The equity premium puzzle and decreasing relative risk aversion

Maurice J. Roche

The National University of Ireland, Maynooth, Co. Kildare, Ireland.

Abstract
Agents are assumed to have a power risk aversion utility function in an otherwise standard asset pricing model. These preferences are shown to be capable of eliminating one version of the equity premium and risk free rate puzzles when they display decreasing relative risk aversion.

Keywords: asset pricing; equity premium; risk aversion

JEL classification: G10, G12

---

1. Introduction

It has been twenty years since the seminal paper of Mehra and Prescott (1985) that first articulated the equity premium puzzle. In a recent critical review of the literature Mehra and Prescott (2003) point out that many of the resolutions to solve this puzzle have failed. The assumption that agents have preferences that exhibit decreasing relative risk aversion has recently been shown to eliminate one version of the equity premium and risk-free rate puzzles (see Meyer and Meyer (2005)). However their utility function is too cumbersome to use in many other applications and lacks intuition. In this paper we show that a power risk aversion utility function that displays decreasing relative risk aversion is also capable of eliminating the puzzles.

2. A standard asset pricing model

Mehra and Prescott (1985) derive two Euler equations from a standard asset pricing model where all agents choose consumption so as to maximize the present discounted value of future expected utility arising from random consumption streams. The two equations are given by

$$E_t\left(\frac{U'(c_{t+1})}{U'(c_t)}\left(R^e_{t+1}-R^b_{t+1}\right)\right)=0 \tag{1}$$

and

$$E_t\left(\beta\frac{U'(c_{t+1})}{U'(c_t)}R^b_{t+1}-1\right)=0 \tag{2}$$

where E_t is the expectations operator conditional on information at time t, $U'(c_t)$ is the marginal utility of real consumption per capita, R^e_{t+1} is the gross real return on equity, R^b_{t+1} is the gross real return on bonds and β is a constant discount factor. Kocherlakota (1996) uses the law of iterated expectations to replace the conditional expectation in equations (1) and (2) with an unconditional expectation and estimates the population means of

$$e_{t+1}^{e} = \left(\frac{U'(c_{t+1})}{U'(c_t)} \left(R_{t+1}^{e} - R_{t+1}^{b} \right) \right) \tag{3}$$

and

$$e_{t+1}^{b} = \left(\beta \frac{U'(c_{t+1})}{U'(c_t)} R_{t+1}^{b} - 1 \right) \tag{4}$$

using annual U.S. data from 1889-1978. Kocherlakota (1996) assumes that agents have utility functions that exhibit constant relative risk aversion and finds that one or both of individual null hypotheses $\overline{e}_{t+1}^{e} = 0$ and $\overline{e}_{t+1}^{b} = 0$ are rejected for the same parameter of constant relative risk aversion which he varied from 0 to 10. The null hypothesis $\overline{e}_{t+1}^{e} = 0$ is not rejected, at the 95% significance level, for values of the parameter of constant relative risk aversion greater than seven, while the null hypothesis $\overline{e}_{t+1}^{b} = 0$ is not rejected, at the 95% significance level, for values of the parameter of constant relative risk aversion less than one. This finding characterizes one version of the equity premium puzzle.

3. Decreasing relative risk aversion

Meyer and Meyer (2005) point out that a reason why preferences that allow habit formation, as in Campbell and Cochrane (1999), can "reduce or eliminate the equity premium puzzle" is because their utility function displays decreasing relative risk aversion. Meyer and Meyer (2005) consider a set of utility functions where marginal utility is given by

$$U'(c_t) = \frac{\delta}{\lambda c_t^{\lambda}} \tag{5}$$

for $\lambda > 0$ and $\delta > 0$. Relative risk aversion is given by δ / c_t^{λ}. The parameter λ governs the rate of decrease in relative risk aversion. Using the same data set as Kocherlakota (1996) they normalize the level of real consumption per capita in 1889 to unity and calculate relative risk aversion for average consumption as $\delta / 2.3^{\lambda}$. Meyer and Meyer (2005) choose values for λ ranging from 0.5 to 2 and adjust δ so as relative risk aversion for average consumption is in the 0.5-10 range. A sample of their results is presented in Table 1. They show that for a large range of values of λ and δ the t-statistics for testing whether the null hypotheses $\overline{e}_{t+1}^{e} = 0$ and $\overline{e}_{t+1}^{b} = 0$ are not rejected when relative risk aversion for average consumption is in the 6-10 range and $\beta = 0.99$. Assuming that $\beta = 0.99$ we estimated the parameters λ and δ using equations (1) and (2) by generalized method of moments[1] using the data set from Kocherlakota (1996). We estimated λ to be 2.18 with a standard error of 1.23 and estimated δ to be 36.32 with a standard error of 17.02. The probability value of the J-test for overidentifying restrictions is 0.45. The parameter estimates of λ and δ are significant at the 10% and 5% levels respectively. Most values of λ and δ used by Meyer and Meyer (2005) which were presented in Table 1 are within the 95% confidence interval of our estimates of λ and δ.

One problem with Meyer and Meyer (2005) preference specification is that the underlying utility function is rather cumbersome. They do not actually present the utility function in their paper and state "the exact form of the utility function is unknown". Using a mathematics software program such as Maple one can integrate equation (5) with respect to consumption and this gives the following utility function

$$U(c_t) = -\frac{c_t}{\lambda}\left(-\frac{\delta}{\lambda c_t^{\lambda}}\right)^{\frac{1}{\lambda}}\left(\Gamma\left(-\frac{1}{\lambda}\right) - \Gamma\left(-\frac{1}{\lambda}, -\frac{\delta}{\lambda c_t^{\lambda}}\right)\right) \tag{6}$$

where $\Gamma\left(-\frac{1}{\lambda}\right)$ is a gamma function and $\Gamma\left(-\frac{1}{\lambda},-\frac{\delta}{\lambda c_t^{\lambda}}\right)$ is an incomplete gamma function. We believe that using this function in other applications would be difficult and lack intuition.

In this paper we offer an alternative functional form for utility, the power risk aversion utility function (see Xie (2000)). This function has the property of decreasing relative risk aversion under certain parameter values. The function is given by

$$U(c_t)=\frac{1}{\gamma}\left(1-\exp\left(-\gamma\left(\frac{c_t^{1-\alpha}}{1-\alpha}-1\right)\right)\right) \tag{7}$$

Note when $\alpha>0$ and $\gamma=0$ then equation (7) is the commonly used constant relative risk aversion utility function. Marginal utility is given by

$$U'(c_t)=c_t^{-\alpha}\exp\left(-\gamma\left(\frac{c_t^{1-\alpha}}{1-\alpha}-1\right)\right) \tag{8}$$

and relative risk aversion is $\alpha+\gamma c_t^{1-\alpha}$. Thus equation (7) exhibits decreasing relative risk aversion when $\alpha>1$ and $\gamma>0$. We calculate the t-statistics for testing the asset pricing Euler equations (1) and (2). Similar to Meyer and Meyer (2005) we show that for a large range of values of α and γ the t-statistics for testing whether the null hypotheses $\overline{e}_{t+1}^{e}=0$ and $\overline{e}_{t+1}^{b}=0$ are not rejected when relative risk aversion for average consumption is in the 6-10 range. We present our results in Table 2. Since α governs the rate of decrease in relative risk aversion we choose values for α ranging from 2 to 5. Then is γ calculated so that relative risk aversion for average consumption takes on values 6, 8 or 10.

Assuming that $\beta=0.99$ we estimated the parameters α and γ using equations (1) and (2) by generalized method of moments.[2] We estimated α to be 3.95 with a standard error of 2.98 and estimated γ to be 30.93 with a standard error of 16.14. The probability value of the J-test for overidentifying restrictions is 0.39. The parameter estimates of α and γ are significant at the 20% and 10% levels respectively. Most values used in Table 2 are within the 95% confidence interval of our estimates of α and λ.

4. Conclusions

In a recent paper Meyer and Meyer (2005) show that preferences that display decreasing relative risk aversion are capable of eliminating one version of the equity premium and risk free rate puzzles. We suggest that their utility function is too cumbersome to use in many other applications and lacks intuition. We show that a power risk aversion utility function that displays decreasing relative risk aversion is also capable of eliminating the puzzles. This function is relatively straightforward to use and has been employed in other applications (see Xie (2000).

References

Meyer, D. and J. Meyer, 2005. Risk preferences in multi-period consumption models, the equity premium puzzle, and habit formation utility. Journal of Monetary Economics, forthcoming.

Kocherlakota, N., 1996. The equity premium: It's still a puzzle. Journal of Economic Literature 34:42-71.

Mehra, R. and Prescott, E., 1985. The equity premium: A puzzle. Journal of Monetary Economics 15:145-61.

Mehra, R. and Prescott, E., 2003. The equity premium in retrospect. The Handbook of the Economics of Finance, Eds. Constantinides, G., Harris, M., and Stulz, R., North-Holland: Amsterdam, 888-936.

Xie, D., (2000). Power risk aversion utility functions. Annals of Economics and Finance 1, 265-282.

Table 1
Testing asset pricing Euler equations using the Meyer and Meyer (2005) utility function

Relative risk aversion at the average consumption level	λ	δ	t-statistic $H_0 : \overline{e}^{e}_{t+1} = 0$	t-statistic $H_0 : \overline{e}^{b}_{t+1} = 0$
6	1	14.04	1.85	-1.92
	1.5	21.48	1.64	-1.21
	2	32.88	1.34	-0.34
8	1	18.72	1.33	-1.42
	1.5	28.64	1.07	-0.60
	2	43.84	0.68	0.34
10	1	23.40	0.09	-0.92
	1.5	35.80	0.55	-0.03
	2	54.80	0.09	0.84

Source: Table 3 in Meyer and Meyer (2005)

Table 2
Testing asset pricing Euler equations using the power risk aversion utility function

Relative risk aversion at the average consumption level	α	γ	t-statistic $H_0 : \overline{e}^{e}_{t+1} = 0$	t-statistic $H_0 : \overline{e}^{b}_{t+1} = 0$
6	3	20.12	1.71	-2.10
	4	24.43	1.63	-1.61
	5	28.13	1.66	-1.50
8	3	26.52	1.17	-1.16
	4	48.86	0.97	-0.01
	5	84.40	0.72	0.83
10	3	37.12	0.69	-0.35
	4	73.29	0.39	0.84
	5	140.67	-0.52	1.06

Endnotes

[1] The instruments chosen were a constant, consumption lagged one period, the equity premium lagged one period and the real return on treasury bills lagged one period.

[2] The instruments were the same as in footnote 1.