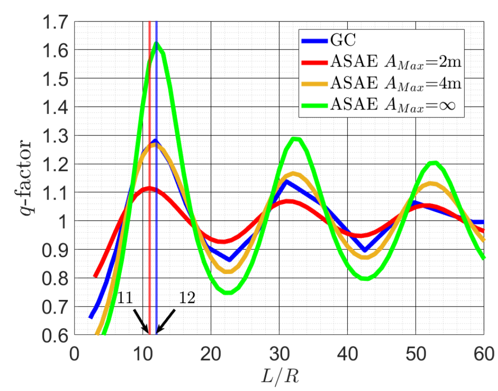

Fig. 2: Validation of q-factor changes with the increase of distances between WECs, obtained using GC in the time domain in [12] and ASAE (different displacement constraints) in the frequency domain, for regular waves with $H = 1$m and $T = 9$s. The vertical lines show the hydrodynamically optimal spacing between WECs.

A comparison of the results is presented in Fig. 2 show that the optimal distance between WECs L/R (highlighted by vertical lines) obtained with the use of GC and ASAE are close to each other, while the fluctuations noted in the q-factor of the ASAE method are in agreement with the time domain results acquired through the GC approach. The ASAE method provides very close to GC results in the case when A_{Max} = 4m which is the declared draft of the devices in [12] (while the constraints have not been specified in [12], the referenced and applied method [26] assumes a possibility of their implementation).

Case 2: Validation for Irregular waves

The second case focuses on validating the ASAE method against results obtained for irregular waves using comparable time domain approaches [12]. The study is carried out using the same array as in the previous case in (Fig. 1). The irregular waves modeled in [12] have a significant wave height of $H_s = 1$m and a peak period of $T_p = 9$s, simulated using the Bretschneider wave spectrum, whose distribution function $p_{ss}(\omega)$ is depicted in Fig. 3.

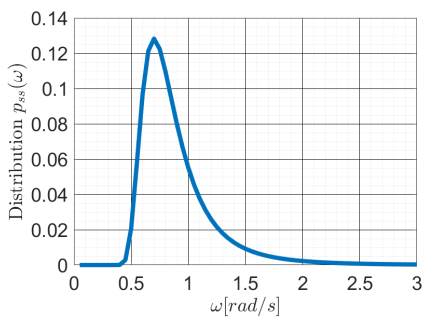

Fig. 3: The Bretschneider wave spectrum distribution for H_s=1m and T_p=9s.

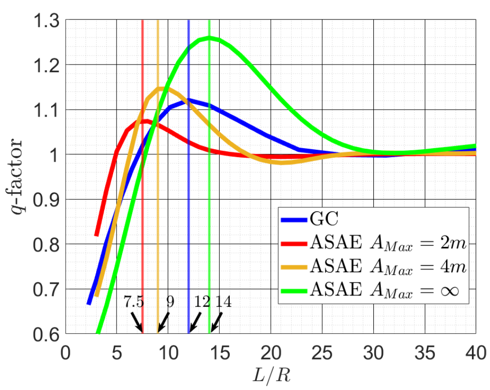

Fig. 4: Validation of q-factor changes with the increase of distances between WECs, obtained using GC in time domain in [12] and ASAE (different displacement constraints) in the frequency domain, for irregular waves with $H_s = 1$m and $T_p = 9$s. The vertical lines show the hydrodynamically optimal spacing between WECs.

The comparative results for the irregular wave case are depicted in Fig. 4. The high sensitivity of the results of the ASAE method to the maximum allowed magnitude for the buoy hull displacement A_{Max} is visible. Thus, for the case of a relatively small magnitude constraint A_{Max} =2m (red line), the max q-factor and optimal distance between devices is quite small compared to the results where obtained for the non-constrained magnitude of the buoy hull $A_{Max} = \infty$ (green line). It can be concluded that the q-factor and optimal L/R shift with changes in the displacement constraint A_{max}, since larger displacements of the buoy hull radiate stronger waves, which alter the optimal distance between the devices and increase the q-factor.

Case 3: Example of Heterogeneous arrays assessments

This case illustrates the capabilities of the proposed frequency domain-based ASAE method for heterogeneous array control performance evaluation. The scenario depicted in Fig. 5 shows four WECs arranged in a square array. For simplicity of illustration, the distance between devices is parameterised with a single parameter L, and their shapes are scaled using the single proportionality parameters a_i, such that $R_i = a_i R_0$ and $H_i = a_i H_0$. As an illustrative example, we consider a rose with two potential directions of ocean wave propagation, with probability d_1= 0.6 to be in the direction of the x axis and probability d_2= 0.4 from the direction which forms 60^o with the x axis, as illustrated in Fig.5.

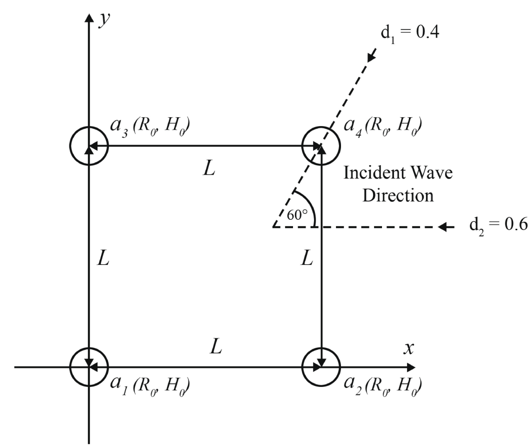

Fig. 5: The parametrisation method for optimization variables for the square array of four WECs with cylindrical hulls.

The reference array is homogeneous, consisting of WECs with semi-submerged cylindrical hull shapes, with dimensions R_0=3m and H_0=6m. Conversely, the other arrays are heterogeneous, with various scaling parameters a_1, a_2, a_3, a_4. The optimisation parameter (variable) is the distance between WEC centers, denoted as L, normalised by the average radius, to L/R_0. The optimisation objective is to maximise the q-factor, as per equations (2) and (3). It is assumed that waves are panchromatic, and are described by the density presented in Fig. 3. The maximum displacement of the buoy hulls are limited to A_{Max}=4m.

Fig. 6 illustrates the sensitivity of the q-factor for a square array of various cylindrical WECs, as depicted in Fig. 5, to the distance between the devices. It is evident that when the devices are positioned too closely ($L/R_0 < 7$), the solution yields an unrealistic assessment value. This can be attributed to overestimation of the positive interaction between closely located devices using linear theory. However, for distances where $L/R_0 > 7$, the method provides more reliable results.

It is evident that, when smaller devices are placed upwave of larger ones, the heterogeneous array significantly outperforms the homogeneous array, as indicated by the purple and green lines in Fig. 6. This aligns with some results

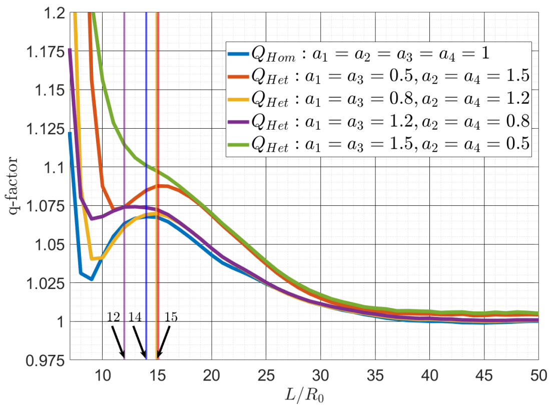

Fig. 6: q-factor for the homogeneous and heterogeneous arrays as a function of L/R_0, the distances between the devices centers normalised by the average radius. The vertical lines show the hydrodynamically optimal spacing between WECs.

published in [16]. Conversely, in cases where slightly larger devices precede slightly smaller ones, the performance of the heterogeneous array is inferior to that of the homogeneous array, for some distances (yellow line). However, when the difference between device sizes is significant (red line), the heterogeneous array again outperforms the homogeneous array, despite the larger devices being positioned closer to the wavefront.

VI. Conclusions

The presented frequency domain-based centralised constrained control method offers a computationally efficient approach for controlling and optimising the shapes and layouts of WECs for heterogeneous arrays. The method allows the assessment of the array performance in the frequency domain, potentially providing faster results compared to time-domain simulations. Additionally, the newly developed ASAE method is versatile and can be applied for centralised control across various WEC arrays. However, the real-time implementation of the proposed optimal control method necessitates the development of a reliable excitation force estimator for each WEC within the array.

The application of the proposed frequency-domain-based ASAE methodology to assess the performance of WECs operating in an array, in terms of q-factor, has been validated against the published results of various control assessments conducted in the time domain. This validation adds credibility to the designed approach and motivates further application and development of the proposed method in future studies.

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