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Market power, ambiguity, and market participation☆

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ABSTRACT

We investigate how market power or price impact of market makers affects the participation decisions of investors with ambiguity aversion. Limited participation exists because some investors are ambiguous about the asset fundamental, but the market power of market makers mitigates limited participation. As a result, when market makers become less competitive, the non-participation range decreases, while return volatility increases; thus, market makers and ambiguity-averse investors are better off, but investors with liquidity needs are worse off. However, the non-participation range and uninformed investors' welfare can increase or decrease when information is more asymmetric, depending on the importance of liquidity demand.

1. Introduction

Limited participation has long been one of the puzzling phenomena in the financial markets (Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995; Bertaut and Starr, 2000; Guiso et al., 2002; Campbell, 2006; Christelis et al., 2010, 2013). For instance, Giannetti and Koskinen (2010) show that the participation rates in most countries are below 30%, and the fraction of investors who participate in the financial market is much lower than expected (Campbell, 2006; Giannetti and Koskinen, 2010; Christelis et al., 2013). High participation is welfare-enhancing for almost all investors, so market incumbents who already trade in the market may have an incentive to induce more participation when they are large and have market power.[1] While limited market participation has been widely analyzed in the literature, the analysis from the market power perspective is missing.[2]

In recent years, a prominent trend in the financial markets is that traders have become more and more concentrated (Bebchuk et al., 2017; Kacperczyk et al., 2022). For example, BlackRock, Vanguard, and State Street ("Big Three") manage over $15 trillion and possess over 82% of the total capitalization of the S&P 500, which has drawn much attention from both regulators and academia (Rock and Rubinfeld, 2017; Bebchuk and Hirst, 2019). These gigantic traders, with enormous market power, could significantly influence asset prices.[3] Moreover, market makers in many financial markets are likely to have market power (E.g., Shachar, 2012;

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[1] Individual investors outside the market can diversify their portfolios by market participation, which is beneficial for market incumbents because risk-sharing is enhanced with more market participants.

[2] Previous studies have proposed several explanations for limited participation and its impact on asset pricing and welfare (Constantinides, 2002; Hong et al., 2004; Alan, 2006; Bogan, 2008; Easley and O'Hara, 2009; Almenberg and Dreber, 2015; Conlin et al., 2015).

[3] Most of the recent research has focused on the trading behaviors of large investors and their impacts on market efficiency, systematic risk, and price informativeness (Gabaix et al., 2006; Cuoco and Kaniel, 2011; Basak and Pavlova, 2013; Koijen and Yogo, 2019; Kacperczyk et al., 2022; Ben-David et al., 2021).

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Ang et al., 2013). Therefore, given the fact of limited market participation, it is appealing to study whether and how large traders, especially large market makers, influence small traders' participation decisions. In this paper, we address this problem by developing an asset pricing model with non-competitive market makers and limited participation due to investors' ambiguity aversion.

We develop a two-period non-competitive model with a single risky asset and three different types of investors: sophisticated investors, naïve investors, and market makers. Each sophisticated investor is subject to a liquidity shock, which generates trading demand for liquidity (Vayanos and Wang, 2012). Naïve investors, however, are ambiguity-averse, so they only participate when asset prices are favorable enough (Easley and O'Hara, 2009). There is a countable number K of market makers with market power who actively trade in the market and provide liquidity. Both market makers and naïve investors (if they participate) essentially provide liquidity to sophisticated investors and share the risk (Grossman and Miller, 1988). Moreover, to analyze the effect of information asymmetry, we also consider the case in which sophisticated investors have more information advantages than market makers and naïve investors.

Our primary objective is to explore how market power of market makers affects the participation decisions of naïve investors and the risk-sharing outcomes. The competitiveness among market makers is measured by the number of market makers K, and market makers become more competitive and have less market power as K increases. When K is one, there is one monopolistic market maker, and when K goes to infinity, market makers are fully competitive and have no market power. Market makers consider the demand functions (or the best responses) of other investors before they post the price (e.g., Kyle, 1989; Liu and Wang, 2016). The naïve and sophisticated investors choose their optimal orders, and then market makers provide liquidity and clear the market.

We first solve a model with symmetric information, and we find several observations. The equilibrium price is piece-wise linear and consists of five ranges conditional on the varied liquidity shock. The five price ranges include whether naïve investors participate or not. When the liquidity shock is large (either positive or negative), naïve investors participate in the market. We thus have two bordering price ranges in which the price is either very high or very low. We have three middle price ranges when the magnitude of liquidity shock is small, in which naïve investors do not participate, and only sophisticated investors and market makers trade in the market. Among the three price ranges, when the liquidity shock is somewhere in the middle, the price is sensitive to the liquidity shock. For the other two price ranges, however, we have "flat-price ranges" in which the prices are insensitive to the liquidity shock.

Given those observations, our first result is that the market power of market makers decreases the non-participation range of naïve investors. The price is determined by the market clearing condition, which varies conditional on the participation decisions of naïve investors. Because naïve investors are ambiguity-averse and do their best to avoid the worst-case outcomes, they only trade when the price is below the minimum possible mean payoff or above the maximum possible mean payoff (e.g., Easley and O'Hara, 2009). The participation of naïve investors can provide additional liquidity, which is beneficial to market makers. Thus, if market makers have market power, they would like to affect prices to induce more participation by naïve investors. For this reason, the non-participation range of naïve investors decreases as market makers become less competitive.

Our second result is that the return volatility increases with the market power of market makers. The changes of return volatility relies on how market power affects price volatility, and there exist two competing forces. First, price impact, as market friction, increases price volatility. Second, we identify some "flat-price ranges" where the asset price is insensitive to liquidity shock in the symmetric information case. Although the flat-price ranges disappear when information is asymmetric, they decrease price volatility. It turns out that the second force is dominated by the first so that when market makers are less competitive, the price volatility becomes higher. As a result, the return volatility increases with market power.

Third, we conduct a welfare analysis and find that as market makers have more market power, sophisticated investors are worse off, while market makers and naïve investors are better off. When market makers have more market power, naïve investors are more likely to participate in the market, and hence their ex-ante expected utility increases. The participation of naïve investors provides additional liquidity, which alleviates the market friction of price impact, increasing the ex-ante expected utility of market makers. However, it would be more difficult for sophisticated investors to hedge the liquidity shock because market makers trade less aggressively with market power. Even though naïve investors are more prone to participate, ambiguity aversion keeps them from trading too much. Therefore, sophisticated investors have to bear more risks, which reduces their ex-ante expected utility.

Moreover, we analyze how information asymmetry affects the participation decisions of naïve investors and risk-sharing outcomes by considering an economy in which sophisticated investors can observe a private signal but market makers and naïve investors cannot. We find that asymmetric information has two opposing effects on the participation decisions of naïve investors. On the one hand, information asymmetry strengthens the motivation of market makers to seduce naïve investors to participate in the market and provide liquidity, leading to more naïve investor participation. On the other hand, information asymmetry decreases the incentive of naïve investors to participate because they are uninformed. Which effect dominates the other depends on the demand for liquidity provision. When the demand for liquidity provision is low, the compensation for liquidity provision is low; thus, the second effect dominates the first, leading the non-participation range of naïve investors to increase with the information asymmetry. On the contrary, when the demand for liquidity provision is high, the liquidity premium is high; thus, the non-participation range of naïve investors decreases with the information asymmetry.

We also explore how information asymmetry affects return volatility and the welfare of all investors. Because both market makers and naïve investors are uninformed liquidity providers in the asymmetric information case, they ask for higher compensation for liquidity provision as information asymmetry increases. In this case, the price deviates more from the fundamental value of the risky asset, resulting in higher return volatility. For welfare analysis, sophisticated investors, as the liquidity demanders, are always worse off with information asymmetry because the liquidity premium increases. Moreover, when the demand for liquidity provision is low, both market makers and naïve investors are worse off due to less participation by naïve investors and less risk-sharing. When the demand for liquidity provision is high, the welfare of naïve investors and market makers increases with information asymmetry because naïve investors are more likely to participate in the market, leading to more liquidity provision and risk-sharing. This situation is likely to happen when there are many naïve investors in the market.

Our paper relates to several strands of literature. First, our paper relates to the literature on the impact of demands of large institutional investors or market makers on asset prices. Koijen and Yogo (2019) and Ben-David et al. (2021) explore whether the trading behaviors of large institutional investors increase market volatility. Kacperczyk et al. (2022) prove that the total size of large institutional investors and the concentration of their ownership affect price informativeness in different directions by developing a general equilibrium model. Our paper features the price impact of market makers in the market and emphasizes how their market power over the price influences the participation decisions of other investors.

Second, our paper also contributes to the literature on non-participation in the financial market. Previous studies show that participation costs such as borrowing constraints (Constantinides, 2002), trading costs (Alan, 2006), information costs (Bogan, 2008), and individual factors, such as social interaction (Hong et al., 2004), financial literacy (Almenberg and Dreber, 2015), and personal traits (Conlin et al., 2015) can lead to limited participation. We focus on how limited participation originated from ambiguity aversion and study how market power can affect it.

Third, we add to a growing literature on ambiguity and asset pricing. Many asset pricing puzzles, such as limited participation, equity premium puzzle, and excess volatility, can be explained by embedding ambiguity in an otherwise standard model. For example, Huang et al. (2017) suggest that limited participation arises when investors are ambiguous about the correlation between the payoffs of assets. Epstein and Schneider (2008) theoretically prove that ambiguity leads to underreaction by investors toward good information and overreaction to bad one. In this paper, naïve investors are ambiguity-averse, and we endogenize their participation decisions to investigate how an imperfect competition affects non-participation.

Moreover, our paper relates to the literature about market-making in which market makers supply liquidity for immediacy and ask for a risk premium to compensate for their cost (e.g., Grossman and Miller, 1988; Vayanos and Wang, 2012). Market power, as one of the most important features of market makers, is well studied in the literature both theoretically (e.g., Kyle, 1989; Liu and Wang, 2016; Chen and Wang, 2020) and empirically (e.g., Bellia et al., 2020). Our paper, complementary to the literature, studies how the market power of market makers affects participation decisions of ambiguity-averse investors.

We employ a similar modeling choice as Grossman and Miller (1988), Kyle (1989), and Vayanos and Wang (2012) and analyze the general risk-sharing problem. Thus, our model is applicable to many centralized financial markets where large traders with market power essentially play the role of liquidity providers (market makers). Moreover, although we do not explicitly consider the bid and ask prices in some OTC markets (e.g., Liu and Wang, 2016; Chen and Wang, 2020), we believe the general idea can still be applied to the OTC markets with designated market makers.

The rest of the paper proceeds as follows. In Section 2, we present the model. In Section 3, we examine market participation under symmetric information. In Section 4, we extend the model to the situation under asymmetric information. We conclude in Section 5. All proofs are provided in Appendix A.

2. Model

We consider an economy with two periods, $t = 0, 1$. There are two assets in the financial market: a risky asset and a risk-free asset. The risky asset has a payoff v at $t = 1$, where v is normally distributed with mean μ and variance σ^2. The price of the risky asset, p, is endogenously determined by the financial market equilibrium at $t = 0$. The risk-free asset has an infinitely elastic supply, so it has a constant price of 1.

There is a continuum with mass one of agents, who are classified into three groups: a fraction $1-\rho-\lambda$ of agents are sophisticated investors (S), a fraction ρ of agents are naïve investors (N), and a fraction λ of agents are market makers (M). The sophisticated investors are standard expected utility maximizers with rational expectations about the parameters of the risky asset payoff. To be specific, they believe that $\mathbf{E}[v] = \hat{\mu}$ and $\mathbf{Var}[v] = \sigma^2$. The naïve investors are ambiguity-averse and are uncertain about the mean of the risky asset payoff. They believe that the mean belongs to some intervals, $\mu \in [\underline{\mu}, \overline{\mu}]$, and we use $\Delta\mu \equiv \overline{\mu} - \underline{\mu}$ to measure their ambiguity on the mean.

We consider the non-competitive behaviors of market makers. In particular, there are K identical market makers, and each has market power λ/K. The total fraction of market makers λ is unchanged with K. In this setting, K measures the market competitiveness. The market becomes more competitive when K increases. There are two special cases: Case $K = 1$ means that there is one monopolistic market maker with market power λ, and Case $K = +\infty$ indicates that market makers are fully competitive and have no market power. Thus, our model is general enough to capture the competitiveness of market makers in the financial market.

Moreover, all agents have CARA utility with the risk aversion parameter τ:

$$U(W_i) = -\exp(-\tau W_i), \tag{1}$$

where W_i $(i = S, N, M)$ is the final wealth for each group of agents.

Each sophisticated investor is subject to a liquidity shock $z(v - E[v])$ at $t = 1$, where $z \sim N(0, \sigma_z^2)$ and independent of the payoff v. Sophisticated investors observe the liquidity shock z before trading, while other agents cannot. Because the liquidity shock perfectly correlates with the risky asset payoff, it generates trading demand for liquidity (Vayanos and Wang, 2012). For this reason, market makers and naïve investors essentially provide liquidity to sophisticated investors and share the risk. However, naïve investors are ambiguity-averse and only participate when the asset prices are favorable enough (e.g., Easley and O'Hara, 2009). Thus, in this setting, we explore how the non-competitive behaviors of market makers affect the participation decisions of naïve investors and the risk-sharing outcomes.

All agents trade at $t = 0$ and consume at $t = 1$. The trading mechanism is similar to those in Kyle (1989) and Liu and Wang (2016), in which market makers take into account the demand functions (or the best responses) of other investors before they post the price. Naïve and sophisticated investors choose their optimal orders, and then market makers provide liquidity and clear the market.

In our model, market makers are the primary liquidity providers facilitating trading and risk-sharing. In addition, they also possess market power and have price impact. Both are the crucial features of market makers in the financial market. Our model follows previous seminal work about market makers, such as Grossman and Miller (1988) and Kyle (1989), and focuses on the risk-sharing problem. The setting in our model could apply to many centralized financial markets with market makers, such as foreign exchange markets, stock markets like the NYSE and NASDAQ, and OTC markets with influential market makers.

We model asymmetric information by assuming that sophisticated investors observe a private signal s, while all agents, including naïve investors and market makers, observe a public signal ξ. To be specific, the private signal is a noisy signal about asset payoff:

$$s = v - \mathbf{E}_{\mu}[v] + \epsilon, \tag{2}$$

where $\epsilon \sim N(0, \sigma_{\epsilon}^2)$, and ϵ and v are independent. Moreover, following Liu and Wang (2016), we assume that the public signal ξ is a noisy signal of the private signal:

$$\xi = s + \eta, \tag{3}$$

where $\eta \sim N(0, \sigma_{\eta}^2)$, and η and s are independent. Sophisticated investors observe s before trading, but other agents cannot. Since sophisticated investors are also subject to the liquidity shock $z(v - E[v])$, the liquidity shock prevents the information from being fully revealed by the price, which is the standard modeling choice in the rational expectations equilibrium (REE) literature.

In the following sections, we display the equilibria under symmetric and asymmetric information and explore the impacts of market power of market makers on the participation decisions of naïve investors and asset prices.

3. Market participation with symmetric information

In this section, we consider the case with symmetric information and examine how the non-competitive behaviors of market makers affect the market participation decisions of naïve investors and risk-sharing outcomes. In an economy with symmetric information, sophisticated investors do not observe any private signals, and all agents have the same information set.

Sophisticated investors choose holdings x_S of the risky asset to maximize the expected utility of the final wealth:

$$W_S = W_{S,0} + (v - p)x_S + z(v - \mathbf{E}_{\hat{\mu}}[v]), \tag{4}$$

where p is the price of the risky asset and $W_{S,0}$ is the initial wealth of sophisticated investors. $\mathbf{E}_{\mu}[\cdot]$ is the expectation operator taken under the assumption that $\mathbf{E}[v] = \mu$. Given the CARA-normal assumption, the demand function of sophisticated investors is:

$$X_S(p, z) = \frac{\mathbf{E}_{\hat{\mu}}[v] - p}{\tau \mathbf{Var}[v]} - z. \tag{5}$$

The final wealth of naïve investors is:

$$W_N = W_{N,0} + (v - p)x_N, \tag{6}$$

but they are uncertain about the mean μ of the risky asset payoff. Following Gilboa and Schmeidler's (1989) axiomatic foundation of ambiguity aversion, naïve investors maximize their minimum expected utility over the set of possible distributions. For this reason, they choose holdings x_N to maximize:

$$\min_{\mu \in [\underline{\mu}, \overline{\mu}]} \mathbf{E}_{\mu}\left[-\exp\left(-\tau W_N\right)\right] = -\exp\left[-\tau \min_{\mu \in [\underline{\mu}, \overline{\mu}]} \left(\mathbf{E}_{\mu}[W_N] - \frac{\tau}{2}\mathbf{Var}[W_N]\right)\right]. \tag{7}$$

According to Gilboa and Schmeidler's (1989) maxmin utility function, the demand function of naïve investors is:

$$X_N(p) = \begin{cases} \dfrac{\mathbf{E}_{\underline{\mu}}[v] - p}{\tau \mathbf{Var}[v]}, & p < \mathbf{E}_{\underline{\mu}}[v], \\ 0, & \mathbf{E}_{\underline{\mu}}[v] \leqslant p \leqslant \mathbf{E}_{\overline{\mu}}[v], \\ \dfrac{\mathbf{E}_{\overline{\mu}}[v] - p}{\tau \mathbf{Var}[v]}, & \mathbf{E}_{\overline{\mu}}[v] < p. \end{cases} \tag{8}$$

The demand function implies that the naïve investors will not participate in the financial market when the price is between the minimum possible and maximum possible means ($\mathbf{E}_{\underline{\mu}}[v] \leqslant p \leqslant \mathbf{E}_{\overline{\mu}}[v]$).

Market makers, like sophisticated investors, hold rational expectations about the payoff of the risky asset.[4] They set the price schedules given the demand functions of naïve and sophisticated investors and clear the market after receiving the orders from investors. The market clearing condition is given by:

$$\sum_{k=1}^{K}\frac{\lambda}{K}x_{M,k}+(1-\lambda-\rho)X_S(p,z)+\rho X_N(p)=0, \tag{9}$$

which determines the equilibrium price, where $x_{M,k}$ is the demand of the k_{th} market maker. Then, the k_{th} market maker chooses holdings $x_{M,k}$ of the risky asset to maximize the expected utility:

$$\mathbf{E}_{\hat{\mu}}\left[-\exp\left(-\tau W_{M,k}\right)\right]=-\exp\left[-\tau\left(\mathbf{E}_{\hat{\mu}}[W_{M,k}]-\frac{\tau}{2}\mathbf{Var}[W_{M,k}]\right)\right],$$

given $X_S(p,z)$ and $X_N(p)$. Denoting $Z(\mu)\equiv\dfrac{\hat{\mu}-\mu}{\tau\sigma^2}$ for simplicity, we derive Theorem 1 to provide the equilibrium prices and equilibrium demands of all agents in closed-form.

Theorem 1. *When there are K market makers, each with market power λ/K, the equilibrium price is piece-wise linear in liquidity shock z and consists of five ranges:*

$$p=\begin{cases}\hat{\mu}-\dfrac{\rho Z(\overline{\mu})+(1-\lambda-\rho)z}{\frac{1-\lambda}{\tau\sigma^2}+\lambda b_p}, & z<\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu}),\\ \overline{\mu}, & \left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\leqslant z<\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu}),\\ \hat{\mu}-\dfrac{(1-\lambda-\rho)z}{\frac{1-\lambda-\rho}{\tau\sigma^2}+\lambda b_{np}}, & \left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\leqslant z\leqslant\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu}),\\ \underline{\mu}, & \left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})<z\leqslant\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu}),\\ \hat{\mu}-\dfrac{\rho Z(\underline{\mu})+(1-\lambda-\rho)z}{\frac{1-\lambda}{\tau\sigma^2}+\lambda b_p}, & \left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})<z,\end{cases} \tag{10}$$

and the corresponding demand of the k_{th} market maker is:

$$x_{M,k}=\begin{cases}b_p(\hat{\mu}-p), & z<\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu}),\\ \dfrac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}, & \left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\leqslant z<\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu}),\\ b_{np}(\hat{\mu}-p), & \left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\leqslant z\leqslant\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu}),\\ \dfrac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, & \left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})<z\leqslant\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu}),\\ b_p(\hat{\mu}-p), & \left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})<z,\end{cases} \tag{11}$$

where the coefficient b_p and b_{np} are given by:

(1) When $K=1$,

$$b_p=\frac{1-\lambda}{\tau\sigma^2},\qquad b_{np}=\frac{1-\lambda-\rho}{(1-\rho)\tau\sigma^2}. \tag{12}$$

(2) When $K\geqslant 2$,

$$b_p=\frac{1}{\tau\sigma^2}\left[1-\frac{1-\sqrt{1-\frac{4(K-1)\lambda^2}{K^2}}}{\frac{2(K-1)\lambda}{K}}\right], \tag{13a}$$

$$b_{np}=\frac{1}{\tau\sigma^2}\left[1-\frac{1-\rho-\sqrt{(1-\rho)^2-\frac{4(K-1)\lambda^2}{K^2}}}{\frac{2(K-1)\lambda}{K}}\right]. \tag{13b}$$

(3) When $K\to+\infty$,

$$b_p=b_{np}=\frac{1}{\tau\sigma^2}. \tag{14}$$

[4] They also believe that $\mathbf{E}[v]=\hat{\mu}$ and $\mathbf{Var}[v]=\sigma^2$.

Theorem 1 displays the equilibrium price and the demand of market makers under varied liquidity shock z. In general, the price is piece-wise linear and consists of five different ranges with varied z. When the liquidity shock z is very low or very high, we have price ranges (1) and (5) in which naïve investors participate and trade with sophisticated investors and market makers. Range (1) means the price is high enough, and range (5) means the price is low enough. Thus, naïve investors participate in those two price ranges. When the liquidity shock z is neither too high nor too low, we have price ranges (2), (3), and (4) in which naïve investors do not participate, and only sophisticated investors and market makers trade in the market. Among the three price ranges, price range (3) happens when the liquidity shock z is somewhere in the middle, and the price decreases with z. For price ranges (2) and (4), the prices are insensitive to the liquidity shock z ($p = \overline{\mu}$ in range (2) and $p = \underline{\mu}$ range (4)). In other words, we have "flat-price ranges".

Our first result is about how the non-competitive behaviors of market makers affect the participation decisions of naïve investors. The price is determined by the market clearing condition (9), which varies conditional on the participation decisions of naïve investors. Because naïve investors are ambiguity-averse and do their best to avoid the worst-case outcomes, they only trade when the price is below the minimum possible mean payoff or above the maximum possible mean payoff (e.g., Easley and O'Hara, 2009). For this reason, we have price ranges (ranges (1) and (5)) with naïve investors' participation and price ranges (ranges (2), (3), and (4)) without naïve investors' participation. The participation decisions of naïve investors crucially depend on the asset price, and their participation provides additional liquidity, which is beneficial to market makers. Thus, if market makers have price impact, they would like to affect the prices to induce more participation by naïve investors. For this reason, the non-participation range of naïve investors changes with the competitiveness of market makers, and the result is formally shown in Proposition 1.

Proposition 1. *The non-participation range of naïve investors decreases as the market makers become less competitive.*

To fully understand the result in Proposition 1, we need to look at the parameters b_p and b_{np}, which measure the trading aggressiveness of market makers with (subscript p) and without (subscript np) the participation by naïve investors.[5] First, as K decreases or market makers have larger price impact, both b_p and b_{np} become smaller, which implies that market makers tend to bear less risk. Second, b_{np} is smaller than b_p, which means that market makers can trade more aggressively and bear more risk when naïve investors participate.[6] Because market makers are risk-averse, they ask for a lower price to bear the risk (Grossman and Miller, 1988), so bearing more risk means making more money. Thus, more participation of naïve investors essentially brings more profits so that market makers have an incentive to induce naïve investors to participate when they have market power.

Fig. 1 numerically illustrates the results in Proposition 1. We consider several cases including the monopolistic market markets ($K = 1$), the fully competitive market markets ($K = +\infty$), and the oligopolistic market markets ($K = 2, 5$). Clearly, we have the lowest non-participation range when $K = 1$, and the highest non-participation range when $K = +\infty$.

The second result with symmetric information is the existence of "flat-price ranges" (ranges (2) and (4)), which is also due to the non-competitive behaviors of market makers. In fact, when we have the fully competitive equilibrium ($K = +\infty$), the flat-price ranges disappear, and the price only has three ranges.[7] Note that naïve investors only participate when the price is favorable enough, but their participation will cause an adverse movement of the price. For example, naïve investors only buy the risky asset when the price is low enough, but the buying behaviors will boost the price. Thus, the market makers face a trade-off for the participation by naïve investors. On the one hand, more participation can alleviate the price impact of market makers, which leads them to trade more aggressively. On the other hand, direct participation leads to an adverse movement of the price, which increases the trading cost of market makers. Thus, when the price (or liquidity shock z) moves to the borders where naïve investors are about to participate, market makers change their positions to take varying risks until the benefits exceed the costs. For this reason, there exist flat-price ranges. Moreover, as shown by Fig. 1, the flat-price ranges decrease with K.[8]

The existence of flat-price ranges raises the question of how non-competitive behaviors affect return volatility. Because the price is "flat" and does not fluctuate with liquidity shock, this leads to less volatile prices in the particular price range. However, the impact on the overall return volatility is not clear. To investigate this issue, we define return volatility as:

$$RetVol \equiv \mathbf{Var}[v - p], \tag{15}$$

and show the result in Proposition 2.

Proposition 2. *The return volatility increases as the market makers become less competitive.*

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[5] To solve the model, we conjecture that the demand of market makers has the form $x_{M,k} = a\mathbf{E}_{\hat{\mu}}[v] - bp$ and solve b_p and b_{np} with and without naïve investors' participation. Moreover, in the equilibrium, b_p and b_{np} change with the number of market makers K, while other parameters stay unchanged.

[6] Because naïve investors provide additional liquidity, which essentially decreases the price impact of market makers ($\frac{\partial p}{\partial x_{M,k}}$), market makers can trade more aggressively and make more profits.

[7] When the market is fully competitive, we have $b_p = b_{np}$, so ranges (2) and (4) disappear. For example, when the liquidity shock satisfies $\left(1 + \frac{\lambda b_p \tau \sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}) \leqslant z < \left(1 + \frac{\lambda b_{np} \tau \sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu})$, we have range (2). Clearly, range (2) will disappear if $b_p = b_{np}$.

[8] Note that the flat-price ranges only exist in the case with symmetric information. In the next section, we show that flat-price ranges disappear when information is asymmetric.

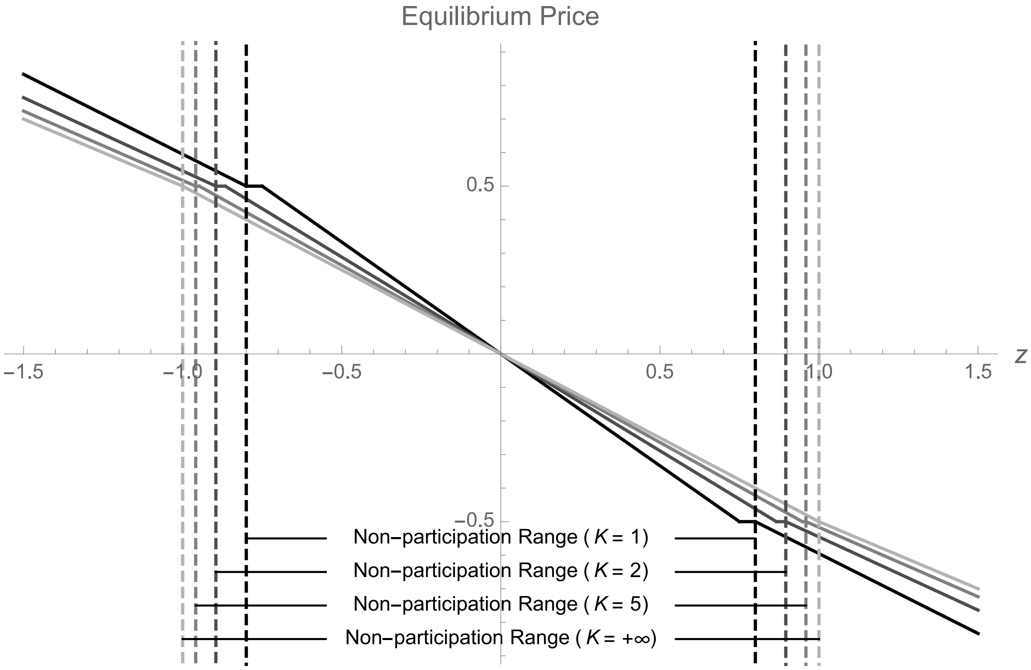

Fig. 1. Equilibrium prices and non-participation ranges under symmetric information. This figure shows both equilibrium prices and the non-participation ranges of naïve investors under symmetric information. The lines from dark to light represent the cases for $K = 1$, $K = 2$, $K = 5$, and $K = +\infty$. Parameter values: $\hat{\mu} = 0$, $\overline{\mu} = 0.5$, $\underline{\mu} = -0.5$, $\sigma = 1$, $\sigma_z = 1$, $\tau = 1$, $\rho = 0.2$, and $\lambda = 0.4$.

Proposition 2 shows that the non-competitive behaviors of market makers generally increase the return volatility. This is because price volatility increases as market makers are less competitive.[9] The price volatility comes from the coefficients of liquidity shock in the price functions, and there exist two competing forces of non-competitive behaviors in the model with symmetric information. First, the existence of flat-price ranges decreases the price volatility in price ranges (2) and (4) because the flat price is insensitive to the liquidity shock. Second, price impact is a market friction, which increases price volatility. In particular, as K decreases, or market makers have a large price impact, both b_p and b_{np} become smaller, which leads the price to be more sensitive to the liquidity shock.[10] As a result, the price volatility is higher. It turns out that the second force dominates the first so that even though the flat-price ranges are larger when market makers are less competitive, the price volatility becomes higher and return volatility increases. Therefore, the non-competitiveness of market makers leads to higher return volatility. Fig. 2 numerically illustrates the results in Proposition 2.

Next, we explore how the market power of market makers affects the welfare of investors. We conduct the welfare analysis by comparing the expected certainty equivalent $\mathbf{E}[CE_i]$ $(i = S, N, M)$, where CE_i is defined as:

$$CE_i \equiv \frac{1}{\tau} \log \mathbf{E}[e^{\tau W_i} | \mathcal{I}_i], \tag{16}$$

and $\mathcal{I}_i$ is the information set of type i investors. Proposition 3 shows the result.

Proposition 3. *As market makers become less competitive, market makers and naïve investors have higher ex-ante expected certainty equivalent, while sophisticated investors have lower ex-ante expected certainty equivalent.*

Proposition 3 demonstrates results from the welfare analysis. First, as market makers have more market power, sophisticated investors are worse off, while both market makers and naïve investors are better off. When market makers have more market power, naïve investors are more likely to participate in the market, and hence their ex-ante expected utility increases. The participation by naïve investors provides additional liquidity, which alleviates the market friction of price impact, and hence increases the ex-ante expected utility of market makers. For sophisticated investors, however, it would be more difficult to hedge the liquidity shock because market makers trade less aggressively with price impact. Even though naïve investors are more prone to participate, ambiguity aversion keeps them from trading too much. Therefore, sophisticated investors have to bear more risks, which reduces their ex-ante expected utility. Fig. 3 numerically illustrates the result in Proposition 3.

[9] Note that, in the model with symmetric information, the return volatility and price volatility satisfy that $\mathbf{Var}[v - p] = \mathbf{Var}[v] + \mathbf{Var}[p]$ because v and z are independent. When information is asymmetric, v and z are not independent any more.

[10] In the Appendix A, we show that b_p and b_{np} are in the denominators of the coefficients of the liquidity shock.

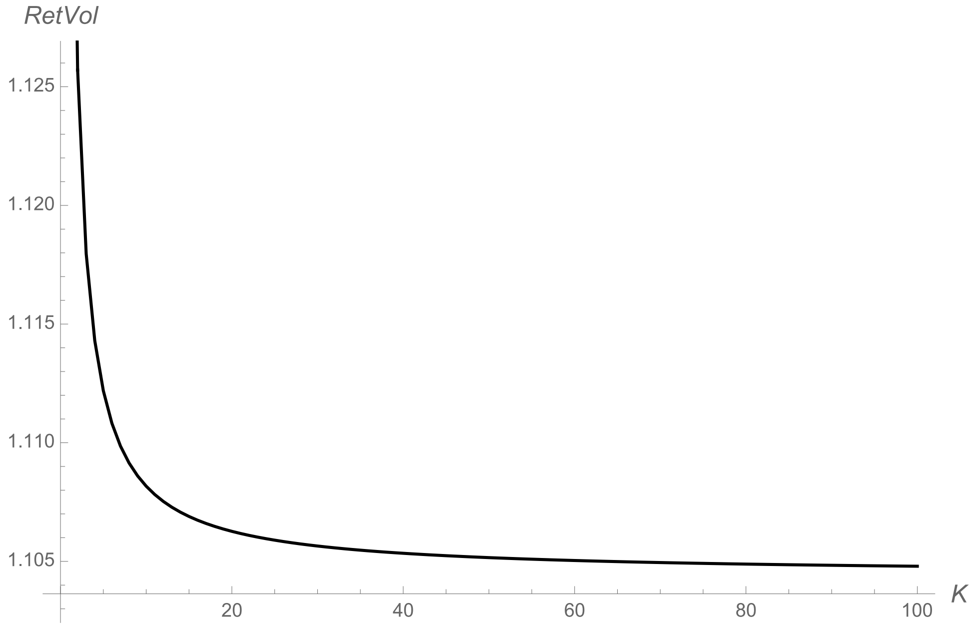

Fig. 2. Return volatility under symmetric information. The figure shows return volatility under symmetric information with varied numbers of market makers. Parameter values: $\hat{\mu} = 0$, $\overline{\mu} = 0.5$, $\underline{\mu} = -0.5$, $\sigma = 1$, $\sigma_z = 1$, $\tau = 1$, $\rho = 0.2$, and $\lambda = 0.4$.

4. Market participation with asymmetric information

In this section, we analyze how asymmetric information affects the participation decisions of naïve investors and risk-sharing outcomes. In an economy with asymmetric information, sophisticated investors receive a private signal s as in Eq. (2), and all agents, including market makers and naïve investors, observe a public signal ξ as in Eq. (3). Given the private signal s, liquidity shock z, and the price, the demand function of sophisticated investor is:

$$X_S(p, s, z) = \frac{\mathbf{E}_{\hat{\mu}}[v|s] - p}{\tau \mathbf{Var}[v|s]} - z. \tag{17}$$

By the projection theorem, given the private signal s, the conditional mean and variance of payoff v are:

$$\mathbf{E}_{\mu}[v|s] = \mu + \beta_I s, \tag{18}$$

$$\mathbf{Var}[v|s] = (1 - \beta_I)\sigma^2, \tag{19}$$

where $\beta_I \equiv \sigma^2/\sigma_s^2$, is the weight that the sophisticated investors put on the private signal s, and $\sigma_s^2 \equiv \sigma^2 + \sigma_\epsilon^2$ is the variance of private signal s.

We conjecture that the price function has the form $P(s, z, \xi) = P(\kappa, \xi)$, where κ is a compound signal of private signal s and liquidity shock z:

$$\kappa = s + \frac{h}{\beta_I} z,$$

where $h \equiv -\tau(1 - \beta_I)\sigma^2$ represents the hedging premium per unit of liquidity shock.

By the projection theorem, the conditional mean and variance of payoff v given the compound signal κ and the public signal ξ are:

$$\mathbf{E}_{\mu}[v|\kappa, \xi] = \mu + \beta_U \left[(1 - \beta_X)\kappa + \beta_X \xi\right], \tag{20}$$

$$\mathbf{Var}[v|\kappa, \xi] = (1 - \beta_U)\sigma^2, \tag{21}$$

where

$$\beta_X \equiv \frac{h^2\sigma_z^2}{h^2\sigma_z^2 + \beta_I^2\sigma_\eta^2}, \qquad \beta_U \equiv \frac{\sigma^2}{\sigma^2 + \beta_X \beta_I \sigma_\eta^2}\beta_I.$$

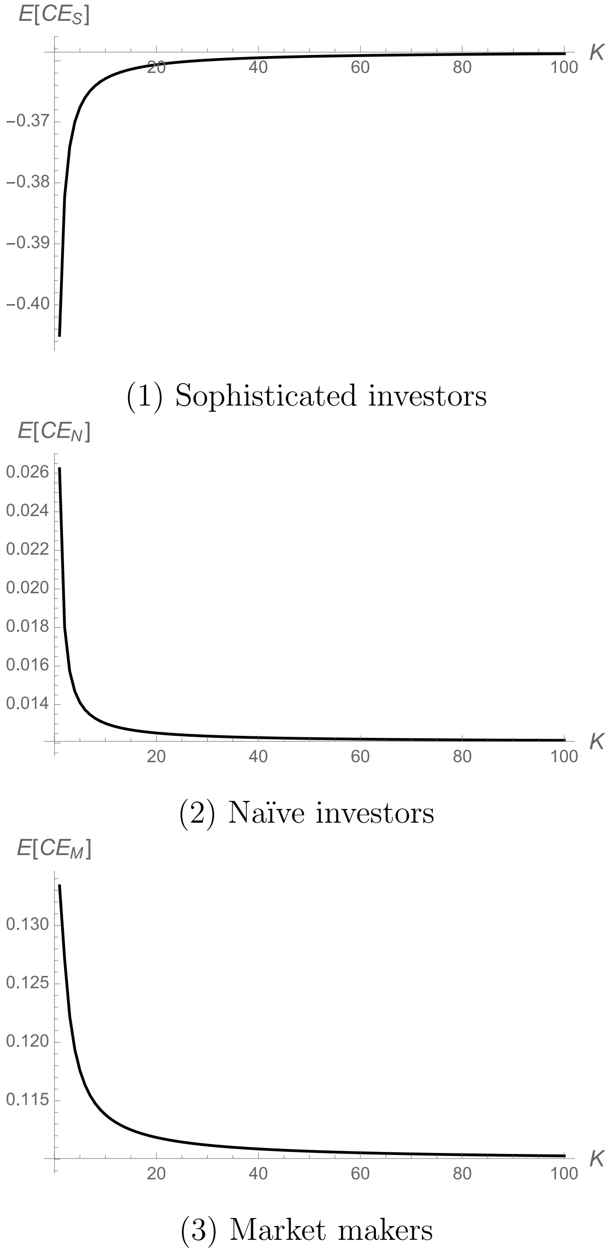

Fig. 3. Welfare analysis under symmetric information. These figures show the ex-ante expected certainty equivalent of different agents under symmetric information with varied numbers of market makers. Parameter values: $\hat{\mu} = 0$, $\overline{\mu} = 0.5$, $\underline{\mu} = -0.5$, $\sigma = 1$, $\sigma_z = 1$, $\tau = 1$, $\rho = 0.2$, and $\lambda = 0.4$.

Market makers and naïve investors cannot observe the private signal s and liquidity shock z, but only the price p and the public signal ξ. Because the compound signal κ can be inferred from the price, the information set of market makers and naïve investors is $\{\kappa, \xi\}$.

Naïve investors choose portfolio x_N to maximize their expected utility given the compound signal κ and public signal ξ:

$$\min_{\mu \in [\underline{\mu}, \overline{\mu}]} \mathbf{E}_\mu \left[-\exp\left(-\tau W_N\right) | \kappa, \xi \right],$$

which is equivalent to:

$$\min_{\mu \in [\underline{\mu}, \overline{\mu}]} \left(\mathbf{E}_\mu \left[v | \kappa, \xi \right] - p \right) x_N - \frac{1}{2} \tau \mathbf{Var}[v | \kappa, \xi] x_N^2. \tag{22}$$

According to Gilboa and Schmeidler's (1989) maxmin utility function, the demand function of naïve investors is:

$$X_N(p,\kappa,\xi)=\begin{cases}\dfrac{\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]-p}{\tau\mathbf{Var}[v|\kappa,\xi]}, & p<\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi],\\ 0, & \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]\leqslant p\leqslant \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi],\\ \dfrac{\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]-p}{\tau\mathbf{Var}[v|\kappa,\xi]}, & \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]<p.\end{cases} \tag{23}$$

Similar to the case with symmetric information, naïve investors will not participate in the market when the price is between the minimum and maximum possible conditional means given the signals.

Given the demand functions of sophisticated investors and naïve investors and the information set $\{\kappa,\xi\}$, market makers determine how much liquidity they would provide. The market clearing condition is:

$$\sum_{k=1}^{K}\frac{\lambda}{K}x_{M,k}+(1-\lambda-\rho)X_S(p,s,z)+\rho X_N(p,\kappa,\xi)=0. \tag{24}$$

Denote $F(\kappa,\xi)\equiv\beta_I\kappa-\beta_U\left[(1-\beta_X)\kappa+\beta_X\xi\right]=\mathbf{E}_\mu[v|s]+hz-\mathbf{E}_\mu[v|\kappa,\xi]$, which equals to the difference between conditional means of payoff given private signal $\{s\}$ and $\{\kappa,\xi\}$ plus the hedging demand for liquidity shock. Theorem 2 provides the equilibrium prices and demands of market makers in closed-form.

Theorem 2. *When there are K market makers and sophisticated investors have private information, the equilibrium price is piece-wise linear in the compound signal κ and public signal ξ, and consists of five ranges:*

$$p=\begin{cases}\mathbf{E}_{\hat\mu}[v|\kappa,\xi]+\dfrac{\frac{(1-\lambda-\rho)F(\kappa,\xi)}{\tau\mathbf{Var}[v|s]}+\frac{\rho(\underline{\mu}-\hat\mu)}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}, & F(\kappa,\xi)<-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{1-\lambda-\rho},\\ \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi], & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{1-\lambda-\rho}\leqslant F(\kappa,\xi)<-\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{1-\lambda-\rho},\\ \mathbf{E}_{\hat\mu}[v|\kappa,\xi]+\dfrac{\frac{(1-\lambda-\rho)F(\kappa,\xi)}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\lambda b_{np}}, & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{1-\lambda-\rho}\leqslant F(\kappa,\xi)\leqslant-\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{1-\lambda-\rho},\\ \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi], & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{1-\lambda-\rho}<F(\kappa,\xi)\leqslant-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{1-\lambda-\rho},\\ \mathbf{E}_{\hat\mu}[v|\kappa,\xi]+\dfrac{\frac{(1-\lambda-\rho)F(\kappa,\xi)}{\tau\mathbf{Var}[v|s]}+\frac{\rho(\overline{\mu}-\hat\mu)}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}, & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{1-\lambda-\rho}<F(\kappa,\xi),\end{cases} \tag{25}$$

and the corresponding demand of k_{th} market maker is:

$$x_{M,k}=\begin{cases}b_p(\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p), & F(\kappa,\xi)<-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{1-\lambda-\rho},\\ -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat\mu-\underline{\mu}\right)}{\lambda\tau\mathbf{Var}[v|s]}, & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{1-\lambda-\rho}\leqslant F(\kappa,\xi)<-\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{1-\lambda-\rho},\\ b_{np}(\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p), & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{1-\lambda-\rho}\leqslant F(\kappa,\xi)\leqslant-\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{1-\lambda-\rho},\\ -\frac{(1-\lambda-\rho)(F(\kappa,\xi)+\hat\mu-\overline{\mu})}{\lambda\tau\mathbf{Var}[v|s]}, & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{1-\lambda-\rho}<F(\kappa,\xi)\leqslant-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{1-\lambda-\rho},\\ b_p(\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p), & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{1-\lambda-\rho}<F(\kappa,\xi),\end{cases} \tag{26}$$

where the coefficients b_p and b_{np} are given by:

(1) When $K=1$,

$$b_p=\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left(1-\frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}}\right), \tag{27a}$$

$$b_{np}=\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left(1-\frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}\right). \tag{27b}$$

(2) When $K\geqslant 2$,

$$b_p=\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left[1-\frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}-\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2-\frac{4(K-1)}{K^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}\right], \tag{28a}$$

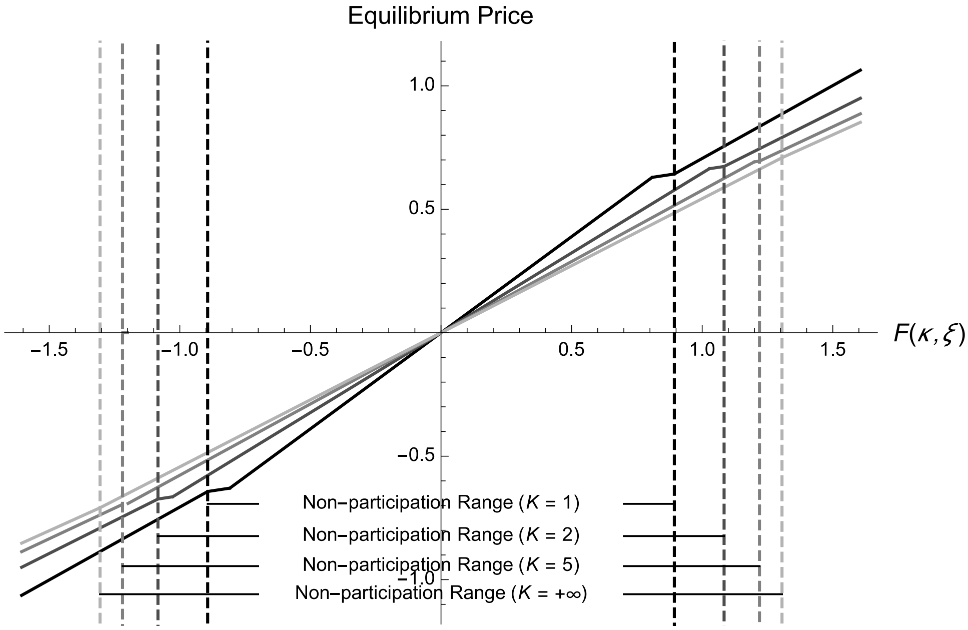

Fig. 4. Equilibrium prices and non-participation ranges under asymmetric information. This figure shows both equilibrium prices and the non-participation ranges of naïve investors under asymmetric information. The lines from dark to light represent the cases for $K = 1$, $K = 2$, $K = 5$ and $K = +\infty$. The public noise η and liquidity shock z are assumed to satisfy that $\eta - \frac{h}{\beta_I} z = 0$. Other parameter values: $\hat{\mu} = 0$, $\overline{\mu} = 0.5$, $\underline{\mu} = -0.5$, $\sigma = 1$, $\sigma_z = 5$, $\sigma_\epsilon = 1$, $\sigma_\eta = 5$, $\tau = 1$, $\rho = 0.2$, and $\lambda = 0.6$.

$$b_{np} = \frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]} \left[1 - \frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]} - \sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]} \right)^2 - \frac{4(K-1)}{K^2} \frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}{\frac{2(K-1)}{K} \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}} \right]. \tag{28b}$$

(3) When $K \to +\infty$,

$$b_p = b_{np} = \frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]}. \tag{29}$$

Theorem 2 shows the equilibrium price and the demand of market makers under asymmetric information. Similar to the discussions under asymmetric information, we first consider how market power affects the non-participation range, return volatility, and agents' welfare. The definitions of return volatility and agents' welfare are the same as those in the symmetric information model. The non-participation range (NPR) is defined differently. We define the non-participation range of naïve investors as follows:

$$\text{NPR} \equiv \frac{\left(1 - \lambda - \rho + \lambda b_p \tau \mathbf{Var}[v|s]\right) \Delta\mu}{(1 - \lambda - \rho)\sigma_F},$$

where σ_F is the standard variance of $F(\kappa,\xi)$.

From the results in Theorem 2, we first find that the effects of the market power of market makers on the non-participation range, return volatility, and agents' welfare are qualitatively the same as those in the case with symmetric information. To be specific, with asymmetric information, when market makers become less competitive, the non-participation range of naïve investors decreases, return volatility increases, and market makers and naïve investors are better off while sophisticated investors are worse off. To save space, we use an Online Appendix for the proofs of those results. Fig. 4 numerically shows the examples of equilibrium price and non-participation ranges when $K = 1$, $K = 2$, $K = 5$, and $K = +\infty$. In Fig. 4, we fix the public noise η and liquidity shock z, and the price is piece-wise linear and consists of five different ranges with $F(\kappa,\xi)$.[11] Note that with asymmetric information, the flat-price ranges in Fig. 1 are not flat anymore. In fact, the prices in those ranges become conditional expectations and change with the compound signal κ and public signal ξ.

Given the above results, our focus here is on how asymmetric information affects the non-participation range, return volatility, and agents' welfare. To conduct a meaningful analysis, we need to find a proper measure of information asymmetry. With asymmetric

[11] Given the public noise η and liquidity shock z, we can rewrite the compound signal κ and the public signal ξ as functions of $F(\kappa,\xi)$: $\kappa = \frac{F(\kappa,\xi)+\beta_U \beta_X \left(\eta - \frac{h}{\beta_I} z\right)}{\beta_I - \beta_U}$ and $\xi = \frac{F(\kappa,\xi)+\left[\beta_I - \beta_U (1-\beta_X)\right]\left(\eta - \frac{h}{\beta_I} z\right)}{\beta_I - \beta_U}$. By inserting them back into the equilibrium price, we can obtain the equilibrium price as a function of $F(\kappa,\xi)$ as shown in Fig. 4.

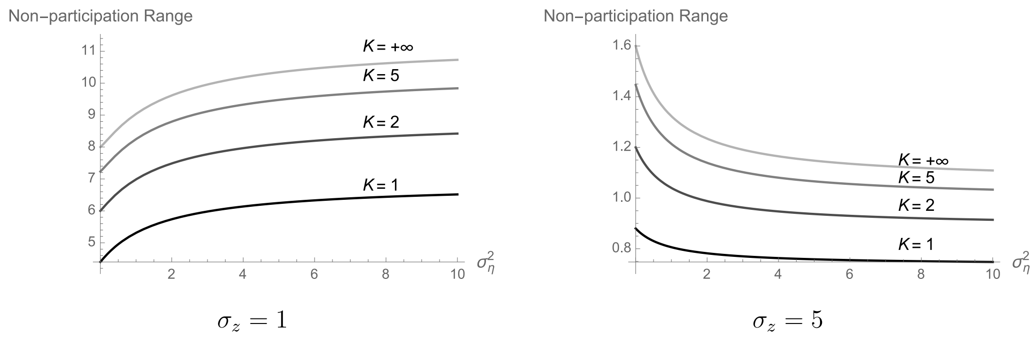

Fig. 5. Non-participation range under asymmetric information. These figures show non-participation range under asymmetric information with varied σ_η^2 under different σ_z. The lines from dark to light represent the cases for $K = 1$, $K = 2$, $K = 5$ and $K = +\infty$. Parameter values: $\hat{\mu} = 0$, $\overline{\mu} = 0.5$, $\underline{\mu} = -0.5$, $\sigma = 1$, $\sigma_\epsilon = 1$, $\tau = 1$, $\rho = 0.2$, and $\lambda = 0.6$.

information, sophisticated investors have private information about their future liquidity shock z and asset payoff v, while market makers and naïve investors can only observe the price and a public signal ξ. Therefore, sophisticated investors have more information advantages over both market makers and naïve investors. The difference between the conditional variances for the uninformed market makers and naïve investors, and the informed the sophisticated investors is:

$$\mathbf{Var}[v|\kappa,\xi] - \mathbf{Var}[v|s] = \frac{\sigma^4\sigma_\eta^2\sigma_z^2\sigma_\epsilon^4\tau^2}{\left(\sigma^2+\sigma_\epsilon^2\right)\left[\sigma_z^2\sigma_\epsilon^4\tau^2\left(\sigma^2+\sigma_\eta^2+\sigma_\epsilon^2\right)+\sigma_\eta^2\left(\sigma^2+\sigma_\epsilon^2\right)\right]},$$

which is monotonically increasing with σ^2, σ_z^2, and σ_η^2, but not monotonically increasing with σ_ϵ^2. Note that changing σ^2 or σ_ϵ^2 changes the quality of aggregate information,[12] and changing σ_z^2 alters the liquidity shock uncertainty. Therefore, following Liu and Wang (2016), we use the σ_η^2 to measure information asymmetry and discuss how it affects the non-participation range, return volatility, and agents' welfare.

We first look at how information asymmetry affects non-participation ranges, and Proposition 4 shows the result.

Proposition 4. *When the uncertainty of liquidity shock σ_z^2 is small enough, the non-participation range of naïve investors increases with the information asymmetry; when the uncertainty of liquidity shock σ_z^2 is large enough, the non-participation range of naïve investors decreases with the information asymmetry.*

Proposition 4 shows that how the non-participation range changes with information asymmetry depends on the uncertainty of the liquidity shock. When the liquidity shock σ_z^2 is small, the non-participation range becomes wider with information asymmetry. However, if the liquidity shock σ_z^2 is large, the non-participation range narrows as information asymmetry increases. In the asymmetric information case, information asymmetry has two opposite effects on the participation decisions of naïve investors. First, with market power, market makers want to seduce naïve investors into participating in the market and providing liquidity, and information asymmetry strengthens their motivation to do so. Second, naïve investors are uninformed, so information asymmetry decreases their incentives to participate. When the liquidity shock σ_z^2 is small, liquidity provision is less important and the liquidity premium for liquidity provision is low. Therefore, the second effect dominates the first, so the non-participation range of naïve investors increases with the information asymmetry. On the contrary, when σ_z^2 is large, liquidity provision is more important and the liquidity premium is high, so the non-participation range of naïve investors decreases with the information asymmetry. Fig. 5 numerically illustrates our results.

Second, we examine the return volatility of the risky asset, and we show the result in Proposition 5.

Proposition 5. *Whenever the uncertainty of liquidity shock σ_z^2 is small or large, return volatility always increases with information asymmetry.*

Proposition 5 demonstrates that the return volatility increases as information asymmetry increases. In the case of information asymmetry, both market makers and naïve investors are uninformed liquidity providers. Therefore, as information becomes more asymmetric, they ask for higher compensation for liquidity provision, which leads the price to deviate more from the fundamental, resulting in higher return volatility. Fig. 6 numerically illustrates our results.

Finally, we explore how information asymmetry affects investors' welfare, and Proposition 6 shows the result.

Proposition 6. *When the uncertainty of liquidity shock σ_z^2 is small, all investors are worse off with information asymmetry; when the uncertainty of liquidity shock σ_z^2 and the fraction of naïve investors ρ are large, naïve investors and market makers are better off with information asymmetry, while sophisticated investors are worse off.*

[12] The quality of aggregate information is defined as $1/\mathbf{Var}[v|s,\kappa,\xi]$.

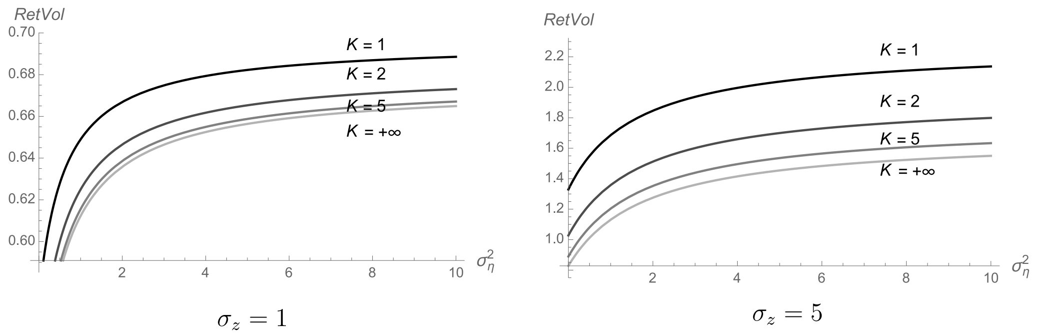

Fig. 6. Return volatility under asymmetric information. These figures show return volatility under asymmetric information with varied numbers of market makers and varied σ_η^2. The lines from dark to light represent the cases for $K = 1$, $K = 2$, $K = 5$ and $K = +\infty$. Parameter values: $\hat{\mu} = 0$, $\overline{\mu} = 0.5$, $\underline{\mu} = -0.5$, $\sigma = 1$, $\sigma_\epsilon = 1$, $\tau = 1$, $\rho = 0.2$, and $\lambda = 0.6$.

Proposition 6 shows the results of welfare analysis under asymmetric information. First, whenever σ_z^2 is small or large, sophisticated investors' welfare decreases with information asymmetry. As information becomes more asymmetric, market makers and naïve investors ask for more compensation for liquidity provision. Although sometimes more naïve investors participate, sophisticated investors are worse off in general when information is more asymmetric. Second, naïve investors and market makers become worse off with information asymmetry when σ_z^2 is small. From Proposition 4, we have shown that the non-participation range increases with information asymmetry when σ_z^2 is small. Therefore, both investors' welfare decreases due to less participation by naïve investors and less risk-sharing. Finally, when σ_z^2 is large and there are many naïve investors in the market, both naïve investors and market makers' welfare increases with information asymmetry. From Proposition 4, when σ_z^2 is large, naïve investors are more likely to participate in the market, leading to more liquidity provision and risk-sharing. Fig. 7 numerically illustrates our results.

5. Conclusion

How do market makers with market power affect market participation and asset prices in the financial market? In this paper, we develop an asset pricing model with both imperfect competition and ambiguity aversion. In the model, we assume that market makers have market power and set the price after considering the best responses of other investors. We also endogenize the participation decision by assuming that naïve investors are ambiguity-averse. This leads naïve investors to participate in the market only when the price is attractive enough. Therefore, market makers could affect asset prices and influence naïve investors' participation decisions.

We find that the imperfect competition of market makers renders a narrower non-participation range. Because the participation by naïve investors could help traders share the risk, market makers have incentives to induce naïve investor participation by influencing the price. From the analysis of equilibrium price, we also find that the return volatility of the risky asset increases with market power. We also conduct a welfare analysis of all agents. The results show that both naïve investors and market makers are better off when there is less competition from market makers, but the welfare of sophisticated investors decreases. The reason is that even though more naïve investors participate in the market, market makers trade less when they have more market power. In addition, naïve investors are ambiguity-averse, which also prohibits them from trading more aggressively. All these contribute to the reduction in sophisticated investors' welfare since it becomes more difficult to hedge the future liquidity shock.

We also explore the situation when sophisticated investors are informed about asset payoff while other agents are not. We find that market power has the same effects on the non-participation range, return volatility, and agents' welfare as those in the symmetric information model, but the impacts of information asymmetry on the market depend on the uncertainty of liquidity shock. When the uncertainty in liquidity shock is low, the demand for liquidity hedging is low, thus liquidity provision is less important. In this case, the non-participation range of naïve investors increases, and the uninformed investors are worse off with the information asymmetry due to information shortage. When the uncertainty in liquidity shock is high, the demand for liquidity hedging is high, thus liquidity provision is more important, the non-participation range of naïve investors decreases, and uninformed investors are better off with the information asymmetry.

Data availability

No data was used for the research described in the article.

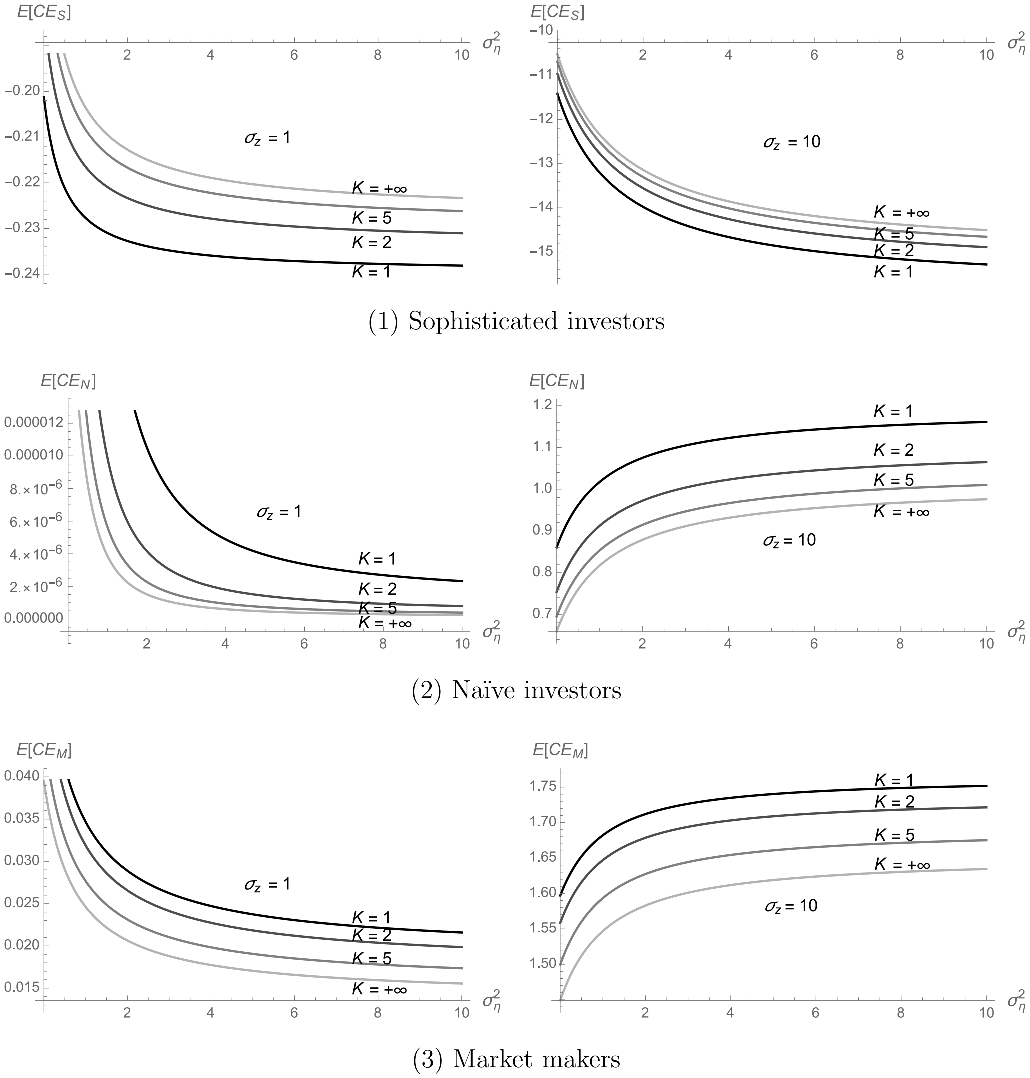

Fig. 7. Welfare analysis under asymmetric information. These figures show the ex-ante expected certainty equivalent of different agents under varied σ_η^2 with different σ_z. The lines from dark to light represent the cases for $K = 1$, $K = 2$, $K = 5$ and $K = +\infty$. Parameter values: $\hat{\mu} = 0$, $\overline{\mu} = 0.5$, $\underline{\mu} = -0.5$, $\sigma = 1$, $\sigma_\epsilon = 1$, $\tau = 1$, $\rho = 0.5$, and $\lambda = 0.3$.

Appendix A

A.1. Proof of Theorem 1

Proof. Substituting the demand function of sophisticated investors from (5) and the one of naïve investors from (8) into market clearing condition (9), we have:

$$P(x_{M,k}) = \begin{cases} \hat{\mu} + \dfrac{\tau\sigma^2}{1-\lambda}\left[\lambda x_{M,k} - \rho Z(\underline{\mu}) - (1-\lambda-\rho)z\right], & p < \underline{\mu}, \\ \hat{\mu} + \dfrac{\tau\sigma^2}{1-\lambda-\rho}\left[\lambda x_{M,k} - (1-\lambda-\rho)z\right], & \underline{\mu} \leqslant p \leqslant \overline{\mu}, \\ \hat{\mu} + \dfrac{\tau\sigma^2}{1-\lambda}\left[\lambda x_{M,k} - \rho Z(\overline{\mu}) - (1-\lambda-\rho)z\right], & \overline{\mu} < p, \end{cases} \tag{A.1}$$

where $x_{M,k}$ denotes the k_{th} market maker's demand.

Given the price p's intervals, we can compute the corresponding intervals on $x_{M,k}$, that is:

$$\begin{aligned} p < \underline{\mu} \quad &\Leftrightarrow \quad x_{M,k} < \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, \\ \underline{\mu} \leqslant p \leqslant \overline{\mu} \quad &\Leftrightarrow \quad \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}, \\ \overline{\mu} < p \quad &\Leftrightarrow \quad \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x_{M,k}, \end{aligned} \tag{A.2}$$

where $Z(\mu) \equiv \dfrac{\hat{\mu}-\mu}{\tau\sigma^2}$. In the following discussions, the conditions on the price will be replaced by the corresponding conditions on $x_{M,k}$.

The final wealth for the k_{th} market maker is:

$$W_{M,k} = W_{M,0} + (v - P(x_{M,k}))x_{M,k}.$$

Given CARA utility, the optimization problem of market makers is equivalent to:

$$\max_{x_{M,k}} \quad CE_{M,k} \equiv \left(\mathbf{E}_{\hat{\mu}}[v] - P(x_{M,k})\right) x_{M,k} - \frac{1}{2}\tau \mathbf{Var}[v] x_{M,k}^2,$$

where $CE_{M,k}$ is the certainty equivalent of the k_{th} market maker. Since the demand function of naïve investors ($X_N(p)$) takes different forms as p changes, $CE_{M,k}$ will also change accordingly, that is:

$$CE_{M,k} = \begin{cases} CE_{M,k}^{(1)} \equiv CE_{M,k}\big|_{X_N(p)=\frac{\underline{\mu}-p}{\tau\sigma^2}}, & x_{M,k} < \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, \\ CE_{M,k}^{(2)} \equiv CE_{M,k}\big|_{X_N(p)=0}, & \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}, \\ CE_{M,k}^{(3)} \equiv CE_{M,k}\big|_{X_N(p)=\frac{\overline{\mu}-p}{\tau\sigma^2}}, & \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x_{M,k}. \end{cases}$$

Note that $CE_{M,k}$ is continuous given that $CE_{M,k}^{(1)} = CE_{M,k}^{(2)}$ when $x_{M,k} = \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}$ and $CE_{M,k}^{(2)} = CE_{M,k}^{(3)}$ when $x_{M,k} = \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}$.

Taking derivative over $x_{M,k}$ of $CE_{M,k}$, we obtain:

$$\frac{dCE_{M,k}}{dx_{M,k}} = \mathbf{E}_{\hat{\mu}}[v] - p - \left(\frac{dp}{dx_{M,k}} + \tau \mathbf{Var}[v]\right) x_{M,k}, \tag{A.3}$$

where $\dfrac{dp}{dx_{M,k}} > 0$ from Eq. (A.1).

Next, we show how to calculate $\frac{\partial p}{\partial x_{M,k}}$. For the k_{th} market maker, the residual demand $D(p)$ equals to:

$$D(p) = \sum_{j\neq k} \frac{\lambda}{K} x_{M,j} + (1-\lambda-\rho)X_S(p,z) + \rho X_N(p). \tag{A.4}$$

From market clearing condition (9) and (A.4), we have

$$D(p) + \frac{\lambda}{K} x_{M,k} = 0. \tag{A.5}$$

Conjecture that the demand of the k_{th} market maker has the following form:

$$x_{M,k} = a\mathbf{E}_{\hat{\mu}}[v] - bp. \tag{A.6}$$

Taking derivative over p in Eqs. (A.4) and (A.5), and given that $x_{M,j} = x_{M,k}$ for $j \neq k$, we have:

$$\frac{\partial D(p)}{\partial p} = -\frac{(K-1)\lambda}{K} b + (1-\lambda-\rho)\frac{\partial X_S(p,z)}{\partial p} + \rho \frac{\partial X_N(p)}{\partial p},$$

and

$$\frac{\partial D(p)}{\partial p} = -\frac{\lambda}{K}\frac{\partial x_{M,k}}{\partial p}.$$

Therefore,

$$\frac{\partial p}{\partial x_{M,k}} = -\frac{\frac{\lambda}{K}}{-\frac{(K-1)\lambda}{K} b + (1-\lambda-\rho)\frac{\partial X_S(p,z)}{\partial p} + \rho\frac{\partial X_N(p)}{\partial p}}. \tag{A.7}$$

Because the demand function of naïve investors $X_N(p)$ takes different forms under varied p, $\frac{\partial p}{\partial x_{M,k}}$ will change accordingly. Replacing $\frac{\partial p}{\partial x_{M,k}}$ from Eq. (A.7) into (A.3), we have:

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}}=\begin{cases}\frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}}=\left(\mathbf{E}_{\hat{\mu}}[v]-p\right)-\left(\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b+\frac{1-\lambda}{\tau\sigma^2}}+\tau\mathbf{Var}[v]\right)x_{M,k}, & x_{M,k}<\frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda};\\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}}=\left(\mathbf{E}_{\hat{\mu}}[v]-p\right)-\left(\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b+\frac{1-\lambda-\rho}{\tau\sigma^2}}+\tau\mathbf{Var}[v]\right)x_{M,k}, & \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}\leqslant x_{M,k}\leqslant\frac{(1-\lambda-\rho)\left[z-Z(\overline{\mu})\right]}{\lambda};\\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}}=\left(\mathbf{E}_{\hat{\mu}}[v]-p\right)-\left(\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b+\frac{1-\lambda}{\tau\sigma^2}}+\tau\mathbf{Var}[v]\right)x_{M,k}, & \frac{(1-\lambda-\rho)\left[z-Z(\overline{\mu})\right]}{\lambda}<x_{M,k}.\end{cases}$$

The optimal demand of the market maker $x^*_{M,k}$ should maximize $CE_{M,k}$. Therefore, we have to find how $\frac{\partial CE^j_{M,k}}{\partial x_{M,k}}$ $(j=1,2,3)$ changes in the corresponding intervals.

(1) When $x_{M,k}<\frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}$ or $p<\underline{\mu}$.

Replacing $CE^{(1)}_{M,k}$ into (A.3), we can obtain the global optimal demand $x^{(1)}_{M,k}$ that maximizes $CE^{(1)}_{M,k}$:

$$x^{(1)}_{M,k}=\frac{\mathbf{E}_{\hat{\mu}}[v]-p}{\frac{\partial p}{\partial x_{M,k}}+\tau\mathbf{Var}[v]}=\frac{\mathbf{E}_{\hat{\mu}}[v]-p}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b+\frac{1-\lambda}{\tau\sigma^2}}+\tau\sigma^2}.$$

Given that $x^{(1)}_{M,k}$ takes the form:

$$x^{(1)}_{M,k}=a_p\mathbf{E}_{\hat{\mu}}[v]-b_p p, \tag{A.8}$$

the coefficients a_p and b_p satisfy the following equations:

$$a_p=b_p,$$
$$b_p=\frac{1}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b_p+\frac{1-\lambda}{\tau\sigma^2}}+\tau\sigma^2}.$$

Solving the second equation about b_p, we derive that:

$$b_p=\begin{cases}\frac{1-\lambda}{\tau\sigma^2}, & K=1,\\ \frac{1}{\tau\sigma^2}\left[1-\frac{1-\sqrt{1-\frac{4(K-1)\lambda^2}{K^2}}}{\frac{2(K-1)\lambda}{K}}\right], & K\geqslant 2,\\ \frac{1}{\tau\sigma^2}, & K=+\infty.\end{cases} \tag{A.9}$$

Substituting the demand function of sophisticated investors from (5), the one of naïve investors from (8), and the one of market makers from (A.8) into market clearing condition (9), we can obtain the equilibrium price under $x_{M,k}=x^{(1)}_{M,k}$:

$$p=\hat{\mu}-\frac{\frac{\rho\left(\hat{\mu}-\underline{\mu}\right)}{\tau\sigma^2}+(1-\lambda-\rho)z}{\lambda b_p+\frac{1-\lambda}{\tau\sigma^2}}. \tag{A.10}$$

In addition, when $\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})<z$, we have $x^{(1)}_{M,k}<\frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}$. Therefore, when $x_{M,k}<\frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}$, for the $\frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}}$, we have the following results:

$$\frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}}\begin{cases}>0, \quad x_{M,k}<\frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, & z\leqslant\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu}),\\ \left.\begin{cases}>0, & x_{M,k}<x^{(1)}_{M,k},\\ =0, & x_{M,k}=x^{(1)}_{M,k},\\ <0, & x^{(1)}_{M,k}<x_{M,k}<\frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda};\end{cases}\right\} & \left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})<z.\end{cases} \tag{A.11}$$

(2) When $\frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}\leqslant x_{M,k}\leqslant\frac{(1-\lambda-\rho)\left[z-Z(\overline{\mu})\right]}{\lambda}$ or $\underline{\mu}\leqslant p\leqslant\overline{\mu}$.

Replacing $CE^{(2)}_{M,k}$ into (A.3), we can obtain the optimal demand $x^{(2)}_{M,k}$ that maximizes $CE^{(2)}_{M,k}$:

$$x^{(2)}_{M,k} = \frac{\mathbf{E}_{\hat{\mu}}[v] - p}{\frac{\partial p}{\partial x_{M,k}} + \tau \mathbf{Var}[v]} = \frac{\mathbf{E}_{\hat{\mu}}[v] - p}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K} b + \frac{1-\lambda-\rho}{\tau\sigma^2}} + \tau\sigma^2}.$$

Given that $x^{(2)}_{M,k}$ takes the form:

$$x^{(2)}_{M,k} = a_{np}\mathbf{E}_{\hat{\mu}}[v] - b_{np} p, \tag{A.12}$$

the coefficients a_{np} and b_{np} satisfy the following equations:

$$\begin{aligned} a_{np} &= b_{np}, \\ b_{np} &= \frac{1}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K} b_{np} + \frac{1-\lambda-\rho}{\tau\sigma^2}} + \tau\sigma^2}. \end{aligned}$$

Solving the second equation about b_{np}, we derive that:

$$b_{np} = \begin{cases} \frac{1-\lambda-\rho}{(1-\rho)\tau\sigma^2}, & K = 1, \\ \frac{1}{\tau\sigma^2}\left(1 - \frac{1-\rho-\sqrt{(1-\rho)^2 - \frac{4(K-1)\lambda^2}{K^2}}}{\frac{2(K-1)\lambda}{K}}\right), & K \geqslant 2, \\ \frac{1}{\tau\sigma^2}, & K = +\infty. \end{cases}$$

Substituting the demand function of sophisticated investors from (5), the one of naïve investors from (8), and the one of market makers from (A.12) into market clearing condition (9), we can obtain the equilibrium price under $x_{M,k} = x^{(2)}_{M,k}$:

$$p = \hat{\mu} - \frac{(1-\lambda-\rho)z}{\lambda b_{np} + \frac{1-\lambda-\rho}{\tau\sigma^2}}. \tag{A.13}$$

In addition, when $\left(1 + \frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}) \leqslant z \leqslant \left(1 + \frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right) Z(\underline{\mu})$, we have $\frac{(1-\lambda-\rho)\left[z - Z(\underline{\mu})\right]}{\lambda} \leqslant x^{(2)}_{M,k} \leqslant \frac{(1-\lambda-\rho)[z - Z(\overline{\mu})]}{\lambda}$. Therefore, when $\frac{(1-\lambda-\rho)\left[z - Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z - Z(\overline{\mu})]}{\lambda}$, for the $\frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}}$, we have the following results:

$$\frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} \begin{cases} > 0, \quad \frac{(1-\lambda-\rho)\left[z - Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z - Z(\overline{\mu})]}{\lambda}, & z < \left(1 + \frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}), \\ \begin{Bmatrix} > 0, & \frac{(1-\lambda-\rho)\left[z - Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} < x^{(2)}_{M,k}, \\ = 0, & x_{M,k} = x^{(2)}_{M,k}, \\ < 0, & x^{(2)}_{M,k} < x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z - Z(\overline{\mu})]}{\lambda}, \end{Bmatrix} & \left(1 + \frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}) \leqslant z \leqslant \left(1 + \frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right) Z(\underline{\mu}) \\ < 0, \quad \frac{(1-\lambda-\rho)\left[z - Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z - Z(\overline{\mu})]}{\lambda}, & \left(1 + \frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right) Z(\underline{\mu}) < z. \end{cases} \tag{A.14}$$

(3) When $\frac{(1-\lambda-\rho)[z - Z(\overline{\mu})]}{\lambda} < x_{M,k}$ or $p > \overline{\mu}$.

Replacing $CE^{(3)}_{M,k}$ into (A.3), we can obtain the optimal demand $x^{(2)}_{M,k}$ that maximizes $CE^{(3)}_{M,k}$:

$$x^{(3)}_{M,k} = \frac{\mathbf{E}_{\hat{\mu}}[v] - p}{\frac{\partial p}{\partial x_{M,k}} + \tau \mathbf{Var}[v]} = \frac{\mathbf{E}_{\hat{\mu}}[v] - p}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K} b + \frac{1-\lambda}{\tau\sigma^2}} + \tau\sigma^2}.$$

Then $x^{(3)}_{M,k}$ takes the same form as $x^{(1)}_{M,k}$.

Substituting the demand function of sophisticated investors from (5), the one of naïve investors from (8), and the one of market makers from (A.8) into market clearing condition (9), we can obtain the equilibrium price under $x_{M,k} = x^{(3)}_{M,k}$:

$$p = \hat{\mu} - \frac{\frac{\rho(\hat{\mu} - \overline{\mu})}{\tau\sigma^2} + (1-\lambda-\rho)z}{\lambda b_p + \frac{1-\lambda}{\tau\sigma^2}}. \tag{A.15}$$

In addition, when $z < \left(1+\frac{\lambda b_p \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu})$, we have $\frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x^{(3)}_{M,k}$. Therefore, when $x_{M,k} > \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}$, for the $\frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}}$, we have the following results:

$$\frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} \begin{cases} \left.\begin{cases} >0, & \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x^{(3)}_{M,k}, \\ =0, & x_{M,k} = x^{(3)}_{M,k}, \\ <0, & x^{(3)}_{M,k} < x_{M,k}; \end{cases}\right\} & z < \left(1+\frac{\lambda b_p \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}), \\ <0, \quad \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x_{M,k}, & \left(1+\frac{\lambda b_p \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}) \leqslant z. \end{cases} \tag{A.16}$$

Finally, given the discussions above, we figure out the optimal $x_{M,k}$ that maximizes $CE_{M,k}$.

Note that $\left(1+\frac{\lambda b_p \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}) < \left(1+\frac{\lambda b_{np} \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}) < \left(1+\frac{\lambda b_{np} \tau\sigma^2}{1-\lambda-\rho}\right) Z(\underline{\mu}) < \left(1+\frac{\lambda b_p \tau\sigma^2}{1-\lambda-\rho}\right) Z(\underline{\mu})$ and $CE_{M,k}$ is continuous.

(1) When $z < \left(1+\frac{\lambda b_p \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu})$, we have:

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} > 0, & x_{M,k} < \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} > 0, & \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}, \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} & \begin{cases} >0, & \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x^{(3)}_{M,k}, \\ =0, & x_{M,k} = x^{(3)}_{M,k}, \\ <0, & x^{(3)}_{M,k} < x_{M,k}. \end{cases} \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $x^{(3)}_{M,k}$. And the equilibrium price is shown in (A.15).

(2) When $\left(1+\frac{\lambda b_p \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}) \leqslant z < \left(1+\frac{\lambda b_{np} \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu})$, we have:

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} > 0, & x_{M,k} < \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} > 0, & \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}, \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} < 0, & \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x_{M,k}. \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $\frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}$. And the corresponding equilibrium price equals to $\overline{\mu}$.

(3) When $\left(1+\frac{\lambda b_{np} \tau\sigma^2}{1-\lambda-\rho}\right) Z(\overline{\mu}) \leqslant z \leqslant \left(1+\frac{\lambda b_{np} \tau\sigma^2}{1-\lambda-\rho}\right) Z(\underline{\mu})$, we have:

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} > 0, & x_{M,k} < \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} & \begin{cases} >0, & \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} < x^{(2)}_{M,k}, \\ =0, & x_{M,k} = x^{(2)}_{M,k}, \\ <0, & x^{(2)}_{M,k} < x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}; \end{cases} \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} < 0, & \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x_{M,k}. \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $x^{(2)}_{M,k}$. And the corresponding equilibrium price is shown in (A.13).

(4) When $\left(1+\frac{\lambda b_{np} \tau\sigma^2}{1-\lambda-\rho}\right) Z(\underline{\mu}) < z \leqslant \left(1+\frac{\lambda b_p \tau\sigma^2}{1-\lambda-\rho}\right) Z(\underline{\mu})$, we have

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} > 0, & x_{M,k} < \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} < 0, & \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}, \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} < 0, & \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x_{M,k}. \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $\frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}$. And the corresponding equilibrium price equals to $\underline{\mu}$.

(5) When $\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu}) < z$, we have

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} & \begin{cases} >0, & x_{M,k} < x^{(1)}_{M,k}, \\ =0, & x_{M,k} = x^{(1)}_{M,k}, \\ <0, & x^{(1)}_{M,k} < x_{M,k} < \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda}, \end{cases} \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} < 0, & \frac{(1-\lambda-\rho)\left[z-Z(\underline{\mu})\right]}{\lambda} \leqslant x_{M,k} \leqslant \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda}, \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} < 0, & \frac{(1-\lambda-\rho)[z-Z(\overline{\mu})]}{\lambda} < x_{M,k}. \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $x^{(1)}_{M,k}$. And the corresponding equilibrium price is shown in (A.10).
Theorem 1 summarizes the results. □

A.2. Proof of Proposition 1

Proof. Given b_p in Theorem 1, it is easy to prove that b_p is increasing with K. Then, given the non-participation range which equals to:

$$\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)\left(Z(\underline{\mu})-Z(\overline{\mu})\right) = \left(\frac{1}{\tau\sigma^2}+\frac{\lambda b_p}{1-\lambda-\rho}\right)\Delta\mu,$$

we can easily obtain the result in Proposition 1. □

A.3. Proof of Proposition 2

Proof. Since v and z are independent, and price is a piece-wise function of z, return volatility can be computed as:

$$RetVol = \mathbf{Var}[v] + \mathbf{Var}[p] = \sigma^2 + \left[\int_z \left(p(z)-\mathbf{E}_{\hat{\mu}}[p]\right)^2 f_z(z)dz\right]^{\frac{1}{2}}, \tag{A.17}$$

where $f_z(\cdot)$ is the density function of the z.

From Theorem 1, $\mathbf{E}_{\hat{\mu}}[p]$ equals to:

$$\begin{aligned}\int_z p(z)f_z(z)dz = \hat{\mu} &- \int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} \frac{\left[\rho Z(\overline{\mu})+(1-\lambda-\rho)z\right]\tau\sigma^2}{1-\lambda+\lambda b_p\tau\sigma^2} f_z(z)dz \\ &- \int_{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} \left(\hat{\mu}-\overline{\mu}\right) f_z(z)dz \\ &- \int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} \frac{(1-\lambda-\rho)z\tau\sigma^2}{1-\lambda-\rho+\lambda b_{np}\tau\sigma^2} f_z(z)dz \\ &- \int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} \left(\hat{\mu}-\underline{\mu}\right) f_z(z)dz \\ &- \int_{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})}^{+\infty} \frac{\left[\rho Z(\underline{\mu})+(1-\lambda-\rho)z\right]\tau\sigma^2}{1-\lambda+\lambda b_p\tau\sigma^2} f_z(z)dz.\end{aligned}$$

Suppose that $\underline{\mu} = \hat{\mu} - \frac{1}{2}\Delta\mu$ and $\overline{\mu} = \hat{\mu} + \frac{1}{2}\Delta\mu$, we have $Z(\underline{\mu}) = -Z(\overline{\mu})$. Then, $\mathbf{E}_{\hat{\mu}}[p] = \hat{\mu}$.
Given the $\mathbf{E}_{\hat{\mu}}[p]$, price volatility can be rewritten as:

$$\mathbf{Var}[p] = 2I_1 + \frac{\Delta\mu^2}{2}I_2 + I_3,$$

where:

$$I_1 \equiv \int_{-\infty}^{-\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} \left\{\frac{\left[\rho Z(\overline{\mu})+(1-\lambda-\rho)z\right]\tau\sigma^2}{1-\lambda+\lambda b_p\tau\sigma^2}\right\}^2 f_z(z)dz,$$

$$I_2 \equiv \int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} f_z(z)dz,$$

$$I_3 \equiv \int_{-\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} \left[\frac{(1-\lambda-\rho)z\tau\sigma^2}{1-\lambda-\rho+\lambda b_{np}\tau\sigma^2}\right]^2 f_z(z)dz.$$

Then, we have:

$$\frac{\partial I_1}{\partial b_p} = \frac{-2\lambda\tau\sigma^2 I_1}{1-\lambda+\lambda b_p\tau\sigma^2} - \frac{\Delta\mu^3}{8}\left(\frac{\lambda}{1-\lambda-\rho}\right) f_z\left(\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\right),$$

$$\frac{\partial I_2}{\partial K} = \frac{\Delta\mu}{2}\left(\frac{\lambda}{1-\lambda-\rho}\right)\left[f_z\left(\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})\right)\frac{\partial b_p}{\partial K} - f_z\left(\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})\right)\frac{\partial b_{np}}{\partial K}\right],$$

$$\frac{\partial I_3}{\partial b_{np}} = \frac{-2\lambda\tau\sigma^2 I_3}{1-\lambda-\rho+\lambda b_{np}\tau\sigma^2} + \frac{\Delta\mu^3}{4}\left(\frac{\lambda}{1-\lambda-\rho}\right) f_z\left(\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})\right).$$

Given the above results, we obtain that:

$$\begin{aligned}\frac{\partial \mathbf{Var}[p]}{\partial K} &= 2\frac{\partial I_1}{\partial b_p}\frac{\partial b_p}{\partial K} + \frac{\Delta\mu^2}{2}\frac{\partial I_2}{\partial K} + \frac{\partial I_3}{\partial b_{np}}\frac{\partial b_{np}}{\partial K} \\ &= -\frac{4\lambda\tau\sigma^2 I_1}{1-\lambda+\lambda b_p\tau\sigma^2}\frac{\partial b_p}{\partial K} - \frac{2\lambda\tau\sigma^2 I_3}{1-\lambda-\rho+\lambda b_{np}\tau\sigma^2}\frac{\partial b_{np}}{\partial K}.\end{aligned}$$

Since I_1 and I_3 are both positive, and that b_p and b_{np} increase as K increases, we have $\frac{\partial \mathbf{Var}[p]}{\partial K} < 0$. Thus, the return volatility decreases as K increases. □

A.4. Proof of Proposition 3

A.4.1. Welfare analysis of sophisticated investors

Proof. Given z, the equilibrium price from Theorem 1, and the demand function of sophisticated investors from (5), the certainty equivalent of sophisticated investors is:

$$CE_S = -(\hat{\mu} - P(z))\,z + \frac{1}{2}\frac{(\hat{\mu}-P(z))^2}{\tau\sigma^2}. \tag{A.18}$$

Then, the ex-ante expected certainty equivalent of sophisticated investors $\mathbf{E}[CE_S]$ equals to:

$$\mathbf{E}[CE_S] \equiv \int_z \left[-(\hat{\mu}-P(z))\,z + \frac{1}{2}\frac{(\hat{\mu}-P(z))^2}{\tau\sigma^2}\right] f_z(z)dz, \tag{A.19}$$

and we prove that $\frac{\partial \mathbf{E}[CE_S]}{\partial K} > 0$.

Suppose that $\underline{\mu} = \hat{\mu} - \frac{1}{2}\Delta\mu$ and $\overline{\mu} = \hat{\mu} + \frac{1}{2}\Delta\mu$, we have $Z(\underline{\mu}) = -Z(\overline{\mu})$. Then, $\mathbf{E}[CE_S]$ can be rewritten as:

$$\mathbf{E}[CE_S] = 2CE_{S,1} + 2CE_{S,2} + CE_{S,3},$$

where:

$$CE_{S,1} = \int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}\left[-\left(\frac{\left[\rho Z(\overline{\mu})+(1-\lambda-\rho)z\right]\tau\sigma^2}{1-\lambda+\lambda b_p\tau\sigma^2}\right)z + \frac{1}{2}\frac{\left(\frac{\left[\rho Z(\overline{\mu})+(1-\lambda-\rho)z\right]\tau\sigma^2}{1-\lambda+\lambda b_p\tau\sigma^2}\right)^2}{\tau\sigma^2}\right] f_z(z)dz,$$

$$CE_{S,2} = \int_{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}\left[-\left(\hat{\mu}-\overline{\mu}\right)z + \frac{1}{2}\frac{\left(\hat{\mu}-\overline{\mu}\right)^2}{\tau\sigma^2}\right] f_z(z)dz,$$

$$CE_{S,3} = \int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})}\left[-\left(\frac{(1-\lambda-\rho)z\tau\sigma^2}{1-\lambda-\rho+\lambda b_{np}\tau\sigma^2}\right)z + \frac{1}{2}\frac{\left(\frac{(1-\lambda-\rho)z\tau\sigma^2}{1-\lambda-\rho+\lambda b_{np}\tau\sigma^2}\right)^2}{\tau\sigma^2}\right] f_z(z)dz.$$

Then, we have:

$$\frac{\partial CE_{S,1}}{\partial b_p} = \left[\frac{\lambda(1-\lambda-\rho)\tau^2\sigma^4\left(\rho+\lambda b_p\tau\sigma^2\right)}{\left(1-\lambda+\lambda b_p\tau\sigma^2\right)^3}\right]\int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} z^2 f_z(z)dz$$

$$
\begin{aligned}
&+\left[\frac{\lambda\rho\Delta\mu\tau\sigma^2\left(1-\lambda-2\rho-\lambda b_p\lambda\tau\sigma^2\right)}{2\left(1-\lambda+\lambda b_p\tau\sigma^2\right)^3}\right]\int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} z f_z(z)dz \\
&+\left[-\frac{\lambda\rho^2\Delta\mu^2}{4\left(1-\lambda+\lambda b_p\tau\sigma^2\right)^3}\right]\int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} f_z(z)dz \\
&+\frac{\lambda\Delta\mu^3(1-\lambda-\rho+2\lambda b_p\tau\sigma^2)}{16(1-\lambda-\rho)^2\tau\sigma^2} f_z\left(\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\right),
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial CE_{S,2}}{\partial K} =& \frac{\lambda\Delta\mu^3(1-\lambda-\rho+2\lambda b_{np}\tau\sigma^2)}{16(1-\lambda-\rho)^2\tau\sigma^2} f_z\left(\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\right)\frac{\partial b_{np}}{\partial K} \\
&-\frac{\lambda\Delta\mu^3(1-\lambda-\rho+2\lambda b_p\tau\sigma^2)}{16(1-\lambda-\rho)^2\tau\sigma^2} f_z\left(\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\right)\frac{\partial b_p}{\partial K},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial CE_{S,3}}{\partial b_{np}} =& \left[\frac{b_{np}\lambda^2(1-\lambda-\rho)\tau^3\sigma^6}{(1-\lambda-\rho+\lambda b_{np}\tau\sigma^2)^3}\right]\int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} z^2 f_z(z)dz \\
&-\left[\frac{\lambda\Delta\mu^3(1-\lambda-\rho+2\lambda b_{np}\tau\sigma^2)}{8\tau\sigma^2(1-\lambda-\rho)^2}\right] f_z\left(\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})\right).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\frac{\partial \mathbf{E}[CE_S]}{\partial K} =& 2\frac{\partial CE_{S,1}}{\partial b_p}\frac{\partial b_p}{\partial K}+\frac{\partial CE_{S,2}}{\partial K}+\frac{\partial CE_{S,3}}{\partial b_{np}}\frac{\partial b_{np}}{\partial K} \\
=& \int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} \frac{\lambda(2z\tau\sigma^2(1-\lambda-\rho)-\rho\Delta\mu)(2z\tau\sigma^2(\rho+\lambda b_p\tau\sigma^2)+\rho\Delta\mu)}{2\left(1-\lambda+\lambda b_p\tau\sigma^2\right)^3} f_z(z)dz\frac{\partial b_p}{\partial K} \\
&+\int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})}\left[\frac{b_{np}\lambda^2(1-\lambda-\rho)\tau^3\sigma^6}{(1-\lambda-\rho+\lambda b_{np}\tau\sigma^2)^3}\right] z^2 f_z(z)dz\frac{\partial b_{np}}{\partial K}.
\end{aligned}
$$

Because $\frac{\lambda(2z\tau\sigma^2(1-\lambda-\rho)-\rho\Delta\mu)(2z\tau\sigma^2(\rho+\lambda b_p\tau\sigma^2)+\rho\Delta\mu)}{2\left(1-\lambda+\lambda b_p\tau\sigma^2\right)^3} > 0$ when $z \leqslant \left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})$, and b_p and b_{np} increase as K grows, we have $\frac{\partial \mathbf{E}[CE_S]}{\partial K} > 0$. □

A.4.2. Welfare analysis of naïve investors

Proof. Given z, the equilibrium price from Theorem 1, and the demand function of naïve investors from (8), the certainty equivalent of naïve investors $\mathbf{E}[CE_N]$ is:

$$
\mathbf{E}[CE_N] \equiv \int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}\frac{\left(\overline{\mu}-P(z)\right)^2}{2\tau\sigma^2} f_z(z)dz+\int_{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})}^{+\infty}\frac{\left(\underline{\mu}-P(z)\right)^2}{2\tau\sigma^2} f_z(z)dz.
$$

Suppose that $\underline{\mu}=\hat{\mu}-\frac{1}{2}\Delta\mu$ and $\overline{\mu}=\hat{\mu}+\frac{1}{2}\Delta\mu$, we have:

$$
\mathbf{E}[CE_N] = 2\int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}\frac{\left[\Delta\mu(1-\lambda-\rho+\lambda b_p\tau\sigma^2)+2(1-\lambda-\rho)\tau\sigma^2 z\right]^2}{8\tau\sigma^2\left(1-\lambda-\lambda b_p\tau\sigma^2\right)^2} f_z(z)dz.
$$

Then, $\frac{\partial \mathbf{E}[CE_N]}{\partial b_p}$ equals to:

$$
\int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}\frac{-\lambda\left[2(1-\lambda-\rho)\tau\sigma^2 z-\rho\Delta\mu\right]\left[2(1-\lambda-\rho)\tau\sigma^2 z-\Delta\mu\left(1-\lambda-\rho+\lambda b_p\tau\sigma^2\right)\right]}{2\left(1-\lambda-\lambda b_p\tau\sigma^2\right)^3} f(z)dz.
$$

Because $\frac{\lambda\left[2(1-\lambda-\rho)\tau\sigma^2 z-\rho\Delta\mu\right]\left[2(1-\lambda-\rho)\tau\sigma^2 z-\Delta\mu\left(1-\lambda-\rho+\lambda b_p\tau\sigma^2\right)\right]}{2\left(1-\lambda-\lambda b_p\tau\sigma^2\right)^3} \geqslant 0$ when $z \leqslant \left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})$, we have $\frac{\partial \mathbf{E}[CE_N]}{\partial K} < 0$. □

A.4.3. Welfare analysis of market makers

Proof. Given z, the equilibrium price, and the demand of market makers from Theorem 1, the certainty equivalent of market makers $\mathbf{E}[CE_M]$ is:

$$\mathbf{E}[CE_M] \equiv \int_z \left[(\hat{\mu} - P(z))x_{M,k} - \frac{1}{2}\tau\sigma^2 x_{M,k}^2\right] f_z(z)dz.$$

Suppose that $\underline{\mu} = \hat{\mu} - \frac{1}{2}\Delta\mu$ and $\overline{\mu} = \hat{\mu} + \frac{1}{2}\Delta\mu$, $\mathbf{E}[CE_M]$ can be rewritten as:

$$\mathbf{E}[CE_M] = 2CE_{M,1} + 2CE_{M,2} + CE_{M,3},$$

where:

$$\begin{aligned}
CE_{M,1} &= \int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} \frac{b_p(2-b_p\tau\sigma^2)\left[2z(1-\lambda-\rho)\tau\sigma^2-\rho\Delta\mu\right]^2}{8\left(1-\lambda+\lambda b_p\tau\sigma^2\right)^2} f_z(z)dz,\\
CE_{M,2} &= \int_{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} \frac{-(1-\lambda-\rho)(2z\tau\sigma^2+\Delta\mu)\left[2z(1-\lambda-\rho)\tau\sigma^2+\Delta\mu(1+\lambda-\rho)\right]}{8\lambda^2\tau\sigma^2} f_z(z)dz,\\
CE_{M,3} &= \int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} \frac{b_{np}(2-b_{np}\tau\sigma^2)(1-\lambda-\rho)^2\tau^2\sigma^4}{2(1-\lambda-\rho+\lambda b_{np}\tau\sigma^2)^2} z^2 f_z(z)dz.
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{\partial CE_{M,1}}{\partial b_p} &= \int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} \frac{(1-\lambda-b_p\tau\sigma^2)\left[2z(1-\lambda-\rho)\tau\sigma^2-\rho\Delta\mu\right]^2}{4\left(1-\lambda+\lambda b_p\tau\sigma^2\right)^3} f_z(z)dz\\
&\quad - \frac{b_p\lambda\Delta\mu^3(2-b_p\tau\sigma^2)}{16(1-\lambda-\rho)} f_z\left(\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\right),\\
\frac{\partial CE_{M,2}}{\partial K} &= -\frac{b_{np}\lambda\Delta\mu^3(2-b_{np}\tau\sigma^2)}{16(1-\lambda-\rho)} f_z\left(\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\right)\frac{\partial b_{np}}{\partial K}\\
&\quad + \frac{b_p\lambda\Delta\mu^3(2-b_p\tau\sigma^2)}{16(1-\lambda-\rho)} f_z\left(\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\right)\frac{\partial b_p}{\partial K},\\
\frac{\partial CE_{M,3}}{\partial b_{np}} &= \int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} \frac{(1-\lambda-\rho)^2\tau^2\sigma^4\left[1-\lambda-\rho-b_{np}(1-\rho)\tau\sigma^2\right]}{\left(1-\lambda-\rho+\lambda b_{np}\tau\sigma^2\right)^3} z^2 f_z(z)dz\\
&\quad + \frac{b_{np}\lambda\Delta\mu^3(2-b_{np}\tau\sigma^2)}{8(1-\lambda-\rho)} f_z\left(\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial \mathbf{E}[CE_M]}{\partial K} &= \int_{-\infty}^{\left(1+\frac{\lambda b_p\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})} \frac{(1-\lambda-b_p\tau\sigma^2)\left[2z(1-\lambda-\rho)\tau\sigma^2-\rho\Delta\mu\right]^2}{2\left(1-\lambda+\lambda b_p\tau\sigma^2\right)^3} f_z(z)dz\frac{\partial b_p}{\partial K}\\
&\quad + \int_{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\overline{\mu})}^{\left(1+\frac{\lambda b_{np}\tau\sigma^2}{1-\lambda-\rho}\right)Z(\underline{\mu})} \frac{(1-\lambda-\rho)^2\tau^2\sigma^4\left[1-\lambda-\rho-b_{np}(1-\rho)\tau\sigma^2\right]}{\left(1-\lambda-\rho+\lambda b_{np}\tau\sigma^2\right)^3} z^2 f_z(z)dz\frac{\partial b_{np}}{\partial K}.
\end{aligned}$$

Because $b_p \geqslant \dfrac{1-\lambda}{\tau\sigma^2}$ and $b_{np} \geqslant \dfrac{1-\lambda-\rho}{(1-\rho)\tau\sigma^2}$, we have $\dfrac{\partial \mathbf{E}[CE_M]}{\partial K} < 0$.

Proposition 3 summarizes the results. □

A.5. Proof of Theorem 2

Proof. Substituting the demand function of sophisticated investors from (17) and the one of naïve investors from (23) into market clearing condition (24), we have the price as a function of the demand of the k_{th} market maker:

$$P(x_{M,k}) = \begin{cases} \frac{\lambda x_{M,k}+\rho\frac{\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]}{\tau\mathbf{Var}[v|\kappa,\xi]}+(1-\lambda-\rho)\frac{\mathbf{E}_{\hat{\mu}}[v|s]+hz}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}}, & p < \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi], \\ \mathbf{E}_{\hat{\mu}}[v|s] + hz + \frac{\tau\mathbf{Var}[v|s]\lambda x_{M,k}}{1-\lambda-\rho}, & \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi] \leqslant p \leqslant \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi], \\ \frac{\lambda x_{M,k}+\rho\frac{\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]}{\tau\mathbf{Var}[v|\kappa,\xi]}+(1-\lambda-\rho)\frac{\mathbf{E}_{\hat{\mu}}[v|s]+hz}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}}, & \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi] < p. \end{cases}$$

Given CARA utility and the information set of market makers $\{\kappa, \xi\}$, the optimization problem of the k_{th} market maker is equal to:

$$\max_{x_{M,k}} \quad CE_{M,k} \equiv \left(\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi] - P(x_{M,k})\right) x_{M,k} - \frac{1}{2}\tau\mathbf{Var}[v|\kappa,\xi]x_{M,k}^2. \tag{A.20}$$

By taking derivative over $x_{M,k}$ of $CE_{M,k}$, we obtain that:

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \left(\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi] - p\right) - \left(\frac{\partial p}{\partial x_{M,k}} + \tau\mathbf{Var}[v|\kappa,\xi]\right) x_{M,k}. \tag{A.21}$$

Note that the demand function of naïve investors $X_N(p,\kappa,\xi)$ takes different forms under varied p, $CE_{M,k}$ will change accordingly, that is:

$$CE_{M,k} = \begin{cases} CE_{M,k}^{(1)} \equiv CE_{M,k}\big|_{X_N(p,\kappa,\xi)=\frac{\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]-p}{\tau\mathbf{Var}[v|\kappa,\xi]}}, & p < \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi], \\ CE_{M,k}^{(2)} \equiv CE_{M,k}\big|_{X_N(p,\kappa,\xi)=0}, & \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi] \leqslant p \leqslant \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi], \\ CE_{M,k}^{(3)} \equiv CE_{M,k}\big|_{X_N(p,\kappa,\xi)=\frac{\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]-p}{\tau\mathbf{Var}[v|\kappa,\xi]}}, & \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi] < p. \end{cases}$$

Similar to the proof of Theorem 1, by conjecturing that the demand of the k_{th} market maker takes the form as in (A.6), we can derive that:

$$\frac{\partial p}{\partial x_{M,k}} = -\frac{\frac{\lambda}{K}}{-\frac{(K-1)\lambda}{K}b + (1-\lambda-\rho)\frac{\partial X_S(p,s,z)}{\partial p} + \rho\frac{\partial X_N(p,\kappa,\xi)}{\partial p}}.$$

Then, we solve the demand of market makers when p is in different ranges.
(1) When $p < \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]$ and $X_N(p,\kappa,\xi) = \frac{\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]-p}{\tau\mathbf{Var}[v|\kappa,\xi]}$.
Since $p < \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]$, we need:

$$x_{M,k} < \frac{(1-\lambda-\rho)\left[\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi] - \left(\mathbf{E}_{\hat{\mu}}[v|s] + hz\right)\right]}{\lambda\tau\mathbf{Var}[v|s]} = -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi) + \hat{\mu} - \underline{\mu}\right)}{\lambda\tau\mathbf{Var}[v|s]},$$

where

$$F(\kappa,\xi) \equiv \mathbf{E}_{\mu}[v|\kappa,\xi] - \left(\mathbf{E}_{\mu}[v|s] + hz\right).$$

We then solve the global solution $x_{M,k}^{(1)}$ which maximizes $CE_{M,k}^{(1)}$. By setting $\frac{\partial CE_{M,k}^{(1)}}{\partial x_{M,k}} = 0$, we derive that:

$$x_{M,k}^{(1)} = \frac{\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi] - p}{\frac{\partial p}{\partial x_{M,k}} + \tau\mathbf{Var}[v|\kappa,\xi]} = \frac{\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi] - p}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b+\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}} + \tau\mathbf{Var}[v|\kappa,\xi]}.$$

Given the form of $x_{M,k}$ in (A.6) and by using subscript p to denote the case when naïve investors participate in the market, we can derive that the coefficients a_p and b_p satisfy the following equations:

$$\begin{aligned} a_p &= b_p, \\ b_p &= \frac{1}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b_p+\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}} + \tau\mathbf{Var}[v|\kappa,\xi]}. \end{aligned}$$

Solving the equation about b_p, we derive that:

$$b_p = \begin{cases} \frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]}\left(1 - \frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}}\right), & K=1, \\ \frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]}\left[1 - \frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}-\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2-\frac{4(K-1)}{K^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}\right], & K \geqslant 2, \\ \frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]}, & K=+\infty. \end{cases}$$

Substituting the demand function of market makers $x^{(1)}_{M,k}$, the one of sophisticated investors from (17) and the one of naïve investors from (23) into market clearing condition (24), we can obtain the equilibrium price when $x_{M,k} = x^{(1)}_{M,k}$:

$$p = \mathbf{E}_{\hat{\mu}}[v|\kappa,\xi] + \frac{\frac{(1-\lambda-\rho)F(\kappa,\xi)}{\tau\mathbf{Var}[v|s]} + \frac{\rho(\underline{\mu}-\hat{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}.$$

Because $p < \mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]$, we need:

$$F(\kappa,\xi) < -\frac{\left(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho}.$$

Then, when $x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau\mathbf{Var}[v|s]}$, for the $\frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}}$, we have the following results:

$$\frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} \begin{cases} \left\{\begin{cases} >0, & x_{M,k} < x^{(1)}_{M,k}, \\ =0, & x_{M,k} = x^{(1)}_{M,k}, \\ <0, & x^{(1)}_{M,k} < x_{M,k} < -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau\mathbf{Var}[v|s]}; \end{cases}\right\} & F(\kappa,\xi) < -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho}, \\ >0, \quad x_{M,k} < -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau\mathbf{Var}[v|s]}, & F(\kappa,\xi) \geqslant -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho}. \end{cases} \tag{A.22}$$

(2) When $\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi] \leqslant p \leqslant \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]$ and $X_N(p,\kappa,\xi) = 0$.
Since $\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi] \leqslant p \leqslant \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]$, we need:

$$-\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau\mathbf{Var}[v|s]} \leqslant x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau\mathbf{Var}[v|s]}.$$

We then solve the global solution $x^{(2)}_{M,k}$ which maximizes $CE^{(2)}_{M,k}$. By setting $\frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} = 0$, we derive that:

$$x^{(2)}_{M,k} = \frac{\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p}{\frac{\partial p}{\partial x_{M,k}} + \tau\mathbf{Var}[v|\kappa,\xi]} = \frac{\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b_{np}+\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}} + \tau\mathbf{Var}[v|\kappa,\xi]}.$$

Then, by using subscript np to denote that case when naïve investors do not participate, we derive the coefficients a_{np} and b_{np} satisfy the following equations:

$$a_{np} = b_{np},$$
$$b_{np} = \frac{1}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b_{np}+\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}} + \tau\mathbf{Var}[v|\kappa,\xi]}.$$

Solving the equation about b_{np}, we derive that:

$$b_{np} = \begin{cases} \frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]}\left(1 - \frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}\right), & K=1, \\ \frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]}\left[1 - \frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}-\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)^2-\frac{4(K-1)}{K^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}\right], & K \geqslant 2, \\ \frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]}, & K=+\infty. \end{cases}$$

Substituting the demand function of market makers $x_{M,k}^{(2)}$, the one of sophisticated investors from (17) and the one of naïve investors from (23) into market clearing condition (24), we can obtain the equilibrium price when $x_{M,k} = x_{M,k}^{(2)}$:

$$p = \mathbf{E}_{\hat{\mu}}[v|\kappa,\xi] + \frac{\frac{(1-\lambda-\rho)F(\kappa,\xi)}{\tau \mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \lambda b_{np}}.$$

Because $\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi] \leqslant p \leqslant \mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]$, we need:

$$-\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho} \leqslant F(\kappa,\xi) \leqslant -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho}.$$

Therefore, when $-\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} \leqslant -\frac{(1-\lambda-\rho)(F(\kappa,\xi)+\hat{\mu}-\overline{\mu})}{\lambda\tau \mathbf{Var}[v|s]}$, for the $\frac{\partial CE_{M,k}^{(2)}}{\partial x_{M,k}}$, we have the following results:

$$\frac{\partial CE_{M,k}^{(2)}}{\partial x_{M,k}} \begin{cases} <0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \quad F(\kappa,\xi) < -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho}, \\ \left.\begin{cases} >0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} < x_{M,k}^{(2)}, \\ =0, & x_{M,k} = x_{M,k}^{(2)}, \\ <0, & x_{M,k}^{(2)} < x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}; \end{cases}\right\} & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho} \leqslant F(\kappa,\xi) \leqslant -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho}, \\ >0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \quad -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho} < F(\kappa,\xi). \end{cases}$$

(3) When $\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi] < p$ and $X_N(p) = \frac{\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]-p}{\tau \mathbf{Var}[v|\kappa,\xi]}$.
Since $\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi] < p$, we need:

$$x_{M,k} > -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}.$$

We then solve the global solution $x_{M,k}^{(3)}$ which maximizes $CE_{M,k}^{(3)}$. By setting $\frac{\partial CE_{M,k}^{(3)}}{\partial x_{M,k}} = 0$, we derive that:

$$x_{M,k}^{(3)} = \frac{\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p}{\frac{\partial p}{\partial x_{M,k}} + \tau \mathbf{Var}[v|\kappa,\xi]} = \frac{\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p}{\frac{\frac{\lambda}{K}}{\frac{(K-1)\lambda}{K}b + \frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}} + \tau \mathbf{Var}[v|\kappa,\xi]}.$$

Then, $x_{M,k}^{(3)}$ has the same form as $x_{M,k}^{(1)}$.

Substituting the demand function of market makers $x_{M,k}^{(3)}$, the one of sophisticated investors from (17) and the one of naïve investors from (23) into market clearing condition (24), we can obtain the equilibrium price when $x_{M,k} = x_{M,k}^{(3)}$:

$$p = \mathbf{E}_{\hat{\mu}}[v|\kappa,\xi] + \frac{\frac{(1-\lambda-\rho)F(\kappa,\xi)}{\tau \mathbf{Var}[v|s]} + \frac{\rho(\overline{\mu}-\hat{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}.$$

Because $\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi] < p$, we need:

$$F(\kappa,\xi) > -\frac{\left(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s]\right)(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho}.$$

Therefore, when $x_{M,k} > -\frac{(1-\lambda-\rho)(F(\kappa,\xi)+\hat{\mu}-\overline{\mu})}{\lambda\tau \mathbf{Var}[v|s]}$, for the $\frac{\partial CE_{M,k}^{(3)}}{\partial x_{M,k}}$, we have the following results:

$$\frac{\partial CE_{M,k}^{(3)}}{\partial x_{M,k}} \begin{cases} <0, & -\frac{(1-\lambda-\rho)(F(\kappa,\xi)+\hat{\mu}-\overline{\mu})}{\lambda\tau \mathbf{Var}[v|s]} < x_{M,k}, \quad F(\kappa,\xi) \leqslant -\frac{(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho}, \\ \left.\begin{cases} >0, & -\frac{(1-\lambda-\rho)(F(\kappa,\xi)+\hat{\mu}-\overline{\mu})}{\lambda\tau \mathbf{Var}[v|s]} < x_{M,k}^{(3)}, \\ =0, & x_{M,k} = x_{M,k}^{(3)}, \\ <0, & x_{M,k}^{(3)} < x_{M,k}; \end{cases}\right\} & -\frac{(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho} < F(\kappa,\xi). \end{cases} \tag{A.23}$$

Finally, given on the discussions above, we find $x_{M,k}$ that maximizes $CE_{M,k}$.

Note that $-\frac{(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho} < -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho} < -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho} < -\frac{(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho}$ since $b_p > b_{np}$.

(1) When $F(\kappa,\xi) < -\frac{(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho}$, we have:

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} \begin{cases} >0, & x_{M,k} < x^{(1)}_{M,k}, \\ =0, & x_{M,k} = x^{(1)}_{M,k}, \\ <0, & x^{(1)}_{M,k} < x_{M,k} < -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \end{cases} \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} < 0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} < 0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} < x_{M,k}. \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $x^{(1)}_{M,k}$. And the equilibrium price is $p^{(1)}$.

(2) When $-\frac{(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho} \leqslant F(\kappa,\xi) < -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho}$, we have

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} > 0, & x_{M,k} < -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} < 0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} < 0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} < x_{M,k}. \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $-\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}$. And the corresponding equilibrium price equals to $\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]$.

(3) When $-\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho} \leqslant F(\kappa,\xi) < -\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho}$, we have

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} > 0, & x_{M,k} < -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} \begin{cases} >0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} < x^{(2)}_{M,k}, \\ =0, & x_{M,k} = x^{(2)}_{M,k}, \\ <0, & x^{(2)}_{M,k} < x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}; \end{cases} \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} < 0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} < x_{M,k}. \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $x^{(2)}_{M,k}$. And the corresponding equilibrium price is $p^{(2)}$.

(4) When $-\frac{(1-\lambda-\rho+\lambda b_{np}\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho} \leqslant F(\kappa,\xi) < -\frac{(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho}$, we have

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} > 0, & x_{M,k} < -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} > 0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} < 0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} < x_{M,k}. \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $\frac{(1-\lambda-\rho)\left(\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]-\hat{\mu}-\beta_I\kappa\right)}{\lambda\tau \mathbf{Var}[v|s]}$. And the corresponding equilibrium price equals to $\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]$.

(5) When $-\frac{(1-\lambda-\rho+\lambda b_p\tau \mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho} \leqslant F(\kappa,\xi)$, we have:

$$\frac{\partial CE_{M,k}}{\partial x_{M,k}} = \begin{cases} \frac{\partial CE^{(1)}_{M,k}}{\partial x_{M,k}} > 0, & x_{M,k} < -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \\ \frac{\partial CE^{(2)}_{M,k}}{\partial x_{M,k}} > 0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\underline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} \leqslant x_{M,k} \leqslant -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]}, \\ \frac{\partial CE^{(3)}_{M,k}}{\partial x_{M,k}} \begin{cases} >0, & -\frac{(1-\lambda-\rho)\left(F(\kappa,\xi)+\hat{\mu}-\overline{\mu}\right)}{\lambda\tau \mathbf{Var}[v|s]} < x^{(3)}_{M,k}, \\ =0, & x_{M,k} = x^{(3)}_{M,k}, \\ <0, & x^{(3)}_{M,k} < x_{M,k}. \end{cases} \end{cases}$$

Therefore, $CE_{M,k}$ is maximized at $x^{(3)}_{M,k}$. And the corresponding equilibrium price is $p^{(3)}$.

Theorem 2 summarizes the results. □

A.6. Lemma 1

Lemma 1. *Given the b_p and b_{np} in Theorem 2, we have the following results:*

1. *b_p, b_{np} and $b_p - b_{np}$ are decreasing with σ_η^2.*
2. *$\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]} + \frac{b_p}{\mathbf{Var}[v|\kappa,\xi]} \geqslant 0$ and $\frac{\partial b_{np}}{\partial \mathbf{Var}[v|\kappa,\xi]} + \frac{b_{np}}{\mathbf{Var}[v|\kappa,\xi]} \geqslant 0$.*

Proof. When $K = 1$, given the definitions of b_p from Theorem 2, we have:

$$\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]} = -\frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]^2}\left[1 - \frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\left(\frac{2(1-\lambda-\rho)}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)}{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2}\right] < 0,$$

and

$$\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]} + \frac{b_p}{\mathbf{Var}[v|\kappa,\xi]} = \frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}}{\tau \mathbf{Var}[v|\kappa,\xi]^2\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2} > 0.$$

Similarly,

$$\frac{\partial b_{np}}{\partial \mathbf{Var}[v|\kappa,\xi]} = -\frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]^2}\left[1 - \frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\left(\frac{2(1-\lambda-\rho)}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)}{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)^2}\right] < 0,$$

and

$$\frac{\partial b_{np}}{\partial \mathbf{Var}[v|\kappa,\xi]} + \frac{b_{np}}{\mathbf{Var}[v|\kappa,\xi]} = \frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}}{\tau \mathbf{Var}[v|\kappa,\xi]^2\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)^2} > 0.$$

Then,

$$\begin{aligned}\frac{\partial (b_p - b_{np})}{\partial \mathbf{Var}[v|\kappa,\xi]} = -\frac{\lambda}{\tau \mathbf{Var}[v|\kappa,\xi]^3}&\left\{\frac{1}{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}} - \frac{1}{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}}\right.\\ &\left.+\frac{(1-\lambda-\rho)}{\mathbf{Var}[v|s]}\left[\frac{1}{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)^2} - \frac{1}{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2}\right]\right\} < 0.\end{aligned}$$

Note that $\frac{\partial b_p}{\partial \sigma_\eta^2} = \frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]}\frac{\partial \mathbf{Var}[v|\kappa,\xi]}{\partial \sigma_\eta^2}$ and $\frac{\partial \mathbf{Var}[v|\kappa,\xi]}{\partial \sigma_\eta^2} > 0$. Therefore, we have $\frac{\partial b_p}{\partial \sigma_\eta^2}$, $\frac{\partial b_{np}}{\partial \sigma_\eta^2}$, and $\frac{\partial (b_p - b_{np})}{\partial \sigma_\eta^2}$ are all negative.

When $K \geqslant 2$ and finite, from the definitions of b_p and b_{np} in Theorem 2, we have:

$$\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]} = -\frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]^2}\left[1 - \frac{\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]} - \frac{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)\left(\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right) - \frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}{\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2 - \frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}\right] < 0,$$

and

$$\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]} + \frac{b_p}{\mathbf{Var}[v|\kappa,\xi]} = \frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}}{\tau \mathbf{Var}[v|\kappa,\xi]^2}\frac{\left[\frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}}{\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2 - \frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}} - 1\right]}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}} > 0.$$

Similarly,

$$\frac{\partial b_{np}}{\partial \mathbf{Var}[v|\kappa,\xi]} = -\frac{1}{\tau \mathbf{Var}[v|\kappa,\xi]^2}\left[1 - \frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]} - \frac{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)\left(\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right) - \frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}{\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)^2 - \frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}\right] < 0,$$

and

$$\frac{\partial b_{np}}{\partial \mathbf{Var}[v|\kappa,\xi]}+\frac{b_{np}}{\mathbf{Var}[v|\kappa,\xi]}=\frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}}{\tau\mathbf{Var}[v|\kappa,\xi]^2}\frac{\left[\frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}{\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)^2-\frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}-1\right]}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}>0.$$

Then,

$$\begin{aligned}&-\tau\mathbf{Var}[v|\kappa,\xi]^2\frac{\partial(b_p-b_{np})}{\partial\mathbf{Var}[v|\kappa,\xi]}\\&=\left[\frac{\frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}{\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)^2-\frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}-\frac{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}}{\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2-\frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}}}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}\right]\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}\right)\\&+\frac{\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2-\frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}-\sqrt{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)^2-\frac{4(K-1)}{N^2}\frac{\lambda^2}{\mathbf{Var}[v|\kappa,\xi]^2}}-\frac{\rho}{\mathbf{Var}[v|\kappa,\xi]}}{\frac{2(K-1)}{K}\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}>0.\end{aligned}$$

Therefore, when $K \geqslant 2$ and finite, $\frac{\partial b_p}{\partial\sigma_\eta^2}<0$, $\frac{\partial b_{np}}{\partial\sigma_\eta^2}<0$, and $\frac{\partial(b_p-b_{np})}{\partial\sigma_\eta^2}<0$ still hold.

When $K=+\infty$, $b_p=b_{np}=\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}$. Then, it is easy to prove that $\frac{\partial b_p}{\partial\sigma_\eta^2}<0$ and $\frac{\partial b_{np}}{\partial\sigma_\eta^2}<0$, while $\frac{\partial(b_p-b_{np})}{\partial\sigma_\eta^2}=0$. In addition, $\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]}+\frac{b_p}{\mathbf{Var}[v|\kappa,\xi]}=\frac{\partial b_{np}}{\partial\mathbf{Var}[v|\kappa,\xi]}+\frac{b_{np}}{\mathbf{Var}[v|\kappa,\xi]}=0.$ □

A.7. Proof of Proposition 5

Proof. Since $\mathbf{Var}[X]=\mathbf{Var}[\mathbf{E}[X|Y]]+\mathbf{E}[\mathbf{Var}[X|Y]]$, the return volatility equals to:

$$\begin{aligned}\mathbf{Var}[v-p] &= \mathbf{Var}[\mathbf{E}_{\hat\mu}[v-p|\kappa,\xi]]+\mathbf{E}_{\hat\mu}[\mathbf{Var}[v-p|\kappa,\xi]]\\&= \mathbf{Var}[\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p]+\mathbf{Var}[v|\kappa,\xi].\end{aligned}$$

To prove that $\frac{\partial\mathbf{Var}[v-p]}{\partial\sigma_\eta^2}>0$, we need:

$$\frac{\partial\left(\mathbf{Var}[\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p]+\mathbf{Var}[v|\kappa,\xi]\right)}{\partial\sigma_\eta^2}>0.$$

Note that $\frac{\partial\mathbf{Var}[v|\kappa,\xi]}{\partial\sigma_\eta^2}>0$, and

$$\frac{\partial\mathbf{Var}[v|\kappa,\xi]}{\partial\sigma_\eta^2}=-\frac{\partial\sigma_F^2}{\partial\sigma_\eta^2}.$$

Next, we prove that $\frac{\partial\mathbf{Var}[\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p]}{\partial\sigma_\eta^2}>\frac{\partial\sigma_F^2}{\sigma_\eta^2}$ when σ_z^2 is small enough, and $\frac{\partial\mathbf{Var}[\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p]}{\partial\sigma_\eta^2}>0$ when σ_z^2 is large enough.

We denote $\zeta\equiv\frac{F(\kappa,\xi)}{\sigma_F}$, which is standard normally distributed. $\phi(\cdot)$ and $\Phi(\cdot)$ are the density function and cumulative distribution function of ζ. Then, $\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p$ can be written as a piecewise function of ζ as follows:

$$\mathbf{E}_{\hat\mu}[v|\kappa,\xi]-p=\begin{cases}\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}+\frac{\rho(\hat\mu-\underline{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}, & \zeta<-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{(1-\lambda-\rho)\sigma_F},\\ \hat\mu-\underline{\mu}, & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}\leqslant\zeta<-\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{(1-\lambda-\rho)\sigma_F},\\ \frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\lambda b_{np}}, & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}\leqslant\zeta<-\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{(1-\lambda-\rho)\sigma_F},\\ \hat\mu-\overline{\mu}, & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{(1-\lambda-\rho)\sigma_F}\leqslant\zeta<-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{(1-\lambda-\rho)\sigma_F},\\ \frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}+\frac{\rho(\hat\mu-\overline{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}, & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat\mu-\overline{\mu})}{(1-\lambda-\rho)\sigma_F}\leqslant\zeta.\end{cases}$$

Then, by taking derivative of $\mathbf{Var}[\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]$ over σ_η^2, we have:

$$\frac{\partial \mathbf{Var}[\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]}{\partial \sigma_\eta^2} = 2(I_1+I_2),$$

where:

$$I_1 = \int_{-\infty}^{-\frac{(1-\lambda-\rho+\lambda b_p \tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}} \frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F \zeta}{\tau \mathbf{Var}[v|s]} + \frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}\right]^2 d\Phi(\zeta),$$

$$I_2 = \int_{-\frac{(1-\lambda-\rho+\lambda b_{np} \tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}^{0} \frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\sigma_F}{\tau \mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \lambda b_{np}}\right]^2 \zeta^2 d\Phi(\zeta).$$

(1) When $\sigma_z^2 \to 0$.

Because $\lim_{\sigma_z^2\to 0} \sigma_F = 0$, we have $I_1 \to 0$. In addition,

$$\frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\sigma_F}{\tau \mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \lambda b_{np}}\right]^2 > \frac{\partial \sigma_F^2}{\partial \sigma_\eta^2},$$

because we have $\frac{\partial b_{np}}{\partial \sigma_\eta^2} < 0$ from Lemma 1. Then,

$$2I_2 > 2\int_{-\infty}^{0} \frac{\partial \sigma_F^2}{\partial \sigma_\eta^2} \zeta^2 d\Phi(\zeta) = \frac{\partial \sigma_F^2}{\partial \sigma_\eta^2}.$$

Therefore, when σ_z^2 is close to zero, we have:

$$\frac{\partial \mathbf{Var}[\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]}{\partial \sigma_\eta^2} > \frac{\partial \sigma_F^2}{\partial \sigma_\eta^2}.$$

(2) When $\sigma_z^2 \to +\infty$.

Because $\lim_{\sigma_z^2\to +\infty} \sigma_F = +\infty$, we have $I_2 \to 0$. Then, we prove that $I_1 > 0$, which is equivalent to:

$$\frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F \zeta}{\tau \mathbf{Var}[v|s]} + \frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}\right] > 0, \tag{A.24}$$

when $\zeta < -\frac{(1-\lambda-\rho+\lambda b_p \tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$.

Note that $\frac{\partial \sigma_F^2}{\partial \sigma_\eta^2} < 0$, and $\lim_{\sigma_F^2\to +\infty} b_p + 2\sigma_F^2 \frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]} = -\infty$ since $\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]} < 0$ from Lemma 1. Hence, we have:

$$\frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F}{\tau \mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}\right] = -\frac{\frac{(1-\lambda-\rho)}{2\sigma_F \tau \mathbf{Var}[v|s]}\frac{\partial \sigma_F^2}{\partial \sigma_\eta^2}\left[\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}\left(1-\frac{2\sigma_F^2}{\mathbf{Var}[v|\kappa,\xi]}\right) + \lambda\left(b_p + 2\sigma_F^2 \frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]}\right)\right]}{\left(\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]} + \lambda b_p\right)^2} < 0.$$

Then, when $\zeta < -\frac{(1-\lambda-\rho+\lambda b_p \tau \mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$ and $\sigma_F \to +\infty$, we have:

$$\frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F \zeta}{\tau \mathbf{Var}[v|s]} + \frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}\right] > \frac{(\hat{\mu}-\underline{\mu})\frac{\partial \sigma_F^2}{\partial \sigma_\eta^2}\left(\lambda \frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]}\right)}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]} + \frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]} + \lambda b_p},$$

which is positive because $\frac{\partial \sigma_F^2}{\partial \sigma_\eta^2} < 0$ and $\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]} < 0$.

Therefore, $\frac{\partial \mathbf{Var}[\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]}{\partial \sigma_\eta^2} > 0$ when σ_z^2 is large enough.

Proposition 5 summarizes the result. □

A.8. Proof of Proposition 6

A.8.1. Welfare analysis of sophisticated investors

Proof. Given the demand function of sophisticated investors in (17) and equilibrium price p, we can calculate that the certainty equivalent of sophisticated investors CE_S equals to:

$$CE_S = \frac{(\mathbf{E}_{\hat{\mu}}[v|s]-p)^2}{2\tau\mathbf{Var}[v|s]} - (\mathbf{E}_{\hat{\mu}}[v|s]-p)z.$$

Then, the expected certainty equivalent $\mathbf{E}\left[CE_S\right]$ is:

$$\mathbf{E}\left[CE_S\right] = \frac{\mathbf{Var}[\mathbf{E}_{\hat{\mu}}[v|s]-p]}{2\tau\mathbf{Var}[v|s]} - \mathbf{Cov}\left[\mathbf{E}_{\hat{\mu}}[v|s]-p,z\right].$$

Since

$$\begin{aligned}\mathbf{E}_{\hat{\mu}}[v|s]-p &= \mathbf{E}_{\hat{\mu}}[v|s]-\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]+\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p\\ &= F(\kappa,\xi)-hz+\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p,\end{aligned}$$

we have:

$$\begin{aligned}\mathbf{Var}[\mathbf{E}_{\hat{\mu}}[v|s]-p] &= \sigma_F^2+h^2\sigma_z^2+\mathbf{Var}[\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]+2\mathbf{Cov}\left(F(\kappa,\xi),\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p\right)\\ &\quad -2h\mathbf{Cov}\left(F(\kappa,\xi),z\right)-2h\mathbf{Cov}\left(\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p,z\right),\end{aligned}$$

and

$$\mathbf{Cov}\left(\mathbf{E}_{\hat{\mu}}[v|s]-p,z\right) = \mathbf{Cov}\left(F(\kappa,\xi),z\right)+\mathbf{Cov}\left(\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p,z\right)-h\sigma_z^2.$$

Thus, $\mathbf{E}\left[CE_S\right]$ can be rewritten as:

$$\mathbf{E}\left[CE_S\right] = \frac{\mathbf{Var}[F(\kappa,\xi)+\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]-h^2\sigma_z^2}{2\tau\mathbf{Var}[v|s]}.$$

From the proof of Proposition 5, $F(\kappa,\xi)+\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p$ can be written as a piecewise function of ζ as follows:

$$F(\kappa,\xi)+\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p =$$

$$\begin{cases}\dfrac{\frac{\rho(\hat{\mu}-\underline{\mu}+\sigma_F\zeta)}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\sigma_F\zeta}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}, & \zeta < -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F},\\ \sigma_F\zeta+\hat{\mu}-\underline{\mu}, & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}\leqslant\zeta< -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F},\\ \dfrac{\lambda b_{np}\sigma_F\zeta}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\lambda b_{np}}, & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}\leqslant\zeta< -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{(1-\lambda-\rho)\sigma_F},\\ \sigma_F\zeta+\hat{\mu}-\overline{\mu}, & -\frac{(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{(1-\lambda-\rho)\sigma_F}\leqslant\zeta< -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{(1-\lambda-\rho)\sigma_F},\\ \dfrac{\frac{\rho(\hat{\mu}-\overline{\mu}+\sigma_F\zeta)}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\sigma_F\zeta}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}, & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{(1-\lambda-\rho)\sigma_F}\leqslant\zeta,\end{cases}$$

(1) When $\sigma_z^2\to 0$.
As $\sigma_z^2\to 0$, we have $\sigma_F^2\to 0$. Then:

$$\frac{\partial\mathbf{Var}[F(\kappa,\xi)+\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]}{\partial\sigma_\eta^2} = \left(\frac{\lambda b_{np}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\lambda b_{np}}\right)^2\frac{\partial\sigma_F^2}{\partial\sigma_\eta^2}+\sigma_F^2\frac{\partial}{\partial\sigma_\eta^2}\left(\frac{\lambda b_{np}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\lambda b_{np}}\right)^2<0,$$

because $\frac{\partial\sigma_F^2}{\partial\sigma_\eta^2}<0$, and $\frac{\partial}{\partial\sigma_\eta^2}\left(\frac{\lambda b_{np}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\lambda b_{np}}\right)^2$ is finite. Therefore, $\frac{\partial\mathbf{E}[CE_S]}{\partial\sigma_\eta^2}<0$ when σ_z^2 is close to zero.
(2) When $\sigma_z^2\to+\infty$.
As $\sigma_z^2\to+\infty$, $\sigma_F^2\to+\infty$. Then,

$$\frac{\partial\mathbf{Var}[F(\kappa,\xi)+\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]}{\partial\sigma_\eta^2} = \int_{-\infty}^{-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{\rho(\hat{\mu}-\underline{\mu}+\sigma_F\zeta)}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\sigma_F\zeta}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]^2 d\Phi(\zeta).$$

When $\sigma_F\to+\infty$, we have:

$$\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{\rho\sigma_F}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\sigma_F}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]$$

$$= \frac{\partial \mathbf{Var}[v|\kappa,\xi]}{\partial \sigma_\eta^2}\left[\frac{\sigma_F\left(-\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]^2}+\lambda\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]}\right)\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}\right)}{\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\right)^2}-\left(\frac{\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right)\frac{1}{2\sigma_F}\right]<0,$$

because $\frac{\partial \mathbf{Var}[v|\kappa,\xi]}{\partial \sigma_\eta^2}>0$, and $\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]}<0$ from Lemma 1.

Thus, when $\zeta < -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$,

$$\frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{\rho(\hat{\mu}-\underline{\mu}+\sigma_F\zeta)}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\sigma_F\zeta}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right] > -\frac{\partial \mathbf{Var}[v|\kappa,\xi]}{\partial \sigma_\eta^2}\frac{\lambda\frac{\partial b_p}{\partial \mathbf{Var}[v|\kappa,\xi]}\left(\frac{1-\lambda}{\tau\mathbf{Var}[v|s]}+\lambda b_p\right)(\hat{\mu}-\underline{\mu})}{\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\right)^2}>0.$$

Therefore, when σ_z^2 is large enough, $\frac{\partial \mathbf{Var}[F(\kappa,\xi)+\mathbf{E}_{\hat{\mu}}[v|\kappa,\xi]-p]}{\partial \sigma_\eta^2}<0$, so $\frac{\partial \mathbf{E}[CE_S]}{\partial \sigma_\eta^2}<0$. □

A.8.2. Welfare analysis of naïve investors

Proof. Given the demand function of naïve investors from (23) and price p, we can calculate the certainty equivalent of naïve investors (CE_N) equals to:

$$CE_N = \begin{cases} \frac{(\mathbf{E}_{\underline{\mu}}[v|\kappa,\xi]-p)^2}{2\tau\mathbf{Var}[v|\kappa,\xi]}, & F(\kappa,\xi) < -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho}, \\ 0, & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{1-\lambda-\rho} \leqslant F(\kappa,\xi) \leqslant -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho}, \\ \frac{(\mathbf{E}_{\overline{\mu}}[v|\kappa,\xi]-p)^2}{2\tau\mathbf{Var}[v|\kappa,\xi]}, & -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\overline{\mu})}{1-\lambda-\rho} < F(\kappa,\xi). \end{cases}$$

By inserting the equilibrium price from Theorem 2 into the expression above, normalizing $F(\kappa,\xi)$ by ζ, and taking integral over ζ, we obtain that the expected certainty equivalent of naïve investors ($\mathbf{E}[CE_N]$) is:

$$\mathbf{E}[CE_N] = \int_{-\infty}^{-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}} \frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]^2 d\Phi(\zeta).$$

The derivative of $\mathbf{E}[CE_N]$ with respect to σ_η^2 is:

$$\frac{\mathbf{E}[CE_N]}{\partial\sigma_\eta^2} = \int_{-\infty}^{-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}} \frac{\partial}{\partial\sigma_\eta^2}\left\{\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]^2\right\} d\Phi(\zeta).$$

Note that:

$$\frac{\partial}{\partial\sigma_\eta^2}\left\{\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]^2\right\} = \frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]$$
$$\left\{-\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\frac{\frac{\partial\mathbf{Var}[v|\kappa,\xi]}{\partial\sigma_\eta^2}}{\mathbf{Var}[v|\kappa,\xi]}+2\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\right\},$$

and

$$\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}<0,$$

when $\zeta < -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$.

(1) When $\sigma_z^2 \to 0$.

Then, we prove that $\frac{\mathbf{E}[CE_N]}{\partial\sigma_\eta^2} < 0$ by showing that:

$$\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right] > 0,$$

when $\sigma_z^2 \to 0$.

Because $\lim_{\sigma_z^2\to 0}\sigma_F = 0$, we have:

$$\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\sigma_F}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right] = \frac{\frac{(1-\lambda-\rho)}{2\sigma_F\tau\mathbf{Var}[v|s]}\frac{\partial\sigma_F^2}{\partial\sigma_\eta^2}\left[\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}\left(1-\frac{2\sigma_F^2}{\mathbf{Var}[v|\kappa,\xi]}\right)+\lambda\left(b_p+2\sigma_F^2\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]}\right)\right]}{\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\right)^2} < 0.$$

In addition,

$$\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right] = -\frac{(\hat{\mu}-\underline{\mu})\frac{\partial\sigma_F^2}{\partial\sigma_\eta^2}\left[\frac{\lambda\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}\left(\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]}+\frac{b_p}{\mathbf{Var}[v|\kappa,\xi]}\right)+\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}\right)\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]^2}\right]}{\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\right)^2} > 0,$$

because $\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]}+\frac{b_p}{\mathbf{Var}[v|\kappa,\xi]} \geqslant 0$ from Lemma 1.

Then, when $\zeta < -\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$, we have:

$$\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right] > 0.$$

Therefore, when $\sigma_z^2 \to 0$, we have $\frac{\mathbf{E}[CE_N]}{\partial\sigma_\eta^2} < 0$.

(2) When $\sigma_z^2 \to +\infty$ and $\rho \to 1$.

Finally, we prove that $\frac{\mathbf{E}[CE_N]}{\partial\sigma_\eta^2} > 0$ by showing that:

$$-\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\frac{\frac{\partial\mathbf{Var}[v|\kappa,\xi]}{\partial\sigma_\eta^2}}{\mathbf{Var}[v|\kappa,\xi]}+2\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]}+\lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right] < 0,$$

when $\sigma_z^2 \to +\infty$ and ρ is large enough.

We first prove that:

$$-\left[\frac{\frac{(1-\lambda-\rho)\sigma_F}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\frac{\frac{\partial\mathbf{Var}[v|\kappa,\xi]}{\partial\sigma_\eta^2}}{\mathbf{Var}[v|\kappa,\xi]}+2\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\sigma_F}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right] > 0, \tag{A.25}$$

which is equivalent to:

$$\frac{1}{\sigma_F}+\frac{\frac{\sigma_F}{\mathbf{Var}[v|\kappa,\xi]}\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}-\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p+2\lambda\mathbf{Var}[v|\kappa,\xi]\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]}\right)}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p} < 0. \tag{A.26}$$

When $\rho \to 1$, because $b_p \leqslant b_{p,K=+\infty}$ and $\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]} \leqslant \frac{\partial b_{p,K=1}}{\partial\mathbf{Var}[v|\kappa,\xi]}$, from the proof of Lemma 1, we have:

$$\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}-\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p+2\lambda\mathbf{Var}[v|\kappa,\xi]\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]}$$

$$< \frac{\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} - \frac{\lambda+\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}\right)\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2 + \frac{2\lambda^2}{\tau\mathbf{Var}[v|\kappa,\xi]^2}\left(\frac{2(1-\lambda-\rho)}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)}{\left(\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]} + \frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}\right)^2} < 0.$$

Then, the inequality (A.26) surely holds as $\lim_{\sigma_z^2\to+\infty}\sigma_F = +\infty$.

Thus, when $\zeta < -\frac{\left(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$, we have:

$$-\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]} + \lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}\right]\frac{\frac{\partial\mathbf{Var}[v|\kappa,\xi]}{\partial\sigma_\eta^2}}{\mathbf{Var}[v|\kappa,\xi]} + 2\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{(1-\lambda-\rho)\left((\hat{\mu}-\underline{\mu})+\sigma_F\zeta\right)}{\tau\mathbf{Var}[v|s]} + \lambda b_p(\hat{\mu}-\underline{\mu})}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}\right]$$

$$< -\frac{2\frac{\left(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{2\sigma_F^2\tau\mathbf{Var}[v|s]}\frac{\partial\sigma_F^2}{\partial\sigma_\eta^2}\left[\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}\left(1-\frac{2\sigma_F^2}{\mathbf{Var}[v|\kappa,\xi]}\right) + \lambda\left(b_p + 2\sigma_F^2\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]}\right)\right]}{\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]} + \lambda b_p\right)^2}$$

$$-\frac{2(\hat{\mu}-\underline{\mu})\frac{\partial\sigma_F^2}{\partial\sigma_\eta^2}\left[\frac{\lambda\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}\left(\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]} + \frac{b_p}{\mathbf{Var}[v|\kappa,\xi]}\right) + \left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}\right)\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]^2}\right]}{\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]} + \lambda b_p\right)^2} < 0.$$

Therefore, $\frac{\mathbf{E}[CE_N]}{\partial\sigma_\eta^2} > 0$ when σ_z^2 and ρ are large enough. □

A.8.3. Welfare analysis of market makers

Proof. Given the equilibrium demand of market makers and equilibrium price in Theorem 2, by normalizing $F(\kappa,\xi)$ by ζ and taking integral over ζ, we obtain that the expected certainty equivalent of market makers ($\mathbf{E}[CE_M]$) equals to:

$$\mathbf{E}[CE_M] = 2CE_{M1} + 2CE_{M2} + 2CE_{M3},$$

where:

$$CE_{M1} = \int_{-\infty}^{-\frac{\left(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}} b_p\left(1-\frac{b_p}{2}\tau\mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]} + \frac{\rho(\hat{\mu}-\underline{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}\right]^2 d\Phi(\zeta),$$

$$CE_{M2} = \int_{-\frac{\left(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}^{-\frac{\left(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}\left[-\frac{(1-\lambda-\rho)\left(\hat{\mu}-\underline{\mu}\right)\left(\hat{\mu}-\underline{\mu}+\sigma_F\zeta\right)}{\lambda\tau\mathbf{Var}[v|s]} - \frac{1}{2}\mathbf{Var}[v|\kappa,\xi]\frac{(1-\lambda-\rho)^2\left(\hat{\mu}-\underline{\mu}+\sigma_F\zeta\right)^2}{\lambda^2\tau\mathbf{Var}[v|s]^2}\right]d\Phi(\zeta),$$

$$CE_{M3} = \int_{-\frac{\left(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}^{0} b_{np}\left(1-\frac{b_{np}}{2}\tau\mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \lambda b_{np}}\right]^2 d\Phi(\zeta).$$

By taking derivative of $\mathbf{E}[CE_M]$ over σ_η^2, we have:

$$\frac{\partial\mathbf{E}[CE_M]}{\partial\sigma_\eta^2} = 2\int_{-\infty}^{-\frac{\left(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}\frac{\partial}{\partial\sigma_\eta^2}\left\{b_p\left(1-\frac{b_p}{2}\tau\mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]} + \frac{\rho(\hat{\mu}-\underline{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]} + \lambda b_p}\right]^2\right\}d\Phi(\zeta)$$

$$+2\int_{-\frac{\left(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}^{-\frac{\left(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}\frac{\partial}{\partial\sigma_\eta^2}\left[-\frac{(1-\lambda-\rho)\left(\hat{\mu}-\underline{\mu}\right)\left(\hat{\mu}-\underline{\mu}+\sigma_F\zeta\right)}{\lambda\tau\mathbf{Var}[v|s]} - \frac{1}{2}\mathbf{Var}[v|\kappa,\xi]\frac{(1-\lambda-\rho)^2\left(\hat{\mu}-\underline{\mu}+\sigma_F\zeta\right)^2}{\lambda^2\tau\mathbf{Var}[v|s]^2}\right]d\Phi(\zeta)$$

$$+2\int_{-\frac{\left(1-\lambda-\rho+\lambda b_{np}\tau\mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}}^{0}\frac{\partial}{\partial\sigma_\eta^2}\left\{b_{np}\left(1-\frac{b_{np}}{2}\tau\mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]} + \lambda b_{np}}\right]^2\right\}d\Phi(\zeta)$$

(1) When $\sigma_z^2 \to 0$.

Because $\sigma_F \to 0$, $\frac{\partial \mathbf{E}[CE_M]}{\partial \sigma_\eta^2}$ equals to:

$$\frac{\partial \mathbf{E}[CE_M]}{\partial \sigma_\eta^2} = \frac{\partial}{\partial \sigma_\eta^2}\left\{ b_{np}\left(1-\frac{b_{np}}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F}{\tau \mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\lambda b_{np}}\right]^2\right\}$$
$$= \frac{(1-\lambda-\rho)^2}{\tau^2 \mathbf{Var}[v|s]^2}\left\{\frac{b_{np}\left(1-\frac{b_{np}}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)}{\left(\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\lambda b_{np}\right)^2}\frac{\partial \sigma_F^2}{\partial \sigma_\eta^2}+\sigma_F^2\frac{\partial}{\partial \sigma_\eta^2}\left[\frac{b_{np}\left(1-\frac{b_{np}}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)}{\left(\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\lambda b_{np}\right)^2}\right]\right\},$$

which is negative since $\frac{\partial \sigma_F^2}{\partial \sigma_\eta^2} < 0$. Therefore, $\frac{\partial \mathbf{E}[CE_M]}{\partial \sigma_\eta^2} < 0$ when σ_z^2 is small enough.

(2) When $\sigma_z^2 \to +\infty$ and $\rho \to 1$.

Because $\lim_{\sigma_z^2 \to +\infty} \sigma_F = +\infty$, $\frac{\partial \mathbf{E}[CE_M]}{\partial \sigma_\eta^2}$ is equal to:

$$2\int_{-\infty}^{-\frac{\left(1-\lambda-\rho+\lambda b_p \tau \mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}} \frac{\partial}{\partial \sigma_\eta^2}\left\{ b_p\left(1-\frac{b_p}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]^2\right\} d\Phi(\zeta).$$

We show that $\frac{\partial \mathbf{E}[CE_M]}{\partial \sigma_\eta^2} > 0$ by proving that:

$$\frac{\partial}{\partial \sigma_\eta^2}\left\{ b_p\left(1-\frac{b_p}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]^2\right\} > 0, \tag{A.27}$$

when $\zeta < -\frac{\left(1-\lambda-\rho+\lambda b_p \tau \mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$.

Note that:

$$\frac{\partial}{\partial \sigma_\eta^2}\left\{ b_p\left(1-\frac{b_p}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]^2\right\}$$
$$= \left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\left\{\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\frac{\partial}{\partial \sigma_\eta^2}\left[b_p\left(1-\frac{b_p}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)\right]\right.$$
$$\left.+\left(2b_p - b_p^2\tau \mathbf{Var}[v|\kappa,\xi]\right)\frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\right\}.$$

When $\zeta < -\frac{\left(1-\lambda-\rho+\lambda b_p \tau \mathbf{Var}[v|s]\right)(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$, we have:

$$\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p} > 0.$$

Then, the inequality (A.27) holds if:

$$\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\frac{\partial}{\partial \sigma_\eta^2}\left[b_p\left(1-\frac{b_p}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)\right]$$
$$+\left(2b_p - b_p^2\tau \mathbf{Var}[v|\kappa,\xi]\right)\frac{\partial}{\partial \sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau \mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau \mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right] > 0. \tag{A.28}$$

We first show that:

$$\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F}{\tau \mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau \mathbf{Var}[v|s]}+\frac{\rho}{\tau \mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\frac{\partial}{\partial \sigma_\eta^2}\left[b_p\left(1-\frac{b_p}{2}\tau \mathbf{Var}[v|\kappa,\xi]\right)\right]$$

$$+\left(2b_p - b_p^2\tau\mathbf{Var}[v|\kappa,\xi]\right)\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F}{\tau\mathbf{Var}[v|s]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]<0,$$

which is equivalent to:

$$\left(b_p - \frac{b_p^2}{2}\tau\mathbf{Var}[v|\kappa,\xi]\right)\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\right)$$
$$<\sigma_F^2\left\{b_p\left[\left(2-b_p\tau\mathbf{Var}[v|\kappa,\xi]\right)\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]^2}-\frac{b_p}{2}\tau\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\right)\right]\right.$$
$$\left.-\frac{\partial b_p}{\partial\mathbf{Var}[v|\kappa,\xi]}\left[\lambda b_p-\left(1-b_p\tau\mathbf{Var}[v|\kappa,\xi]\right)\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}\right)\right]\right\}. \tag{A.29}$$

From Lemma 1, we know that b_p is increasing with K, so we have:

$$\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left(1-\frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}}\right)\leqslant b_p\leqslant\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}.$$

Thus, we can obtain that:

$$\lambda b_p-\left(1-b_p\tau\mathbf{Var}[v|\kappa,\xi]\right)\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}\right)$$
$$=\tau\mathbf{Var}[v|\kappa,\xi]\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}\right)\left[b_p-\frac{1}{\tau\mathbf{Var}[v|\kappa,\xi]}\left(1-\frac{\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\mathbf{Var}[v|s]}+\frac{\lambda+\rho}{\mathbf{Var}[v|\kappa,\xi]}}\right)\right]\geqslant 0,$$

and when $\rho\to 1$,

$$\left(2-b_p\tau\mathbf{Var}[v|\kappa,\xi]\right)\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]^2}-\frac{b_p}{2}\tau\left(\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p\right)$$
$$\geqslant\frac{1}{2\mathbf{Var}[v|\kappa,\xi]}\left[\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\frac{\rho}{\tau\mathbf{Var}[v|s]}-\left(\frac{1-\lambda}{\tau\mathbf{Var}[v|s]}+\frac{\lambda}{\mathbf{Var}[v|\kappa,\xi]}\right)\right]>0.$$

Therefore, when σ_F^2 and ρ are large enough, inequality (A.29) holds.

When $\zeta<-\frac{(1-\lambda-\rho+\lambda b_p\tau\mathbf{Var}[v|s])(\hat{\mu}-\underline{\mu})}{(1-\lambda-\rho)\sigma_F}$, we have:

$$\frac{\partial}{\partial\sigma_\eta^2}\left\{b_p\left(1-\frac{b_p}{2}\tau\mathbf{Var}[v|\kappa,\xi]\right)\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]^2\right\}$$
$$>\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]\left(2b_p-b_p^2\tau\mathbf{Var}[v|\kappa,\xi]\right)\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right],$$

which is positive because $\frac{\partial}{\partial\sigma_\eta^2}\left[\frac{\frac{-(1-\lambda-\rho)\sigma_F\zeta}{\tau\mathbf{Var}[v|s]}+\frac{\rho(\hat{\mu}-\underline{\mu})}{\tau\mathbf{Var}[v|\kappa,\xi]}}{\frac{1-\lambda-\rho}{\tau\mathbf{Var}[v|s]}+\frac{\rho}{\tau\mathbf{Var}[v|\kappa,\xi]}+\lambda b_p}\right]>0$ when $\sigma_z^2\to+\infty$.

Therefore, when σ_z^2 and ρ are large enough, we have $\frac{\partial CE_M}{\partial\sigma_\eta^2}>0$. □

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.finmar.2022.100761.

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