FDTD-Equivalent Neural Network Model for Electromagnetic Simulations

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***Abstract*—This paper introduces a novel three-dimensional electromagnetic (EM) solver, grounded in theory and enhanced by integrating a convolutional neural network (CNN) with the finite-difference time-domain (FDTD) method. The proposed solver is designed for efficient and precise full-wave simulation of large-scale structures. By substituting the traditional operators in the FDTD method with convolutional operators, our approach maintains the accuracy and stability inherent to the FDTD method, while also being ideally suited for parallel computations. Compared to existing models, our proposed CNN-FDTD solver demonstrates improved accuracy, efficiency, and flexibility. Numerical validations confirm its superior stability and computational efficiency.**

I. Introduction

Numerical methods play a crucial role in electromagnetic (EM) simulations, enabling precise modeling and analysis of wave propagation interactions across a range of complex applications, including antenna design, medical imaging techniques, and wireless communications. Among these methods, the finite-difference time-domain (FDTD) method stands out for its simplicity, adaptability to complex media, and exceptional parallel computational efficiency, making it a popular choice in the field.

However, when dealing with electrically large objects, the FDTD method encounters limitations. The requirement for fine meshes leads to prolonged simulation time and substantial computational memory demands. To overcome these obstacles, various strategies have been explored, including hybrid FDTD methods, subgridding techniques, and parallel algorithms [1], [2]. Concurrently, the advent of machine learning (ML) technologies like convolutional neural networks (CNN) and recurrent neural networks (RNN) has spurred interest in merging ML with EM simulation [3], [4]. This integration could elevate EM engineering through advanced frameworks and extensive matrix operations in specialized libraries. However, this approach faces its own set of challenges, particularly in terms of interpretability and generalization, as most ML-EM solutions are often highly specialized.

In response to these challenges, we propose a stable, interpretable, and flexible physics-oriented CNN-FDTD solver, suitable for various differential schemes. By integrating CNN operators into the classic FDTD framework, our proposed CNN-FDTD scheme not only achieves high parallel efficiency in massively parallel computing architectures but also retains the inherent accuracy and stability of the FDTD method.

II. Methodology

A. The FDTD Method

Here, we focus on a rectangular system at a specific time step, denoted as t. In the finite-difference time-domain (FDTD) framework, the electric and magnetic field components are discretized in both spatial and temporal domains. Second-order differential operators are used for approximation. To briefly demonstrate iterative formulas in the FDTD method, we consider E_x and H_y as a representative set of electromagnetic components since the remaining field components follow a similar methodology. The updated formulas are written as

$$\begin{aligned} E_x^{n+1}\left(i+\tfrac{1}{2},j,k\right) &= CA(m)\cdot E_x^{n}\left(i+\tfrac{1}{2},j,k\right) \\ &+CB(m)\cdot\left[\begin{array}{c} \dfrac{H_z^{n+1/2}\left(i+\frac{1}{2},j+\frac{1}{2},k\right) - H_z^{n+1/2}\left(i+\frac{1}{2},j-\frac{1}{2},k\right)}{\Delta y} \\ -\dfrac{H_y^{n+1/2}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right) - H_y^{n-1/2}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}{\Delta z} \end{array}\right] \end{aligned} \tag{1a}$$

$$\begin{aligned} &H_y^{n+1/2}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right) = \\ &CP(m)\cdot H_y^{n-1/2}\left(i+\tfrac{1}{2},j,k+\tfrac{1}{2}\right) \\ &-CQ(m)\cdot\left[\begin{array}{c} \dfrac{E_x^{n}\left(i+\frac{1}{2},j,k+1\right) - E_x^{n}\left(i+\frac{1}{2},j,k\right)}{\Delta z} \\ -\dfrac{E_z^{n}\left(i+1,j,k+\frac{1}{2}\right) - E_z^{n}\left(i,j,k+\frac{1}{2}\right)}{\Delta x} \end{array}\right] \end{aligned} \tag{1b}$$

where $E_{x,y,z}$ and $H_{x,y,z}$ denote electric fields and magnetic fields in the x, y and z direction, respectively. ε is the permittivity, σ is the electric conductivity and μ denotes the permeability of the medium. Besides, $m=(i,j,k+1/2)$, CA, CB, CP, and CQ are defined as

$$\begin{aligned} CA(m) &= \frac{1-\sigma(m)\Delta t/\left(2\varepsilon(m)\right)}{1+\sigma(m)\Delta t/\left(2\varepsilon(m)\right)} \\ CB(m) &= \frac{\Delta t/\varepsilon(m)}{1+\sigma(m)\Delta t/\left(2\varepsilon(m)\right)} \\ CP(m) &= \frac{1-\sigma_m(m)\Delta t/\left(2\mu(m)\right)}{1+\sigma_m(m)\Delta t/\left(2\mu(m)\right)} \\ CQ(m) &= \frac{\Delta t/\mu(m)}{1+\sigma_m(m)\Delta t/\left(2\mu(m)\right)} \end{aligned} \tag{2}$$

B. The CNN-FDTD Framework

Noticing a similarity in the local operations performed on components at each pixel within Yee's grids and those in a convolutional layer, the framework of the CNN is naturally integrated with the iterative process of the FDTD method. Our approach involves a set of convolutional operations consisting of the shift & add operator, the Hadamard product, and the summation operator. These operations are strategically employed to replace differential operators, material-related parameters, and summation elements in the FDTD method, respectively. The implementation is as follows:

- Shift & Add Layer: This convolutional layer is designed to shift input feature maps by a specified number of pixels. With particular parameters, the shifting can be in any direction. Subsequently, each pixel of the shifted map is added to its corresponding pixel in either the original or another shifted map. Considering the consistency of the field components makes this layer the perfect choice to do the spatial iteration in our proposed scheme.
- Hadamard Product Layer: In this layer, the Hadamard operator is employed to perform the element-wise multiplication of two matrices of identical dimensions. By incorporating material-related parameters, we construct the Hadamard product layer with kernels CA, CB, CP, and CQ, effectively embedding material properties into the computational process.
- Summation Layer: Playing a pivotal role in integrating information from different layers, this layer utilizes weighted coefficients to process inputs. Taking the $(1, 1/\Delta y, -1/\Delta x)$ as an example, E_x^t yields E_x^{t+1} through the summation layer, preparing it for the subsequent time step. Unlike pooling layers that reduce the spatial resolution, the summation operator preserves the spatial dimensions of feature maps.

In the proposed scheme, a distinct advantage is that no training process is needed, as each convolutional operator is rigorously derived from updating equations. This also stands in contrast to other CNN-FDTD methods, which utilize convolutional layers at a cellular level, for our operator-level scheme offers enhanced flexibility, which means that it can be easily applied to any differential framework.

III. Numerical Results and Discussion

The validation of the proposed CNN-FDTD scheme uses the simulation of the specific absorption rate (SAR) of a human head [1], which is exposed to a 900 MHz Gaussian pulse. The setup of this simulation is depicted in Fig. 1. The computational domain for this study is discretized into a grid of $280 \times 380 \times 215$ cells along the x, y, and z directions, respectively. This discretization results in a total of 22.876 million cells in the computational domain. The simulation was carried out over 3000 time steps.

The SAR calculated by the CNN-FDTD method and the proposed SBP-SAT FDTD method is depicted in Fig. 2, which verifies the consistency with the proposed CNN-FDTD scheme

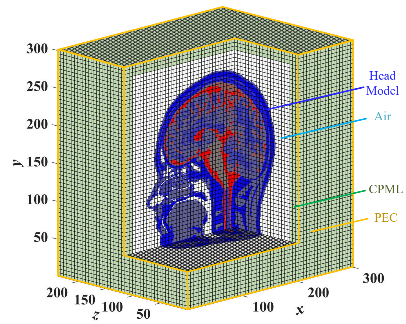
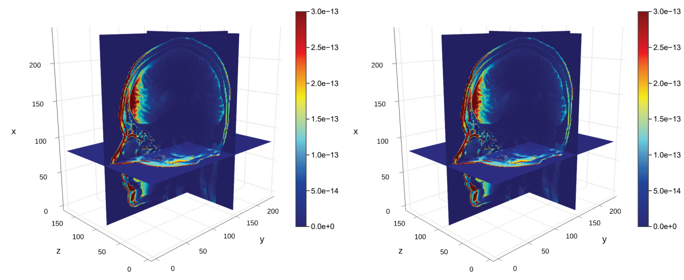


Fig. 1. The mesh configuration of the human head model in the simulation.



(a) (b)

Fig. 2. The SAR calculated by (a) the FDTD method and (b) the CNN-FDTD method.

TABLE I
Performance Comparison: FDTD on CPU vs. CNN-FDTD on GPU.

	FDTD on a CPU	CNN-FDTD on a GPU
Iterations per Second (it/s)	0.116	5.47

and the FDTD method. A significant enhancement of the efficiency is clearly shown in Table. 1 where the proposed method demonstrates an impressive 47-time increase in the simulation speed compared to the traditional approach.

IV. Conclusion

In this paper, we have implemented the FDTD method within a CNN framework. Through the rigorous correspondence at the operator level between the CNN and the FDTD method, we have developed a fully interpretable and adaptable machine learning-based FDTD method. In future research, the proposed scheme is expected to be extended with modules for applications in complex scenarios.

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