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CAPM and Skewness Pricing Under Probability Weighting: Based on the Generalised Wang Transform

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ABSTRACT

In this paper, we examine the conditions under which the capital asset pricing model (CAPM) holds with heterogeneous probability weighting. Using the generalised Wang transform within rank-dependent expected utility, we show that CAPM holds for heterogeneous risk-averse investors, while the security market line theorem (SMLT) applies to heterogeneous loss-averse investors. However, CAPM under loss aversion requires homogeneous investors. Revisiting skewness pricing, we find that probability weighting, rather than the S-shaped value function, drives skewness overpricing. The preference for skewed assets stems from the high distorted mean under probability weighting.

JEL Classification: G12, G40

1 | Introduction

Probability weighting, also referred to as probability distortion, is a key aspect that distinguishes non-expected utility theory from traditional expected utility theory. This concept explains how individuals assign decision weights to events rather than relying solely on objective ones. Experimental studies consistently show that individuals tend to overweight the probabilities of extreme events while underweighting those of moderate events (Kahneman and Tversky 1979; Tversky and Kahneman 1992; Dimmock et al. 2021). In financial markets, probability weighting has been linked to various asset pricing anomalies, such as skewness pricing, under-diversification in household portfolios, and the equity premium puzzle (Polkovnichenko 2005; Barberis and Huang 2008; Bordalo et al. 2013; Dimmock et al. 2021).

A crucial question when applying non-expected utility to finance is whether the Capital Asset Pricing Model (CAPM), a fundamental framework in asset pricing, remains valid under probability distortion. In their seminal work, Barberis and Huang (2008) show that CAPM holds within the cumulative prospect theory (CPT) framework, especially when investors' preferences align with the value function and weighting function introduced by Tversky and Kahneman (1992). However, two important issues have often been overlooked in Barberis and Huang (2008) and subsequent studies. First, most research assumes homogeneous probability weighting across individuals, although several studies have revealed significant heterogeneity in weighting functions (Gonzalez and Wu 1999; Bleichrodt and Pinto 2000; Bruhin et al. 2010; Dimmock et al. 2021; Andrikogiannopoulou and Papakonstantinou 2021). For example, Bruhin et al. (2010) find that nearly 20% of individuals weight probabilities linearly and 80% do so non-linearly, demonstrating considerable variation within the probability distortion groups. Second, individuals may exhibit diverse value functions and do not consistently follow the S-shaped value function. Levy et al. (2003) employ the stochastic dominance approach to test the S-shaped value function, finding that at least 50%–86% of the choices reject the assumption of the S-shaped value function. Similarly, Malul et al. (2013) find that only one-third of the individuals behave as predicted by the S-shaped value function. Given these findings, the heterogeneity in both probability weighting functions and value functions should be considered when testing the validity of CAPM.

This paper aims to contribute to the literature by developing a CAPM that incorporates heterogeneous probability weighting and varied value functions. To address the aforementioned issues, our model introduces three distinct features that differentiate it from previous research. First, we employ the generalised Wang transform (GWT), proposed by Sun et al. (2023), as the probability weighting function, providing greater flexibility in capturing diverse probability weighting shapes and enhanced tractability in theoretical models compared to conventional functions (Tversky and Kahneman 1992; Prelec 1998). By adjusting its parameters, GWT accommodates both linear and non-linear probability weighting, including S-shaped, inverse S-shaped, and diagonal weighting functions. The unique normality invariance property of GWT ensures that asset returns remain normally distributed after distortion, a feature critical for extending CAPM. Second, we account for heterogeneity in value functions and incorporate reference points and loss aversion without imposing specific functional forms, thereby preserving generality. Our model does not prescribe a particular structure for the value function; instead, it assumes that investors exhibit greater sensitivity to changes in the loss domain than in the gain domain. This flexibility allows our model to accommodate a wide range of value functions within the framework of non-expected utility. Third, we adopt the rank-dependent expected utility (RDEU) rather than CPT as our framework, allowing us to leverage the GWT's normality invariance while maintaining core elements of prospect theory, such as loss aversion and probability weighting. Although our model operates within an alternative framework, our results offer insights into CAPM under prospect theory. We find that when investors exhibit risk aversion and hold heterogeneous probability weighting, the CAPM can still hold. However, for loss-averse investors, while the Security Market Line Theorem (SMLT) remains valid under heterogeneous probability weighting functions and value functions, CAPM requires homogeneity in both functions. This suggests a substitution effect between risk attitude and probability attitude. Our results shed light on the mechanism proposed by Yaari (1987), indicating that probability weighting endogenously represents the risk attitude in preferences.

Additionally, as an extension of the results, our paper relaxes the assumption of normal distribution for asset returns and revisits skewness pricing. Building on our CAPM model, we employ GWT as the probability weighting function and consider both risk-averse and loss-averse investor models. Our study contributes to the existing literature in three key ways. First, we demonstrate that the preferences for skewness are primarily driven by probability weighting rather than the shape of the value function. Our results show that even for risk-averse investors, probability weighting will lead to the overpricing of skewed assets. Second, we analyse the different effects of likelihood insensitivity and probability attitudes (optimism/pessimism) on skewness pricing. We find that optimistic attitudes amplify the overpricing of positively skewed securities, whereas sufficiently pessimistic attitudes can rationalise pricing, even in the presence of high probability distortion. Finally, we compare the original and transformed distributions of the market portfolio and the skewed portfolio, finding that the overpricing of skewed assets is driven by the high distorted mean of the skewed portfolio.

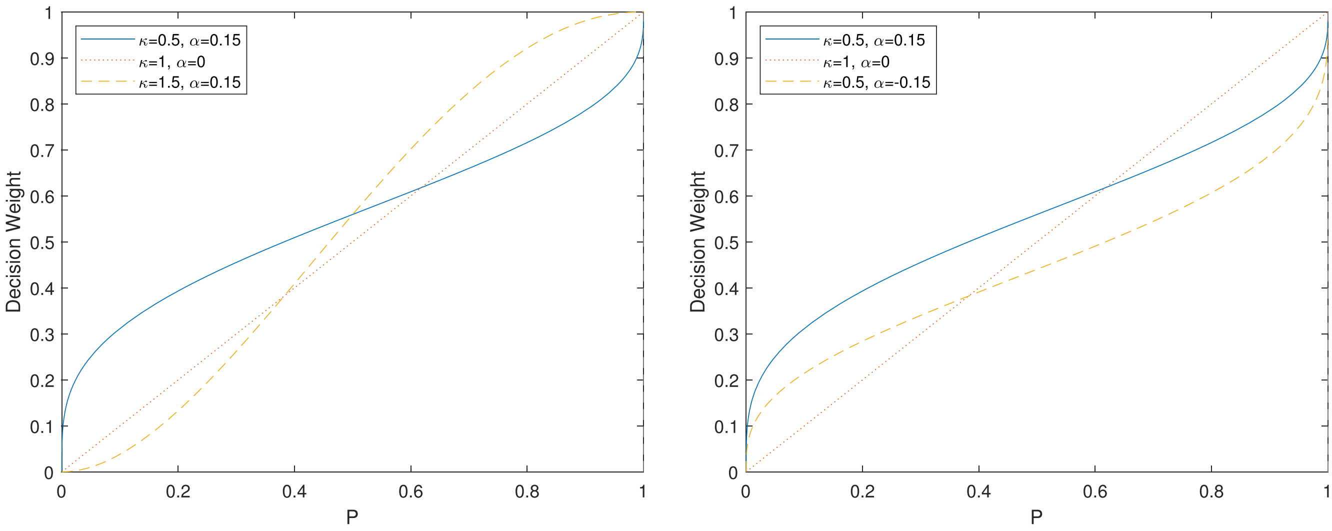
This paper mainly contributes to two strands of literature. The first thread is the theoretical studies in asset pricing within non-expected utility theories. Prior research focuses on whether and how the pricing principle can be affected by the behaviour assumptions proposed by prospect theory (Levy et al. 2003; Levy and Levy 2004; Barberis and Huang 2008; Driessen et al. 2021). These studies suggest that the CAPM, or SMLT, can be compatible with probability weighting under specific weighting functions and value functions. However, there is no clear answer to the conditions required for the CAPM to hold under probability distortion. Our paper extends this line of research by employing the GWT as the probability weighting function in the CAPM. We show that with multivariate normally distributed securities, the CAPM can hold with risk-averse investors with heterogeneous weighting and value functions. Additionally, we demonstrate that the SMLT can hold under heterogeneous probability weighting and loss-averse value functions, nesting the CPT value function as a special case.

Second, our paper adds to the literature on skewness pricing. Empirical studies have consistently shown that investors favour positively skewed, lottery-like securities (Mitton and Vorkink 2007; Kumar 2009; Boyer et al. 2010; Bali et al. 2011; Blau et al. 2020; Jiang et al. 2020). This phenomenon has been interpreted through different perspectives, such as prospect theory (Brunnermeier and Parker 2005; Brunnermeier et al. 2007; Barberis and Huang 2008), salience theory (Bordalo et al. 2013) and social network features and transmission bias (Han et al. 2022). Compared to prior literature, our paper highlights the influence of probability weighting over the value function in explaining the overpricing of skewness. Of the two main components of probability weighting—likelihood sensitivity and probabilistic attitudes, we find that the latter factor, especially the optimistic attitude, contributes to the skewness pricing. Lastly, we attribute the preference for skewness to the differences between the original asset distribution and the transformed distribution after probability weighting.

The structure of the paper is as follows. Section 2 introduces the probability weighting function GWT and the framework of our RDEU model, and discusses the differences and similarities between our model and CPT. Section 3 presents the proof of CAPM with GWT as the probability weighting function. Section 4 revisits the overpricing of skewed securities under the framework provided in Section 3. Section 5 concludes the paper.

2 | Theoretical Background

In this section, we present the theoretical background of the paper. In Section 2.1, we introduce a new class of



(a) Curvature: κ (b) Elevation: α

FIGURE 1 | These figures illustrate how GWT changes with variations in κ and α. Figure (a) shows an inverse S-shaped weighting function when $\kappa < 1$ and an S-shaped weighting function when $\kappa > 1$, with a lower κ resulting in more overweighting at probabilities near 0 and 1. Figure (b) demonstrates that an increase in α elevates the function across all probabilities. As depicted in both figures, the function becomes linear when $\kappa = 1$ and $\alpha = 0$. [Colour figure can be viewed at wileyonlinelibrary.com]

distortion operators, the Generalised Wang Transformation. In Section 2.2, we review the classic model RDEU and incorporate the key elements of cumulative prospect theory (CPT) into the model.

2.1 | Generalised Wang Transform

Identifying a probability weighting function that accurately captures individuals' sensitivity to probabilities has been a central issue in NEU. One widely recognised characteristic of probability weighting is differential sensitivity to extreme versus moderate events, known as likelihood insensitivity. Although the inverse S-shaped probability weighting function proposed by Tversky and Kahneman (1992) has gained significant support, some studies indicate that individuals can also exhibit S-shaped probability distortions (Humphrey and Verschoor 2004; Polkovnichenko and Zhao 2013). The inverse S-shaped weighting suggests higher sensitivity to extreme events compared to intermediate ones, whereas the S-shaped weighting implies the opposite. Another important aspect of probability weighting is the asymmetric distortion of extreme events. For instance, in lottery scenarios, individuals often overweight the best possible outcomes relative to the worst, while in the context of significant risks, they tend to overweight the worst outcomes over the best. This characteristic, known as probability attitudes, was defined by Quiggin (1982) and further examined by Abdellaoui et al. (2010). Specifically, probability attitudes are defined as follows:

Definition 1. (*Probability attitude*). If, for all p, a weighting function $g(\cdot)$ satisfies $g(p) > (<) 1 - g(1-p)$, an individual is pessimistic (optimistic) since the worst (best) outcomes are, on average, overweighted.

In this paper, we use the generalised Wang transform (GWT) as the probability weighting function to characterise both likelihood sensitivity and probability attitudes in probability weighting. GWT, which extends the original Wang transform (Wang 2000), was first introduced by Wang (2002) and thoroughly analysed by Sun et al. (2023). The GWT can be expressed as follows:

$$g_{\kappa,\alpha}(\cdot) = \Phi\left(\kappa\Phi^{-1}(\cdot) + \alpha\right) \tag{1}$$

where, $\kappa \in (0, +\infty)$, $\alpha \in \mathbb{R}$, and $\Phi(\cdot)$ represents the cumulative distribution function of standard normal distribution.

GWT has two parameters, κ and α, which control its curvature and elevation, respectively (see Figure 1). These parameters provide flexibility in the shape of the probability weighting function, capturing likelihood insensitivity and probabilistic attitude independently.

The curvature parameter, κ, reflects likelihood insensitivity by indicating the degree of probability distortion between extreme and moderate events. A value of $\kappa < 1$ results in overweighting extreme events and underweighting moderate ones, producing an inverse S-shaped probability weighting function. In contrast, a value of $\kappa > 1$ leads to underweighting extreme events and overweighting moderate events, yielding an S-shaped weighting function. When $\kappa = 1$ and $\alpha = 0$, the GWT forms a diagonal line, representing no probability distortion. Figure 1a illustrates the GWT for different values of κ.

The elevation parameter, α, represents the probabilistic attitude, ranging from optimism to pessimism. As α increases, the decision weights on the probability of the worst outcomes become higher than that of the best outcomes (see Figure 1b).

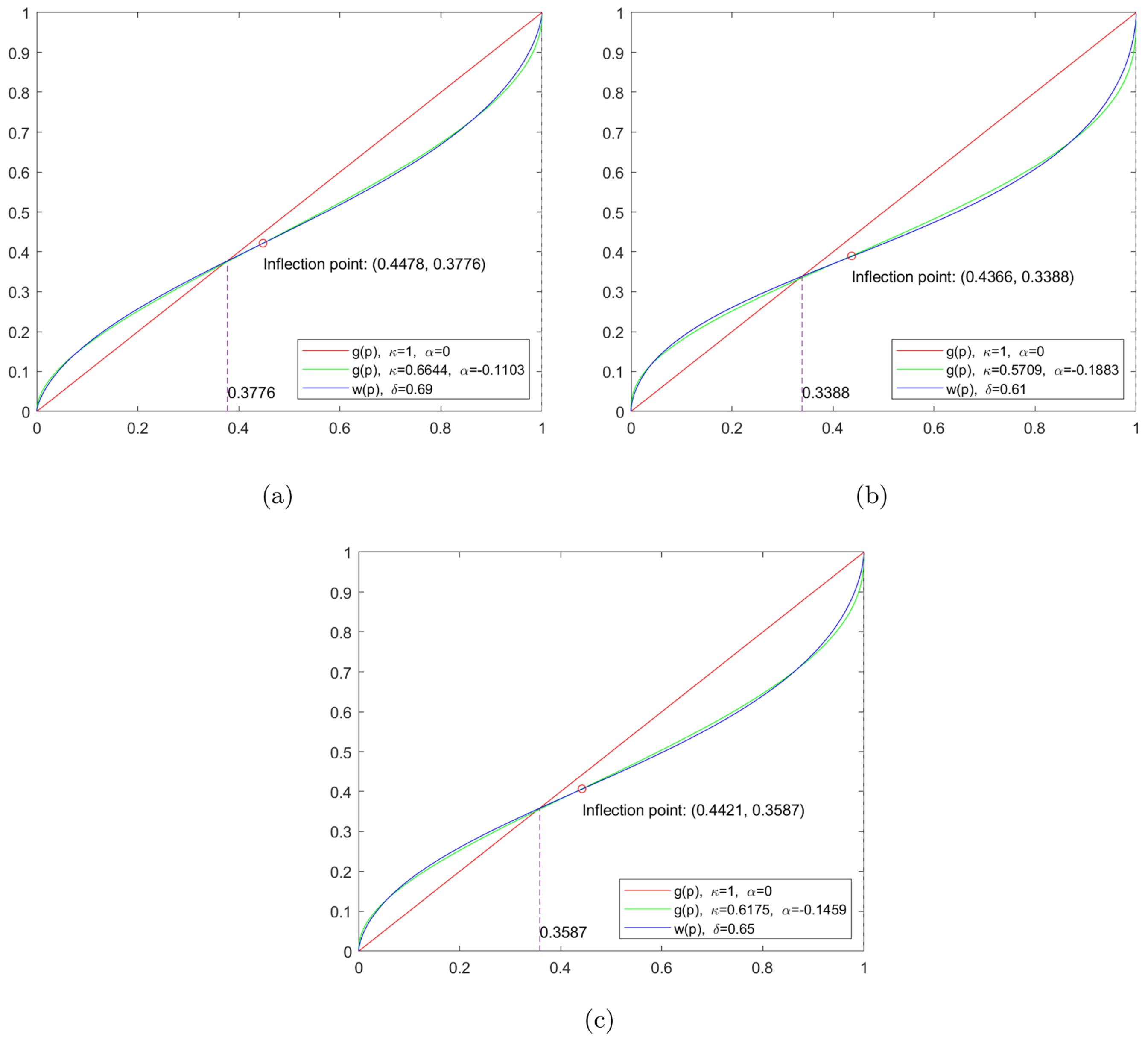


FIGURE 2 | These figures present the scenario where the GWT fits the probability weighting function $w(p)=\frac{x^{\delta}}{\left[x^{\delta}+(1-x)^{\delta}\right]^{\frac{1}{\delta}}}$ in Cumulative Prospect Theory (CPT). We choose the appropriate parameters κ and α to minimise $\int_0^1 \left(g_{\kappa,\alpha}(p)-w(p)\right)^2 dp$. [Colour figure can be viewed at wileyonlinelibrary.com]

According to Definition 1, a negative α indicates an optimistic probabilistic attitude, while a positive α corresponds to a pessimistic attitude. When $\alpha = 0$, the probability attitude is considered neutral.

A unique feature of the GWT, distinguishing it from other probability weighting functions, is its normality invariance; specifically, GWT transforms any normal distribution into another normal distribution. This property is formalised in Proposition 1.

Proposition 1. (*Normality invariance*). *If $X \sim \mathbf{N}\left(\mu, \sigma^2\right)$, the cumulative distribution function of the random variable X^* is given by $F_{X^*}(x) = g_{\alpha,\kappa}\left(F_X(x)\right)$, then X^* is also normally distributed $X^* \sim \mathbf{N}\left(\mu - \frac{\alpha}{\kappa}\sigma, \frac{\sigma^2}{\kappa^2}\right)$.*

Proposition 1 grants GWT a computational advantage over traditional probability weighting functions in financial analyses where normality is a common assumption. In our model, this property facilitates the derivation of closed-form solutions. Sun et al. (2023) provide an in-depth discussion of the properties of GWT.

Additionally, with specific values for the parameters α and κ, GWT can closely approximate distortion functions established in the classic literature, such as those by Tversky and Kahneman (1992). Figure 2 illustrates how the GWT model replicates the probability weighting function of CPT, that is, $w(p)=\frac{x^{\delta}}{\left[x^{\delta}+(1-x)^{\delta}\right]^{\frac{1}{\delta}}}$, proposed in previous literature. Figure 2a,b shows the fitting of the GWT model to the probability weighting function suggested in Tversky and Kahneman (1992) for the loss domain ($\delta = 0.69$) and the gain domain ($\delta = 0.61$), respectively. As depicted, the intersection of the GWT function with the diagonal line falls within the interval (0.3,0.4). To facilitate comparisons with the findings in Barberis and Huang (2008), we adopt $\delta = 0.65$ in subsequent sections. Figure 2c presents the fitting with $\delta = 0.65$, corresponding to parameter values $\kappa = 0.6175$ and $\alpha = -0.1459$.[1]

2.2 | A Review and Discussion of RDEU

In this paper, we adopt the Rank-Dependent Expected Utility (RDEU) model as proposed by Quiggin (1982). To provide a clearer understanding, we begin by reviewing the fundamental structure of the RDEU model. Suppose an agent i is evaluating a gamble $\mathbb{X}$ with support (a, b), where $-\infty \leqslant a < b \leqslant +\infty$. The cumulative and decumulative distribution functions of $\mathbb{X}$ are denoted by $F_{\mathbb{X}}(\cdot)$ and $\overline{F}_{\mathbb{X}}(\cdot)$, respectively. Agent i's sensitivities to outcomes and probabilities are described by a continuously increasing utility function $u_i: \mathbb{R} \to \mathbb{R}$ and a non-decreasing, continuous probability weighting function $g_i: [0,1] \to [0,1]$, respectively, with $g_i(0) = 0$ and $g_i(1) = 1$. The RDEU of the gamble $\mathbb{X}$ for the agent i is then given by:

$$RDEU_i[\mathbb{X}] = \int_a^b u_i(x) dg_i\left(F_{\mathbb{X}}(x)\right) \quad (2)$$

In the subsequent sections, we adopt GWT as the probability weighting function for agents, allowing for heterogeneity in the probability distortion operator of different investors. Regarding utility functions, we consider two scenarios: (1) investors are risk-averse (RA), and (2) investors are loss averse (LA). The preferences of risk-averse investors are characterised as follows:

$$RDEU_i^{RA}[\mathbb{X}] = \int_a^b u_i(x) dg_{\alpha_i, \lambda_i}\left(F_{\mathbb{X}}(x)\right) \quad (3)$$

where, $u_i(x)$ is increasing and strictly concave. Regarding the preferences of loss-averse investors, we need to incorporate certain elements of the Cumulative Prospect Theory (CPT). We replace the utility function with a value function to incorporate key elements of CPT, specifically reference points and loss aversion. We define $\widehat{\mathbb{X}} \equiv \mathbb{X} - x_0$ as the relative gain and loss, where x_0 represents the reference point such that $a \leqslant x_0 \leqslant b$. Let $v_i(x)$ denote the value function of agent i, noting that agents may have distinct value functions. The adapted RDEU of the gamble $\mathbb{X}$ for the agent i is as follows:

$$RDEU_i^{LA}\left[\widehat{\mathbb{X}}\right] = \int_a^b v_i(x) dg_{\alpha_i, \lambda_i}\left(F_{\widehat{\mathbb{X}}}(x)\right) \quad (4)$$

The advantage of the RDEU model is that it addresses violations of first-order stochastic dominance, a common issue in traditional probability weighting models. Additionally, the RDEU model applies a single probability weighting function across both loss and gain domains, avoiding the need for separate functions. This unified approach allows us to leverage the normality invariance of GWT, facilitating the derivation of closed-form solutions in the model. Specifically, if the gamble $\mathbb{X}$ is normally distributed, the distorted cumulative distribution function $g_{\kappa,\alpha}\left(F_{\mathbb{X}}(x)\right)$ will also be normal, as will $g_{\kappa,\alpha}\left(F_{\widehat{\mathbb{X}}}(x)\right)$.

3 | CAPM Under Probability Weighting

The Capital Asset Pricing Model (CAPM) is regarded as one of the most essential models in finance theory. However, its underlying assumptions, particularly the Expected Utility (EU) hypothesis, have been widely debated. A key question in behavioural finance is whether CAPM holds when the EU assumption is violated. In this section, we examine CAPM under probability weighting by applying the GWT as the probability weighting function. Additionally, we introduce heterogeneity in both probability weighting functions and value functions across investors and consider two scenarios: first, where investors are risk-averse toward all outcomes, and second, where they are loss-averse. Security payoffs are assumed to be multivariate normally distributed, an assumption that will be relaxed in Section 4.

3.1 | Risk-Averse Investors With Probability Weighting

We begin by retaining most of the traditional CAPM assumptions as outlined by Sharpe (1964), Lintner (1965), and Mossin (1966), including a one-period investment horizon, risk-averse investors, and multivariate normally distributed asset payoffs, except the assumption of homogeneous beliefs. We allow investors' preferences to be represented by RDEU with varying probability weighting functions. This approach enables investors to hold diverse expectations about risky securities.

We consider a one-period economy consisting of two dates, date 0 and date 1. All investors trade at $t = 0$ and consume at $t = 1$. The economy is frictionless and has no trading constraints. There is one risk-free asset with a constant return R_f and J risky assets. There are $i = 1, \dots, I$ investors in the market.

Assumption 1. *Asset return*. The return of the jth risky security in the market at date 1 is denoted as $\widetilde{R}_j$. We denote $\widetilde{R} \equiv \left(\widetilde{R}_1, \widetilde{R}_2, \cdots, \widetilde{R}_J\right)^\top$. The return vector of risky assets $\widetilde{R}$ is multivariate normally distributed, that is $\widetilde{R} \sim N(\mu, \Sigma)$, where $\mu \equiv \left(\mu_1, \mu_2, \cdots, \mu_J\right)^\top$ with μ_j as the *objective* mean of $\widetilde{R}_j$, and Σ represents the *objective* variance–covariance matrix. The securities are highly divisible and liquid, and the supply of each risky security is strictly positive.

Assumption 2. *Risk aversion*. All investors are strictly risk-averse. The utility function $u_i(\cdot)$ of investor i is increasing and strictly concave.

Our approach diverges from the traditional CAPM by incorporating probability weighting into investor behaviour. Unlike the classic model, which assumes homogeneous beliefs and that investors form expectations of terminal wealth based on objective risk distributions, our model posits that investors evaluate prospects using distorted probabilities rather than objective ones. This shift acknowledges the role of subjective probability assessments in shaping investment decisions.

Assumption 3. *Probability weighting and heterogeneous perceptions*. The decision weights over the terminal wealth of investor i are determined by the probability weighting function in the form of GWT in Equation (1) with parameters $\kappa_i > 0$ and $\alpha_i \in \mathbb{R}$. These weighting functions may vary across investors and are not necessarily identical for all.

Assumption 3 allows investors to distort probabilities in different ways. Under Assumption 3, investors are not required to hold identical, accurate beliefs about the returns on risky assets. Instead, they may make decisions based on a perceived distribution that does not necessarily match the actual distribution.

Each investor is endowed with a strictly positive initial wealth $W_{0i} > 0$. Let $\widetilde{W}_i$ denote the terminal wealth of investor i. Investor i's investment strategy is given by the wealth allocation vector $w_i \equiv \left(w_{i1}, w_{i2}, \cdots, w_{iJ}\right)^{\top}$, where w_{ij} denotes portion of wealth allocated to the jth risky security by investor i. Thus, the final wealth of investor i can be expressed as

$$\widetilde{W}_i = W_{0i}\left(1 + R_f\right) + w_i^{\top}\left(\widetilde{R} - R_f\mathbf{1}\right) \tag{5}$$

Investor i needs to select the optimal investment strategy w_i to maximise her rank-dependent utility. Accordingly, the optimization problem is formulated as follows:

$$\max_{w_i} \; RDEU_i^{RA}\left[\widetilde{W}_i\right] = \int_{-\infty}^{\infty} u_i(x) dg_{\kappa_i,\alpha_i}\left(F_{\widetilde{W}_i}(x)\right) \tag{6}$$

where, $F_{\widetilde{W}_i}(\cdot)$ represents the cumulative distribution function of $\widetilde{W}_i$.

By solving the optimisation problem, we demonstrate in Theorem 1 that CAPM still holds.

Theorem 1. *Under the Assumptions 1–3, CAPM holds, that is*

$$E\left(\widetilde{R}_j\right) - R_f = \beta_j\left[E\left(\widetilde{R}_M\right) - R_f\right], \quad j = 1, \ldots, J$$

where, $\beta_j \equiv \frac{\mathrm{Cov}\left(\widetilde{R}_j, \widetilde{R}_M\right)}{\mathrm{Var}\left(\widetilde{R}_M\right)}$, *and* $\widetilde{R}_M$ *denotes the market return.*

Remark 1. Theorem 1 does not require the weighting function to follow an inverse-S shape. Depending on the value of the parameter κ, our model's weighting function can exhibit various forms: an inverse-S shape ($0 < \kappa < 1$), be linear ($\kappa = 1, \alpha = 0$), concave ($\kappa = 1, \alpha > 0$), convex ($\kappa = 1, \alpha < 0$), or even an S shape ($\kappa > 1$).

Remark 2. Theorem 1 does not impose any specific requirement on α_i, which represents the probabilistic attitude. The parameter α_i can be either negative or positive, indicating that an investor assigns greater weight to either the best or worst outcomes, respectively. Additionally, investors can hold heterogenous probabilistic attitudes, allowing for a scenario where some investors maintain an optimistic attitude while others adopt a pessimistic one.

Remark 3. Specifically, assuming $\kappa_i = 1$ yields the CAPM under the Wang transform, while setting $\alpha_i = 0$ reduces Theorem 1 to the traditional CAPM. Our framework encompasses both as special cases.

Theorem 1 extends the traditional CAPM by relaxing the assumptions of homogeneous beliefs and replacing objective probabilities with distorted probabilities. Despite investors' altered perceptions due to probability weighting, CAPM remains valid. Furthermore, there are no restrictions on the parameters of GWT; in other words, both the shape and the probabilistic attitudes represented by GWT are unrestricted. Each investor can overweight or underweight the tail event and may adopt either an optimistic or pessimistic probability attitude. Therefore, Theorem 1 demonstrates that under the assumption of normality in asset returns and risk-averse investors whose perceptions of final wealth remain normally distributed, CAPM holds regardless of how probabilities are distorted.

The reason CAPM still holds is that the market portfolio remains unchanged after the probability distortion introduced by GWT and the efficient frontier is also preserved. Given the expression of GWT in Equation (1), it can be verified that, for a given level of risk, a portfolio with a higher return than others will still achieve the highest transformed expected return for the defined level of transformed risk. When combined with a risk-free asset, this result implies the same tangent line, as the portfolios along this line continue to outperform others, even under the transformed distribution. Thus, the market portfolio, or the tangent portfolio, is uniquely determined, leading to CAPM.

3.2 | Loss-Averse Investors With Probability Weighting

In this subsection, we further relax the assumption of risk-averse investors by replacing the utility function with a value function that incorporates key features of the value function in CPT, including the reference point and loss aversion. We allow flexibility in the functional form of the value function to preserve generality.

We consider the same economic setting as in the risk-aversion case. The setup for investors is modified as follows: Investor i's preference is characterised by their value function $v_i(x)$ and probability weighting function $g_{\kappa_i,\alpha_i}(\cdot)$. The value function $v_i(\cdot)$ captures the investor's loss aversion, as presented in the following assumption:

Assumption 2′. *Loss aversion*. The value function of investor i, that is, $v_i(\cdot)$, is increasing and concave over the gain, and satisfies that $v_{i'}(-x) \geqslant v_{i'}(x)$ for $x \in (0, \infty)$ and $v_i(0) = 0$.

Assumption 2′ establishes the framework for investors' loss aversion. When investors are loss-averse, they experience greater pain from a loss than pleasure from an equivalent gain. As a result, investors display higher sensitivity to losses than to gains of the same magnitude, leading to a steeper slope in the loss domain compared to the gain domain in the value function. Notably, risk aversion is a special case under Assumption 2′.

In addition, Assumption 2′ implies that investors take zero as the reference point. If the reference point were not zero, it could be shifted to zero through transformations in the RDEU framework. Moreover, Assumption 2′ does not request a specific form on the value function, allowing it to accommodate most value functions in NEU that characterise loss aversion, such as the S-shaped value function in CPT.

Moreover, we introduce an additional assumption regarding the investors' probabilistic attitudes.

Assumption 4. *Pessimistic or neutral probabilistic attitude.* Investors hold pessimistic or neutral probabilistic attitudes, that is, $\kappa_i > 0$ and $\alpha_i \geqslant 0$ for $i = 1, \cdots, I$.

Assumption 4 presents the setup for investors' probability attitudes. We assume that investors' elevation parameter, denoted by α, is non-negative. This implies that investors assign greater weight to worst outcomes, exhibiting a form of probabilistic pessimism. While Assumption 2′ relaxed the requirements for the value function, here we impose stricter conditions on the probability weighting function.

Because the utility is defined over gains and losses, we denote the measure of gain and loss as $\widehat{W}_i$ as follows

$$\widehat{W}_i \equiv \widetilde{W}_i - W_{0i}\left(1 + R_f\right)$$

where, $W_{0i}\left(1 + R_f\right)$ is the reference wealth level.

The investor needs to select her optimal trading strategy w_i to maximise her utility

$$\max_{w_i} V_i\left(\widehat{W}_i\right) \equiv RDEU_i^{LA}\left[\widehat{W}\right] = \int_{-\infty}^{\infty} v_i(x) dg_{\kappa_i,\alpha_i}\left(F_{\widehat{W}_i}(x)\right) \quad (7)$$

where, $F_{\widehat{W}_i}(x)$ represents the cumulative distribution function of $\widehat{W}_i$.

Given the above assumptions, the adapted RDEU of the final wealth $\widehat{W}_i$ can be represented as follows.[2]

$$V_i\left(\widehat{W}_i\right) = v_i\left(\widehat{W}_{0i}\right) - \int_{-\infty}^{\widehat{W}_{0i}} g_{\kappa_i,\alpha_i}\left(F_{\widehat{W}_i}(t)\right) dv_i(t) + \int_{\widehat{W}_{0i}}^{\infty} \left[1 - g_{\kappa_i,\alpha_i}\left(F_{\widehat{W}_i}(t)\right)\right] dv_i(t), \quad \widehat{W}_{0i} \in \mathbb{R} \quad (8)$$

Equation (8) resembles the Choquet utility in Schmeidler (1986, 1989), suggesting how expected utility can be represented with nonadditive probability. The position of $\widehat{W}_{0i}$ does not affect the utility, indicating that a random choice of reference point is feasible in our model.

With the alternative Assumption 2′, we prove the Security Market Line Theorem for loss-averse investors in Theorem 2 (see proof in Appendix A.2).

Theorem 2. *Under the Assumptions 1, 2′, 3, 4, and the utility expression in Equation (8), if $\alpha_i = 0$ and $v_{i'}(-x) = v_{i'}(x)$ for $x \in (0, \infty)$ do not hold simultaneously, Security Market Line Theorem (SMLT) stands, that is:*

$$E\left(\widetilde{R}_j\right) - R_f = \beta_j\left[E\left(\widetilde{R}_M\right) - R_f\right], \quad j = 1, \ldots, J$$

where, $\beta_j \equiv \frac{\mathrm{Cov}\left(\widetilde{R}_j, \widetilde{R}_M\right)}{\mathrm{Var}\left(\widetilde{R}_M\right)}$, and $\widetilde{R}_M$ denotes the market return.

Remark 4. Theorem 2 suggests that investors with heterogeneous probability weightings under GWT agree on the same mean–variance efficient frontier, thereby implying the existence of SMLT in equilibrium. However, unlike Theorem 1, the optimization problem for Theorem 2 is not formally defined, and the investor's decision problem may lack a solution. As a result, the CAPM does not hold.[3]

Remark 5. The requirement $\alpha_i \geqslant 0$ *for* $i = 1, \cdots, I$ is introduced in Theorem 2. This implies that all investors are required to assign greater weight to potentially worst outcomes than to the best outcomes. Since Theorem 2 differs from Theorem 1 only by relaxing the risk-aversion assumption, the additional requirement on probability attitudes serves as a compensatory adjustment. This suggests that a pessimistic probabilistic attitude functions similarly to risk aversion in deriving SMLT.

In classical literature, the value function in CPT captures the characteristic of loss aversion among investors, whereas the utility function in Rank-Dependent Expected Utility (RDEU) assumes risk aversion. By incorporating the concepts of loss aversion and reference points into RDEU, we find that the SMLT still holds. This result extends the findings of Levy et al. (2003), who proved the SMLT under an S-shaped value function. Notably, Theorem 2 requires the elevation parameter in GWT to be non-negative, whereas Theorem 1 imposes no such restriction. This is because, when the concavity of the value function is restricted to the gain domain only, the probabilistic attitude must become more pessimistic. The finding implies that a pessimistic probabilistic attitude and a risk-averse attitude are interchangeable, consistent with the dual theory.

The SMLT holds under heterogenous value functions; however, the CAPM necessitates additional assumptions on the value function.

Assumption 2″. The value function of all investors, that is, $v(\cdot)$, is increasing and concave over the gain, and satisfies that $v'(-x) \geqslant v'(x)$ for $x \in (0, \infty)$ and $v(0) = 0$. In addition, the value function is divisible, that is:

$$v\left(a\hat{R}\right) = m(a)v\left(\hat{R}\right)$$

where, a is a constant and $m(a)$ is increasing with a.

Assumption 5. *Homogeneous investors*. All investors are homogeneous. That is, they share the same utility function or value function and have the same parameters in GWT as the probability weighting function.

Corollary 1. *Under the Assumptions 1, 2″, 3, 4, and 5, and the utility expression in Equation (8), CAPM holds with excess market return* $\hat{R}_M \equiv \tilde{R}_M - R_f$ *satisfying* $V\left(\hat{R}_M\right) = 0$. *In addition, when* $\alpha = 0$, $v'(-x) > v'(x)$ *for* $x \in (0, \infty)$; *when* $v'(-x) = v'(x)$ *for* $x \in (0, \infty)$, $\alpha > 0$.

Corollary 1 suggests that when the value function is divisible and homogeneous, CAPM holds. Barberis and Huang (2008) proved CAPM using the value function and weighting functions proposed by Tversky and Kahneman (1992). In this paper, we show that CAPM can also hold under RDEU with GWT without imposing a specific functional form on the value function. However, the homogenous assumption on the value function cannot be relaxed.

3.3 | Implications of CAPM With Generalised Wang Transform

Theorems 1 and 2 prove that CAPM and SMLT remain valid under heterogeneous beliefs. Compared to the traditional CAPM, our paper indicates that such pricing principles do not necessarily rely on homogeneous correct beliefs among all investors. Theorem 1 demonstrates that CAPM will hold, and the probability weighting function can adopt various shapes and heterogeneous probabilistic attitudes, provided that investors are risk-averse. Therefore, our model not only nests the traditional CAPM as a special case but also extends its applicability to scenarios with probability weighting. Theorem 2 further shows that when investors are loss-averse, SMLT holds as long as they maintain a pessimistic probabilistic attitude. Compared to Theorem 1, Theorem 2 relaxes the risk-aversion assumption while imposing a stricter condition on the probability weighting function. This aligns with the dual theory proposed by Yaari (1987), which suggests that the utility function and probability weighting function both capture the risk attitudes of people and can be interchangeable.

A critical assumption in our model is that the objective distributions of asset returns are normally distributed, and the probability weighting function takes the form of GWT. These assumptions are equivalent to stating that both the objective and the transformed distributions of asset returns follow a normal distribution. This is because GWT is the only probability weighting function that preserves normality after distortion. Therefore, if both the original and distorted distributions are normal, the probability weighting function must necessarily take the form of GWT. Notably, GWT can also be applied to scenarios where the objective distribution is not normal, which will be discussed in the next section.

4 | Preference for Skewness: Pricing Implications and Transformed Distributions

In this section, we relax the assumption of normal distribution and attempt to explore how non-normally distributed securities are priced under probability weighting. Previous literature has shown that lottery-like securities tend to be overpriced in the financial market (Kumar 2009; Kumar et al. 2011; Han and Kumar 2013; Gao and Lin 2015; An et al. 2020; Blau et al. 2020; Dimmock et al. 2021). Among these studies, Barberis and Huang (2008) proposed a framework based on CPT to explain skewness overpricing from the perspective of probability weighting. In their framework, the preferences of investors are characterised by an S-shaped value function and two different probability weighting functions toward gains and losses. However, it remains challenging to distinguish whether the overpricing of skewed assets arises primarily from the value function or the probability weighting. To address this, our paper examines two scenarios: in the first, we assume investors are risk-averse, and in the second, investors are loss-averse. In both cases, investors are subject to probability weighting.

The model setup is similar to Section 3 but with a few modifications. In addition to the risk-free asset and those normally distributed risky assets, we incorporate one positively skewed asset into the framework. Following Barberis and Huang (2008), the following assumption about the skewed asset is introduced.

Assumption 6. *Independent skewed asset*. The payoff of the skewed asset is binomial distributed with $(L, q; 0, 1 - q)$.[4] Denote the price of the skewed asset as p_n. The return of the skewed asset conforms to a binomial distribution as follows

$$\tilde{R}_n \sim \left(\frac{L}{p_n}, q; 0, 1 - q\right)$$

The excess return of the skewed asset is defined as $\hat{R}_n \equiv \tilde{R}_n - \left(1 + R_f\right)$. The return of the skewed asset is independent of the return of the J risky securities. The supply of the skewed security is infinitesimal relative to the total supply of the risky securities.

In addition, to simplify the framework, we assume identical investors, as specified in Assumption 5 and impose the short-selling constraint.

Assumption 7. *Short-selling constraint*. Short-selling is not allowed.

4.1 | Risk-Averse Investors With Probability Weighting

We first consider risk-averse investors and discuss how the skewed asset is priced. Before introducing the skewed security, under Assumptions 1–3, 5 and Theorem 1, all investors in this economy are expected to hold the same market portfolio with its return denoted by $\tilde{R}_M$. The market excess return of the market portfolio, defined as $\hat{R}_M \equiv \tilde{R}_M - R_f$, follows a normal distribution $N\left(\mu_M, \sigma_M^2\right)$.

The decision problem is described by whether investors are willing to hold an additional positively skewed asset. We assume that investors hold the same amount of market portfolio as before. A fair price should ensure that investors are indifferent in including or excluding the skewed security in their portfolios. This framework characterises the heterogeneous holdings of investors in the market and facilitates the comparison with the loss aversion case. The constrained optimisation problem for investors is given by:

$$U\left(W_0(1+R_f)+x_M^*\hat{R}_M\right)=U\left(W_0(1+R_f)+x_M^*\hat{R}_M+x_n^*\hat{R}_n\right)$$
$$U\left(W_0(1+R_f)+x_M\hat{R}_M\right)<U\left(W_0(1+R_f)+x_M^*\hat{R}_M\right) \text{ for } x_M \neq x_M^*$$
$$U\left(W_0(1+R_f)+x_M^*\hat{R}_M+x_n\hat{R}_n\right)<U\left(W_0(1+R_f)+x_M^*\hat{R}_M+x_n^*\hat{R}_n\right) \text{ for } 0<x_n \neq x_n^*$$

where, x_M and x_n are the wealth allocated to the market portfolio and the skewed asset, respectively, the superscript $*$ denotes the optimal allocation, W_0 represents the initial wealth and $U(\cdot)$ represents the rank-dependent expected utility, that is:

$$U\left(\tilde{W}\right) \equiv RDEU\left(\tilde{W}\right) = \int_{-\infty}^{\infty} u(w) dg_{\kappa,\alpha}(F(w))$$

where, $F(\cdot)$ is the cumulative distribution function of terminal wealth $\tilde{W}$. The utility function $u(\cdot)$ takes the form of CARA utility function, that is:

$$u(W) = -e^{-\eta W}$$

where, $\eta > 0$ represents the risk aversion parameter.

The optimised wealth allocations for the market portfolio and the skewed asset can be determined sequentially. With the CARA utility function and GWT, the optimisation problem for the wealth allocation to the market portfolio is equivalent to the following maximisation problem:

$$\max_{x_M>0} \quad W_0 + \left(x_M\mu_M - |x_M|\frac{\alpha}{\kappa}\sigma_M\right) - \frac{1}{2}\eta x_M^2 \frac{\sigma_M^2}{\kappa^2} \qquad (9)$$

Note that $x_M > 0$ since the market portfolio is in positive supply.

The optimization problem above establishes a one-to-one correspondence between the optimal holding x_M^* and the expected market excess return μ_M. For simplicity, we set $x_M^* = 1$. In this case, $x \equiv x_n / x_M^* = x_n$ represents the fraction of wealth allocated to the skewed security relative to the optimal wealth allocated to the market portfolio. With the CARA utility function, the constrained optimization problem for the skewed security can be rewritten as follows:

$$U\left(\hat{R}_M\right)=U\left(\hat{R}_M+x^*\hat{R}_n\right)$$
$$U\left(\hat{R}_M+x\hat{R}_n\right)<U\left(\hat{R}_M+x^*\hat{R}_n\right) \quad \text{for} \quad 0<x\neq x^*$$

where, x^* is the optimal wealth allocation ratio of the skewed security relative to the market portfolio.

We solve this constrained optimization problem numerically. First, we specify the parameters in the utility function and probability weighting function. For the utility function $u(\cdot)$, we set $\eta = 0.16$, where a positive η reflects the risk-averse attitude of investors. For the probability weighting function GWT, we use the parameters $\alpha = 0.1459$ and $\kappa = 0.6175$, representing a pessimistic probabilistic attitude. These parameter values approximate the weighting function proposed by Tversky and Kahneman (1992) over the gain domain, as implied by the dual operator. Regarding the market portfolio, the standard deviation of the excess return is set to $\sigma_M = 0.15$, and the risk-free rate is $R_f = 0.02$. Given these values, the expected excess return is $\mu_M = 0.0449$ at $x_M^* = 1$. Lastly, for the skewed security, we take $L = 10$ and $q = 0.1$, meaning this skewed security offers a significant payoff at a low probability. As probability q decreases, the skewness of this security strictly increases.

We use numerical integration to solve the price of the skewed security. We find that a price of $p_n = 1.4061$ satisfies this requirement of this constrained optimization problem, with the optimal strategy occurring at $x^* = 0.1379$. Then, the expected excess return for the skewed security is calculated as follows:

$$E\left(\hat{R}_n\right) = \frac{qL}{p_n} - \left(1 + R_f\right) = \frac{0.1 \times 10}{1.4061} - 1.02 = -0.3088$$

So the expected return of the skewed asset is $E\left(\tilde{R}_n\right) - 1 = -0.2888$.

The negative expected excess return of skewed security implies a strong preference for skewness by investors. Thus, even when investors are risk-averse, probability weighting will lead to the overpricing of the skewed asset.

4.2 | Loss-Averse Investors With Probability Weighting

In this section, we investigate the pricing of skewed security when investors are loss-averse. Following Tversky and Kahneman (1992), we suppose that all investors share a value function defined as follows:

$$v(w) = \begin{cases} w^\gamma, & w \geqslant 0 \\ -\lambda(-w)^\gamma, & w < 0 \end{cases}$$

where parameters $\gamma \in (0, 1)$ and $\lambda > 1$. The parameter γ measures the degree of risk aversion, with $\gamma < 1$ indicating that investors are risk-averse over gains and risk-seeking over losses. The parameter λ represents the degree of loss aversion, where $\lambda > 1$ reflects a higher sensitivity to changes in losses compared to gains. The value function kinks at the reference point $w = 0$, with concavity over gains and convexity over losses.

Following Barberis and Huang (2008), the equilibrium is defined as follows:

$$V\left(\hat{R}_M\right) = V\left(\hat{R}_M + x^*\hat{R}_n\right) = 0 \tag{10}$$

$$V\left(\hat{R}_M + x\hat{R}_n\right) < V\left(\hat{R}_M + x^*\hat{R}_n\right) \quad \text{for} \quad 0 < x \neq x^* \tag{11}$$

$$V\left(\hat{R}_n\right) < V\left(\hat{R}_M\right) \tag{12}$$

where, $V(\cdot)$ is given in Equation (8). The optimization problem stated in Equations (9–12) implies a heterogeneous holdings equilibrium. In the equilibrium, two groups of identical investors achieve maximum utility with their respective positions. The first group holds a combination of the risk-free asset and the market portfolio, which consists of the J normally distributed assets. The second group holds a combination of the risk-free asset, the market portfolio, and a long position in the skewed asset. Therefore, there is no incentive for any investor to alter the strategy. The key feature of the equilibrium is that the prices of the original J risky assets remain unaffected after the introduction of the new skewed asset.[5]

As in the previous scenario, we first specify the parameters in the value function and probability weighting function and then solve the equilibrium numerically. For the value function $v(\cdot)$, we take $\gamma = 0.88$ and $\lambda = 2.25$. For the probability weighting function GWT, the parameters are the same as in the risk-averse scenario: $\alpha = 0.1459$ and $\kappa = 0.6175$. The variance of the excess return of the market portfolio is the same as in the risk-averse scenario, that is, $\sigma_M = 0.15$. Different from the risk-averse model, the mean of the expected excess return is $\mu_M = 0.1169$, determined by solving the equilibrium condition $V\left(\hat{R}_M\right) = 0$. The risk-free rate and the parameters for the skewed asset are the same as in the risk-averse model.

In this equilibrium, the price of the skewed asset, $p_n = 0.6357$, satisfies the equilibrium conditions for the skewed security with $q = 0.1$. Investors holding the market portfolio achieve the same utility as those who hold an amount $x = 0.0695$ of skewed securities. The expected excess return for the skewed security is

$$E\left(\hat{R}_n\right) = \frac{qL}{p_n} - \left(1 + R_f\right) = \frac{0.06 \times 10}{0.6357} - 1.02 = -0.0761$$

The expected net return is $E\left(\tilde{R}_n\right) - 1 = -0.0561$. Therefore, the loss-averse investors also show a strong preference for positively skewed assets.

4.3 | Discussions on Skewness Pricing

In this section, we analyse the factors influencing skewness pricing, measured by the expected excess return. A negative expected excess return indicates that investors are overpricing the skewed security. We begin by examining the asset's skewness and the risk attitude embedded in the value function, followed by an exploration of two key characteristics of probability weighting—likelihood insensitivity and probabilistic attitudes.

Figure 3 illustrates how the expected excess return changes with the asset's skewness and the risk parameters in the risk-aversion model and the loss-aversion model. From Figure 3a,a′, we can find that as the security becomes more positively skewed, that is, when q is smaller, the skewed asset is more overpriced. As q increases, the skewness decreases, and the expected excess return is back to zero. The risk attitude is represented by the parameter η in the risk-aversion model, and by γ and λ in the loss-aversion model. An increase in η indicates a stronger risk-averse attitude in the risk-aversion model, while an increase in γ reflects a less risk-averse attitude toward gains and a less risk-seeking attitude toward losses. The parameter λ captures the degree of loss aversion. As λ increases, investors become more sensitive to changes in losses compared to gains. Both Figure 3b,b′ demonstrates that as investors become more risk-averse, the overpricing of skewness diminishes. This occurs because higher risk aversion toward gains reduces the marginal utility of large gains, making the skewed asset less appealing, thereby lowering its price. Figure 3b″ shows that greater loss aversion further reduces the overpricing of the skewed asset.

As observed in the risk-averse model, the overpricing of skewed assets is primarily driven by probability weighting rather than the S-shaped value function. We now examine the effects of different factors in probability weighting on skewness pricing. Given the characteristics of GWT, we focus on likelihood insensitivity and probabilistic attitudes, which are captured by the parameters κ or α in GWT. Figure 4 shows how these characteristics influence the return of the skewed security in both risk-averse and loss-averse models. From these panels, we can find that as κ or α drops, the skewed asset becomes more overpriced. Specifically, a decrease in the curvature parameter κ indicates that investors are more overweighting the extreme events and underweighting the middle events, thus investors are more drawn to the positively skewed asset. Similarly, a decrease in α leads investors to overweight the best outcomes compared to the worst outcomes, further enhancing their preference for positively skewed assets. Interestingly, as α increases, the overpricing of skewness diminishes even when $\kappa > 0$. This suggests that the skewness pricing is a joint result of both overall probability distortion and probabilistic attitude. When investors have a pessimistic attitude, it offsets the effect of distortion toward extreme events, leading to more rational pricing of a positively skewed asset.

4.4 | Implications of Transformed Distribution

From the above results, we can see that whether investors are risk-averse or loss-averse does not alter the main results. A positively skewed asset, traded by either a risk-averse or a loss-averse agent who overweights extreme payoffs, will be overpriced. Therefore, probability weighting plays a crucial role in the pricing of skewness. Note that probability weighting bridges the original risk distribution and the transformed risk distribution. Therefore, the transformed distribution of a skewed asset is critical in determining its price. However, existing literature has seldom discussed how probability weighting affects the transformed distribution of a skewed asset. In this section, we attempt to address this question by exploring how probability weighting changes the transformed distribution and its implications for the skewness pricing.

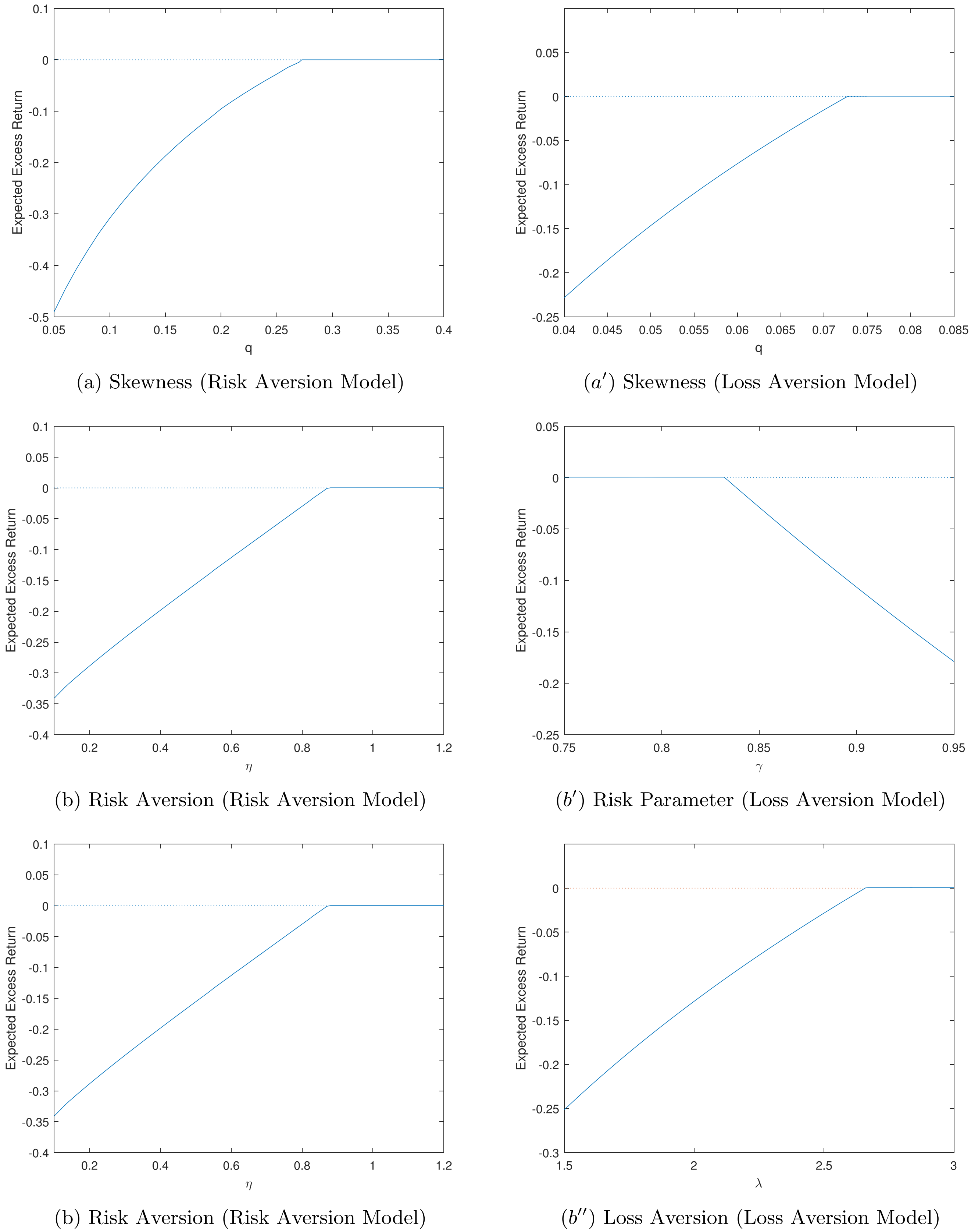


FIGURE 3 | These panels show how the expected excess return varies with the changes in the asset's skewness and the risk parameters in the risk-aversion model and the loss-aversion model. Panels (a, a′) present the expected excess return $E(\widehat{R}_n)$ of the skewed security with respect to probability q in the risk-aversion model and loss-aversion model. As q decreases, the skewness increases, leading to lower and negative expected excess returns in both models. Panel (b) is presented twice in the second and third rows for comparison with panels (b′, b″). Panels (b, b′) demonstrate that the expected excess return increases with risk aversion. Panel (b″) shows that the expected excess increases with loss aversion. These panels confirm consistent pricing implications of skewness and risk attitudes across both models. [Colour figure can be viewed at wileyonlinelibrary.com]

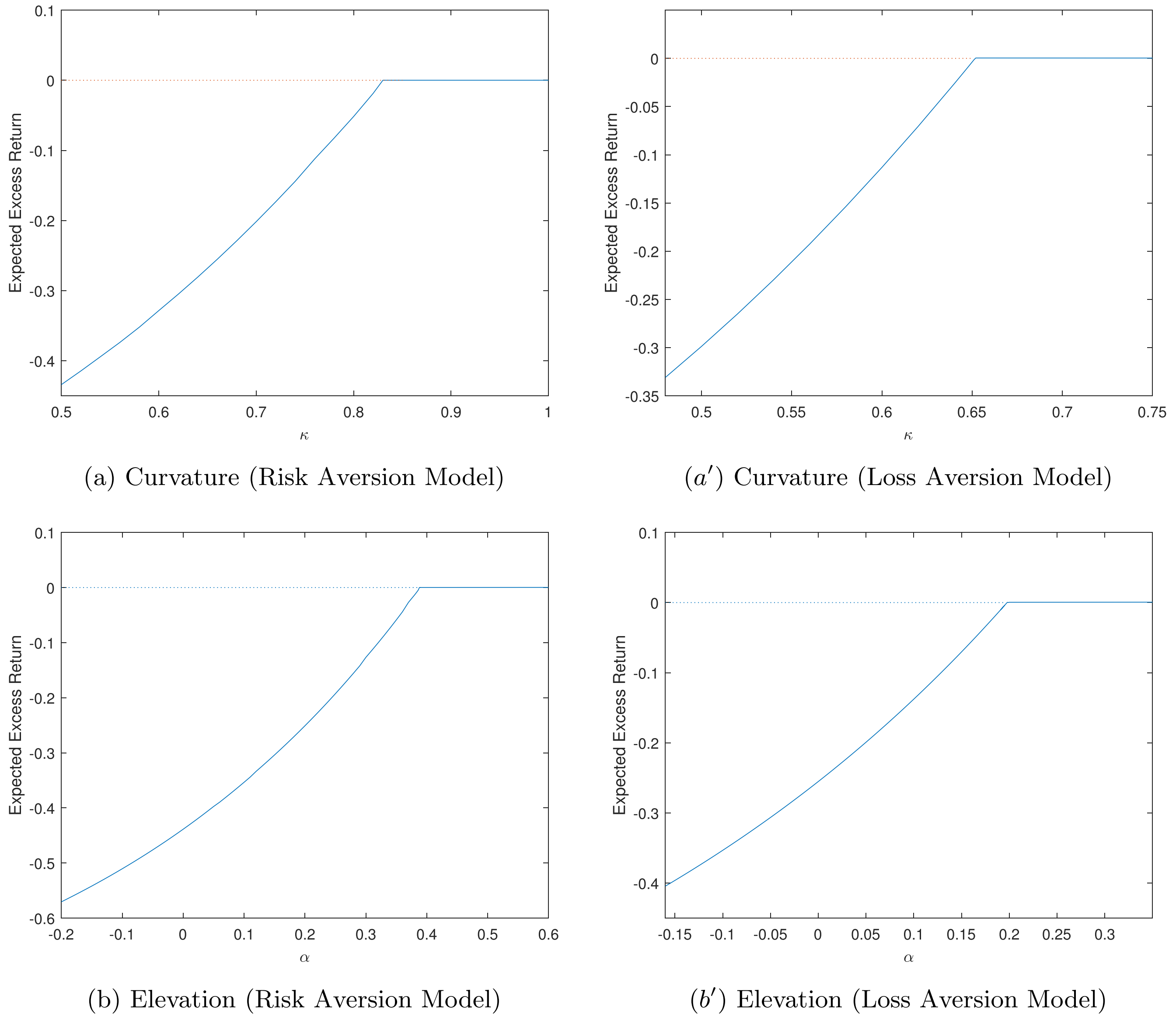


FIGURE 4 | These panels illustrate how the expected excess return varies with the changes in GWT parameters. Panels (a, a′) show that a higher curvature (lower κ) results in a lower expected excess return. Panels (b, b′) display that a more optimistic attitude, represented by a lower α, leads to a lower expected excess return. [Colour figure can be viewed at wileyonlinelibrary.com]

We examine two portfolios in the heterogeneous equilibrium: the *market portfolio* and the *skewed portfolio*, which is constructed by combining the market portfolio with the skewed security. We then compare the objective and transformed cumulative distributions of these two portfolios. Figure 4 illustrates the objective cumulative distributions and transformed cumulative distributions for these two portfolios with $x = 0.0695$ and $p_n = 0.6357$.

According to the EU, investors prefer the market portfolio over the skewed portfolio in a mean–variance framework because adding an independent security with a negative excess expected return to the market portfolio reduces the expected payoff and increases risk. However, if investors are subject to probability weighting, their preferences may shift. This is because probability weighting has a more significant effect on the distribution of the skewed portfolio than on the market portfolio. Specifically, the skewed portfolio has a higher probability for extreme high payoff than the market portfolio. Therefore, as the probability of extreme high payoff is amplified, the skewed portfolio achieves a higher distorted mean than the market portfolio, thus increasing its attractiveness to investors. We can observe the change in mean from Figure 5. In fact, the difference between distorted mean and objective mean can be expressed as follows

$$E_{g_{\kappa,\alpha}}(R) - E(R) = \int_{-\infty}^{\infty} \left[F(R) - g_{\kappa,\alpha}(F(R))\right] dR$$

Therefore, the difference in the area between the transformed distribution and the objective distribution illustrates the disparity between the distorted mean and the objective mean. As shown in Figure 5, the mean change for the skewed portfolio is larger than that for the market portfolio. For a more accurate calculation, we denote the excess return of the skewed portfolio as $\hat{R}_p$. The distorted mean and distorted standard deviation of the skewed portfolio are:

$$E_{g_{\kappa,\alpha}}\left[\hat{R}_p\right] = \int_{-\infty}^{\infty} R dg_{\kappa,\alpha}\left(F_x(R)\right) = 0.1397$$

$$\sigma_{g_{\kappa,\alpha}}\left[\hat{R}_p\right] \equiv \int_{-\infty}^{\infty} \left(R - E_{g_{\kappa,\alpha}}\left[\hat{R}_p\right]\right)^2 dg_{\kappa,\alpha}\left(F_x(R)\right) = 0.4872$$

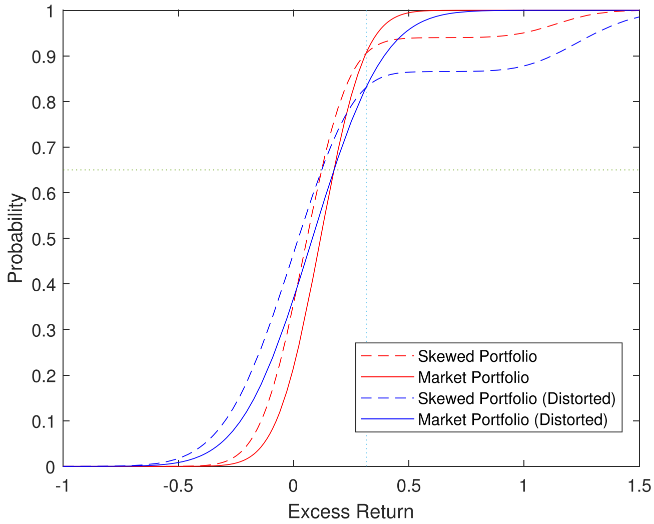


FIGURE 5 | This figure plots the original cumulative distribution functions and distorted cumulative distribution functions for the market portfolio and the skewed portfolio. Both distorted cumulative distribution functions are lower over larger returns and higher over smaller returns compared to the original ones, indicating the increased volatility of distorted distributions. The cumulative distribution of the skewed portfolio is much lower over larger returns after distortion compared to that of the market portfolio, reflecting a greater distortion of large payoffs in the skewed portfolio. [Colour figure can be viewed at wileyonlinelibrary.com]

where, $F_x(R) \equiv P\left\{\hat{R}_M + \hat{R}_n \leqslant R\right\}$ represents the cumulative distribution function of the skewed portfolio's excess return with a proportion x in the skewed security. Meanwhile, the distorted mean and distorted standard deviation of the market portfolio are:

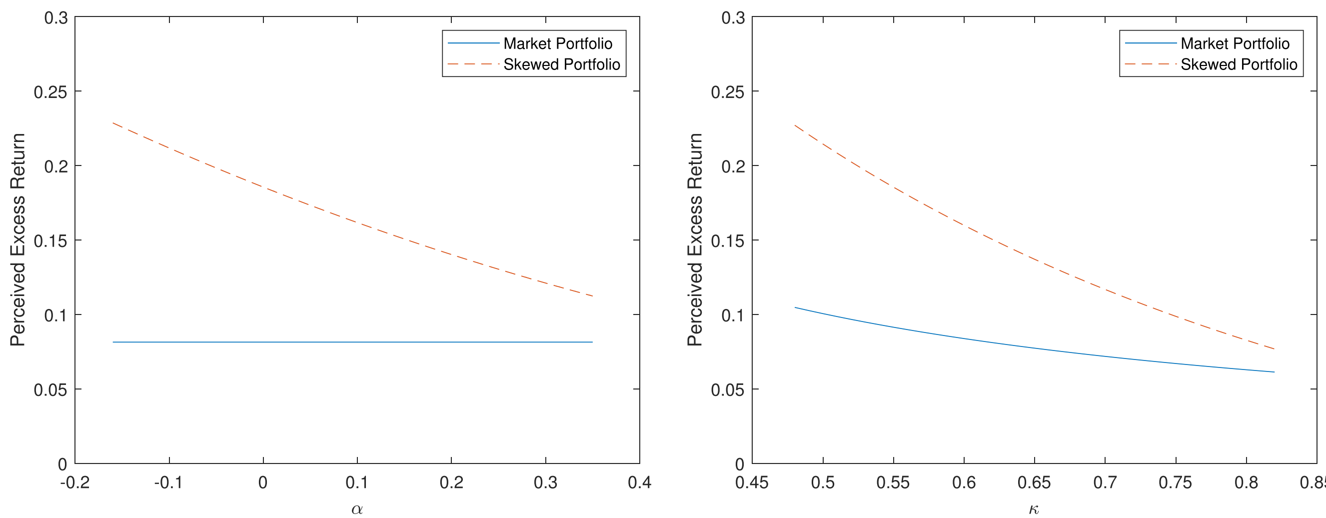
$$E_{g_{\kappa,\alpha}}\left[\hat{R}_M\right] = \mu_M - \frac{\alpha}{\kappa}\sigma_M = 0.0815$$

$$\sigma_{g_{\kappa,\alpha}}\left[\hat{R}_M\right] = \frac{1}{\kappa}\sigma_M = 0.2429$$

Even though the skewed portfolio has a larger volatility than the market portfolio, the mean of the skewed portfolio is higher. Therefore, the overpricing of the skewed asset is driven by the high distorted mean of the skewed portfolio.

Our previous results have shown that investors with lower risk aversion or lower loss aversion exhibit stronger preferences for skewed securities. This phenomenon can be understood from the perspective of transformed distributions. When investors are less risk-averse or less loss-averse, they care more about the mean rather than the volatility. Therefore, they prefer the skewed portfolio to the market portfolio.

Finally, the characteristics of probability weighting significantly influence the transformed distribution characteristics. Figure 6 shows the distorted expected return as the parameters α and κ of the weighting function are varied. The price is set such that the skewed portfolio has the same expected return as the market portfolio. We can see that both α and κ are negatively correlated with the distorted mean, implying a stronger appeal of the skewed portfolio under a more optimistic or more distorted probabilistic attitude. This occurs because greater overall distortion, represented by a decrease in κ, leads to a more severe overweighting on extreme payoffs, thereby increasing the distorted mean of the skewed portfolio. Similarly, a more optimistic



(a) Distorted Excess Return (α changes) (b) Distorted Excess Return (κ changes)

FIGURE 6 | These two panels plot the distorted expected excess return of the market portfolio and the skewed portfolio. The price of the skewed security is set at $p_n = 0.5882$ to ensure that both portfolios have the same expected return. Investors are assumed to hold a positive position in the skewed asset, with $x = 0.0695$. The left panel shows the changes in the distorted returns with varying κ, while the right panel displays the distorted returns with varying α. In both panels, the skewed portfolio demonstrates a higher distorted expected return and exhibits greater sensitivity to probability weighting. [Colour figure can be viewed at wileyonlinelibrary.com] attitude, reflected by a reduction in α, implies that the decision weights on the best outcomes are more heavily overweighted compared to the worst outcomes, further elevating the distorted mean of the skewed portfolio. Therefore, the overpricing of the skewed asset reflects a higher perceived mean of the skewed portfolio by investors.

5 | Conclusions

In this paper, we investigate the pricing implications of probability weighting on CAPM and skewness pricing, employing the GWT within the framework of RDEU. CAPM and SMLT are derived under two distinct scenarios: risk aversion and loss aversion. In both cases, we assume heterogeneous probability weighting and value functions among investors. There is no restriction on the shape of the probability weighting function in the risk aversion model to ensure CAPM, while in the loss-aversion scenario, a probabilistic pessimistic attitude is necessary to ensure SMLT.

Our analysis of skewness pricing emphasises the critical role of probability weighting in skewness overpricing, as opposed to the influence of an S-shaped value function. We further illustrate investors' preference for skewness through the perspective of transformed distribution, highlighting that the distorted mean is a key determinant of investors' preference.

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Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Endnotes

[1] In the following chapters, the parameter values we use are $\kappa = 0.6175, \alpha = -0.1459$, that is, we have taken the dual operator of the original operator $g_{-\alpha,\kappa}(p) = 1 - g_{\alpha,\kappa}(1-p)$.

[2] For the proof, please refer to Appendix A.2. In the proof, we employed a technical assumption: $x \infty \lim v_i(x) g_{\kappa_i,\alpha_i}(F(x)) + x \infty \lim v_i(x)\left[1 - g_{\kappa_i,\alpha_i}(1-F(x))\right] = 0$.

[3] Drawing on Levy et al. (2003), we explain the potential failure of equilibrium in the Appendix S1.

[4] A binomial distribution $(L, q; 0, 1-q)$ generates L with a probability q and generates 0 with a probability $1-q$.

[5] This feature has been illustrated in the appendix in Barberis and Huang (2008).

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Supporting Information

Additional supporting information can be found online in the Supporting Information section.

Appendix A

Proof of Theorem 1

Given Assumptions 1–3, the budget constraint for investor i is:

$$\widetilde{W}_i = W_{0i}(1+R_f) + w_i^\top\left(\widetilde{R} - R_f\right) \sim N\left(W_{0i}(1+R_f) + w_i^\top(\mu - R_f\mathbf{1}), w_i^\top\Sigma w_i\right)$$

which conforms to a normal distribution.

We construct $\widetilde{W}_i^*$ as follows:

$$\widetilde{W}_i^* = \frac{1}{\kappa_i}\left[\widetilde{W}_i - \left(W_{0i}(1+R_f) + w_i^\top(\mu - R_f\mathbf{1})\right)\right] + \left(W_{0i}(1+R_f) + w_i^\top(\mu - R_f\mathbf{1})\right)$$
$$-\frac{\alpha_i}{\kappa_i}\sqrt{w_i^\top\Sigma w_i} = \frac{1}{\kappa_i}\widetilde{W}_i + \left(1 - \frac{1}{\kappa_i}\right)\left[W_{0i}(1+R_f) + w_i^\top(\mu - R_f\mathbf{1})\right] - \frac{\alpha_i}{\kappa_i}\sqrt{w_i^\top\Sigma w_i}$$

Then, we have:

$$\widetilde{W}_i^* \sim N\left(E\left(\widetilde{W}_i\right) - \frac{\alpha_i}{\kappa_i}\sqrt{Var\left(\widetilde{W}_i\right)}, \frac{1}{\kappa_i^2}\mathrm{Var}\left(\widetilde{W}_i\right)\right)$$

and

$$\mathrm{Cov}\left(\widetilde{W}_i^*, \widetilde{R}_j\right) = \frac{1}{\kappa_i}\mathrm{Cov}\left(\widetilde{W}_i, \widetilde{R}_j\right)$$

Note that the distribution of $\widetilde{W}_i^*$ is the same as the distorted distribution of the terminal wealth $\widetilde{W}_i$ under $g_{\kappa_i,\alpha_i}(\cdot)$, and $\widetilde{W}_i^*$ maintains the features of the original covariance. So we obtain the following equation between the expectation and distorted expectation:

$$E_{g_{\kappa_i,\alpha_i}}\left[u_i\left(\widetilde{W}_i\right)\right] = E\left[u_i\left(\widetilde{W}_i^*\right)\right]$$

for a monotonic utility function u_i.

Therefore, the investor's optimal problem is equivalent to:

$$\max_{w_i} \quad E\left[u_i\left(\widetilde{W}_i^*\right)\right] \tag{A1}$$

The first order condition of Equation (A1) is:

$$\frac{\partial E\left[u_i\left(\widetilde{W}_i^*\right)\right]}{\partial w_i} = E\left[u_i'\left(\widetilde{W}_i^*\right)\left(\frac{1}{\kappa_i}\left(\widetilde{R} - R_f\mathbf{1}\right) + \left(1 - \frac{1}{\kappa_i}\right)(\mu - R_f\mathbf{1}) - \frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top\Sigma w_i}}\right)\right] = 0$$

Because $u(\cdot)$ is increasing and strictly concave, the second order condition must be satisfied.

Notice that:

$$E\left[\frac{1}{\kappa_i}\left(\widetilde{R} - R_f\mathbf{1}\right) + \left(1 - \frac{1}{\kappa_i}\right)(\mu - R_f\mathbf{1}) - \frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top\Sigma w_i}}\right] = (\mu - R_f\mathbf{1}) - \frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top\Sigma w_i}}$$

Using the definition of a covariance and the first-order condition, we have:

$$E\left[u_i'\left(\widetilde{W}_i^*\right)\right]E\left[\frac{1}{\kappa_i}\left(\widetilde{R} - R_f\mathbf{1}\right) + \left(1 - \frac{1}{\kappa_i}\right)(\mu - R_f\mathbf{1}) - \frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top\Sigma w_i}}\right]$$
$$= -\mathrm{Cov}\left(u_i'\left(\widetilde{W}_i^*\right), \frac{1}{\kappa_i}\left(\widetilde{R} - R_f\right) + \left(1 - \frac{1}{\kappa_i}\right)(\mu - R_f\mathbf{1}) - \frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top\Sigma w_i}}\right)$$

which is equivalent to:

$$E\left[u_i'\left(\widetilde{W}_i^*\right)\right]\left[\left(\mu-R_f\mathbf{1}\right)-\frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top \Sigma w_i}}\right]$$
$$=-\mathrm{Cov}\left(u_i'\left(\widetilde{W}_i^*\right),\frac{1}{\kappa_i}\left(\widetilde{R}-R_f\right)+\left(1-\frac{1}{\kappa_i}\right)\left(\mu-R_f\mathbf{1}\right)-\frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top \Sigma w_i}}\right) \tag{A2}$$

From the construction of $\widetilde{W}_i^*$, we know that $\widetilde{W}_i^*$ and $\widetilde{R}$ are multivariate normally distributed. Then from Stein's lemma, we obtain:

$$\mathrm{Cov}\left(u_i'\left(\widetilde{W}_i^*\right),\frac{1}{\kappa_i}\left(\widetilde{R}-R_f\mathbf{1}\right)+\left(1-\frac{1}{\kappa_i}\right)\left(\mu-R_f\mathbf{1}\right)-\frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top \Sigma w_i}}\right)$$
$$=E\left[u_i''\left(\widetilde{W}_i^*\right)\right]\frac{1}{\kappa_i^2}\mathrm{Cov}\left(\widetilde{W}_i,\widetilde{R}\right) \tag{A3}$$

Combing the Equations (A2) and (A3), we have:

$$E\left[u_i'\left(\widetilde{W}_i^*\right)\right]\left[\left(\mu-R_f\mathbf{1}\right)-\frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top \Sigma w_i}}\right]=-\frac{1}{\kappa_i^2}E\left[u_i''\left(\widetilde{W}_i^*\right)\right]\mathrm{Cov}\left(\widetilde{W}_i,\widetilde{R}\right)$$

then:

$$\left(\mu-R_f\mathbf{1}\right)-\frac{\alpha_i}{\kappa_i}\frac{\Sigma w_i}{\sqrt{w_i^\top \Sigma w_i}}=-\frac{1}{\kappa_i^2}\frac{E\left[u_i''\left(\widetilde{W}_i^*\right)\right]}{E\left[u_i'\left(\widetilde{W}_i^*\right)\right]}\mathrm{Cov}\left(\widetilde{W}_i,\widetilde{R}\right)$$

Notice that $\mathrm{Cov}\left(\widetilde{W}_i,\widetilde{R}\right)=\mathrm{Cov}\left(w_i^\top\widetilde{R},\widetilde{R}\right)=\Sigma w_i$, so we have:

$$\mu-R_f\mathbf{1}=\left\{\frac{\alpha_i}{\kappa_i}\frac{1}{\sqrt{w_i^\top \Sigma w_i}}-\frac{1}{\kappa_i^2}\frac{E\left[u_i''\left(\widetilde{W}_i^*\right)\right]}{E\left[u_i'\left(\widetilde{W}_i^*\right)\right]}\right\}\mathrm{Cov}\left(\widetilde{W}_i,\widetilde{R}\right)$$

Define $\theta_i \equiv \frac{\alpha_i}{\kappa_i}\frac{1}{\sqrt{w_i^\top \Sigma w_i}}-\frac{1}{\kappa_i^2}\frac{E\left[u_i''\left(\widetilde{W}_i^*\right)\right]}{E\left[u_i'\left(\widetilde{W}_i^*\right)\right]}$. When $\theta_i \neq 0$, the above equation could be rewritten as:

$$\theta_i^{-1}\left(\mu-R_f\mathbf{1}\right)=\mathrm{Cov}\left(\widetilde{W}_i,\widetilde{R}\right)$$

We consider the situation when $\mu \neq R_f\mathbf{1}$. Then, $\theta_i \neq 0$ for any i. Summarising the equations over $i=1,\cdots,I$, we have:

$$\left(\sum_{i=1}^{I}\theta_i^{-1}\right)\left(\mu-R_f\mathbf{1}\right)=\mathrm{Cov}\left(\widetilde{M},\widetilde{R}\right)$$

where, $\widetilde{M}\equiv\sum_{i=1}^{I}\widetilde{W}_i=W_{M0}\left(1+\widetilde{R}_M\right)$. Therefore,

$$\mu-R_f\mathbf{1}=W_{M0}\left(\sum_{i=1}^{I}\theta_i^{-1}\right)^{-1}\mathrm{Cov}\left(\widetilde{R}_M,\widetilde{R}\right)$$

when $\sum_{i=1}^{I}\theta_i^{-1}\neq 0$.

Notice that $\mathrm{Cov}\left(\widetilde{R}_M,\widetilde{R}\right)=\Sigma w_M$. If $\sum_{i=1}^{I}\theta_i^{-1}=0$, $w_M=0$, which means that the market value of any risky security is 0. It cannot hold with a positive supply and positive price of each risky security. Then,

$$E\left[\widetilde{R}_M-R_f\right]=E\left[w_M^\top\left(\widetilde{R}-R_f\mathbf{1}\right)\right]=w_M^\top E\left[\widetilde{R}-R_f\mathbf{1}\right]$$
$$=w_M^\top\left(\mu-R_f\mathbf{1}\right)=W_{M0}\left(\sum_{i=1}^{I}\kappa_i^2\theta_i^{-1}\right)^{-1}\sigma^2\left(\widetilde{R}_M\right)$$

where, $w_M^\top\mathrm{Cov}\left(\widetilde{R}_M,\widetilde{R}\right)=\sigma^2\left(\widetilde{R}_M\right)$. Then, we have

$$\mu-R_f\mathbf{1}=W_{M0}\left(\sum_{i=1}^{I}\theta_i^{-1}\right)^{-1}\mathrm{Cov}\left(\widetilde{R}_M,\widetilde{R}\right)=\frac{\mathrm{Cov}\left(\widetilde{R}_M,\widetilde{R}\right)}{\sigma^2\left(\widetilde{R}_M\right)}\left(E\left[\widetilde{R}_M\right]-R_f\right)$$

Therefore, CAPM holds.

Proof of Theorem 2

Proof of Equation (8):

Since $g_{\kappa,\alpha}(p)$ is right-continuous, the dual operator $1-g_{\kappa,\alpha}(p)=g_{\kappa,-\alpha}(1-p)$, and the decumulative distribution function $\overline{F}(x)=1-F(x)$, we rearrange the total utility $V\left(\widehat{W}\right)$ as,

$$V\left(\widehat{W}\right)=\int_{-\infty}^{\widehat{W}_0}v(x)dg_{\kappa,\alpha}(F(x))-\int_{\widehat{W}_0}^{+\infty}v(x)dg_{\kappa,-\alpha}\left(\overline{F}(x)\right),\qquad \widehat{W}_0\in\mathbb{R}$$

We integrate by parts to get

$$\int_{-\infty}^{\widehat{W}_0}v(x)dg_{\kappa,\alpha}(F(x))=\left[v(x)g_{\kappa,\alpha}(F(x))\right]_{x=-\infty}^{x=\widehat{W}_0}-\int_{-\infty}^{\widehat{W}_0}g_{\kappa,\alpha}(F(x))dv(x),\qquad \widehat{W}_0\in\mathbb{R} \tag{A4}$$

and

$$\int_{\widehat{W}_0}^{+\infty}v(x)dg_{\kappa,-\alpha}\left(\overline{F}(x)\right)=\left[v(x)g_{\kappa,-\alpha}\left(\overline{F}(x)\right)\right]_{x=\widehat{W}_0}^{+\infty}-\int_{\widehat{W}_0}^{+\infty}g_{\kappa,-\alpha}\left(\overline{F}(x)\right)du(x),\qquad \widehat{W}_0\in\mathbb{R} \tag{A5}$$

Summing Equations (A4) and (A5) up, we obtain:

$$V\left(\widehat{W}\right)=-\left[x\infty\widehat{\lim}v(x)g_{\kappa,\alpha}(F(x))+x\infty\widehat{\lim}v(x)g_{\kappa,-\alpha}(1-F(x))\right]$$
$$+v\left(\widehat{W}_0\right)-\int_{-\infty}^{\widehat{W}_0}g_{\kappa,\alpha}(F(x))dv(x)+v\left(\widehat{W}_0\right)+\int_{\widehat{W}_0}^{+\infty}\left[1-g_{\kappa,\alpha}(F(x))\right]dv(x),\qquad \widehat{W}_0\in\mathbb{R}$$

Therefore, if $x\infty\widehat{\lim}v(x)g_{\kappa,\alpha}(F(x))+x\infty\widehat{\lim}v(x)g_{\kappa,-\alpha}(1-F(x))=0$, then Equation (8) holds.

Proof of Theorem 2:

Our proof gets the idea from Barberis and Huang (2008) to give the first-order stochastic dominance (FSD) and second-order stochastic dominance (SSD) over normal distributions under probability weighting. Three propositions, A1, A2, and A3, are needed here to reach the conclusion, and we prove them one by one.

Proposition A1. *The preference satisfy the first-order stochastic dominance property. That is, if $\widetilde{W}_1$ first-order stochastically dominates $\widetilde{W}_2$, then $V\left(\widetilde{W}_1\right)\geqslant V\left(\widetilde{W}_2\right)$. Moreover, if $\widetilde{W}_1$ strictly first-order dominates $\widetilde{W}_2$, then $V\left(\widetilde{W}_1\right)>V\left(\widetilde{W}_2\right)$.*

Proof of Proposition A1:

Since $\widetilde{W}_1$ first-order stochastically dominates $\widetilde{W}_2$, $F_1(x)\leqslant F_2(x)$ for all $x\in R$, where $F_k(\cdot)$ is the cumulative distribution function for $\widetilde{W}_k$. Because $g_{\kappa,\alpha}(\cdot)$ is strictly increasing, $g_{\kappa,\alpha}\left(F_1(x)\right)\leqslant g_{\kappa,\alpha}\left(F_2(x)\right)$. We see

$g_{\kappa,\alpha}(F_k(x))$ as a transformed cumulative distribution function, thus the first-order stochastic dominance property holds if the first-order stochastic dominance property holds before the distortion.

Proposition A2. *Take two distributions, $\widehat{W}_1$ and $\widehat{W}_2$, and suppose that*:

i. $E\left(\widehat{W}_1\right) = E\left(\widehat{W}_2\right) \geqslant 0$;
ii. *$\widehat{W}_1$ and $\widehat{W}_2$ are both symmetrically distributed*;
iii. *$\widehat{W}_1$ and $\widehat{W}_2$ satisfy a single-crossing property; so that if $F_k(\cdot)$ is the cumulative distribution function for $\widehat{W}_k$ ($k = 1, 2.$), there exists z such that $F_1(x) \leqslant F_2(x)$ for $x < z$ and $F_1(x) \geqslant F_2(x)$ for $x > z$.*

And we require some weak conditions for the value function $v(\cdot)$:

iv. $x\infty \widehat{\lim} v(x) g_{\kappa,\alpha}(F(x)) + x\infty \widehat{\lim} v(x) g_{\kappa,-\alpha}(1 - F(x)) = 0$, *so that* Equation (8) *holds.*
v. *For $x \in (-\infty, 0)$, $v'(x) \geqslant v'(-x)$.*
vi. *For $x \in [0, \infty)$, $v(x)$ is concave.*
vii. $v(0) = 0$.

Further, for the probability weighting function $g_{\kappa,\alpha}(\cdot)$, we requires that.

viii. *$\alpha > 0$. Or, $\alpha = 0$, if $E\left(\widehat{W}_1\right) = E\left(\widehat{W}_2\right) > 0$, or $v'(x) > v'(-x)$ for $x \in (-\infty, 0)$.*

Then, $V\left(\widehat{W}_1\right) \geqslant V\left(\widehat{W}_2\right)$. If, furthermore, the inequalities in condition (iii) hold strictly for some x, then $V\left(\widehat{W}_1\right) > V\left(\widehat{W}_2\right)$.

Proof of Proposition A2:

For $\widehat{W}_k, k = 1, 2$, with the same mean $\mu \geqslant 0$, we have:

$$V\left(\widehat{W}_k\right) = V\left(\widehat{W}_k^-\right) + V_A\left(\widehat{W}_k^+\right) + V_B\left(\widehat{W}_k^+\right) + V_C\left(\widehat{W}_k^+\right), \qquad k = 1, 2$$

where,

$$V\left(\widehat{W}_k^-\right) = -\int_{-\infty}^{0} g_{\kappa,\alpha}(F_k(x)) v'(x) dx, \qquad V_A\left(\widehat{W}_k^+\right) = \int_{0}^{\mu} \left[1 - g_{\kappa,\alpha}(F_k(x))\right] v'(x) dx$$

$$V_B\left(\widehat{W}_k^+\right) = \int_{\mu}^{2\mu} \left[1 - g_{\kappa,\alpha}(F_k(x))\right] v'(x) dx, \qquad V_C\left(\widehat{W}_k^+\right) = \int_{2\mu}^{\infty} \left[1 - g_{\kappa,\alpha}(F_k(x))\right] v'(x) dx$$

Applying the change of variable $x = 2\mu - x'$ to equations $V_B\left(\widehat{W}_k^+\right)$ and $V_C\left(\widehat{W}_k^+\right)$ and noting that, since the distribution are symmetric, $1 - F_k(x) = F_k(2\mu - x) = F(x')$, we have:

$$V_B\left(\widehat{W}_k^+\right) = \int_{0}^{\mu} g_{\kappa,-\alpha}(F_k(x)) v'(2\mu - x) dx, \quad k = 1, 2$$

and

$$V_C\left(\widehat{W}_k^+\right) = \int_{-\infty}^{0} g_{\kappa,-\alpha}(F_k(x)) v'(2\mu - x) dx, \quad k = 1, 2$$

Thus, we have:

$$V\left(\widehat{W}_k\right) = \int_{0}^{\mu} \left[g_{\kappa,-\alpha}(F_k(x)) v'(2\mu - x) + \left[1 - g_{\kappa,\alpha}(F_k(x))\right] v'(x)\right] dx + \int_{-\infty}^{0} \left[g_{\kappa,-\alpha}(F_k(x)) v'(2\mu - x) - g_{\kappa,\alpha}(F_k(x)) v'(x)\right] dx, \quad k = 1, 2$$

So, using $g_{\kappa,\alpha}(p) = 1 - g_{\kappa,-\alpha}(1 - p)$, we can get the difference of $V\left(\widehat{W}_1\right)$ and $V\left(\widehat{W}_2\right)$ as:

$$V\left(\widehat{W}_1\right) - V\left(\widehat{W}_2\right) = \int_{0}^{\mu} \left[\left[g_{\kappa,\alpha}(F_2(x)) - g_{\kappa,\alpha}(F_1(x))\right] v'(x) - \left[g_{\kappa,-\alpha}(F_2(x)) - g_{\kappa,-\alpha}(F_1(x))\right] v'(2\mu - x)\right] dx + \int_{-\infty}^{0} \left[\left[g_{\kappa,\alpha}(F_2(x)) - g_{\kappa,\alpha}(F_1(x))\right] v'(x) - \left[g_{\kappa,-\alpha}(F_2(x)) - g_{\kappa,-\alpha}(F_1(x))\right] v'(2\mu - x)\right] dx$$

Because $\widehat{W}_1$ and $\widehat{W}_2$ are symmetric, the condition (iii) only holds for $z = \mu$. This means that $F_1(x) \leqslant F_2(x)$ for $x < \mu$, which derives that $g_{\kappa,\alpha}(F_2(x)) - g_{\kappa,\alpha}(F_1(x)) \geqslant 0$ and $g_{\kappa,-\alpha}(F_2(x)) - g_{\kappa,-\alpha}(F_1(x)) \geqslant 0$.

For $x \in [0, \mu]$, since $v(\cdot)$ is concave and increasing, we also have $v'(x) > v'(2\mu - x) > 0$. And for $x \in (-\infty, 0)$, under condition (v) and (vi), we must have $v'(x) \geqslant v'(-x) \geqslant v'(2\mu - x) > 0$. Thus, to show $V\left(\widehat{W}_1\right) \geqslant V\left(\widehat{W}_2\right)$, it would be sufficient to show that:

$$g_{\kappa,\alpha}(F_2(x)) - g_{\kappa,\alpha}(F_1(x)) \geqslant g_{\kappa,-\alpha}(F_2(x)) - g_{\kappa,-\alpha}(F_1(x)), \qquad \text{for} \quad x \in (-\infty, \mu)$$

We define the function as follows:

$$h(p) = g_{\kappa,\alpha}(p) - g_{\kappa,-\alpha}(p), \quad \text{for} \quad p < \frac{1}{2}$$

Then we have:

$$\frac{dh(p)}{dp} = \frac{dg_{\kappa,\alpha}(p)}{dp} - \frac{dg_{\kappa,-\alpha}(p)}{dp} = \frac{\kappa}{\phi(\Phi^{-1}(p))}\left[\phi\left(\kappa\Phi^{-1}(p) + \alpha\right) - \phi\left(\kappa\Phi^{-1}(p) - \alpha\right)\right]$$

Since $p < \frac{1}{2}$ and $\alpha > 0$, we have $\kappa\Phi^{-1}(p) < 0$ and thus $\phi\left(\kappa\Phi^{-1}(p) + \alpha\right) > \phi\left(\kappa\Phi^{-1}(p) - \alpha\right)$. Then we get $\frac{dh(p)}{dp} > 0$, and consequently

$$h(F_2(x)) \geqslant h(F_1(x))$$

$$g_{\kappa,\alpha}(F_2(x)) - g_{\kappa,-\alpha}(F_2(x)) \geqslant g_{\kappa,\alpha}(F_1(x)) - g_{\kappa,-\alpha}(F_1(x))$$

$$g_{\kappa,\alpha}(F_2(x)) - g_{\kappa,\alpha}(F_1(x)) \geqslant g_{\kappa,-\alpha}(F_2(x)) - g_{\kappa,-\alpha}(F_1(x))$$

If $F_1(x) < F_2(x)$, the inequality holds strictly, which complete the strict inequality of $V\left(\widehat{W}_1\right) > V\left(\widehat{W}_2\right)$.

If $\alpha = 0$, the equality holds anyway, $g_{\kappa,\alpha}(F_2(x)) - g_{\kappa,\alpha}(F_1(x)) = g_{\kappa,-\alpha}(F_2(x)) - g_{\kappa,-\alpha}(F_1(x))$ for $x \in (-\infty, \mu)$.

The comparison attributes to value function. Since $v'(x) > v'(2\mu - x)$, proposition A2 holds.

Proposition A3. *Under the preference and probability weighting function, assume the assumptions hold. If $\widetilde{W}$ is normally distributed with mean μ and variance σ_W^2, then $V\left(\widetilde{W}\right)$ can be written as a function of μ_W and σ_W^2, $F\left(\mu_W, \sigma_W^2\right)$. Moreover, for any σ_W^2, $F\left(\mu_W, \sigma_W^2\right)$ is strictly increasing with μ_W; and for any $\mu_W \geqslant 0$, $F\left(\mu_W, \sigma_W^2\right)$ is strictly decreasing in σ_W^2.*

Proof for Proposition A3:

Since every normal distribution is fully specified by its mean and variance, we can write $V\left(\widetilde{W}\right) = F\left(\mu_W, \sigma_W^2\right)$. Proposition A1 implies that $F\left(\mu_W, \sigma_W^2\right)$ is strictly increasing in μ_W. Now consider any pair of normal wealth distributions, $\widetilde{W}_1$ and $\widetilde{W}_2$, with the same nonnegative mean but different variance. These two wealth distributions satisfy conditions (i)–(iii) in Proposition A2. That proposition therefore implies that, for any $\mu_W \geqslant 0$, $F\left(\mu_W, \sigma_W^2\right)$ is strictly decreasing in σ_W^2.

Proof of Theorem 2 (Continued):

Proposition A1–A3 depicts a class of upward indifference curves in mean/standard deviation plane. With a risk-free asset, investors would choose a combined portfolio of risk-free asset and tangency portfolio, which is also the market portfolio. Notice that for any investor, the choice is the same tangency portfolio no matter what utility function and which GWT they hold. So the heterogenous perception does not affect their choice. Thus Security Market Line Theorem holds.

Proof of Corollary 1:

If the value function $v(\cdot)$ satisfies that $v\left(W_0 x_M \hat{R}\right) = m(W_0 x_M) v\left(\hat{R}\right)$, we would have the utility is

$$V\left(\widehat{W}\right) = m(W_0 x_M) V\left(\hat{R}\right)$$

Given that the $m(W_0 x_M)$ is strictly increasing with x_M, the equilibrium, if exists, must satisfy that

$$V\left(\hat{R}\right) = 0$$

Otherwise, investors would choose to long or short infinitely.

If investors are identical, the above equation $V\left(\hat{R}\right) = 0$ and the Security Market Line Theorem construct J non-redundant equations in J non-redundant unknowns prices for each risky security. Hence we can solve the equations to get the market prices.

With these conditions, we solve for the equilibrium prices, and therefore there exists a market equilibrium. For more details, see Barberis and Huang (2008).