Open economies review **16**: 263–281, 2005

Limited Reserves and the Optimal Width of an Exchange Rate Target Zone

SIMON BROOME Simon.J.Broome@may.ie
NUI Maynooth, Maynooth, Co Kildare, Ireland

Key words: stabilisation policy, exchange rate crises, optimal target zones

JEL Classification Numbers: F31, F33

Abstract

This paper analyses the stabilising properties of an exchange rate target zone when the stock of available reserves is limited. In these circumstances it is reasonable to suppose that the optimal bandwidth is affected by the expected lifetime of the zone. Our analysis uses Sutherland's (1995) target zone model to assess the importance of the expected lifetime in determining the optimal width of the zone. We find that the expected lifetime tends to widen the optimal bandwidth considerably but unless the stock of initial reserves is small and/or the fundamentals drift large, the extra lifetime bought is small in percentage terms.

A number of authors, Klein (1990), Miller and Weller (1991), Sutherland (1995) and Beetsma and Van der Ploeg (1998), have analysed the stabilising properties of target zones using versions of the original Krugman (1991) target zone model that incorporate more complex underlying models of exchange rate determination. In each case the target zone is treated as an infinitely lived structure so that the expected lifetime plays no part in the choice of bandwidth. In practice systems of managed exchange rates last for finite periods of time before being realigned or abandoned. This is especially true for countries with limited access to foreign borrowing. Therefore, if some form of managed regime is superior to a pure float, the optimal regime will be determined not just by how much stabilisation a particular regime provides, but also by how long the regime is expected to last. This argument is especially relevant to the target zone system that has the unique feature that foreign exchange interventions are required only at the edges of the zone.

Some support for omitting the lifetime from the determination of the optimal band is to be found in Dumas and Svensson (1994). Using the Krugman target zone model the authors show that for realistic parameters the expected lifetime is extremely long. Although the bandwidth is exogenous the clear implication is that the lifetime is unlikely to have any significant influence on the choice of bandwidth. However, these results depend upon the structural form underlying the Krugman model. Broome (2001) uses the richer underlying structure of the Sutherland (1995) target zone model to show that for an identical bandwidth, size of disturbance and initial reserve stock, there are circumstances in which the expected lifetime is significantly lower than that found by Dumas and Svensson (1994). In particular, there are cases where the lifetime is very short. Unfortunately, because Broome (2001) also treats the bandwidth as exogenous, it is not clear that the expected lifetime has a significant effect on the optimal bandwidth, even if the lifetime is short. The innovation of this paper is to endogenise the lifetime into the bandwidth's objective function. Specifically, the optimal bandwidth is chosen to create a zone of stability of targeted macroeconomic variables over the longer run, reflecting the trade-offs between the variability caused by different shocks, and the endogenously determined expected lifetime. This allows us to explicitly examine the influence of the lifetime on the optimal bandwidth and conversely, it also addresses the question of whether or not the expected lifetime is low when the bandwidth is optimally chosen. The remainder of the paper is organised as follows. Section 2 presents the exchange rate model that is used in the paper. Section 3 presents and discusses the objective function from which the optimal bandwidth is derived. Section 4 draws conclusions.

1. A target zone model

The Krugman (1991) target zone model is based upon a reduced form of the flexible price monetary model of exchange rate determination with random disturbances to money demand. The drawback of this structure is that because the real and monetary sectors are independent of each other it is difficult to rationalise the existence of target zones or any other form of managed exchange rate regime. To overcome this Sutherland (1995) shows how the same basic equation used in Krugman (1987) can be derived from an underlying model in which the nominal and real sectors are not independent, and in which a variety of shocks cause variations in both real and nominal variables. The equations below represent a modified version of this model.

$$y_t^s = \alpha\left[p_t - E_{t-1}\left(p_t \mid I_{t-1}\right)\right] + \varepsilon_t \tag{1}$$

$$y_t^d = \eta(s_t - p_t + p_t^*) + \omega_t \tag{2}$$

$$m_t - p_t = \phi y_t - \lambda i_t - \upsilon_t \tag{3}$$

$$E\frac{(ds_t)}{dt} = i_t - i_t^* \tag{4}$$

$$d\upsilon_t = u_\upsilon dt + \sigma_\upsilon dz_\upsilon \tag{5}$$

$$d\varepsilon_t = u_\varepsilon dt + \sigma_\varepsilon dz_\varepsilon \tag{6}$$

$$d\omega_t = u_\omega dt + \sigma_\omega dz_\omega . \tag{7}$$

Where,

$y =$ domestic output
$p =$ price level
$m =$ nominal money supply
$i =$ nominal interest rate
$s =$ nominal exchange rate
$\varepsilon, \omega, \upsilon =$ goods supply, goods demand and velocity disturbances respectively.

All variables except i and i^* are in logs, and an asterix denotes a foreign variable. The foreign variables are assumed exogenous and omitted from the remainder of the paper.

Equation (1) is a Lucas (1972) surprise supply function, where ε represents the disturbance term. Equation (2) defines aggregate demand to be a function of the real exchange rate and the demand disturbance ω. Equation (3) is the money market equilibrium condition, where real money demand is determined by output, the nominal interest rate and a monetary disturbance υ. Equation (4) is the Uncovered Interest Parity condition. The disturbance terms ε, ω and υ each follow independent Brownian motion processes with constant drifts. In addition to the variety of disturbances the model differs from the flexible price monetary structure of the Krugman model by allowing the real exchange rate to vary if $\eta < \infty$, and for output to respond to surprise movements in the aggregate price level if $\alpha > 0$.

In the original Sutherland model the aggregate supply curve relates output to the price level rather than to surprise movements in the price level. This has the undesirable property that predictable movements in fundamentals, including nominal variables affect output. Sutherland (1995) avoids the problem by assuming the fundamentals to be drift-less but it is necessary to include drifts here because in the absence of any drifts the expected lifetime is extremely long.[1] To ensure that predictable movements in nominal variables do not affect output we use a surprise supply function. In continuous time this implies any deviations from equilibrium last only for an infinitesimal period of time. However the impact effects of the shocks are unchanged from Sutherland (1995), and are identical to uncorrelated shocks in a fully specified discrete time version of the neo-classical exchange rate model. Specifically solving (1) and (2) for the price level and output yields,

$$p = \frac{\eta s + \alpha E(s)}{\alpha + \eta} - \frac{\eta \varepsilon + \alpha E(\varepsilon)}{\eta(\alpha + \eta)} + \frac{\eta \omega + \alpha E(\omega)}{\eta(\alpha + \eta)} \tag{8}$$

$$y = \frac{\alpha}{\alpha + \eta}\left[\eta(s - E(s)) + (\omega - E(\omega)) - (\varepsilon - E(\varepsilon))\right] + \varepsilon. \tag{9}$$

For simplicity the time subscripts are suppressed. Equations (8) and (9) show the effects of the various disturbances on prices and output. Intuitively both predictable and unpredictable movements in demand affect the aggregate price level whilst only surprise movements in demand affect output. Using the same method as in Sutherland (1995) the solution for the exchange rate can be expressed using the same reduced form equation that is used in Krugman (1991).

$$s = f + \lambda E\frac{[ds]}{dt}. \tag{10}$$

The difference between (10) and the Krugman version is that f is a composite fundamental that reflects a richer structural form. The dynamics of the unregulated part of f can be expressed as the following composite Brownian motion process,

$$df = \mu_f dt + \sigma_f dz \tag{11}$$

$$\mu_f = \mu_v + \frac{(1-\phi\eta)}{\eta}\mu_\varepsilon - \frac{1}{\eta}\mu_\omega \tag{12}$$

$$\sigma_f^2 = \left(\frac{\alpha+\eta}{(1+\alpha\phi)\,\eta}\right)^2\sigma_f^2 + \left(\frac{(\phi\eta-1)}{(1+\alpha\phi)\,\eta}\right)^2\sigma_\varepsilon^2 + \left(\frac{1}{\eta}\right)^2\sigma_\omega^2. \tag{13}$$

The coefficients of Equations (12) and (13) should be thought of as defining the amount of exchange rate pressure generated by each individual disturbance. The use of the surprise supply function means that (12) is an improvement on Broome (2001) insofar as α is absent. This is briefly discussed in 3.2. The solution under a target zone can now be obtained in the standard way. This is,

$$s_t = \lambda\mu_f + f_t + A_1 e^{\rho_1 f_t} + A_2 e^{\rho_2 f_t} \tag{14}$$

where, $\rho_1, \rho_2 = (-\mu_f \pm \sqrt{\mu_f^2 + 2\sigma_f^2/\lambda})/\sigma_f^2$. The arbitrary constants are defined by the smooth pasting conditions. The solution is a characteristic S-curve such as depicted in Figure 1.

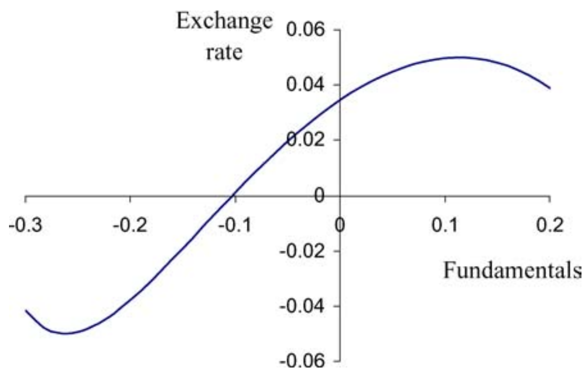

Figure 1. A target zone.

2. The optimal band and the lifetime of the target zone

The novelty of this paper is that the lifetime is an endogenous variable in the objective function determining the optimal bandwidth. The optimal bandwidth is chosen to minimise the variances of the potential target variables, prices, output and the nominal or real exchange rate over an infinite but discounted horizon. Although there are a variety of possibilities we assume the post abandonment regime is a pure float that is expected to be permanent.[2] The objective function is,

$$\begin{aligned}\operatorname*{Min}_{\bar{s}} V = E_t\Bigg\{&\pi_s\left(\int_0^T \sigma_s^2 e^{-\delta\tau} d\tau + \int_T^\infty \sigma_{sf}^2 e^{-\delta\tau} d\tau\right)\\ &+\pi_p\left(\int_0^T \sigma_p^2 e^{-\delta\tau} d\tau + \int_T^\infty \sigma_{pf}^2 e^{-\delta\tau} d\tau\right)\\ &+\pi_y\left(\int_0^T \sigma_y^2 e^{-\delta\tau} d\tau + \int_T^\infty \sigma_{yf}^2 e^{-\delta\tau} d\tau\right)\Bigg\}. \qquad (15)\end{aligned}$$

Where π_s, π_p, π_y are the preferences of the policy maker towards exchange rate, price and output stability. The variances of the exchange rate, price level and output under the target zone regime are represented by $\sigma_s^2, \sigma_p^2, \sigma_y^2$, whilst $\sigma_{sf}^2, \sigma_{pf}^2, \sigma_{yf}^2$ are the corresponding variances under a free float regime. The rate of time preference is δ and T represents the lifetime of the target zone. The difference between (15) and the earlier literature on the optimal bandwidth is that the lifetime is not implicitly assumed to be infinite. If the lifetime were infinite the policymaker narrows the bandwidth if the asymptotic variance of the target variables falls. According to (15) the bandwidth is narrowed only if the fall in the mean variances discounted over the expected lifetime, exceeds the discounted increase in the mean variances caused by the expectation of an earlier transition to the floating regime. The lifetime is calculated following the fundamentals based approach used in Dumas and Svensson (1994) and Broome (2001). This requires assuming that implicit constraints prevent the exclusive focus of monetary policy on the defence of the target zone, or equivalently the stock of foreign reserves is not infinite. The lifetime can then be calculated as the first passage time of initial reserves to a critical lower bound that triggers the collapse of the target zone. The difference between (15) and Broome (2001) is that the lifetime is optimally chosen. Solving (15) requires the policymaker to choose both the optimal bandwidth and the lifetime.

The solution of (15) requires the calculation of the expected variances of the target variables under the target zone regime. For a fixed bandwidth the distribution of the fundamental and the exchange rate are determined by the relative sizes of the drift and variance of the composite fundamental, and by the length of time for which the zone is expected to last. To simplify we use the slope of the secant line between the limits on the exchange rate and the limits on the fundamental, as a simple approximation to the expected average relationship between the exchange rate and fundamental. Because the width on the fundamental's band increases with the drift size this approximation has the correct feature that a larger drift reduces the expected variance of the exchange rate for a given lifetime. If the drift is small and the lifetime short the estimated gap between the free float and within band variability is overestimated and the results overstate the importance of the lifetime. Conversely if the drift is not small and/or the lifetime not short our results understate the importance of the lifetime. For the short to moderate lifetimes in which we are interested the approximation is a reasonable compromise that captures the essential features of the density function.[3]

3. Results

To understand the interaction between the lifetime and optimal bandwidth it is easiest to first analyse the optimal bandwidth when the lifetime is infinite, and the lifetime when the bandwidth is exogenous. Thus subsection 3.1 reviews and extends Sutherland (1995) whilst 3.2 summarises the results of Broome (2001). Subsection 3.3 contains the main results on the relationship between the expected lifetime and the optimal bandwidth.

3.1. The optimal bandwidth with an infinite lifetime

In Figure 2 we plot the variance of output and the contribution of each individual shock against the width of the target zone assuming the lifetime to be infinite. The parameter values are, $\lambda = 3$, $\eta = .5$, $\alpha = .5$, $\phi = 1$, $\mu_v = \mu_\varepsilon = \mu_\omega = 0$, $\sigma_v = .05$, $\sigma_\varepsilon = .05$, $\sigma_\omega = .05$, giving $\sigma_f^2 \approx 0.0002$.

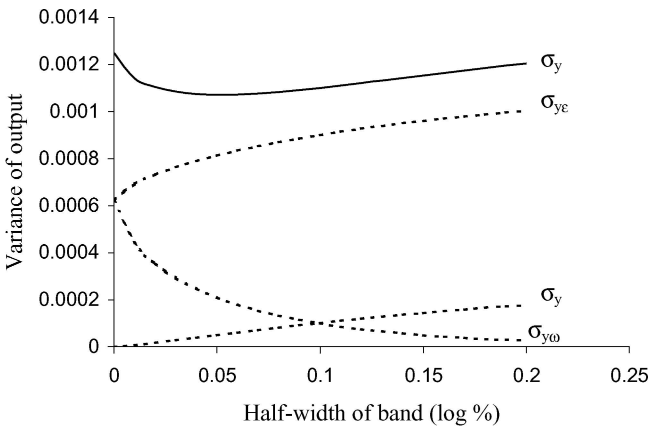

Figure 2. Shocks, output variability and the bandwidth.

The solid line represents the total variance of output whilst the dashed lines show the contribution of the individual shocks to this total. Widening the band increases the amount of output variability caused by the velocity shock but reduces the amount caused by the demand shock. The supply shock has opposing effects on prices and output, causing its effect on money demand and the nominal exchange rate to be ambiguous. In this example the exchange rate depreciates and widening the band increases the amount of output variability attributable to the supply shock. As in Sutherland (1995), in comparison with a fixed or floating regime a target zone reduces the overall variance of output when a variety of shocks are present. The bandwidth that minimises the variance of output is approximately $\pm 5 \ \log\%$.

With respect to velocity or goods demand shocks the variance of the price level is a multiple of the variance of output. Thus whatever the target, the optimal band is narrower the larger is the ratio of velocity to good demand shocks. It also means that in the absence of supply shocks, the optimal band is not affected by the relative weights given to output or price stabilisation. If the exchange rate depreciates (appreciates) in response to supply shocks, the optimal band is narrower (wider) for an output target than for a price target whenever supply shocks are present. The exchange rate depreciates if $\phi\eta < 1$ and appreciates if $\phi\eta > 1$. Table 1 summarises the changes in the bandwidth that are required to minimise the variability of the price level or output in response to the individual shocks.

Other than the bandwidth, the main determinants of the amount of price and output variability created by the individual shocks are the slopes of the aggregate supply and goods demand curves. The slope of the goods demand curve is determined by the elasticity of aggregate demand to the real exchange rate, η. We refer to this loosely as the degree of openness. If the fraction of tradables in domestic output is high and if domestic goods have close substitutes, aggregate demand is sensitive to variations in the real exchange rate and the goods demand curve is relatively flat. In the limit goods are homogenous, purchasing power parity holds and the demand curve for goods is horizontal. In this case shocks to goods demand have no effect on the price level, nominal and real exchange rates or output. Conversely, if η is low goods demand shocks are an important source of price and/or output variability. The slope of the aggregate

Table 1. The optimal bandwidth and the individual shocks.

Shock type	Change in band to stabilise the price level	Change in band to stabilise output
Money demand	Narrow	Narrow
Goods demand	Widen	Widen
Supply & $\phi\eta < 1$	Widen	Narrow
Supply & $\phi\eta > 1$	Narrow	Widen

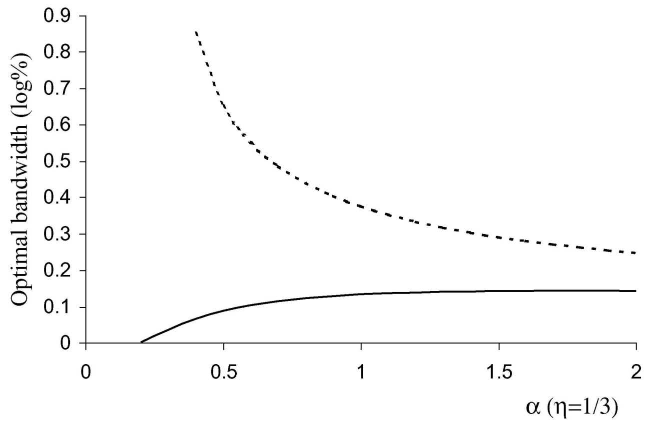

Figure 3. The supply curve and optimal band.

supply curve is determined by the elasticity of supply to unexpected increases in the price level, α. As α increases the supply curve flattens out, and demand shocks have a smaller (larger) effect on the price level (output), whilst supply shocks have a smaller effect on both output and the price level.

To illustrate these effects Figure 3 plots the optimal band for separate price level (shaded line) and output targets (unbroken line) against the slope of the supply curve assuming that $\eta = 1/3$. Figure 4 performs the same operation for $\eta = 3$.

With $\eta = 1/3$ the slope of the goods demand curve is -3. A combination of relatively steep goods demand and supply curves produces very different optimal bands for the price and output targets. There are two reasons for this. Firstly

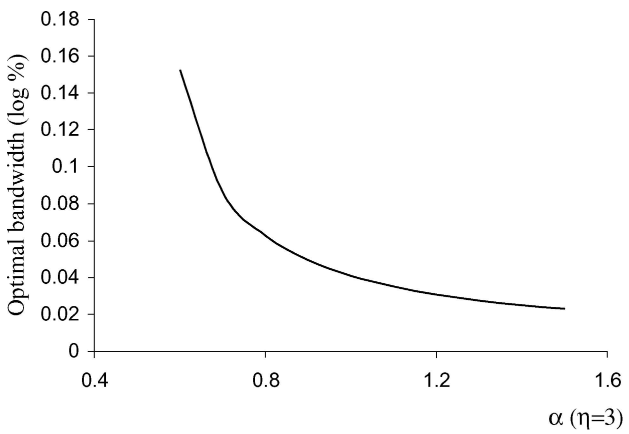

Figure 4. The supply curve and optimal band.

with $\eta = 1/3$ real shocks create a lot of systemic variability. However, the steep aggregate supply curve means that most of the variability emanating from the demand side is concentrated on the price level rather than output, and thus the optimal band for output stability depends mainly on the supply shocks. Since $\eta = 1/3$ the exchange rate depreciates in response to a supply shock and output variability is reduced by having a narrower zone, whilst price level variability is reduced by a wider zone. Consequently, the optimal band for output stability is narrow whilst the optimal band for price level stability is wide. For example if $\alpha = .2$, the optimal band for output stability is $\pm.024$ log% whilst the optimal band for the price level target is a free float regime. Increasing α means that supply shocks become less important whilst demand shocks create more output variability but less price variability. The optimal band for the output target first widens and then narrows. Thus initially the dominant effect of increasing α is the reduction in the contribution of supply shocks, but as this becomes small enough, the dominant effect is on velocity shocks and the optimal band starts to narrow. For α large enough the supply shocks become irrelevant, and the optimal band is identical for the price level and output targets. In both cases with $\eta = 1/3$ and given the assumed distribution of shocks, the optimal band is $\approx \pm 12.5 \ \log\%$.

With $\eta = 3$ the slope of the demand curve is $-1/3$ and goods demand shocks are almost irrelevant. As before, if the aggregate supply curve is steep supply shocks are the dominant source of output variability. Since the exchange rate now appreciates to supply shocks the optimal band for output stability is wide, for $\alpha = .2$ the optimal regime is a free float. In the case of the price level the optimal band is very narrow, both because stabilisation in response to a supply shock calls for a narrower zone, and because goods demand shocks are unimportant. Thus in terms of price stability the optimal band is a fixed exchange rate and this is true for all values of α. As the aggregate supply curve flattens out the supply shock becomes less important, and the optimal band for output narrows. In the limit real shocks do not affect output and the optimal regime is a fixed rate for both price and output stability. An overview of the relationship between the slopes of the supply and goods demand curves and the optimal band is given in Table 2. This table refers to

Table 2. Goods supply and demand curves and the optimal bandwidth.

α, η	Price	Output	Explanation
High, high	Very narrow	Very narrow	Only money shocks matter
High, low	Wide	Wide	Goods demand shocks dominate
Low, high	Very narrow	Wide	σ_ε dominates, with $\phi\eta > 1$ output (price) is stabilised by a narrow (wide) zone
Low, low	Wide	Narrow	All shocks matter. Since $\phi\eta < 1$ σ_ε and σ_v create a narrow zone for output

the values just discussed and assumes that the variances of the shocks are identical.

Ignoring the lifetime the optimal band is determined by the distribution of the shocks, the slopes of the aggregate demand and supply curves and the chosen target in the manner outlined by Tables 1 and 2.

3.2. Review of the lifetime

For a given bandwidth the lifetime depends upon the stock of reserves and size of disturbance in an obvious way. The lifetime also depends on the source of the disturbance and the structural parameters. The main results in Broome (2001) can be summarised as follows. A disturbance originating in the money market is offset by the equal and opposite change in reserves therefore the relationship between the lifetime and velocity drift is unaffected by the parameters of the model. Under a fixed rate a drift of 10% per year will cause reserves to fall at the same rate. If the initial log excess of reserves is 100% above the critical minimum, the lifetime of the fixed rate is 10 years. Thus unless the band is very wide, the extra lifetime time bought by the bandwidth is small compared to the lifetime of a fixed rate. Therefore for the lifetime to significantly alter the bandwidth the underlying drift must be large, and/or the initial log excess of reserves small.

The lifetime that corresponds to disturbances originating in the goods market is strongly influenced by the elasticity of aggregate demand to the real exchange rate, (or openness) parameter η. Openness matters because it determines the amount of exchange rate pressure caused by a real shock. In particular if η is low the lifetime is much lower in the case of a real drift than in the case of a velocity drift. The surprise supply function used in this paper is an improvement on, and strengthens the results contained in Broome (2001). This improvement means that the slope of the supply function has no significant effect on the lifetime. The reason is that Brownian motion shocks tend to cancel on another out, which is why any reasonably short lifetime requires some type of drift. Since the slope of the supply curve relates surprise movements in the price level to output, and the drifts are predictable, the slope of the supply curve has no effect on the size of composite drift and so has only a minimal effect on the lifetime.

3.3. The lifetime and the optimal band

From 3.1 we know when the optimal band will be narrow ignoring the lifetime, and from 3.2 we know when the lifetime of a given bandwidth will be short. In this section we combine these results to assess the influence of the lifetime on the optimal band. Throughout, the optimal band is chosen to stabilise output alone. In addition to the parameter values used for Figure 2 we set the rate of

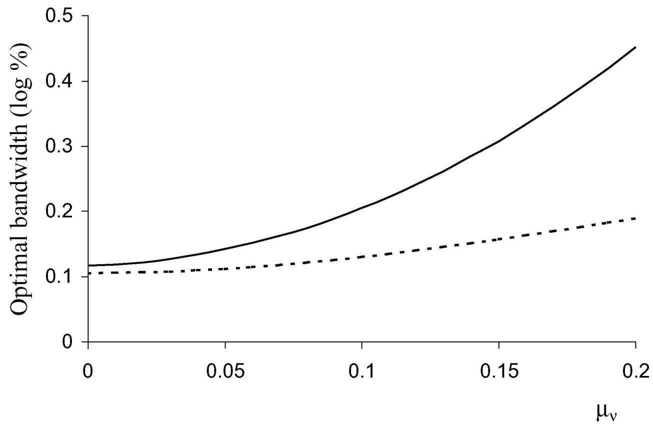

Figure 5. The optimal band and a monetary drift.

time discount equal to 5% per year $\delta = .05$, and the initial log excess of reserves equal to, $r = (p+q)/2 = (0+200)/2 = 100 \ \log$%.

Our first example examines how the size of the fundamental drift affects the optimal band through the lifetime. Increasing the size of the drift alters the position and shape of the S-curve and this changes the instantaneous stabilising properties of the zone. Consequently the optimal band changes irrespectively of any effect via the lifetime.[4] To isolate the effect of the lifetime we also plot the optimal band under an infinite rate of time discount. In the latter case the optimal band is chosen without regard to the lifetime so that the gap between the two bands represents the widening that is caused by the finite lifetime. Specifically Figure 5 plots the optimal band under $\delta = .05$ and under $\delta \to \infty$ against the size of a velocity drift whilst Figure 6 plots the corresponding lifetimes.

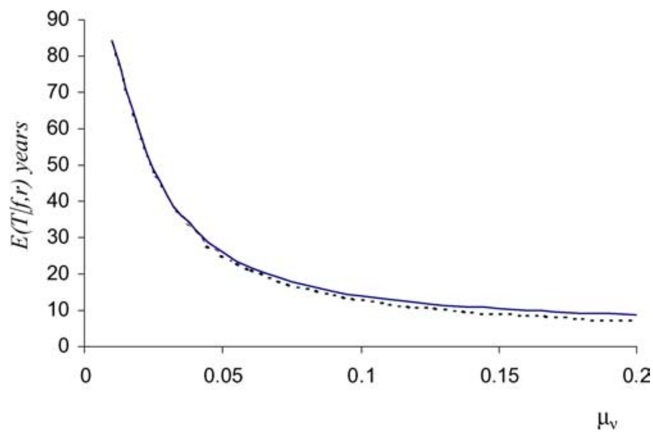

Figure 6. The lifetime and a monetary drift.

The solid line represents the optimal band under the 5% yearly discount rate whilst the dashed line is the optimal band under the infinite rate of time discount. We refer to the optimal band under the 5% rate of time discount as the long horizon band. If the drift of fundamentals is close or equal to zero, the expected lifetime of the target zone is sufficiently long that the long horizon band and infinitely discounted band are close to one another. For example with a velocity drift of 1% per year the infinitely discounted band is $\pm 10.6 \log \%$ and the expected lifetime is 82.7 years, whilst the long horizon band is $\pm 11.9 \log \%$ and the lifetime is 84.5 years. Increasing the drift size widens both bands and the gap between them also widens. A drift of 10% per year results in a long horizon band of $\pm 20.5 \log \%$, whilst the infinitely discounted band is $\pm 13 \log \%$. Percentage-wise including the lifetime as a decision variable widens the bandwidth by 12% when there is a 1% per year annual drift in velocity, and by 58% for a 10% annual drift in velocity. The pure effect of adding a drift is also quite strong. For example, a 10% annual velocity drift widens the infinitely discounted band by 24% from $\pm 10.5 \log \%$ to $\pm 13 \log \%$.

Figure 6 shows that the lifetime falls as the size of the velocity shocks is increased, and that whilst the lifetime significantly alters the optimal bandwidth the reverse is not true. Even for an annual velocity shock equal to 10% per year the lifetime for the long horizon band at 13.9 years is only 7% more than the 13 years under the infinitely discounted band. This is not particularly significant in either percentage or absolute terms. It is also clear that the expected lifetime in this case is quite long even for quite large monetary drifts. This is not surprising, as outlined in 3.2, if the drift were purely monetary then regardless of the slopes of aggregate demand or supply curves, a 100% log excess of reserves and 10% annual drift result in the somewhat long lifetime of 10 years for a fixed exchange rate. Consequently it is unlikely that variations in the bandwidth have a significant effect on the lifetime in percentage terms. Moreover, since the variations in the band are large only when the drift is large this limits the extra lifetime bought in absolute terms.

Although widening the bandwidth increases the lifetime by a seemingly small amount the lifetime significantly influences the optimal band for the following reason. Since the lifetime is not short and the drift is quite large, the average within band variance of the exchange rate is low even for quite wide bands. This means the expected increase in the within band variance caused by widening the band is small, making it worthwhile to widen the band significantly even though the extra lifetime gained is also small. This is not a product of the approximation. A combination of a moderate drift and a long lifetime mean that on average the exchange rate is very close to the limit of the zone in the direction of the drift, even for a very wide target zone. An alternative way of saying this is to note that even for wide bands, the gap between the free float variance and average within band variance is large, and what matters is not so much the width of the zone but the survival time.

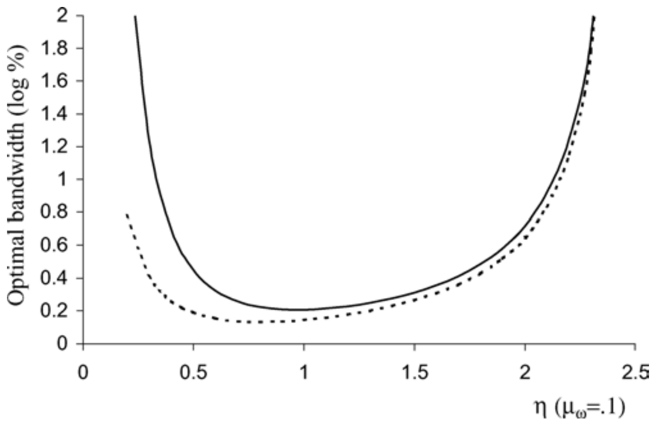

Figure 7. The optimal band and openness.

From Sections 3.1 and 3.2 we know that if the drift is purely monetary, variations in the structural parameters have only a limited effect on the importance of the lifetime via the effect on the incentive to choose a narrow band. With real drifts the structural parameters directly affect the lifetime, and thus affect the importance of the lifetime more significantly. To illustrate this Figures 7 and 8 plot the optimal band and lifetime against the degree of openness, assuming a goods demand drift of 10% per year.

The general patterns of these figures reflect how the degree of openness alters the importance of the individual shocks and the lifetime. With a low degree of openness disturbances to goods demand have very significant effects. This

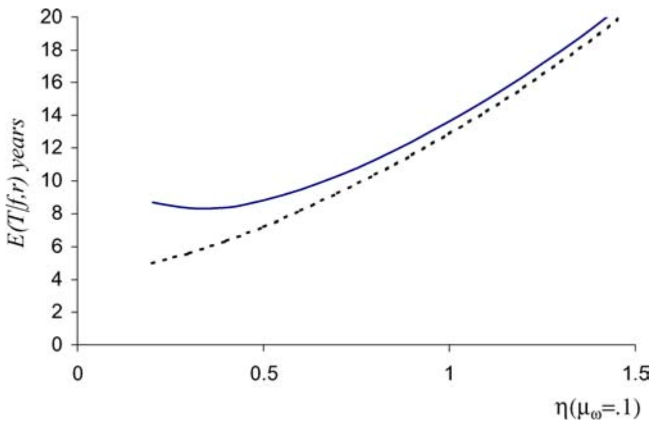

Figure 8. The lifetime and openness.

causes the long horizon optimal band to be significantly wider than the already wide infinitely discounted optimal band. For example, at $\eta = .2$ the long horizon band is ± 250 $\log\%$ and the infinitely discounted band is ± 78 $\log\%$, whilst the lifetime is 8.7 and 5 years respectively. As η increases the goods demand shock become less significant and both bands narrow. Since the goods demand drift also becomes less significant the lifetime increases and the long horizon band and infinitely discounted bands tend to one another. At the benchmark value of $\eta = .5$ the long horizon band is ± 45 $\log\%$ which is 136% wider than the infinitely discounted band of ± 19 $\log\%$. This increases the lifetime by 22% from 7.2 to 8.8 years. As η increases beyond $\phi\eta = 1$ the optimal response to a supply shock changes from narrowing to widening the zone. Given that increasing η also increases the importance of supply shocks relative to velocity shocks the long horizon band starts to widen. At $\eta = 2$ the long horizon band is $\pm 71\log\%$ and the infinitely discounted band ± 64 $\log\%$. The lifetime in this case is very long both because the infinitely discounted band is wide, and because exchange rate variations are highly effective at offsetting the drift in goods demand. If price stability were to be targeted the infinitely discounted optimal band would be wider for $\phi\eta < 1$ but narrower for $\phi\eta > 1$, reflecting the differential effects of supply shocks on prices and output. Consequently the incentive to choose a narrow zone would be reduced when the lifetime is low, but increased when the lifetime is high, as compared to the case of the output target.

The results depend crucially upon the assumed log excess of initial reserves. The effects of changing this log excess are plotted in Figures 9 and 10. The parameter values are as for Figure 5 but assume a 10% annual velocity drift. We also report the values for a 10% drift in goods demand.

With zero excess reserves the infinitely discounted band is ± 13 $\log\%$ and the long horizon band is ± 83 $\log\%$. The corresponding values for the lifetime

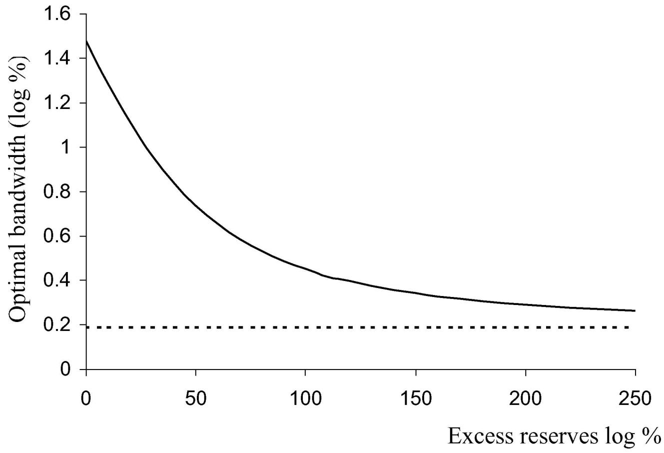

Figure 9. The optimal band and excess reserves.

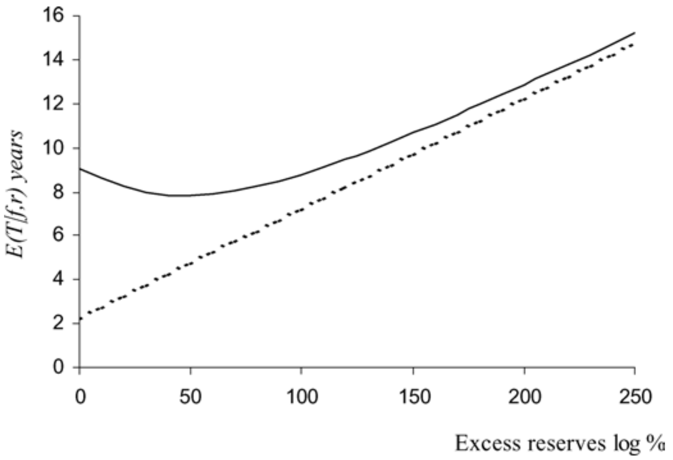

Figure 10. The lifetime and excess reserves.

are 2.9 and 10.5 years. With a 50% log excess of reserves the long horizon band at $\pm 31 \ \log\%$ is 138% wider than the infinitely discounted band and this increases the lifetime by 26%, from 8 to 10.1 years. If the underlying drift were to goods demand the lifetime is more important. For example a 50% log excess of reserves produces a long horizon band of $\pm 71 \ \log\%$ that is 284% wider than the infinitely discounted band of $\pm 19 \ \log\%$, and this increases the lifetime by 66% from 4.7 years to 7.8 years. In the case of goods demand drift even with an excess of log reserves equal to $\pm 250 \ \log\%$ the lifetime still widens the bandwidth by 36% from $\pm 19 \ \log\%$ to $\pm 26 \ \log\%$ even though the lifetime itself increases by just 2.7% from 14.7 to 15.1 years.

It is noticeable that for a very small log excess of reserves, increasing the reserve stock causes the long horizon band to narrow so much that the lifetime actually falls. This is due to non-linearity of the S-curve. The optimal band balances the gain from tightening the band, the difference between within band and free float volatility, against the reduction in the lifetime. Narrowing the band flattens the S-curve and this both increases the gain from tightening the zone, and reduces the loss in lifetime that occurs from a further tightening. If this improvement in the trade off is large enough in terms of a lower variance of the target variable (output) it becomes worthwhile to sacrifice lifetime in exchange for the increased stability on offer. When the initial zone is very wide this improvement in the trade off is quite large but it falls as the band narrows so that in general adding reserves leads to a narrower zone and a higher lifetime.

Thus far the results suggest that unless the drift in fundamentals is small the lifetime will have a significant effect on the optimal bandwidth in both absolute and percentage terms. The reverse is not true unless there is a real drift and a low elasticity of aggregate demand to the real exchange rate, and/or the initial log

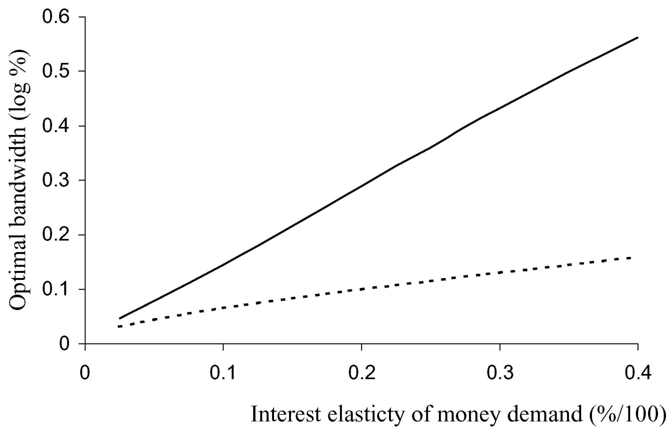

Figure 11. The optimal band and the honeymoon effect.

excess of reserves is much less than the 100% we have used as the benchmark example.

One way of choosing the initial log excess of reserves would be to consider the realism of the estimated lifetime of the fixed rate. The lifetime of the fixed rate depends on just two parameters, the initial log excess of reserves and the size of the composite drift. The latter is determined by the size of the underlying drift and in the case of real drifts, the degree of openness. Our basic parameterisation assumes an initial reserve stock 2.7 times the size of the speculative attack that destroys the zone and a 10% annual drift. With $\eta = .5$ this gives a lifetime for a fixed rate of 10 years for the monetary drift, and 5 years for the drift in goods demand. An underlying drift of 10% implies that $\mu_f = 10\%$ per year for the monetary drift and $\mu_f = 20\%$ per year for the goods demand drift. Since μ_f measures the average annual exchange rate depreciation under a pure free float, for those countries subject to periodic crises these values are plausible. The amount of initial excess reserves depends upon the access to foreign borrowing and the political commitment to the target zone regime. This makes the appropriate initial excess of reserves more difficult to assess. An alternative way to choose the initial excess reserves would be to contrast the calculated lifetime with the study by Klein and Marion (1997) of 61 exchange rate pegs in Latin American countries in which the average peg lasted just 2.7 years.[5] Given a drift such that $\mu_f = 10\%$ per year we choose the initial excess of log reserves such that the lifetime of a fixed rate is 3 years. Given a lifetime of a fixed rate equal to 3 years our final example analyses how altering the interest elasticity of money demand λ, alters the importance of the lifetime.

With λ very low the effect of the zone on the exchange rate is quite weak and both bands are very narrow. In percentage terms the long horizon band at $4.65 \ \log\%$ is 49% wider than the infinitely discounted band of 3.11 log%. Since

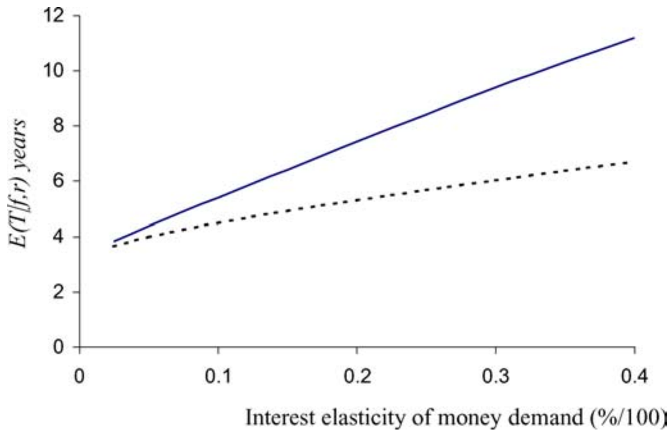

Figure 12. The lifetime and honeymoon effect.

the absolute values are so small this raises the lifetime by just 5% from 3.65 to 3.85 years. Increasing λ widens both bands and the gap between the two grows. Since the old bandwidth was optimally chosen and increasing λ leads to greater exchange rate stability, the old band is now too tight so that the infinitely discounted band widens as λ increases. The long horizon band widens more than proportionately indicating that the lifetime is having a bigger influence the larger is λ. The reason that the lifetime matters more is that the amount of stability offered by the zone increases, and this makes it more important that it survives. Consequently the lifetime has a bigger influence on the optimal band the larger is the value of λ. An interest elasticity of money demand equal to 20% produces a long horizon band of $\pm 29 \ \log\%$ that is 190% wider than the infinitely discounted band of $\pm 10 \ \log\%$. In terms of the lifetime the big variations in the bandwidth do increase the lifetime significantly if the interest elasticity of money demand is not low. For example, if $\lambda = 20\%$ the lifetime under the long horizon band at 7.42 years is 40% more than the 5.3 years under the infinitely discounted band. At $\lambda = 30\%$ these values are 9.4 years, 56%, and 6 years respectively. In this case in which we set the lifetime of the fixed rate equal to 3 years, including the lifetime in the determination of the optimal band causes variations in the bandwidth that are large enough to buy significant amounts of extra lifetime in absolute and percentage terms.

The results depend upon the approximation to the within band variance of the target zone and the assumed rate of time discount. The latter affects our results in an obvious way. Increasing the rate of time discount causes the long horizon band to converge to the infinitely discounted band and the expected lifetime to fall accordingly. The within band approximation almost certainly causes our results to understate the importance of the lifetime. As outlined previously the larger the drift and the longer the lifetime, the lower is the average variance of the exchange rate and the less it responds to changes in the bandwidth. Since the lifetime is only really low when the drift is very large the exchange rate must on average be close to the bands edges and the average within band variance would be lower than implied by our approximation.

4. Conclusions

Several papers analyse the optimality of an exchange rate target zone assuming the zone to be infinitely lived. Dumas and Svensson (1994) and Broome (2001) analyse the lifetime of target zones treating the bandwidth as exogenous. This paper extends the literature on the optimal bandwidth by endogenising the lifetime into the objective function defining the bandwidth. This allows us to examine the influence of the lifetime on the optimal bandwidth. It also extends the literature on the lifetime, insofar as the lifetime is now dependent on the optimally chosen bandwidth. Our main result can be stated as follows. Varying the bandwidth does not usually increase the lifetime significantly in percentage terms. Nonetheless provided there is some form of drift the lifetime is a significant determinant of the optimal bandwidth. The reason is that with a large drift the lifetime will be short. As a consequence of the large drift the distribution of the exchange rate is heavily skewed to the upper edge of the zone where the exchange rate is very flat. This means the within band variance of the exchange rate is much lower than the variance under the free float regime. The larger the gap between the target zone level of variance and the free float level, the more important is the survivability of the zone. On the other hand if the drift is small the lifetime is long. But in this case the amount of time the fundamental has to reach the upper limit is increased, and once again the within band variance is significantly less than the variance under the free float regime. What matters is not so much how wide the band is but how long it survives. That is, the lifetime is an important determinant of the optimal bandwidth. As above the amount of extra lifetime bought is quite small, even for quite large increases in the bandwidth. However if there is a large real drift and a low degree of openness, and/or if access to foreign borrowing is very limited the amount of extra lifetime bought is significant in percentage terms. In particular if the initial reserve stock is chosen to match the average lifetime of fixed rates for countries subject to frequent crises as in Klein and Marion (1997), the extra lifetime bought is a very significant fraction of the total lifetime. Finally it is worth reiterating that stabilisation without intervention, and hence the extra lifetime is the unique feature of the target zone.

Acknowledgments

The author is grateful to Alan Sutherland for comments on an earlier draft of this paper and to research funding from the Institute for International Integration Studies, Trinity College Dublin.

Notes

1. In the classic Krugman (1979) and Flood and Garber (1984) models of speculative attacks, the expected lifetime of the fixed rate is equal to the initial reserve stock divided by the drift in domestic credit minus the time lost due to the attack.
2. A more stable post abandonment regime reduces the gains from having a target zone and the importance of the lifetime. However as in Krugman and Miller (1993), one could equally argue that the post abandonment regime exhibits excess volatility due to speculative behaviour not present in the target zone.
3. To limit the size of the paper we omit the technical details. Full derivations are available on request.
4. This is easily understood by considering the drift-less case in which the fundamental is uniformly distributed between symmetric upper and lower limits. The asymptotic variance of the exchange rate would be most heavily weighted towards the steepest part of the S-curve at the very centre of the target zone. Adding a positive velocity drift shifts the S-curve upward and the asymptotic variance becomes more heavily weighted toward a flatter part of the S-curve. Given that the average variance (of the exchange rate) is now lower the previous optimal band would be too narrow. Thus adding a drift widens the optimal band even if its effect on the lifetime is limited.
5. The standard deviation was 49 months and the median duration just 10 months. That is a lot of pegs collapsed within the first year whilst a significant number of pegs, having gained some credibility survived for considerably longer than 32 months.

References

Beetsma, R.M.W.J. and F. van der Ploeg (1998) "Macroeconomic Stabilisation and Intervention Policy under an Exchange Rate Band." *Journal of International Money and Finance* 17:339–353.

Broome, S.J. (2001) "The Lifetime of a Unilateral Target Zone: Some Extended Results." *Journal of International Money and Finance* 20:419–438.

Cox, D.R. and H.D. Miller (1965) *The Theory of Stochastic Processes.* Chapman and Hall, London.

Dumas, B. and L.E.O. Svensson (1994) "How Long Do Unilateral Target Zones Last?" *Journal of International Economics* 36:467–481.

Karatzas, I., and S.E. Shreve (1998) *Brownian Motion and Stochastic Calculus.* Springer Verlag, New York.

Klein, M.W. (1990) "Playing with the Band: Dynamic Effects of Target Zones in an Open Economy." *International Economic Review* 31:757–772.

Klein, M.W. and N.P. Marion (1997) "Explaining the Duration of Exchange Rate Pegs." *Journal of Development Economics* 54:387–404.

Krugman, P. (1991) "Target Zones and Exchange Rate Dynamics." *Quarterly Journal of Economics* 106:669–682.

Krugman, P. (1979) "A Model of Balance of Payments Crises." *Journal of Money, Credit and Banking* 11:311–325.

Krugman, P. and M. Miller (1993) "Why Have a Target Zone?" *Carnegie-Rochester Series on Public Policy* 35:67–78.

Lucas, R.E. Jr. (1972) "Expectations and the Neutrality of Money." *Journal of Economic Theory* 4:103–124

Miller, M. and P Weller (1991) "Exchange Rate Bands with Price Inertia." *Economic Journal* 101:1380–1399.

Poole, W. (1970) "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model." *Quarterly Journal of Economics* 84:197–216.

Sutherland, A. (1995) "Monetary and Real Shocks and the Optimal Target Zone." *European Economic Review* 39:161–172.