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# Less of a puzzle: a new look at the forward forex market

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**Abstract**

The two-country monetary model is extended to include a consumption externality with habit persistence. The model is simulated using the artificial economy methodology. The 'puzzles' in the forward market are re-examined. The model is able to account for: (a) the low volatility of the forward discount; (b) the higher volatility of expected forward speculative profit; (c) the even higher volatility of the spot return; (d) the persistence in the forward discount; (e) the martingale behavior of spot exchange rates; and (f) the negative covariance between the expected spot return and expected forward speculative profit. It is unable to account for the forward market bias because the volatility of the expected spot return is too large relative to the volatility of the expected forward speculative profit.


*Keywords:* Artificial economy; Forward foreign exchange; Habit persistence

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## 1. Introduction

Campbell and Cochrane (1999) have proposed a preference specification in which there is both an aggregate consumption externality[1] and utility is time-

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[1]See Abel (1990) and Duesenberry (1949).

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inseparable because of habit-persistence. They apply this to the 'equity premium' puzzle and show that their model can replicate the relevant 'stylized facts' for US data. In this paper, we extend the Campbell and Cochrane (1999) preferences to both a monetary and an international setting. The 'puzzles' associated with the forward foreign exchange market are revisited in the light of the new theoretical framework.[2]

Numerous writers (a good example is Backus et al., 1993) have shown that the standard deviation of the forward discount is smaller than that of the expected forward speculative profit[3] and is an order of magnitude less than that of the spot return. We call this ordering of relative standard deviations the *volatility* puzzle. The standard cash-in-advance for goods model is unable to replicate this stylized fact at levels of risk aversion that are commonly used in the literature. Bekaert (1996) has argued that the standard for judging a successful theory of the forward market is its ability to replicate the *volatility* puzzle at plausible absolute levels of volatility. Our model is not only able to replicate these relative volatilities but goes most of the way in matching the absolute volatilities also.

A second feature of the data is that the forward discount is highly persistent while the spot return is virtually white noise (see Bekaert, 1996). The standard model is usually able to explain one of these but not both at the same time (see Moore and Roche, 2001). This is the *persistence* puzzle. Our model succeeds in getting close to the stylized facts for every case we examine.

The best-known deficiency in our understanding of the forward market is the *unbiasedness* puzzle: that the forward discount is a poor predictor of realized future spot return. Indeed, the forward discount typically fails to even forecast the correct direction of spot exchange rate changes. The most challenging[4] task of a good theory is to replicate this 'bias'. Since Fama (1984), it has been understood that two conditions need to be met to replicate the bias. Firstly, there needs to be a negative covariance between expected spot return and expected forward speculative profit. This is the condition that has received most emphasis (see Alvarez et al., 1999) and our model succeeds in satisfying it. Fama's second condition has received less attention in the literature. It requires that the volatility of expected forward speculative profit must exceed the volatility of expected spot returns. In this respect our model falls short. Ironically, its very success in explaining the volatility of spot return is accounted for by an excessive increase in the volatility of expected spot return. It is this feature of the model that prevents it from meeting Fama's second condition.

The plan of the paper is as follows. Section 2 provides a brief introduction to

[2]For discussions of forward market puzzles, see Bekaert (1996), Engel (1996) and Hodrick (1987).

[3]Expected forward speculative profits are often referred to as 'the' risk premium. This is, in general, incorrect because of non-convexities: see Engel (1999).

[4]For a recent attempt to address the forward bias puzzle, see Obstfeld and Rogoff (1998). For a thorough critique of that paper's contribution to resolving the puzzle, see Engel (1999).

Campbell and Cochrane (1999) preferences, develops the theoretical framework and provides intuition for explaining our results. Section 3 summarizes the results of simulating an artificial economy. The conclusion is in Section 4.

## 2. A theoretical framework

### 2.1. Background

Campbell and Cochrane (1999) have proposed a novel solution to the 'equity premium' puzzle in closed economies. They introduce a simple and tractable modification to the utility function in the standard model.[5] An external habit is introduced. In their model the real risk-free rate is constant. This is achieved by ensuring that precautionary savings effects and intertemporal substitution effects cancel each other out. This removes the 'risk-free rate' puzzle (see Weil, 1989). Campbell and Cochrane (1999) preferences overcome the irritating problem with existing habit persistence models that marginal utilities can sometimes take on negative values. The habit adjusts slowly in response to consumption. This differs from the usual habit persistence specifications that model the habit as proportional to past consumption choices. Campbell and Cochrane (1999) habit persistence takes the form of an aggregate consumption externality, i.e., 'Keeping Up with the Jones's' effects along the lines of Abel (1990) and Duesenberry (1949).

Habit persistence preferences have already been proposed before as a solution to the puzzles in the forward market. For example, Bekaert (1996) introduces habit persistence into a conventional Lucas (1982) two-country model where equilibrium consumption in each good equals half of the current endowment. He is able to match the observed volatilities of the spot return, the forward discount and expected forward speculative profit but never at the same configuration of parameters.

The reason why we believe that the Campbell and Cochrane (1999) framework can help to solve the forward market puzzles is as follows. The standard cash in advance model is able to match the stylized facts of the forward discount relatively well. However, it is unable to account for the observed high volatility in spot return. It is even more unsuccessful at explaining the volatility of expected forward speculative profit. The trouble is that existing attempts to improve performance with the latter leads to greatly increased volatility in the forward discount. This is an international analogue to the equity premium and risk-free rate puzzles in closed economy stock markets. Campbell and Cochrane (1999) preferences

[5] The Campbell–Cochrane model, like the one used here, is an exchange economy. Ljungqvist and Uhlig (2000) have pointed out that there are problems in expanding this framework to a production economy. 'Consumption bunching' rather than consumption smoothing becomes welfare optimal. However, this only arises if the habit is internalised.

provide the opportunity to 'anchor' the volatility of the forward premium because it forces real risk-free rates to display low volatility. This is because the intertemporal marginal rate of substitution (IMRS) is more volatile than the standard case, because of the habit specification, while the *expected* IMRS is constant by construction.

### 2.2. A cash in advance model with habit persistence

The basic structure of the model is the well-known Lucas (1982) two-country, two-good, two-money representative agent story. The households in both countries[6] have the same intertemporal utility function. They maximize the discounted expected value of lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t U(C_{it}^1, C_{it}^2) = \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_{it}^1 - H_{it}^1)^{(1-\gamma)}}{1-\gamma} + \frac{(C_{it}^2 - H_{it}^2)^{(1-\gamma)}}{1-\gamma} \right), \quad i = 1,2 \tag{1}$$

where $\beta$ is the discount factor, $1/\gamma$ is the intertemporal elasticity of substitution, $C_{it}^j$ is the consumption of goods and services of country $j$ by the household of country $i$ and $H_{it}^j$ is the subsistence consumption (or habit) of goods and services of country $j$ by the household of country $i$. We reparameterize the utility function in terms of $X_{it}^j$, the surplus consumption ratio of goods and services of country $j$ by the household of country $i$:

$$X_{it}^j = \frac{C_{it}^j - H_{it}^j}{C_{it}^j}, \quad i = 1,2, \quad j = 1,2 \tag{2}$$

When $C_{it}^j = H_{it}^j$, $X_{it}^j = 0$: this is the worst possible state. By contrast, as $C_{it}^j$ rises, the surplus consumption ratio converges on unity.

The agent in the goods market faces the following cash-in-advance constraint:

$$M_{it}^j \geq P_t^j C_{it}^j, \quad i = 1,2, \quad j = 1,2 \tag{3}$$

where $M_{it}^j$ is the amount of money of country $j$ held by the household of country $i$ for transactions in the goods market at time $t$ and $P_t^j$ is the price of country $j$ goods in terms of country $j$ money. If interest rates are positive the cash-in-advance constraint will hold with equality. At the end of period $t$ (or the beginning of period $t+1$), the domestic households holding of domestic currency:

$$M_{1t+1}^1 \geq P_t^1 C_t^1 + B_{1t}^1 - F_t G_t^1 \tag{4}$$

[6] The superscript denotes country of origin and the subscript denotes country of use. Uppercase letters denote variables in levels; lowercase letters denote variables in log levels, including growth and interest rates. Greek letters without time subscripts denote parameters. Bars over variables denote steady states.

is made up of proceeds from the sale of the endowment, the redemption of nominal discount bonds, $B^j_{it}$, and proceeds from forward contracts, where $F_t$ is the one-period ahead forward foreign exchange rate expressed as the home price of foreign currency. Note $G^1_t > 0$ constitutes the number of long forward contracts for foreign currency. The domestic household's holding of foreign currency is:

$$M^2_{1t+1} \geq B^2_{1t} + G^1_t \tag{5}$$

Analogously the foreign households holding of foreign currency is:

$$M^2_{2t+1} \geq P^2_t C^2_t + B^2_{2t} - G^2_t \tag{6}$$

and of domestic currency is:

$$M^1_{2t+1} \geq B^1_{2t} + F_t G^2_t \tag{7}$$

where $G^2_t > 0$ constitutes a short position in forward foreign exchange for the foreign country.

The only role for the government is to have a central bank that engages in open market operations. In each period the central bank of each country changes the money stock by issuing one-period discount bonds. The bonds are redeemed at the end of period $t$ (or the beginning of period $t+1$). Equilibrium in the goods market is given by:

$$C^j_t = C^j_{1t} + C^j_{2t}, \quad j = 1,2 \tag{8}$$

Equilibrium in the money market given by:

$$M^j_t = M^j_{1t} + M^j_{2t}, \quad j = 1,2 \tag{9}$$

Equilibrium in the forward foreign exchange market is given by:

$$G^1_t = G^2_t \tag{10}$$

Each household maximizes Eq. (1) subject to Eqs. (3)–(10). Like Lucas (1982) and Bekaert (1996) we assume that there is perfect international risk pooling in equilibrium. Thus in equilibrium consumption of each good equals half of the current endowment. There are four first-order efficiency conditions to each household's maximization problem. They are all well known in the international asset pricing literature but differ from the standard model in the arguments of the marginal utility function. The first two are Fisher equations and relate the intertemporal marginal rate of substitution for each good to its country's nominal interest rate and inflation:[7]

[7]From this point the country of use subscript is suppressed for ease of notation.

$$E_t\left[\beta(1+i_t^j)\frac{\frac{\delta U/\delta C_{t+1}^j}{P_{t+1}^j}}{\frac{\delta U/\delta C_t^j}{P_t^j}}-1\right]=E_t\left[\beta(1+i_t^j)\frac{\frac{(C_{t+1}^j X_{t+1}^j)^{-\gamma}}{P_{t+1}^j}}{\frac{(C_i^j X_t^j)^{-\gamma}}{P_t^j}}-1\right]=0$$

$$j=1,2 \tag{11}$$

where $i_t^j$, $j=1,2$, are the nominal rates of interest for the home and foreign country, respectively.[8] The intuition is the usual one that if the individual decides to decrease $M_{it}^j$ by one unit of currency this would decrease consumption spending by one unit of currency. This money can be saved in the form of discount bonds and used to buy goods in the next period.

The third efficiency condition is the purchasing power parity relationship. It equates the real exchange rate with the marginal rate of substitution between domestic and foreign goods and can be written as:

$$\frac{S_t P_t^2}{P_t^1}=\frac{\delta U/\delta C_t^2}{\delta U/\delta C_t^1}=\frac{(C_t^2 X_t^2)^{-\gamma}}{(C_t^1 X_t^1)^{-\gamma}} \tag{12}$$

where $S_t$ is the nominal spot exchange rate, measured as the home price of foreign currency at time $t$. Note that, if $X_t^1 = X_t^2 = 1$ in Eqs. (11) and (12), the conditions in a standard (no habit) model with addilog preferences hold.[9] The final efficiency condition is covered interest rate parity and is common to most models in international finance:

$$\frac{F_t}{S_t}=\frac{1+i_t^1}{1+i_t^2} \tag{13}$$

We define exogenous consumption and money growth as:

$$\frac{C_{t+1}^j}{C_t^j}=(1+\mu_{t+1}^j), \quad j=1,2 \tag{14}$$

and

$$\frac{M_{t+1}^j}{M_t^j}=(1+\pi_{t+1}^j), \quad j=1,2 \tag{15}$$

respectively. When we calibrate the model in Section 3 we will assume that consumption and money growth rates, $\mu_t^j$ and $\pi_t^j$, follow vector autoregressive processes with innovations to the stochastic processes denoted by $\nu_t^j$ and $u_t^j$,

[8] As usual, the price of bonds is $1/(1+i_t^j)$ $\quad j=1,2$.

[9] Throughout the remainder of the paper, the phrase ‘standard model’ or ‘standard addilog model’ is used to mean addilog preferences, i.e., Eq. (1) with $H_{it}^j$ for $i=1,2$ and $j=1,2$.

respectively. We closely follow Campbell and Cochrane (1999) by assuming that the log of the surplus consumption ratios evolve as follows:

$$x^j_{t+1} = (1-\phi)\bar{x}^j + \phi x^j_t + \lambda(x^j_t)(v^j_{t+1}), \quad j = 1,2 \tag{16}$$

where $\phi < 1$, is the habit persistence parameter and $\bar{x}^j$ is the steady state value for the logarithm of the surplus consumption ratio for good $j$. The function $\lambda(x^j_t)$ describes the sensitivity of the log surplus consumption ratio to endowment innovations. It depends non-linearly on the current log surplus consumption ratio. The form of the sensitivity function $\lambda(x^j_t)$ is:

$$\begin{aligned} \lambda(x^j_t) &= \frac{\sqrt{1-2(x^j_t - \bar{x}^j)}}{\bar{X}^j} - 1 \quad \text{for} \quad x^j_t \leq x^j_{\max} \\ &= 0 \quad \text{for} \quad x^j_t > x^j_{\max} \\ \text{where } x^j_{\max} &= \bar{x}^j + \frac{1-(\bar{X}^j)^2}{2}, \quad j = 1,2 \end{aligned} \tag{17}$$

$\bar{X}^j$ is the steady state value of the surplus consumption ratio for good $j$ and is defined as:

$$\bar{X}^j = \sigma_{\nu^j}\sqrt{\frac{\gamma}{1-\phi}} \tag{18}$$

We define $\sigma_{\nu^j}$ as the standard deviation of the innovation to the consumption of the $j$th good. The local curvature of the utility function with respect to good $j$ at the steady state is:

$$\frac{\gamma}{\bar{X}^j} = -C^j_{it}\frac{\delta^2 U/\delta(C^j_{it})^2}{\delta U/\delta C^j_{it}} = \frac{\sqrt{\gamma(1-\phi)}}{\sigma_{\nu^j}} \quad i,j = 1,2 \tag{19}$$

Note that this expression is positively related to the coefficient of relative risk aversion for the one good case.[10]

The main difference between our specification and that of Campbell and Cochrane (1999) is that we have two goods in the model. We assume that there is a separate habit in each good.[11] There are three advantages to specifying the habit along the lines of Eqs. (16)–(18). Two of them are sensible technical features. Firstly, the habit is predetermined at the steady state. This means that it takes time

[10] Defining risk aversion in a multi-good model is not trivial (see Engel, 1992; and Moore, 1997). An intertemporal model has as many goods as time periods. In addition, our model has two goods in each time period. We evade this problem by only considering its value at the steady state.

[11] An alternative strategy would be to specify a single habit in a basket of home and foreign goods. We explored this possibility but were unable to develop the Campbell–Cochrane property of constant real interest rates. The main reason for this goes back to Lucas (1982) who pointed out that there is no straightforward definition of the real interest rate in a multi-good context.

for the consumption externality to affect an individual agents habit. The second advantage avoids a possible difficulty with the first. If the habit were always predetermined, a sufficiently low realization of consumption would mean that habit exceeded current consumption. The arguments of the utility function (1) would become negative. Our habit specification prevents this by ensuring that the habit moves non-negatively with consumption everywhere. These two features are illustrated in detail in Campbell and Cochrane (1999). The final and most important advantage of our specification is that the real interest rate is constant. This point is illustrated in the next sub-section.

### 2.3. A benchmark example

Under some simplifying assumptions on the forcing processes, we can derive expressions for the forward discount, spot return and expected forward speculative profit. This will provide some intuition for our simulated results. We assume that consumption growth is a white noise process with a non-zero mean as in Campbell and Cochrane (1999) and money growth follows a simple AR(1) process as in Christiano (1991):

$$\mu^j_{t+1} = \bar{\mu}^j + \nu^j_{t+1}, \quad \nu^j_{t+1} \sim N(0,\sigma^2_{\upsilon^j}), \quad j = 1,2 \tag{20}$$

and

$$\pi^j_{t+1} = (1-\rho)\bar{\pi}^j + \rho\pi^j_t, u^j_{t+1} u^j_{t+1} \sim N(0,\sigma^2_{u^j}), \quad j = 1,2 \tag{21}$$

respectively. The unconditional means of consumption and money growth in country $j$ are defined as $\bar{\mu}^j$ and $\bar{\pi}^j$, respectively. The variances of shocks to consumption and money growth in country $j$ are defined as $\sigma^2_{\nu^j}$ and $\sigma^2_{u^j}$, respectively. We make the following simplifying assumptions:[12] (i) the unconditional mean of consumption growth may differ from the unconditional mean of money growth but the parameters are the same across countries; (ii) the variance of shocks to consumption growth may differ from the variance of shocks to money growth but the parameters are the same across countries; (iii) the covariances between all shocks are zero;[13] (iv) the first-order autocorrelation coefficient for money growth, $\rho$, is the same in both countries.

The assumption on $\sigma^2_{\nu^j}$ implies that the steady state surplus consumption ratio is the same for both goods: $\bar{x}^1 = \bar{x}^2 = \bar{x}$. In what follows we drop the $j$ superscripts on all steady state parameters.

[12] In principle all of the assumptions are relaxed for the calibrations of Section 3.

[13] Engel (1992) argues that the empirical covariance between real and nominal shocks is low or zero.

As shown in Campbell and Cochrane (1999) the real interest rate, $r^j$, for country $j$ is constant and is given by:[14]

$$\ln(1+r^j) \approx r^j = -\ln E_t\left[\beta \frac{\delta U/\delta C^j_{t+1}}{\delta U/\delta C^j_t}\right]$$
$$= -\ln(\beta) + \gamma\bar{\mu} + \frac{1}{2}\gamma(1-\phi) \tag{22}$$

Thus both domestic and foreign real interest rates are constant and equal under the specific assumptions of the last paragraph. The reason for this central feature of the habit specification is as follows. If the surplus consumption ratio is low, the marginal utility of consumption is high. Consequently, households wish to borrow pushing up the real interest rate in the face of exogenous resource constraints, i.e., the intertemporal substitution effect. In contrast, when the surplus consumption ratio is low, uncertainty is high; consumers wish to increase precautionary saving and this pushes interest rates down. These two forces offset each other.

Similarly we can derive an expression for domestic and foreign nominal interest rates. Using Eq. (11) the nominal interest rate for country $j$ is given by:[15]

$$i^j_t = r^j + [(1-\rho)\pi + \rho\pi^j_t - \mu^j] - \frac{1}{2}(\sigma^2_u + (1-2\gamma(1+\lambda(x^j_t)))\sigma^2_\nu),$$
$$j = 1,2 \tag{23}$$

The first term is just the constant real interest rate that we have already presented in Eq. (22). The second term is the expected rate of monetary growth minus the steady state rate of real consumption growth. This term can be interpreted as the expected rate of inflation. The final term is the risk premium. Note that if $\lambda(x^j_t) = 0$, this simplifies to the risk premium in the standard model. The habit specification adds an additional risk premium that has the following property. When surplus consumption is low, uncertainty (in the form of $\lambda(x^j_t)$) is high and the risk premium in the nominal interest rate is higher. In short, the lower the surplus consumption ratio, the more consumers have to be compensated for taking on inflation risk.

Taking logarithms of the covered interest parity Eq. (13) and using Eq. (23), it is easy to obtain the following expression for the forward discount:

$$f_t - s_t = i^1_t - i^2_t = \left[\rho(\pi^1_t - \pi^2_t)\right] + \left[\gamma\sigma^2_\nu(\lambda(x^1_t) - \lambda(x^2_t))\right]$$

[14] The proofs of expressions used throughout the paper are derived in a technical appendix that is available from www.may.ie/academic/economics/pdf/N910799a.pdf.

[15] For proof, see Section B, Eq. (B2) in the technical appendix.

or equivalently

$$f_t - s_t = \left[\rho(\pi_t^1 - \pi_t^2)\right] - \left[\gamma(1-\phi)\frac{\bar{X}}{\gamma}(x_t^1 - x_t^2)\right] \tag{24}$$

The second expression for the forward discount in Eq. (24) is obtained from a linearization of $\lambda(x_t^j)$ in Eq. (17).[16] In the case of standard addilog preferences, the forward discount is simply the term in the first set of square brackets in both expressions in Eq. (24), i.e., $\rho(\pi_t^1 - \pi_t^2)$. This is the home and foreign expected money growth differential. Habits add real influences in the form of the ratio between the home and foreign expected IMRS. When the home country has a lower surplus consumption ratio than the foreign country (i.e., uncertainty is higher in the home country), the home currency is at a deeper forward discount.

Next we derive an explicit expression for spot return. Substituting the cash-in-advance constraints from Eq. (3) into Eq. (12) gives:

$$S_t = \left(\frac{X_t^1}{X_t^2}\right)^{\gamma}\left(\frac{C_t^1}{C_t^2}\right)^{\gamma-1}\frac{M_t^1}{M_t^2} \tag{25}$$

Thus spot return is given by:

$$\frac{S_{t+1}}{S_t} = \left(\frac{C_{t+1}^1}{C_t^1}\right)^{\gamma-1}\left(\frac{C_{t+1}^2}{C_t^2}\right)^{1-\gamma}\left(\frac{M_{t+1}^1}{M_t^1}\right)\left(\frac{M_{t+1}^2}{M_t^2}\right)^{-1}\left(\frac{X_{t+1}^1}{X_t^1}\right)^{\gamma}\left(\frac{X_{t+1}^2}{X_t^2}\right)^{-\gamma} \tag{26}$$

Using Eqs. (14)–(21) in Eq. (26) we derive an expression for the logarithm of the spot return:

$$\begin{aligned} s_{t+1} - 2_t = {} & \rho(\pi_t^1 - \pi_t^2) + (u_{t+1}^1 - u_{t+1}^2) - \gamma(1-\phi)(x_t^1 - x_t^2) \\ & - ((1 - \gamma(1 + \lambda(x_t^1))\nu_{t+1}^1 - (1 - \gamma(1 + \lambda(x_t^2))\nu_{t+1}^2) \end{aligned}$$

or equivalently

$$\begin{aligned} s_{t+1} - 2_t = {} & [\rho(\pi_t^1 - \pi_t^2) + (u_{t+1}^1 - u_{t+1}^2) + (1-\gamma)(\nu_{t+1}^2)] \\ & + [\gamma(\Delta x_{t+1}^1 - \Delta x_{t+1}^2)] \end{aligned} \tag{27}$$

The second expression in Eq. (27) is obtained from the first by noting that Eq. (16) can be reparameterized as:

$$\Delta x_{t+1}^j = (1-\phi)(\bar{x} - x_t^j) + \lambda(x_t^j)(\nu_{t+1}^j), \quad j = 1,2 \tag{28}$$

The contribution of habits to spot return is straightforward. Under standard addilog preferences the spot return is the first square-bracketed term in the second

[16] For proof, see Section A, Eq. (A6) in the technical appendix.

expression in Eq. (27). It consists of the difference between home and foreign money growth as well as a term that is proportional to the difference between home and foreign real innovations. The remainder of the expression arises solely because of the habit specification. It introduces differences in the changes in the growth of home and foreign surplus consumption ratios.

It is clear that one of the main sources of variation in spot return is the ratio of the IMRS of home and foreign goods. The volatility of this is typically very low, leading to the conclusion by Flood and Rose (1999) that macroeconomics is incapable of explaining exchange rate volatility. Our habit specification makes the IMRS dependent on the change in the surplus consumption ratios, which are volatile by construction. Consequently, the spot return is much more variable under our habit specification than under the standard addilog case.

It is useful at this stage to introduce the *expected* spot return conditional on time $t$. Taking expectations of Eq. (27) at time $t$ gives:

$$E_t(s_{t+1} - s_t) = [\rho(\pi_t^1 - \pi_t^2)] - [\gamma(1-\phi)(x_t^1 - x_t^2)] \tag{29}$$

Like the forward discount in Eq. (24), the expected spot return is influenced (in the same direction) by the home and foreign country's relative uncertainty, as measured by the difference between the home and foreign log surplus consumption ratios. The expected forward profit is the expected return from taking a short position in forward foreign exchange, i.e., $f_t - E_t s_{t+1}$. This can be derived as the difference between the forward discount and the expected spot return: $(f_t - s_t) - E_t(s_{t+1} - s_t)$, i.e., Eq. (24) minus Eq. (29).

$$(f_t - E_t s_{t+1}) = \gamma(1-\phi)\left(1 - \frac{\bar{X}}{\gamma}\right)(x_t^1 - x_t^2) \tag{30}$$

It is well known (see, for example, Engel, 1999) that the expected forward profit consists of two elements: a risk premium and a non-convexity term. However under the specific assumptions that we have made about the forcing processes in this example, the non-convexity term is zero. In addition, the risk premium itself is zero under standard addilog preferences without habits.[17] Consequently, Eq. (30) is exclusively a risk premium that is attributable to habits. It is time varying and its sign has two determinants. As in the cases of the forward discount and expected spot return, it depends on relative uncertainty in the home and foreign countries, as measured by the difference between the home and foreign surplus consumption ratios. However, it also depends on the local curvature of the utility function at the steady state function $\gamma/\bar{X}$ and in particular whether it is greater or less than unity.

It is straightforward to provide an interpretation of the time-varying risk premium of Eq. (30) in terms of the properties of the pricing kernels that price

[17] For proof, see Section C in the technical appendix.

nominal assets in each country. Backus et al. (2001)[18] show that for log-normal pricing kernels such as ours, the risk premium is proportional to the difference between the conditional variances of the log of the home and foreign pricing kernels. It is clear that equation can be reparamaterized in those terms. It also provides a new understanding of the surplus consumption ratio in this model: it is proportional to the conditional variance of the pricing kernel.

We define $\sigma_x^2$ as the unconditional variance of $x_t^j$, $j = 1,2$. Given the definition of $\bar{X}$ in Eq. (18) it is easy to show that the unconditional variance of the forward discount, spot return and expected forward profit using Eqs. (24), (27) and (30) are:[19]

$$\mathrm{var}(f_t - s_t) = 2\left(\frac{\rho^2\sigma_u^2}{1-\rho^2} + \gamma(1-\phi)\sigma_v^2\sigma_x^2\right) \tag{31}$$

$$\mathrm{var}(s_{t+1} - s_t) = 2\left(\frac{\sigma_u^2}{1-\rho^2} + (1-\phi)^2\gamma^2\sigma_x^2 + \sigma_v^2\left(1-\frac{\gamma}{\bar{X}}\right)^2\right) \tag{32}$$

and

$$\mathrm{var}(f_t - E_t s_{t+1}) = 2(1+\phi)^2\gamma^2\left(1-\frac{\bar{X}}{\gamma}\right)^2\sigma_x^2 \tag{33}$$

respectively. We now summarize the impact of the habit specification on relative and absolute volatilities in the following proposition:[20]

**Proposition 1.** (Volatility)

*Under the assumptions of (i) addilog utility for the standard case, (ii) the forcing processes in Eqs. (20) and (21), (iii) that the mean surplus consumption ratio $\bar{X} < 1$ and (iv) the local curvature of the utility function at the steady state $\gamma/\bar{X} > 2$, Campbell and Cochrane (1999) habits:*[21]

*(a) increase the absolute volatilities of (i) spot return, (ii) the forward discount and (iii) expected forward profit*

*(b) increase the volatility of spot return by (i) more than the increase in the volatility of the forward discount and (ii) more than the increase in the volatility of expected forward profit*

[18]See their Eq. (14).

[19]For proof see Section D in the technical appendix.

[20]For proof see Section D in the technical appendix.

[21]Assumptions (iii) and (iv) are not very restrictive, as the mean surplus consumption ratio is typically a small positive fraction. For example, in Campbell and Cochrane (1999), it is approximately 0.08.

(c) *increase the volatility of expected forward profit by more than the increase in the volatility of the forward discount.*

Proposition 1 shows that the habit specification moves the three volatilities in the desired absolute and relative directions. Whether this is enough to match the data depends on the relative importance of real and monetary shocks. The less volatile the monetary shocks, the greater the importance of habits.

The next proposition relates to the effect of the habit specification on the persistence of the spot return and the forward discount. We measure persistence for a time series $y_t$ as $\mathrm{cov}(y_t y_{t-1})/\mathrm{var}(y_t)$, its first-order autocorrelation coefficient. The first-order autocorrelation coefficient of the forward discount and the spot return are:[22]

$$\frac{\rho \dfrac{\rho^2 \sigma_u^2}{1-\rho^2} + \phi\gamma(1-\phi)\sigma_\nu^2\sigma_x^2}{\left(\dfrac{\rho^2\sigma_u^2}{1-\rho^2} + \gamma(1-\phi)\sigma_\nu^2\sigma_x^2\right)} \tag{34}$$

and

$$\frac{\rho \dfrac{2\sigma_u^2}{1-\rho^2} + \left(\sigma_\nu^2\gamma(1-\phi)(1-\gamma)\left(\dfrac{1}{\bar{X}} - 1\right) - \gamma^2(1-\phi)^2\sigma_x^2\right)}{\dfrac{2\sigma_u^2}{1-\rho^2} + 2\left(\gamma^2(1-\phi)^2\sigma_x^2 + \sigma_\nu^2\left(1 - \dfrac{\gamma}{\bar{X}}\right)^2\right)} \tag{35}$$

respectively. We are now ready to state Proposition 2:[23]

**Proposition 2.** (Persistence)

*Under the assumptions of (i) addilog utility for the standard case, (ii) the forcing processes in Eqs. (20) and (21), (iii)*$\phi > \rho$ *and (iv) terms in* $\sigma_\nu^2$ *are small in relation to terms in both* $\sigma_{\hat{u}}^2$ *and* $\sigma_x^2$*, Campbell and Cochrane (1999) habits*:

(*a*) *increase the persistence of the forward discount*
(*b*) *decrease the persistence of the spot return.*

Proposition 2 can be explained easily by reference to Eqs. (24) and (27). In the standard addilog case, the only persistent element determining both spot return and the forward premium is expected money growth differentials. Consequently, they both share the first order autocorrelation coefficient of money growth, $\rho$. Under the habit specification, the forward discount in Eq. (24) also depends on the log

[22] For proof see Section E in the technical appendix.
[23] For proof see Section E in the technical appendix.

surplus consumption ratios. This is because the expected IMRS depends on the *level* of the surplus consumption ratios. Since their first-order autocorrelation coefficient $\phi$ is close to unity, the persistence of the forward discount rises. By contrast, under the habit specification, spot return in Eq. (27) depends on the IMRS itself. The IMRS is a function of the *change* in the log surplus consumption ratio, the first-order autocorrelation coefficient of which is negative. Consequently, the persistence of the spot return falls.

Backus et al. (1993) and Bekaert (1996) argue that the forward discount 'bias' can be summarized by the inability of the standard model to explain the high volatility of expected forward speculative profit. The introduction of our habit specification increases this volatility but is it enough? Backus et al. (1995) remind us that the slope coefficient, $b_1$, from a regression of the spot return on the forward discount can be written as

$$b_1 = \frac{\mathrm{cov}(E_t s_{t+1} - s_t, f_1 - E_t s_{t+1}) + \mathrm{var}(E_t s_{t+1} - s_t)}{\mathrm{var}(f_t - s_t)} \tag{36}$$

Many studies report negative estimates of this coefficient and thus the 'bias puzzle'. For this to happen the covariance term must be (i) negative and (ii) larger in absolute value than the variance term in the numerator. Point (i) is Fama's first necessary condition and point (ii) is equivalent to Fama's second condition: that the variance of expected forward profit exceeds the variance of the expected spot return. Most writers concentrate on the first of Fama's necessary conditions (see, for example, Alvarez et al., 1999). In Proposition 3 below, we show that the habit specification, unlike the standard addilog model, indeed delivers this result. The reason for this is straightforward. In the standard model, the expected forward profit is, in fact, a constant and therefore is uncorrelated with expected spot returns. With the habit specification, both expected spot return, Eq. (29) as well as the expected forward profit, Eq. (30), depend on $x_t^1 - x_t^2$, the difference between the home and foreign log surplus consumption ratios. It has already been suggested that this measures relative uncertainty in the home and foreign countries. When uncertainty is higher in the home country, a depreciation is expected. The opposite happens to forward profit. The reason for this is embedded in Proposition 1 (b) (i): spot returns are more sensitive to habits than the forward discount. Consequently, $f_t - E_t(s_{t+1})$ is dominated by movements in $E_t(s_{t+1})$, which enters the expected forward profit with the opposite sign to which it appears in $E_t(s_{t+1}) - s_t$.

However, we also show that the second of Fama's conditions is violated by the habit specification. This violation is so significant that it completely overshadows the success in obtaining the negative covariance to the extent that the coefficient slope $b_1$ becomes *larger* than unity. We are now ready to state Proposition 3:[24]

[24] For proof see Section F in the technical appendix.

**Proposition 3.** (unbiasedness)

*Under the assumptions of (i) addilog utility for the standard case, (ii) the forcing processes in Eqs. (20) and (21), (iii) the local curvature of the utility function at the steady state* $\gamma/\bar{X} > 1$,[25] *Campbell and Cochrane (1999) habits imply that*:

(a) *the unconditional covariance between expected spot return and expected forward profit is negative*
(b) *the unconditional variance of expected spot return is greater than the variance of expected forward profit*
(c) *the slope coefficient from a regression of the spot return on the forward discount exceeds unity.*

The reason why this perverse result occurs is related to our success in increasing the volatility of the spot return: too much of the increase is accounted for by the volatility of the *expected* spot return. We require the volatility of spot return to exceed the volatility of expected forward profit but this must, in turn, exceed the volatility of expected spot return.

All of the results in this sub-section depend on the specific error processes in Eqs. (20) and (21). In reality, home and foreign money and consumption growth do not follow AR(1) processes with scalar variance covariance matrices. In the next section, we calibrate the model to processes estimated from the data and re-evaluate the results.

## 3. Empirical and model evidence

### *3.1. Some stylized facts about the forward exchange market*

Hodrick (1987), Backus et al. (1993) and Bekaert (1996) present the stylized facts for a number of bilateral exchange rates. In order to remind the reader of these facts we present some quarterly statistics over the period 1973:1–2000:4 for bilateral exchange rates between the US dollar, sterling and the yen in Table 1. In Section 1 we discussed three sets of well-known facts about the forward exchange rate market. The first is that the standard deviation of the forward discount is smaller than that of expected forward speculative profit[26] and is an order of magnitude less than that of the spot return. The second is that while the spot return is not very persistent, the AR(1) coefficient of the forward discount is high. The

[25] Note that the condition on the local curvature of the utility function is less stringent than assumption (iv) of Proposition 1.

[26] The expected forward speculative profit is calculated as the fitted value from regressing realized profits on the lagged forward premium.

Table 1
Properties of forward and spot exchange rates

| | US–UK | US–Japan | UK–Japan |
|---|---|---|---|
| *Volatility* | | | |
| Spot return (%) | 5.44 | 6.11 | 6.12 |
| Expected forward profit (%) | 1.75 | 2.85 | 2.58 |
| Forward discount (%) | 0.72 | 0.71 | 0.47 |
| *Persistence* | | | |
| Spot return | 0.09 | 0.13 | 0.19 |
| | (0.09) | (0.09) | (0.09) |
| Forward discount | 0.74 | 0.80 | 0.65 |
| | (0.08) | (0.08) | (0.10) |
| *Unbiasedness* | | | |
| Point estimate of $b_1$ | −1.43 | −3.00 | −4.42 |
| | (0.89) | (0.76) | (0.93) |

The data are quarterly and obtained from Datastream.
The sample period for all spot exchange rates is 1973:1–2000:4.
The sample period for the US–UK forward exchange rates is 1976:2–2000:4.
The sample period for the Japan–US forward exchange rates is 1978:3–2000:4.
Volatility is measured by the standard deviation.
Persistence is measure by the first-order autocorrelation coefficient.
The expected forward profit is calculated as the fitted value from regressing realized profits on the lagged forward premium. The coefficient $b_1$ is estimated from regressing the spot return on the lagged forward premium.
HAC standard errors are in parentheses.

third fact is that the coefficient on the forward discount from a regression for predicting spot return, $b_1$, is usually the wrong sign (negative).

## *3.2. Calibration and results from baseline parameterization*

There is no closed form solution for the first two non-linear stochastic rational expectations efficiency conditions, i.e., Eq. (11). Thus we find an approximate solution using a linear-quadratic method of undetermined coefficients, see Christiano (1991).[27] This yields optimal linearised rules for domestic and foreign bond prices (and thus nominal interest rates) as functions of the consumption and money growth shocks and the log of surplus consumption ratio. Spot and forward exchange rates can easily be calculated using Eqs. (3), (12) and (13).

We calibrate the model discussed in Section 2 and compare the moments generated from the model with those typically found in quarterly data. Two approached are adopted. First, we assume very simple AR(1) time series processes for the exogenous consumption and money growth shocks, as in Section 2.3

[27]The linear-quadratic solution method is explained in Section G of the technical appendix.

above. In this case the baseline parameters are taken straight from existing literature. This allows us to show that the predictions from the simulated linear-quadratic approximation are indeed what are theoretically derived in Section 2.3 above. Second, we use data on consumption and money growth for the US, UK and Japan and estimate bivariate vector autoregressions. We calibrate the model using a VAR from each bilateral relationship and compare the moments generated from the model with those found with each bilateral exchange rate.

We present the baseline parameterization in Table 2. The discount rate, $\beta$, is assumed to be $(1.03)^{-0.25}$ which is based on an annual real rate of interest of 3%; a value commonly used in the literature (see for example Christiano, 1991). This parameter remains constant in the various experiments we simulate. The elasticity of intertemporal substitution, $1/\gamma$, and the AR(1) coefficient of the log of the surplus consumption ratio, $\phi$, have major effects in the habit persistence model. In the baseline parameterization we set the elasticity of intertemporal substitution equal to 5 ($\gamma = 0.2$). The value of $\gamma$ is low compared to Campbell and Cochrane (1999): their lowest value for this parameter is 0.7. However, even with $\underline{\gamma} = 0.2$, it is worth remembering that the local curvature of the utility function is $\gamma/\bar{X} = 13.8$. In Campbell and Cochrane (1999) this is 47.6. The AR(1) coefficient of log surplus consumption equals 0.97, which is used in Campbell and Cochrane (1999). Later in this section we perform sensitivity analysis and examine how the results change when the elasticity of intertemporal substitution varies between 10 and 0.5 and when we change the AR(1) coefficient of the log of the surplus consumption ratio from 0.9 to 0.99.

The parameters of the exogenous endowment and money growth rate AR(1) processes are also taken from the literature. Campbell and Cochrane (1999) use US real consumption expenditure on non-durables and services to proxy for endowments. They assume that consumption growth is a white noise process with a non-zero mean estimated to be 0.44% per quarter. They estimate that the standard deviation of shocks to consumption growth is 0.56%. Christiano (1991) estimates the mean of US base money growth to be 1.6% per quarter. He also estimated the first-order autocorrelation coefficient to be 0.32 and the standard deviation of the shock to money growth to be 0.38%. We simulated the standard

Table 2
Baseline parameterization

| | | |
|---|---|---|
| Discount factor $\beta$ | | 0.9926 |
| Curvature of the utility function $\gamma$ | | 0.20 |
| Persistence of the log surplus-consumption ratio $\phi$ | | 0.97 |
| | Consumption growth | Money growth |
| Unconditional mean % | 0.44 | 1.60 |
| AR(1) coefficient | 0.00 | 0.32 |
| Standard deviation of shock % | 0.56 | 0.38 |

and habit models 1000 times generating 110 observations for each series. The results from the baseline parameterizations for the moments of interest are presented in Table 3. We present the mean of the simulated moment and its standard error (in parentheses).

Compare the results in Table 3 to Proposition 1. The latter claims that the habit specification increases the three volatilities, in absolute terms. A comparison of the columns 'habit preferences' and 'standard preferences' shows that this is indeed the case. In particular, note that the simulated volatilities have very low sampling standard errors. Proposition 1 also claims that habits increase spot return volatility by more than the other two and increases expected forward profit volatility by more than the volatility of the forward discount. This clearly occurs and the ranking of the three volatilities in the habit model is consistent with the stylised facts in Table 1. Proposition 2 claims that Campbell and Cochrane (1999) habits raise the persistence of the forward discount and lower that of the spot return.

Table 3
Moments in the theoretical economy using baseline parameterization

| | Standard preferences | Habit preferences |
|---|---|---|
| *Volatility* | | |
| Spot return (%) | 0.84 | 9.85 |
| | (0.002) | (0.216) |
| Expected forward profit (%) | 0.07 | 1.50 |
| | (0.002) | (0.041) |
| Forward discount (%) | 0.18 | 0.82 |
| | (0.000) | (0.024) |
| *Persistence* | | |
| Spot return | 0.13 | −0.04 |
| | (0.003) | (0.005) |
| Forward discount | 0.30 | 0.77 |
| | (0.003) | (0.006) |
| *Unbiasedness* | | |
| Covariance | 0.00000 | −0.00048 |
| | (0.000) | (0.00004) |
| Negative covariances (%) | 48.50 | 93.50 |
| Mean of $b_1$ | 0.94 | 3.36 |
| | (0.002) | (0.062) |

The results presented are the mean of 1000 replications of 110 observations using the baseline parameterization in Table 2.
The standard error of the mean is reported in parenthesis.
Volatility is measured by the standard deviation.
Persistence is measure by the first-order autocorrelation coefficient.
The expected forward profit is calculated as the fitted value from regressing realized profits on the lagged forward premium. The coefficient $b_1$ is estimated from regressing the spot return on the lagged forward premium.

From Table 3, the results from the simulations demonstrate that this occurs. A comparison with the stylised facts in Table 1 shows that both the simulated AR(1) coefficients are within two standard errors of the estimates. Finally, Proposition 3 argues that in contrast to the standard case where the covariance between forward profit and expected spot return is zero, the habit specification delivers a negative covariance. The row headed 'Covariance' in Table 3 shows the average simulated value of this covariance and its standard error in the sample. The row headed 'Negative covariances' in Table 3 shows the percentage that is negative. In the standard case, about one-half of the covariances are negative. By contrast, the habit preferences deliver negative covariances more than 90% of the time. Finally, as predicted by Proposition 3, Campbell and Cochrane (1999) preferences give rise to a perverse result in the forward rate regression. The standard model suggests that this slope is unity in contrast to the negative values that we see in the data (see Table 1). Because of the fact that the volatility of *expected* spot returns is so high in the habit model, the average simulated value of this slope is greater than unity as demonstrated by Proposition 3.

It is worthwhile showing the effect of varying $\gamma$, the inverse of the intertemporal elasticity of substitution on the exchange rate volatilities. This is illustrated in Fig. 1 where the elasticity varies from 10 ($\gamma=0.1$) to 0.5 ($\gamma=2$). As the elasticity of intertemporal substitution falls, the lack of intertemporal substitution opportunities means that it is more difficult to diffuse the effect of shocks over different time periods. Consequently, all exchange rate volatilities rise.

Consider the effect of varying the habit persistence parameter. This is illustrated in Fig. 2. The AR(1) coefficient of the log of the surplus consumption ratio, $\phi$, is

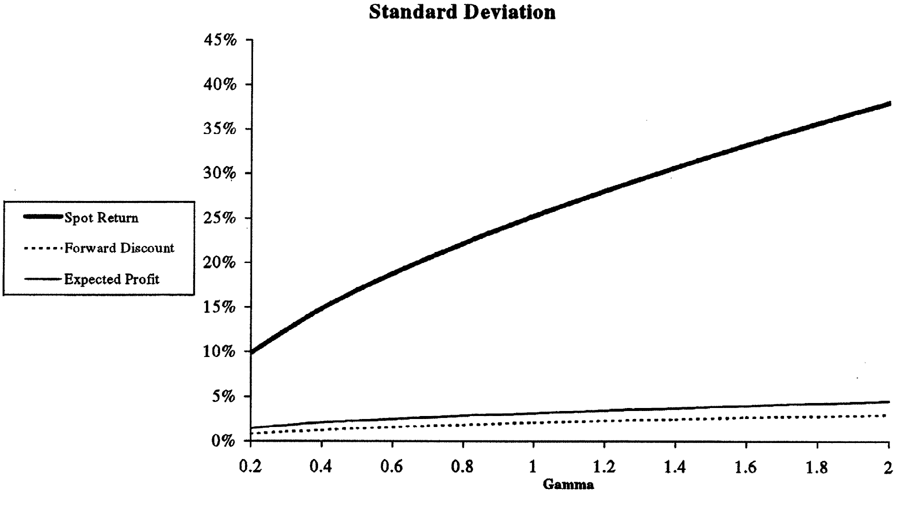


Fig. 1. The effects of changing $\gamma$ on the volatilities of exchange rates.

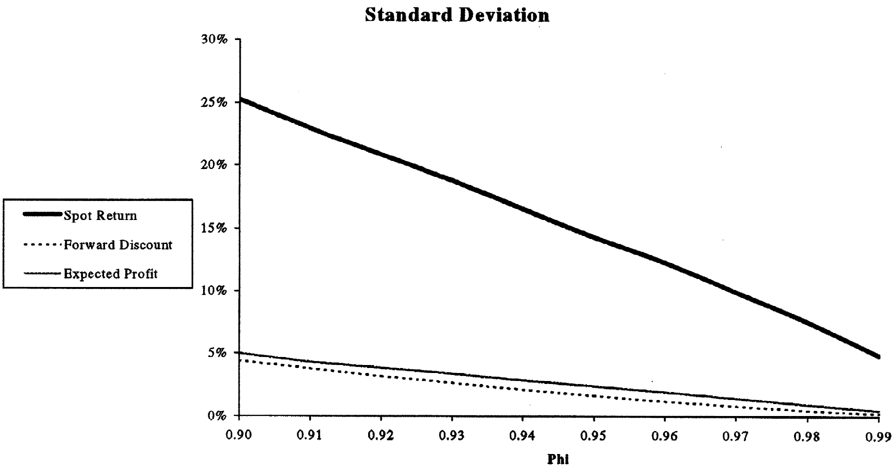


Fig. 2. The effects of changing $\phi$ on the volatilities of exchange rates.

allowed to vary from 0.9 to 0.99. As $\phi$ increases, the variability of the *change* in the surplus consumption ratio declines. Consequently, the variability of the intertemporal marginal rate of substitution, and therefore the three exchange rate volatilities decline as $\phi$ approaches unity.[28] Another way of thinking about it is that the steady state surplus consumption ratio, $\bar{X}$, rises as $\phi$ rises. As the steady state surplus consumption ratio rises, the local curvature of the utility function, $\gamma/\bar{X}$, falls and therefore exchange rate volatilities decline.

### *3.3. Calibration and results from alternative parameterizations*

In this section we use data on consumption and money growth for the US, UK and Japan to estimate parameters from bivariate vector autoregressions. We use quarterly seasonally adjusted data on real consumption expenditure on non-durables and services and the nominal monetary base. The data on consumption and money are obtained from the Federal Reserve Bank of St. Louis for the US and from Datastream for the UK and Japan. The sample period is 1973:1–2000:1. The unconditional means of consumption and money growth for the US, UK and Japan are presented in Table 4. We use these in further simulations of both the standard and habit models. We estimated three versions of the following vector autoregression:

$$W_{t+1} = \Phi_0 + \Phi_1 W_1 + U_{t+1}, \quad U_{t+1} \sim MN(0,\Sigma), \tag{37}$$

[28] At $\phi = 1$, the steady state surplus consumption ratio is not defined. In effect, the habit model converges on the standard model.

Table 4
Unconditional means of consumption and money growth rates

| Growth | US | UK | Japan |
|---|---|---|---|
| Consumption (%) | 0.75 | 0.58 | 0.69 |
| Money (%) | 1.89 | 1.70 | 2.02 |

Quarterly data on real consumption expenditure on non-durables and services and the money base are obtained from the Federal Reserve Bank of St. Louis for the US and from Datastream for the UK and Japan.
The sample period is 1973:1–2000:1.

where $W_t' = (\mu_t^1, \mu_t^2, \pi_t^1, \pi_t^2)$ and $U_t' = (\nu_t^1, \nu_t^2, u_t^1, u_t^2)$. The three versions used data for US–UK, US–Japan and UK–Japan consumption and money growth with one lag in each of the variables. We performed likelihood ratio tests for various restrictions on Eq. (37) for each country pair and present $p$-values in Table 5. We test the following three hypotheses on the coefficients in $\Phi_1$: whether real growth variables affected nominal variables (and vice versa), whether domestic variables affected foreign variables (and vice versa) and whether $\Phi_1$ is diagonal. In the top half of Table 5 we allow $\Sigma$ to be unrestricted and in the bottom half the covariance matrix was restricted to conform to the restrictions on $\Phi_1$.

The UK–US data do not reject the restriction that the slope coefficients on money growth in the consumption growth equations are zero (and vice versa) and that the money growth shocks are uncorrelated with consumption growth shocks. By contrast, the US–Japanese data do not reject the restriction that the slope coefficients on foreign growth variables in the domestic growth equations are zero

Table 5
Probability values for likelihood ratio tests on a VAR(1) model of consumption and money growth

| | US–UK | US–Japan | UK–Japan |
|---|---|---|---|
| *Unrestricted covariance matrix* | | | |
| VAR(1) with no real-nominal coefficients | 0.335 | 0.750 | 0.030 |
| VAR(1) with no cross-country coefficients | 0.001 | 0.796 | 0.008 |
| Simple AR(1) processes | 0.007 | 0.858 | 0.016 |
| *Restricted covariance matrix* | | | |
| VAR(1) with no real-nominal coefficients | 0.140 | 0.005 | 0.000 |
| VAR(1) with no cross-country coefficients | 0.000 | 0.742 | 0.005 |
| Simple AR(1) processes | 0.000 | 0.000 | 0.000 |

Quarterly data on real consumption expenditure on non-durables and services and the money base are obtained from the Federal Reserve Bank of St. Louis for the US and from Datastream for the UK and Japan.
The sample period is 1973:1–2000:1.
VAR(1) with no real-nominal coefficients is a vector autoregression where money growth does not effect consumption growth and vice versa.
VAR(1) with no cross-country coefficients is a vector autoregression where domestic growth variables do not effect foreign growth variables and vice versa.

(and vice versa) and that foreign growth shocks are uncorrelated with domestic growth shocks. Finally, we were unable to find any valid restrictions on the UK–Japanese vector autoregression.

The remaining parameters in the simulations are given the same value as in the baseline parameterisation. We use the estimated $\Phi_1$ and $\Sigma$ based on the restrictions discussed in the above to simulate three versions of the model.[29] The simulated results are presented in Table 6. The results are very similar to those presented for the baseline parameterization even though we use three different sets of parameters for the exogenous processes. We note that the volatility of the forward discount

Table 6
Moments in the theoretical economies

| | Standard preferences | | | Habit preferences | | |
|---|---|---|---|---|---|---|
| | US–UK | US–Japan | UK–Japan | US–UK | US–Japan | UK–Japan |
| *Volatility* | | | | | | |
| Spot return (%) | 1.42 | 3.16 | 2.93 | 9.49 | 9.87 | 9.56 |
| | (0.004) | (0.007) | (0.006) | (0.200) | (0.205) | (0.202) |
| Expected forward profit (%) | 0.10 | 0.24 | 0.21 | 1.09 | 1.23 | 1.21 |
| | (0.002) | (0.006) | (0.005) | (0.039) | (0.041) | (0.041) |
| Forward discount (%) | 0.64 | 0.46 | 0.48 | 1.04 | 1.00 | 1.00 |
| | (0.002) | (0.000) | (0.000) | (0.019) | (0.022) | (0.021) |
| *Persistence* | | | | | | |
| Spot return | 0.36 | 0.11 | 0.08 | −0.01 | −0.02 | −0.04 |
| | (0.003) | (0.003) | (0.003) | (0.005) | (0.004) | (0.001) |
| Forward discount | 0.47 | 0.22 | 0.04 | 0.66 | 0.58 | 0.46 |
| | (0.003) | (0.003) | (0.003) | (0.005) | (0.007) | (0.009) |
| *Unbiasedness* | | | | | | |
| Covariance | 0.000 | 0.000 | 0.000 | −0.0004 | −0.0004 | −0.0004 |
| | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| Negative covariances (%) | 42.70 | 53.70 | 54.10 | 83.80 | 88.50 | 87.60 |
| Mean of $b_1$ | 0.96 | 0.93 | 1.00 | 1.79 | 2.11 | 2.07 |
| | (0.006) | (0.021) | (0.018) | (0.032) | (0.036) | (0.035) |

The results presented are the mean of 1000 replications of 110 observations using parameters for the forcing processes described in Section 3.
The standard error of the mean is reported in parenthesis.
Volatility is measured by the standard deviation.
Persistence is measure by the first-order autocorrelation coefficient.
The expected forward profit is calculated as the fitted value from regressing realized profits on the lagged forward premium. The coefficient $b_1$ is estimated from regressing the spot return on the lagged forward premium.

[29] For the sake of brevity the estimated coefficients and covariance matrix are omitted.

increases relative to the baseline results. This is due to the fact that the real interest rate is not constant.

## 4. Conclusion

This paper has proposed a modelling strategy that makes substantial progress towards resolving many of the outstanding puzzles in respect of the forward foreign exchange market. A model that combines habit persistence and a constant real rate of interest in a monetary framework is capable of explaining most of the volatility, persistence and forward bias puzzles. However, Campbell and Cochrane (1999) habits ultimately fail to explain the forward bias puzzle because they make the volatility of expected spot returns too high.

The main consequence of this failure is that the regression slope in the forward market equation is greater than unity rather than negative as is empirically found. It is interesting that Bekaert (1996) reports that he was unable to generate a single simulation that gave rise to a negative estimate of this slope coefficient. Since he was also modelling habit persistence it would be worth revisiting his work to determine if he was also obtaining our perverse result. Backus et al. (2001) have already noted the possibility that otherwise sensible models can be created where this can occur. Interestingly, they find it in a two-currency version of Cox et al. (1985).

Alvarez et al. (1999) claim to have resolved the forward bias anomaly. They introduce trading frictions by relaxing the assumption of complete markets and by allowing the extent of the trading friction to be endogenously determined. Their strategy is as follows: asset markets and goods markets are segmented in the sense that agents must pay a fixed cost to transfer money between the two markets. This segmentation is endogenous because, in equilibrium, some agents chose to pay the fixed cost while some do not. They derive sufficient conditions for *one* of Fama's necessary conditions for the forward premium anomaly viz., that the covariance between expected forward speculative profits and expected spot rate changes be negative. They do not determine the implication of their model for Fama's second necessary condition: that the variance of expected forward speculative profits exceed the variance of expected spot rate changes. This paper shows the importance of this condition. Overall, it is difficult to come to any definite conclusion about their model at this stage because they do not simulate it.

A few research directions are immediately suggested. It would be interesting to apply this approach to *nominally* denominated asset prices in closed economies. There are many issues unresolved in international real business cycle theory, such as the behaviour of real exchange rates. It would be worthwhile to examine whether this model casts any light on them. A production sector could be included in the models discussed in the paper. This is common in most international real business cycle models. Rather than using binding cash-in-advance constraints money could be introduced into the model in a manner that does not impose the counterfactual unit consumption velocity of money.[30] Finally, any future research must focus explicitly on this issue: how can the high *unconditional* volatility of spot returns be explained without excessively high volatility of expected excess spot returns conditional on the forward discount? In other words, high spot return volatility must come from high volatility in spot return surprises.

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