THE PORTFOLIO INVESTMENT DECISION MODEL: A REVIEW

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Investment has been defined as the making of present sacrifices with a view to obtaining future benefits.[1] The essence of the investment decision is thus intertemporal choice or allocation – the allocation of financial resources for consumption and investment over time.[2] In making such decisions investors are viewed as seeking that allocation which will enable them to maximize their enjoyment of lifetime consumption.

In so far as the present is known, the sacrifices necessary to make an investment – the amount of current consumption foregone – can be ascertained with certainty.[3] The future benefits, however, are uncertain as the future cannot be known with certainty. It is therefore necessary to develop a model of investment behaviour which incorporates both certainty and uncertainty.

CLASSICAL THEORY OF INVESTMENT

The microeconomic theory of investment was fully developed in the work of Irving Fisher.[4] Fisher's work, however, was developed only under the assumption of perfect certainty regarding future events. The core proposition of this theory with perfect capital markets and rational wealth maximizing behavior is that:

> ". . . . a firm should adjust its capital stock until the marginal rate of return on further investment (or dis-investment) is equal to the cost of capital."[5]

The implementation of this decision rule led to the development of such computational techniques as the net present value rule.[6] Thus, an investment should be made (that is, a share of stock should be purchased) if the present value of its future cash dividends, including the liquidating dividend, discounted at the investor's required rate of return, equalled or exceeded its cost. Since the assumptions of perfect certainty and perfect capital markets imply that if all future dividends are non-negative there can exist only one discount rate (required rate of return) the solution to this problem is simple and straightforward: symbolically,

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an investor should buy a share of stock if:

$$C \leqslant \sum_{t=1}^{N} D_t (1+k)^{-t}$$

where C is the current market price of the asset
D_t is the dividend to be paid at the end of period t
k is the investor's required rate of return.

The perfect certainty investment model violates the conditions which obtain in the working world in both its assumptions and implications. If the assumption of perfect certainty is relaxed, the method of relating future dividends to a single rate of return yields results which are incapable of economic interpretation.[7] This is because the estimated future dividends and hence the discount rate can have many possible outcomes. Further, the main implication of the model, which is that each investor will acquire the asset offering the highest rate of return, would lead all investors to hold single investment asset portfolios. Yet such a phenomenon does not obtain in observed investment behaviour. Stated simply, the classical theory of investment fails to explain the prevalent practice of investment diversification.

The inability of classical investment theory to deal with the problem of uncertainty and to explain observed investment behaviour led in the first instance to the development of partial solutions to the investment decision problem.[8] These solutions, however, were also unsatisfactory in that either they predicted single asset portfolios or failed to adequately deal with the relative riskiness of different investments. This inadequacy provided the impetus which led to the development of the portfolio model.

PORTFOLIO THEORY

The origins of portfolio theory date back to the eighteenth century work of Bernoulli on the theory of risk.[9] However, it was in the work of Markowitz and Tobin that it was first applied rigorously to the analysis of the investment decision.[10] The main advantage of the portfolio model over previous models is that it incorporates both the riskiness of an asset and the additional return required from investing in an asset of greater risk.[11]

The portfolio model of investment behaviour is based on a theory of rational choice under uncertainty, the expected utility hypothesis. The hypothesis, developed axiomatically by Von Neuman and Morganstern, states that a rational individual faced with a choice under conditions of uncertainty acts as if he attaches numbers (utilities) to each possible outcome and chooses that strategy with the greatest expected value of utility.[12] The hypothesis implies that a risk averse individual will choose that strategy which provides for the maximum expected utility for a given level of risk and the minimum risk for a given expected utility. Although originally developed in the context of a single time period, Fama has shown that under very general conditions the multiperiod choice problem can be reduced to a series of single period decisions.[13]

RISK IN THE PORTFOLIO MODEL

In the portfolio model of investment, the risk attaching to an investment is represented by the variance of the expected future return around its expected value. Provided investors are risk averse (that is, have a risk averse utility function for wealth) and the return distribution of the individual securities are normally distributed the variance is an appropriate measure of risk.[14] Fama has found that return distributions are adequately characterized as symmetric and that at the portfolio level the variance is highly correlated with other common measures of dispersion.[15]

Thus, in the portfolio model of investment behaviour, a rational risk averse investor will attempt to maximize the expected value of future return for a given variance and minimize the variance of future return for a given expected value. That is, he will choose an investment which is mean-variance efficient. From this, one can rationalize the existence of diversified portfolios, a phenomenon which classical investment theory failed to explain.

The investment decision for an individual asset will depend upon its contribution to the expected future return and the variance of the investor's total portfolio of investments. The expected future return from a portfolio of investment assets is the weighted average of the expected future returns from the assets comprising the portfolio where the weights are the relative amounts invested in each asset. Thus, for a portfolio, P, consisting of N assets with an equal amount invested in each asset, the expected future return on the portfolio ($E(\tilde{R}_p)$) is:

$$E(\tilde{R}_p) = E\left[\frac{1}{N}\tilde{R}_1 + \frac{1}{N}\tilde{R}_2 + \ldots\ldots\ldots\ldots + \frac{1}{N}\tilde{R}_N\right]$$

$$= \frac{1}{N}\sum_{i}^{N} E(\tilde{R}_i)$$

where $E(\tilde{R}_i)$ is the expected future return on asset i, i = 1. . .N.

The contribution of an individual asset to the expected future return of a portfolio is therefore represented by its own expected future return scaled by the amount invested in it relative to the rest of the portfolio.

The variance of a portfolio's return, however, is more complex. It is in effect the sum of two terms, the variances of the individual assets' returns plus the pairwise covariances of those assets' returns. Again, assuming a portfolio, P, consisting of N assets with an equal amount invested in each asset, the variance of the portfolio's return ($\mathrm{Var}(\tilde{R}_p)$) is:

$$\begin{aligned}
\mathrm{Var}(\tilde{R}_p) = {} & \frac{1}{N^2}\mathrm{Var}(\tilde{R}_1) + \frac{1}{N^2}V(\tilde{R}_2) + \ldots\ldots \frac{1}{N^2}\mathrm{Var}(\tilde{R}_N) \\
& + \frac{1}{N^2}\mathrm{Covar}(\tilde{R}_1, \tilde{R}_2) \ldots\ldots + \frac{1}{N^2}\mathrm{Covar}(\tilde{R}_1, \tilde{R}_N) \\
& + \frac{1}{N^2}\mathrm{Covar}(\tilde{R}_2, \tilde{R}_1) \ldots\ldots + \frac{1}{N^2}\mathrm{Covar}(\tilde{R}_2, \tilde{R}_N) \\
& + \ldots\ldots\ldots \\
& + \frac{1}{N^2}\mathrm{Covar}(\tilde{R}_N, \tilde{R}_1) \ldots + \frac{1}{N^2}\mathrm{Covar}(\tilde{R}_N, \tilde{R}_{N-1}) \\
= {} & \frac{1}{N^2}\sum_i^N \mathrm{Var}(\tilde{R}_i) + \frac{1}{N^2}\sum_i^N \sum_{\substack{j \\ i \neq j}}^N \mathrm{Covar}(\tilde{R}_i, \tilde{R}_j) \\
= {} & \frac{1}{N}\overline{\mathrm{Var}(\tilde{R}_i)} + \frac{1}{N^2}(N(N-1))\overline{\mathrm{Covar}(\tilde{R}_i, \tilde{R}_j)} \\
= {} & \frac{1}{N}\overline{\mathrm{Var}(\tilde{R}_i)} + \frac{N-1}{N}\overline{\mathrm{Covar}(\tilde{R}_i, \tilde{R}_j)}
\end{aligned}$$

where $\mathrm{Var}(\tilde{R}_i)$ is the variance of the return on asset i, i = 1 . . N

$\mathrm{Covar}(\tilde{R}_i, \tilde{R}_j)$ is the covariance of the return on asset i with the return on asset j.

$$\overline{\mathrm{Var}(\tilde{R}_i)} = \frac{1}{N}\sum_i^N \mathrm{Var}(\tilde{R}_i)$$

is the average of the variances of the individual securities in the portfolio

$$\overline{\mathrm{Corar}(\tilde{R}_i, \tilde{R}_j)} = \frac{1}{N(N-1)}\sum_i^N \sum_{\substack{j \\ i \neq j}}^N \mathrm{Cov}(\tilde{R}_i, \tilde{R}_j)$$

is the average of the covariance of each individual security in the portfolio with every other security in it.

Thus, the variance of the return of a portfolio is composed of N average variances and $\frac{N-1}{N}$ average covariances. As N increases, that is as the number of securities in the portfolio grows large, the first term $(\frac{1}{N}\ \overline{\text{Var}\ (\tilde{R}_i)})$ converges to zero, and second second term $((N-1)/N$ $\overline{\text{Cov}\ (\tilde{R}_i, \tilde{R}_j)})$ converges to the average of the covariances among the securities making up the portfolio. Thus, for a large portfolio, an individual security's contribution to the risk of the portfolio is measured by its average covariance with all the other securities in the portfolio, not by its variance. For this reason, an investor might reasonably ignore the uncertainty or risk attaching to the expected future return on any single asset and concentrate attention instead on the average covariability of the expected future returns of all assets in a portfolio.

The implication of this is that a risk-averse investor will examine the risk attaching to an investment in terms of his total portfolio of assets rather than in terms of the uncertainty attaching to any one asset. An asset with a large variance but with a small covariance (especially a negative covariance) with other assets held is not a risky asset to acquire as by including it in the portfolio the risk of the portfolio (its variance) will be reduced.

The direct implementation of the portfolio model is made extremely onerous by the large number of parameters which require to be estimated. For a portfolio consisting of N securities it is necessary to compute $(N^2 + 3N)/2$ parameters as follows:

Expected returns	N
Variances	N
Covariances	$N(N-1)/2$
Total	$(N^2 + 3N)/2$

Thus, in order to estimate the risk of a portfolio it would be necessary to estimate $N(N+1)/2$ parameters which for a portfolio of 100 assets would amount to 5,050. In order to overcome this difficulty, a model of the process generating returns on securities was developed. This model, the market or diagonal model, is discussed in the next section.

THE MARKET MODEL

The market model was first suggested by Markowitz and later developed by Sharpe.[16] It defines the process generating security price returns in the following manner:

$$\tilde{R}_{it} = \alpha_i + \beta_i \tilde{R}_{mt} + \tilde{\epsilon}_{it}$$

where

$$E(\tilde{\epsilon}_{it}) = 0$$
$$Cov(\tilde{R}_{mt}, \tilde{\epsilon}_{it}) = 0$$
$$Cov(\tilde{\epsilon}_{it}, \tilde{\epsilon}_{jt}) = 0$$

and

$\tilde{R}_{it}$ is the return on security i in period t

$\tilde{R}_{mt}$ is the return on all other capital assets in the market in period t (the market return)

$\tilde{\epsilon}_{it}$ is an individualistic factor reflecting that part of security i's return in period t which is not a linear function of R_{mt}

α_i, β_i are the intercept and slope respectively associated with the linear relationship.

The model asserts that there is a linear relationship between the expected future return on security i and the expected return on all other securities in the market, the market return which refers to the average weighted return on all securities in the market.[17]

The model posits that the return on a security is composed of two parts: that portion which reflects the co-movement of a single security's return with the average return on all other securities in the market ($\beta_i R_{mt}$), which is referred to as the systematic component, and that portion which reflects the residual part of a security's return which moves independently of the market return ($\alpha_i + \epsilon_{it}$), which is referred to as the unsystematic component.

The rationale behind the model is that events which affect the rate of return on a security can be classified as having either economy-wide impact or an impact only upon one particular security. The first type of event would affect the return on all securities in the market and is reflected in β_i; the second would affect only the return on individual securities and is reflected in ($\alpha_i + \epsilon_{it}$). Classifying events in this way is clearly arbitrary to some extent: for example, another broad class of events affecting the returns on securities would be those with industry wide impact. However, empirical studies conducted in the United States suggest that the omission of other factors is not a serious misspecification of the model.[18] Further, in another American study, King found that approximately 52% of the variation in an individual security's return could be explained by its co-movement with the market return.[19]

In the context of the market model, the variance of the return on a portfolio, P, of N assets with an equal amount invested in each asset is defined as

$$\text{Var}(\tilde{R}_p) = \frac{1}{N}\,\overline{\text{Var}(\epsilon_i)} + (\bar{\beta})^2 \cdot \text{Var}(\tilde{R}_m)$$

where

$\overline{\text{Var}(\tilde{\epsilon}i)}$ is the average of the variances of the unsystematic portion of the returns on the individual securities in the portfolio,

$\bar{\beta}$ is the average of the systematic portions of the returns of the securities in the portfolio, that is

$$\bar{\beta} = \frac{1}{N}\sum_{i}^{N} \beta_i$$

$\text{Var}(\tilde{R}_m)$ is the variance of the market return.

Analogous to the portfolio model, the variance is composed of two parts. As the number of securities in the portfolio (N) increases, the first part (the average unsystematic portion) converges to zero so that the variance of the portfolio approaches $(\bar{\beta})^2$ Var (R_m). Among different portfolios the variance of the return will therefore differ only according to the magnitude of $\bar{\beta}$. As $\bar{\beta} = \frac{1}{N}\sum_{1} \beta_i$, the contribution of a single security to the risk of a portfolio is measured by its β_i, not its Var (ϵ_i).

For a single security, i, the variance of its return is defined as

$$\text{Var}(\tilde{R}_i) = \text{Var}(\tilde{\epsilon}_i) + \beta_i^2\,\text{Var}(\tilde{R}_m)$$

As Var $(\tilde{R}_m)$ is common to the variance of every security, an individual security's variance can differ from the variances of other securities because of either Var $(\tilde{\epsilon}_i)$ or β_i. The first factor, the unsystematic portion, can be driven to zero through increasing the size of the portfolio, that is, by diversifying. For this reason, it is often referred to as the avoidable risk of a security. A risk-averse investor will always select a portfolio where the avoidable risk is zero, that is, an efficient portfolio. The second factor, the systematic portion (β_i), measures the security's sensitivity to market-wide events. It cannot be reduced or eliminated by diversification and is therefore called the unavoidable risk of the security.

MARKET DETERMINED RISK MEASURES

The systematic or unavoidable risk measure (β_i) of the market model of the process generating security returns is directly related to the concept of the covariance as developed in the portfolio model of investment behaviour.[20] If security returns are normally distributed,

$$\beta_i = \text{Covar}\,(\tilde{R}_i, \tilde{R}_m)/\text{Var}\,(\tilde{R}_m)$$

where

Covar $(\tilde{R}_i, \tilde{R}_m)$	is the covariance of return on security i with the return on the market,
Var $(\tilde{R}_m)$	is the variance of the return on the market.

It is therefore consistent to use β_i from the market model as a measure of the riskiness of a security where such riskiness is defined in terms of the covariance of the security's rate of return with the market rate of return. Thus, the greater the value of β_i, the greater the risk of security i, and the smaller the value of β_i the less the security's risk. A value of β_i of one implies that the risk of a security relative to the risk of all other securities in the market is average.

Empirically, α_i and β_i can be estimated from a time series, least squares regression of the form:

$$R_{it} = a_i + b_i R_{mt} + \tilde{e}_{it} \qquad t = 1 \ldots\ldots\ldots\ldots T$$

where

R_{it}	is the return on security i in period t
R_{mt}	is the market return in period t
e_{it}	is the disturbance term in the equation
a_i, b_i	and the empirical estimates of α_i and β_i.

The use of a time series regression to estimate β_i requires first that the resulting equation conforms with the assumptions of the linear regression model of serial independence of the disturbance terms, homoscedasticity and linearity. Empirical evidence suggests that these assumptions are not violated by American data.[21] Second, for β_i to be a valid measure of systematic risk, it is necessary that it be stationary over time. Again evidence obtained in American studies suggests that such stationarity does exist.[22]

CAPITAL ASSET PRICING MODELS

Sharpe and Lintner and later Mossin extended the portfolio model to capital asset pricing models which determine the equilibrium prices for all securities in the market.[23] The models are based on the following assumptions:[24]

(a) All investors are single period expected utility of terminal wealth maximizers who choose among alternative portfolios on the basis of the mean and variance of return.

(b) All investors can borrow or lend an unlimited amount at an exogenously determined risk free rate of interest.

(c) All investors have identical subjective estimates of the means, variances and covariances of return among all assets, that is, they have homogeneous expectations.

(d) The capital markets are perfect in the sense that:

(i) There are no transaction costs;

(ii) There are no taxes;

(iii) All investors have equal and costless access to information;

(iv) Competition is atomistic, that is, all investors are price takers.

Starting from these assumptions, the models show that, in equilibrium capital assets will be priced such that:

$$\begin{aligned} R_{it} &= R_{ft} + \left[E(R_{mt}) - R_{ft}\right] \frac{\text{Cov}\,(R_{it}, R_{mt})}{\text{Var}\,(R_{mt})} \\ &= R_{ft} + \left[E(R_{mt}) - R_{ft}\right] \lambda_{it} \\ &= R_{ft}\,(1 - \lambda_{it}) + \lambda_{it}\,E(R_{mt}) \end{aligned}$$

where

R_{it}	is the return on asset i in period t,
R_{ft}	is the risk free rate of interest,
$E(R_{mt})$	is the expected return on the market portfolio (all the assets in the market) in the period t,
$\text{Cov}(R_{it}, R_{mt})$	is the covariance of the return on asset i with the return on the market portfolio in period t,
$\text{Var}(R_{mt})$	is the variance of the return on the market portfolio,
λ_{it}	$= \text{Cov}\,(R_{it}, R_{mt})/\text{Var}\,(R_{mt})$

Thus, according to the model, the only variable which determines differential expected returns among securities is the risk coefficient, λ_{it}. Also, the relationship between the expected return on an asset, R_{it} and its risk coefficient, λ_{it}, is linear so that the greater the risk, the higher the expected return and vice versa.

RELATIONSHIP BETWEEN THE MARKET MODEL AND THE CAPITAL ASSET PRICING MODELS

The market model is a specification of the stochastic process generating returns in securities while the capital asset pricing models determine the equilibrium prices for all assets in the market. There is thus no necessary relationship between them.[25] As a result, the market model is consistent with several equilibrium models of which the capital asset pricing models are but a subset, while the capital asset pricing models are not dependent upon acceptance of the market model.

However, under certain assumptions the systematic risk coefficient from the market model, β_i, will tend to approximate the risk coefficient from the capital asset pricing models, λ_{it}. These assumptions are that:

(a) the variance of the market return in the market model is equal to the variance of the return on the market portfolio in the capital asset pricing model;

(b) every security in the market portfolio is a small proportion thereof;

(c) β_i and λ_{it} are stationary over time;

(d) the variance of the disturbance term, e_{it}, in the market model is not too much larger than the variance of the return on the market portfolio, R_{mt}, in the capital asset pricing model.

The closeness of the link between the two models would therefore suggest the value to be obtained by connecting them.[26] However, it is important to note that the only assumption underlying the market model is that investors are risk averse, single period, expected utility of terminal wealth maximizers who select their holdings of securities on the basis of the mean and variance of the distribution of returns.[27]

The capital asset pricing model has two major implications.[28] First, in equilibrium, the return on an individual asset will reflect only its systematic risk component, β_i, that is, that portion of its variability resulting from its co-movement with the market rate of return. Thus, only the systematic risk element will command a price in the form of an increased return as the unsystematic risk element can be diversified away and therefore will not be compensated for in the market. Second, higher risk will always be associated with higher returns, that is, investors must be paid a premium to take on additional degrees of risk.

SUMMARY

Portfolio theory is the most advanced attempt to deal adequately with the analysis of the investment decision under conditions of uncertainty. It deals specifically and comprehensively with the problem of risk and it provides a rational explanation of the observed investment behaviour phenomenon of diversification.

Risk in the portfolio model is decomposed into two elements: systematic risk or that part of the risk attaching to a security which derives from its sensitivity to market-wide events and which cannot be eliminated; and unsystematic risk which derives from factors peculiar to the security itself but which can be eliminated through diversification. The individual risk averse investor, basing his decisions on the expected return from an investment and the risk attaching to this expected return, will therefore be primarily concerned with the systematic risk of a security. The market model provides a means of estimating this measure of risk.

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