Pitch response comparison of offshore platforms: a Lagrangian approach

M. M. G. Costa
Centre for Ocean Energy Research
Maynooth University
Maynooth, Ireland
matheus.costa.2024@mumail.ie

J. P. Santos
Department of Mathematics
Federal University of São João del-Rei
São João del-Rei, Brazil
jander@ufsj.edu.br

A. Bertolin
Centre for Ocean Energy Research
Maynooth University
Maynooth, Ireland
Ariadne.Bertolin@mu.ie

J. G. Silva
Centre for Ocean Energy Research
Maynooth University
Maynooth, Ireland
josefredo.silva.2024@mumail.ie

E. Nepomuceno
Centre for Ocean Energy Research
Maynooth University
Maynooth, Ireland
Erivelton.Nepomuceno@mu.ie

2024 35th Irish Signals and Systems Conference (ISSC) | 979-8-3503-5298-6/24/$31.00 ©2024 IEEE | DOI: 10.1109/ISSC61953.2024.10603133

***Abstract*—Wind energy is a renewable source considerably explored worldwide. Currently, attempts have been made to create wind farms on offshore areas, given the higher intensity and consistency of the wind resource. By doing this, conventional fixed foundation platforms become infeasible and floating structures have to be considered. One of the consequences for using these floating structures is the presence of troublesome motions, that increase mechanical stress and reduce the energy production of the wind turbine. Generally in the literature, this problem is evaluated with the use of numeric simulations, producing results for one specific case at a time. This paper proposes a classical mechanics approach, using a Lagrangian framework to describe the dynamics of pitch motion in a analytical formulation. This approach can describe the system response with a general model, instead of only numerical values, facilitating the comprehension of the most important parameters for the dynamic response.**

***Index Terms*—offshore platforms, wind energy, modeling**

I. Introduction

Wind energy is a renewable source that has proven itself, having intensive investments and use in the past decade. A report from the Global Wind Energy Council (GWEC) [1] states that the global energy production associated with wind energy is around 900 GW. In wind energy production rankings, for example the rankings of the International Renewable Energy Agency (IRENA), China, the USA and Germany lead the ranking with the highest power generation.

With the increasing effort to diminish the costs of renewable energy and make it more competitive, multiple improvements keep being proposed. One current trend of research is to move farms towards offshore areas, given the higher quality of the wind resource in terms of being more steady throughout time and stronger [2].

As the offshore wind exploration goes into deeper waters, floating offshore platforms becomes an important solution [3]. According to [1], floating offshore wind (FOW) is rapidly shifting to commercial scale and is expected to rapidly accelerate. China leads the wind and FOW section in terms of machinery production and energy generation, with Europe in second place, mostly represented by projects such as Hywind (Scotland) and Windfloat (Portugal).

In deep sea waters, i.e. more than 50 meters, it is more feasible to consider floating foundations instead of more traditional fixed-foundation structures. In that category, semi-submersible floating platforms presents the highest TRL (technology readiness levels). Some examples of semi-submersible platforms include WindFloat in Portugal (25 MW), the biggest wind turbine platform in the world, and DeepCWind, developed by the National Renewable Energy Laboratory (NREL) [3]. As a consequence of the use of floating platforms, the structure becomes subjected to motions that can influence in multiple important aspects: installation and maintenance, operation, lifespan, energy production and so on [4].

There are multiple approaches to the discussion of offshore platform stabilization, but the common method for authors to evaluate their ideas on this topic is by using one or more numerical tools. This method is common, no matter what aspect is focused on, be it in controller design, structural aspects, hydrodynamic simulation, or a combination of those [5]–[8].

Despite a substantial amount of publications on platform stabilization, analytical proposals to investigate the platform dynamics are not common, generating a struggle to setup and use complex numerical frameworks that are still unable to describe the system in a simpler and general way. This paper aims to introduce a analytical method to compare the dynamics of offshore platforms with the use of Lagrangian mechanics.

This publication has emanated from research conducted with the financial support of Science Foundation Ireland under Grant number 21/FFP-P/10065. Erivelton Nepomuceno was supported by Brazilian Research Agencies: CNPq/INERGE (Grant No. 465704/2014-0), CNPq (Grant No. 425509/2018-4 and Grant No. 311321/2020-8) and FAPEMIG (Grant No. APQ-00870-17). Jander P. Santos would like to thank CNPq, Brazil (No. 304101/2022-2) for the financial support.

979-8-3503-5298-6/24/$31.00 ©2024 IEEE

The paper is organized as follows: in section two, the underlying theory used throughout the paper is presented. Section three proposes three different semi-submersible platform systems and presents their modeling. Section four analyses the obtained models in an example scenario to investigate it's possible outcomes. The main points and contributions of the paper are summarized in the conclusion, where further investigation aspects are also listed.

II. Underlying theory

In classical mechanics theory, Lagrangian mechanics refers to a framework founded in the principle of least action, in which the dynamic equation for a system can be defined with the use of it's energies, potential and kinetic. To obtain the dynamic equation, a procedure has to be followed, as will be described bellow.

First, the Lagrangian equation (L) is defined, as in (1)

$$L = T - U, \tag{1}$$

where T is the sum of kinetic energies of all i bodies that composed the investigated system, calculated by (2)

$$T = \frac{1}{2}\sum_{i=1}^{N} I_i \dot{\theta}_i\left(t\right)^2, \tag{2}$$

in which I_i is the moment of inertia of the i body, $\dot{\theta}_i\left(t\right)$ the angular velocity of the i body and N the total number of bodies in the system. And U is the sum of all potential energies of the system, calculated by (3)

$$U = \sum_{i=1}^{N} m_i g h_i, \tag{3}$$

where h_i is the vertical distance to the origin frame, and g is the gravity acceleration constant.

Once the Lagrangian (L) is defined, the differential equation that describes the system can be obtained using the Euler-Lagrange formulation presented in (4)

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \frac{\partial W}{\partial q}, \tag{4}$$

where t is time, q is the generalized coordinate system chosen for the analysis, and W is the so called virtual work.

To evaluate the rotational motion described by the different bodies in the proposed systems further in the paper, a complementary theorem is required. When bodies rotate along their own axis of rotation there are immediate formulations that can find the resulting moment of inertia. In this paper however, since the bodies are rigidly connected to form systems, they rotate along the axis passing through the center of mass of the system. To compensate for this it's necessary to modify the moment of inertia of each body using the paralel-axis theorem. The modified moment of inertia for a body (I_{PA}) given it's already known moment of inertia (I) is given by (5)

$$I_{PA} = I + md, \tag{5}$$

where m is the mass of that body, and d the distance to the new axis of rotation. This formulation is necessary though model development to obtain the Lagrangian (L) already mentioned.

III. Description and modeling

Three systems are proposed to be evaluated in this paper. They are thought out to be a simple representation of a floating wind turbine on top of a platform. Three bodies compose each system: a platform, a column whose base is at the center of the platform (indicated with subscript C) and a spherical mass centered on the column's top (indicated with subscript S). The difference between each system is the chosen platform that can be of different geometries: a cuboid, a cylinder and a cone with it's face to the sky. To simplify the equations, a greek letter notation is used to identify the platforms: the cuboid is α, the cylinder is β and the cone γ. A illustration of the three systems can be seen in Fig 1.

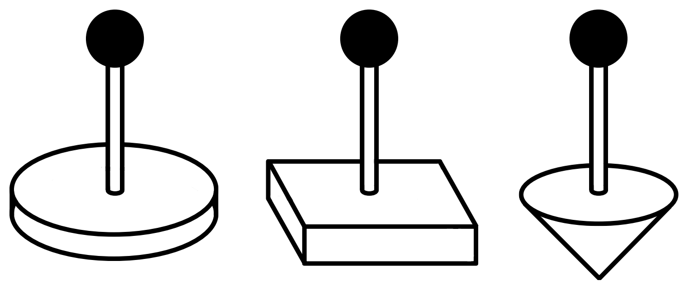

Fig. 1. Illustration of the three proposed systems.

For ease of the calculations that are going to be developed, the origin point of the coordinate system is placed at the central point of the platform top surface, which coincides with the center point of the column base. This definition eases the calculation of potential energies and distances between the center of mass of different bodies. Given that the origin points is fixed to the platform, all translational kinetic energies are equal to zero in this reference frame, and only kinetic energies of rotational nature remain.

For the purposes of facilitating the comprehension of all equations moving forward: the subscript associated with a parameter states the body to whom that parameter belongs to. All parameters are indicated as follows: moment of inertia (I) mass (m), radius (r), height (h), width (w), the distances between the system center of mass and a body center of mass (l), the pitch angle of the system (θ) or angular velocity ($\dot{\theta}$) or angular acceleration ($\ddot{\theta}$).

All movements described by the proposed systems are of rotational nature, therefore, to define total kinetic energy it is necessary to describe the moments of inertia of all bodies withing the system. When a free moving body is subjected to a external force, a rotational motion around it's center of mass may occur. If this rotational motion is on an axis different from one of body axis, an alteration must be made to the moment of inertia using the parallel axis theorem mentioned in (5). Since it's considered that the bodies are rigidly connected in the three proposed systems, they all rotate along the same axis, which is the system center of mass. This implies that each of the three bodies of all three systems must have their moments of inertia adapted. The resulting moments of inertia of all bodies for all systems are shown in the following equations. First, the spherical mass on top of the column in (6)

$$I_S = \frac{2}{5} m_S r_S^2 + m_S l_S^2, \tag{6}$$

for the column (7)

$$I_C = \frac{1}{12} m_C h_C^2 + m_C l_C^2, \tag{7}$$

and for the different platforms, starting with the cylindrical platform (8)

$$I_\beta = \frac{1}{4} m_\beta r_\beta^2 + \frac{1}{12} m_\beta h_\beta^2 + m_\beta l_\beta^2, \tag{8}$$

for the cuboid platform (9)

$$I_\alpha = \frac{m_\alpha \left(h_\alpha^2 + w_\alpha^2\right)}{12} + m_\alpha l_\alpha^2, \tag{9}$$

and lastly, for the conical platform (10)

$$I_\gamma = \frac{3 m_\gamma \left(h_\gamma^2 + 4 r_\gamma^2\right)}{80} + m_\gamma l_\gamma^2. \tag{10}$$

Once all moments of inertia are defined, the total kinetic energy mentioned in (2) can be calculated. For the sake of clarity, the kinetic energy (T) for each body is presented separately in the following equations. First, for the sphere (11)

$$T_S = 0.5 \left(\frac{2}{5} m_S r_S^2 + m_S l_S^2\right) \dot{\theta}\,(t)^2, \tag{11}$$

for the column (12)

$$T_C = 0.5 \left(\frac{1}{12} m_C h_C^2 + m_C l_C^2\right) \dot{\theta}\,(t)^2, \tag{12}$$

the same for the platforms, starting with the cuboid platform (13)

$$T_\alpha = 0.5 \left(\frac{m_\alpha \left(h_\alpha^2 + w_\alpha^2\right)}{12} + m_\alpha l_\alpha^2\right) \dot{\theta}\,(t)^2, \tag{13}$$

for the conical platform (14)

$$T_\gamma = 0.5 \left(\frac{3 m_\gamma \left(h_\gamma^2 + 4 r_\gamma^2\right)}{80} + m_\gamma l_\gamma^2\right) \dot{\theta}\,(t)^2, \tag{14}$$

and the cylindrical platform (15)

$$T_\beta = 0.5 \left(\frac{1}{4} m_\beta r_\beta^2 + \frac{1}{12} m_\beta h_\beta^2 + m_\beta l_\beta^2\right) \dot{\theta}\,(t)^2. \tag{15}$$

The potential energies (U) can also be described: first for the sphere in (16)

$$U_S = m_S g h_C \cos(\theta)\,, \tag{16}$$

the column (17)

$$U_C = 0.5 m_C g h_C \cos(\theta), \tag{17}$$

the cuboid platform (18)

$$U_\alpha = \frac{m_\alpha g h_\alpha \cos(\theta)}{2}, \tag{18}$$

the conical platform (19)

$$U_\gamma = \frac{m_\gamma g h_\gamma \cos(\theta)}{4}, \tag{19}$$

and the cylindrical platform (20)

$$U_\beta = \frac{m_\beta g h_\beta \cos(\theta)}{2}. \tag{20}$$

At this stage the Lagrangian equation for each one of the three systems can be obtained following the general format of (21)

$$\begin{aligned} &L = (T_S + T_C + T_P) - (U_S + U_C + U_P) \\ &P = \{\alpha, \beta, \gamma\}. \end{aligned} \tag{21}$$

Equation (21) defines all three Lagrangian (L) equations, one for each system. To obtain each one, the kinetic and potential energy correspondent to that platform has to be included in place of T_P and U_P.

After all energy components have been described, the Euler-Lagrange formulation (4) can be used to obtain the dynamic equation for each configuration, as shown in (22)

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = \frac{\partial W}{\partial \theta}, \tag{22}$$

where the pitch angle θ was the chosen generalized for the analysis.

At this stage of the modeling process, all the excitation forces acting on the system can be defined. For this study, one force will be considered as the system input. The force acts on the center of mass of the sphere in the horizontal direction and it's intended to mimic a wind perturbation. There is no intention at this stage of the proposal to use sophisticated models of wind excitation forces, therefore, a proposition is made for force F, as indicated in (23)

$$F = A \sin(\omega t), \tag{23}$$

where A is the amplitude of the excitation force, ω is the angular frequency, and t is time. Real wind forces are usually presented as a non-zero mean value with an oscillatory nature as can be seen in [9], however, this simplified model is proposed to get a simpler initial assessment of the concept, while still accounting for the change in wind direction.

With the proposed force (23) acting, the system will rotate along it's own center of mass, and this displacement can be measured with an angle θ. The virtual work (W) done by this force can be defined as $W = Fd$, where the distance d is the x-axis projection of the arc length path described by the spherical mass, described by $d = l_S \sin(\theta)$. Combining these two information the virtual work can be defined as (24)

$$W = A l_S \sin(\omega t) \sin(\theta) . \tag{24}$$

Substituting the results of (21) and (24) in (22) and performing the necessary calculations, a differential equation (DE) that describes the system dynamics is found. To simplify the solution procedure of the DE, a linearization step is made for the equilibrium position of the platform, that is, the upright position. This results in the simplification $\sin(\theta) = \theta$ and $\cos(\theta) = 1$.

The linearized DE is solved symbolically with the aid of Maple 2023 software. The DE solution has a rather inconvenient size, but for the sake of demonstration, the solution for the cuboid platform case is presented in (25), after some effort to simplify the equation:

$$\theta(t) = \frac{60 A l_S \left(\omega \sqrt{\mathrm{B}} \sinh\left(\frac{C}{\sqrt{B}} t\right) - C \sin(\omega t)\right)}{C\left(B\omega^2 + \mathrm{D}\right)}, \tag{25}$$

where the constants B, C and D are defined as follows: constant B is defined in (26)

$$\begin{aligned} B =& 5\left(h_\alpha^2 + 12 l_\alpha^2 + w_\alpha^2\right) m_\alpha + 5\left(h_C^2 + 12 l_C^2\right) m_C \\ &+ 12 m_S \left(5 l_S^2 + 2 r_S^2\right), \end{aligned} \tag{26}$$

constant C on (27)

$$C = \sqrt{30}\,\sqrt{\mathrm{g}}\,\sqrt{\left(m_C + 2 m_S\right) h_C - h_\alpha m_\alpha}, \tag{27}$$

and lastly, constant D on (28)

$$D = 30(g h_C m_C + 2 g h_C m_S - g h_\alpha m_\alpha). \tag{28}$$

All lengths l describe the distances between two points: the body and the system center of mass. To calculate l it is necessary to define those two points. With the origin at the base center of the column, as proposed, the coordinates of each i_{th} body center of mass (CM_i) can be easily described in terms of the geometric parameters of the bodies. As an example, the center of mass of the cuboid platform is found in half it's height, since the origin is centered at the cuboid top, that point would just be $-h_C/2$. The other point, however, is the system center of mass, which changes for each different platform geometry used. To solve this, each of the three systems (j) has it's own center of mass and they can be calculated individually using equation (29)

$$CM_j = \frac{\sum_{i=1}^{N} m_i r_i}{\sum_{i=1}^{N} m_i}, \tag{29}$$

where m_i is the mass of each of the i bodies in the system, and r_i the distance from origin of each one of the i bodies. After that, all lengths l for each body can be calculated with (30)

$$l_i = CM_i - CM_{sys}. \tag{30}$$

After all the algebraic steps, the function $\theta(t)$ can be investigated for all cases of interest by substituting all geometric parameters of the system and the excitation force parameters by it's numerical values. In the next section, a example scenario is proposed to investigate the possible outcomes of the obtained models.

IV. Analysis

Given the complexity of the DE solution shown in (25), it is not a trivial task to evaluate what parameters present a greater impact on the systems pitch response. Only two observations are more evident: first, the excitation force amplitude A is directly proportional to the motion response, which is expected. Secondly, the constant D which has a very similar resemblance to the U component of the Lagrangian, has a inverse relation to the pitch motion. This indicates that the total potential energy for the system has a inversely proportional relation to θ.

For the sake of describing in more detail the possible outcomes of the models, a given set of numerical parameters is set in order to analyze some example scenarios. These parameters are shown in Table I and are used to produce numerical results for all three platforms, whose models were obtained with the presented procedure.

TABLE I
System parameters

Parameter	Value
m_S	150 kg
r_S	1.5 m
m_C	20 kg
h_C	10 m
m_α	1000 kg
h_α	3.5 m
l_α	5 m
w_α	10 m
m_β	1000 kg
r_β	$\frac{5\sqrt{2}}{\sqrt{\pi}}$ m
h_β	3.5 m
m_γ	1000 kg
r_γ	$\frac{5\sqrt{2}}{\sqrt{\pi}}$ m
h_γ	10.5 m
A	50
ω	0.5 rad/s
g	9.81 m/s^2

The dimensions of the cuboid platform are arbitrated by the authors, using values of tens of meters, since those are a reasonable measurements for real applications, as can be verified by looking at any of the two devices mentioned in section I. Once the cuboid is defined, the other geometries were determined. For the sake of trying to create a fair comparison between the systems, two restricting conditions are used. All platforms being compared must be of same volume and have the same top surface area. Two conditions are necessary since the cylinder and cone have two variables (height and radius) that need to be defined. The rationale for those restrictions being that all platforms are built of the same materials and have the same volume, therefore, same density (volume restriction) and must occupy the same area on the ocean (top surface restriction).

Once all geometry and force parameters are defined, a graph of the pitch over time (θ x t) for all three systems is obtained. The cuboid platform response is shown in Fig. 2; the cylindrical platform in Fig. 3 and conical platform in Fig. 4. It's worth mentioning that the graphs already present the conversion from radians to degrees.

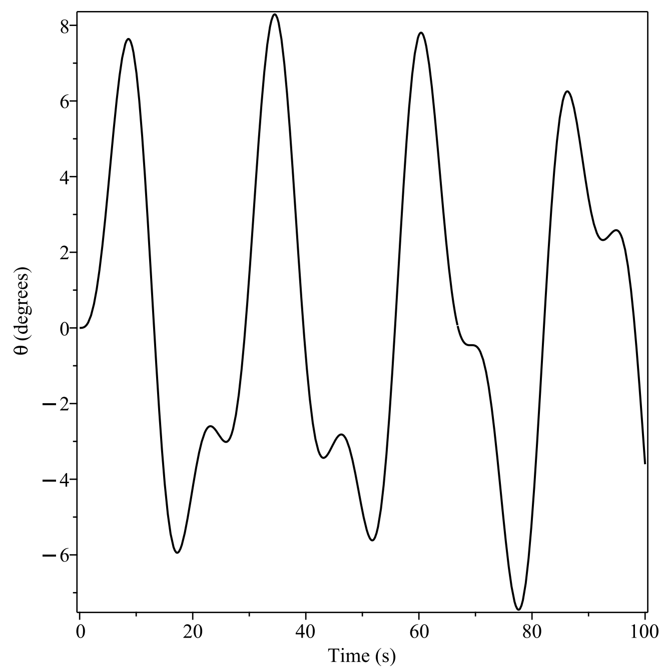

Fig. 2. System dynamic with cuboid platform.

Note that the cuboid and cylindrical platforms reach higher values of θ when compared with the conical system. Also, a observation can be made regarding the period of the oscillations in all cases. The cuboid platform shows the highest periods, followed by the cylindrical platform, while the conical system presents the smallest oscillation periods.

It's also possible to evaluate how each individual parameter change can alter the dynamic response of one particular system. In Fig. 5 it's illustrated how an alteration of the cuboid platform mass can influence the system dynamic response.

As can be seen, when comparing the same system, versions with a higher platform mass present smaller oscillations. This indicates that, for the same system, the total moment of inertia value is a good indication for the oscillation amplitude. That is not the case when comparing different platforms, given that the conical system response presented in Fig. 4 had the smallest total inertia and still outperformed the other platforms in this regard.

Beyond the oscillations observed in this time window, the dynamics of all systems are outlined withing a periodic envelope function that shifts up and down as indicated in Fig. 6, and can be understood as a fast oscillation withing

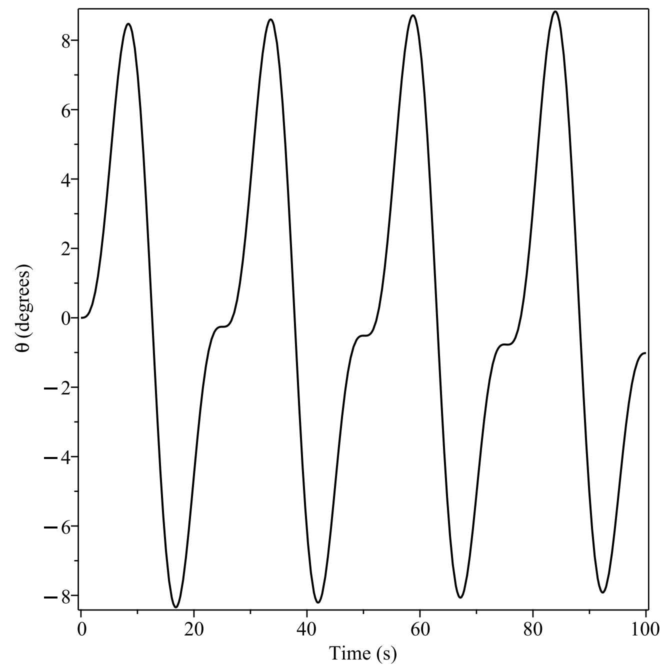

Fig. 3. System dynamic with cylindrical platform.

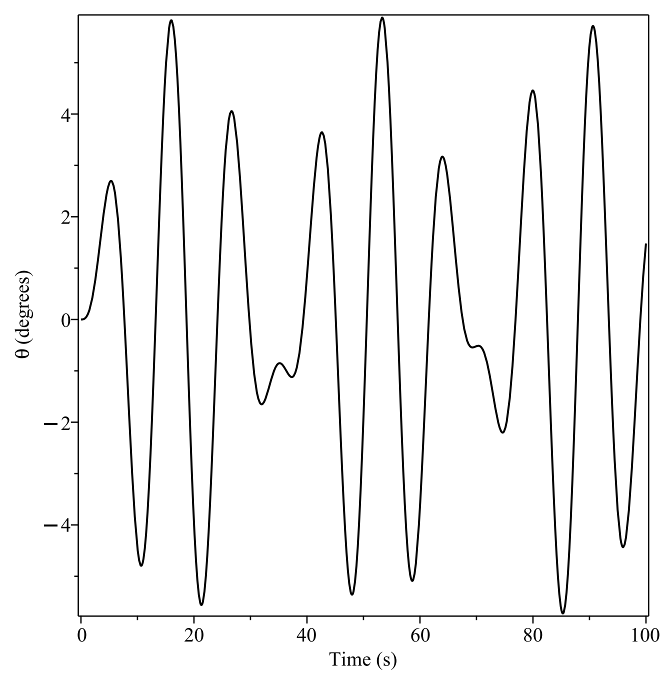

Fig. 4. System dynamic with conical platform.

a slower oscillatory motion, in this case, around two orders of magnitude slower.

The authors would like to make clear that no comparison with other works in the literature are made since, to the best of our efforts, no similar proposals, tackling a simpler analytical formulation of the pitch motion of offshore platforms, was found.

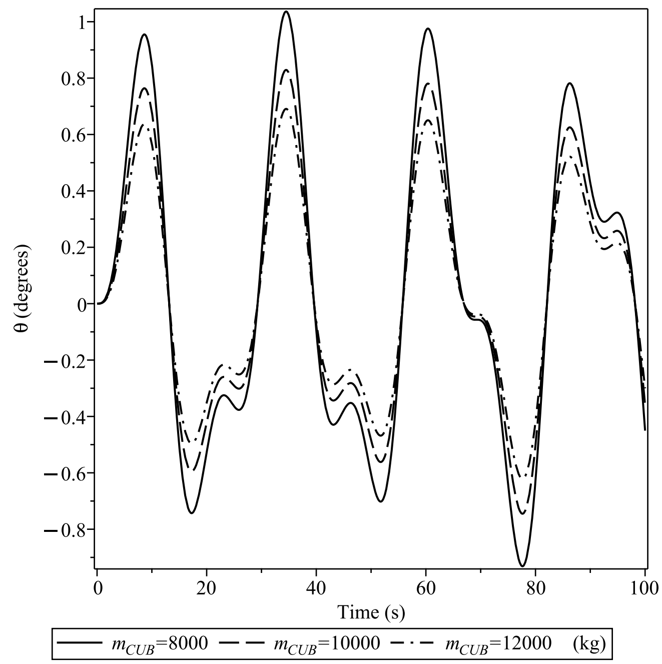

Fig. 5. Change in dynamic response from mass alteration.

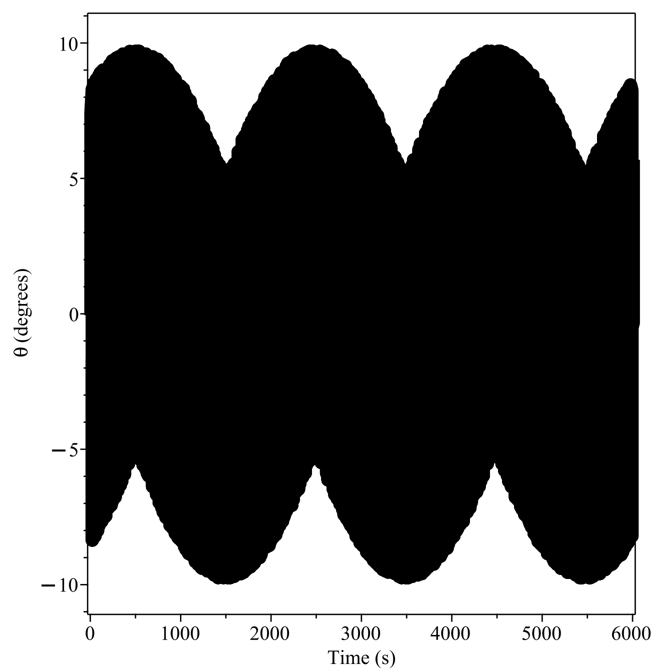

Fig. 6. Envelope silhouette for the cuboid case over a long time.

V. Conclusion

The Lagrangian approach was used to produce an analytical model of the pitch motion of three different platform systems. The DE describing the system dynamics was found, linearized and solved to obtain the pitch equation for each case. General comments are presented for the final dynamic equation and a comparison between the proposed platforms was made in an example case scenario.

The results suggest that the total moment of inertia may be a good metric for oscillation amplitude, but only when used to compare similar systems. In the proposed example scenario, the system associated with the conical platform outperforms the cuboid and conical platform systems reaching the smallest oscillation range, while the cuboid platform system outperforms the other two with a higher oscillation period. While evaluating the systems over a long window frame it was noted that their behavior is described by an periodic enveloping function with non-zero mean.

For future works, the authors intend to expand the analysis to other rotational motions; evaluate more platform shapes and investigate more in depth patterns in the systems response considering changes on it's parameters.

References

[1] Wind report 2023, Global Wind Energy Council (GWEC), 2023, https://gwec.net/globalwindreport2023/, Accessed in March 2024

[2] Jung, Christopher, and Dirk Schindler. "The properties of the global offshore wind turbine fleet." Renewable and Sustainable Energy Reviews 186 (2023): 113667.

[3] Dong, Xiaochen, Yanni Li, Demin Li, Feifei Cao, Xiaoqiang Jiang, and Hongda Shi. "A state-of-the-art review of the hybrid wind-wave energy converter." Progress in Energy 4, no. 4 (2022): 042004.

[4] Meng, Fantai, Nataliia Sergiienko, Boyin Ding, Binzhen Zhou, Leandro Souza Pinheiro Da Silva, Benjamin Cazzolato, and Ye Li. "Co-located offshore wind–wave energy systems: Can motion suppression and reliable power generation be achieved simultaneously?." Applied Energy 331 (2023): 120373.

[5] Koo, WeonCheol, and Hong-Bhin Kim. "Numerical and experimental studies of the effects of WEC motion on a combined wind-wave energy platform." In Proceedings of the European Wave and Tidal Energy Conference, vol. 15. 2023.

[6] Jin, Peng, Zhi Zheng, Zhaomin Zhou, Binzhen Zhou, Lei Wang, Yang Yang, and Yingyi Liu. "Optimization and evaluation of a semi-submersible wind turbine and oscillating body wave energy converters hybrid system." Energy 282 (2023): 128889.

[7] Zhang, Hongjian, Ningchuan Zhang, and Xinyu Cao. "Conceptualization and dynamic response of an integrated system with a semi-submersible floating wind turbine and two types of wave energy converters." Ocean Engineering 269 (2023): 113517.

[8] Skene, David M., Nataliia Sergiienko, Boyin Ding, and Benjamin Cazzolato. "The prospect of combining a point absorber wave energy converter with a floating offshore wind turbine." Energies 14, no. 21 (2021): 7385.

[9] Wei, Da, Dongsheng Li, Tao Jiang, Pin Lyu, and Xiaofei Song. "Load identification of a 2.5 MW wind turbine tower using Kalman filtering techniques and BDS data." Engineering Structures 281 (2023): 115763.