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A Filtering Super-Twisting Controller with Noise Rejection

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Abstract— **This paper addresses the design of a filtering Super-Twisting (FST) controller with noise rejection. To effectively achieve noise rejection and improve ST performance, the proposed control structure includes a zero-phase sliding-mode filter, capable of rejecting unbounded measurement noise. The features of the FST controller reduce the control effort required to steer the sliding variable to zero, without compromising the control robustness. The convergence of the FST is demonstrated, and a numerical example based on wave energy systems is presented to illustrate the effectiveness of the proposal.**

I. Introduction

Robust control plays a fundamental role in guaranteeing control performance standards, even in the presence of model uncertainty or external disturbances. In particular, among the variety of robust control strategies, sliding mode (SM) algorithms offer an interesting solution. By resorting to high-frequency switching, SM forces the system to remain on a user-defined state-dependent surface despite non-parametric uncertainty and unmodeled dynamics. However, to apply SM algorithms, a relative degree condition on the sliding variable $\sigma(\boldsymbol{x},t)$ must be satisfied.

Specifically, for systems where $\sigma(\boldsymbol{x},t)$ is of relative degree one with respect to the control action, u, the super-twisting (ST) control may be used. Since ST was first proposed in [1], its properties, including stability conditions, convergence time, and robustness, have been extensively studied in [2][3][4][5] and references therein. One of the main advantages of ST is that it provides a smooth control action, with the discontinuous action appearing on the first-time derivative of u.

However, a disadvantage of ST control is that, in noisy environments, its performance is considerably degraded. This occurs because the ST structure cannot disaggregate the noise, ν, from the measured sliding variable: $\sigma_m = \sigma(\boldsymbol{x},t) + \nu$, and, as a result of the ST robustness to keep $\sigma_m = 0$, large and/or highly varying control actions are applied. Naturally, such effects in the control action challenges ST implementation due to wind-up and/or slew-rate limitations. Typically, in practice, to reduce the effect of noisy measurements, linear filters are employed. However, linear filters affect the relative degree of the measured variable and introduce phase delay.

In order to address existing limitations of ST control, this paper presents two contributions. The first and main contribution consists of the design of an ST-like controller, termed a Filtering-ST (FST) controller, that incorporates a zero-phase filtering structure, capable of robustly rejecting *small in average* (formally introduced in Section III) possibly unbounded measurement noise. Filtering structures have been proposed for the design of robust differentiators in [6]; however, to the best of the authors' knowledge, these structures have not been used for the design of SM-based controllers. The second contribution is an analysis of FST convergence when feedforward control action is used in combination with SM control action. Including feedforward control reduces the required ST gains and, hence, diminishes the effects of chattering and high-frequency oscillations due to measurement noise. However, the inclusion of such feedforward action results in state-dependent error dynamics, which subsequently leads to a so-called *algebraic loop* in demonstration of the convergence of the ST control. In this paper, the algebraic loop is solved for homogeneous perturbations with homogeneity degree (HD) $r_\alpha > 1/2$.

In the following, some preliminaries are presented in Section II. Section III presents the main contributions: Subsection III-A introduces the FST control structure, while Subsection III-B presents the FST convergence analysis under state-dependent perturbations. Then, Section IV presents an application case focused on wave energy systems. Finally, conclusions are drawn in Section V.

II. Preliminaries

A. Problem formulation

Consider the nonlinear control-affine system,

$$\Sigma_s : \begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u + \boldsymbol{d}(\boldsymbol{x},t), & \text{(1a)} \\ y = h(\boldsymbol{x}) + \nu, & \text{(1b)} \end{cases}$$

with $\boldsymbol{x} \in \mathcal{X} \subset \mathbb{R}^n$, $u \in \mathcal{U} \subset \mathbb{R}^1$, $y \in \mathcal{Y} \subset \mathbb{R}^1$, $\boldsymbol{f}(\boldsymbol{x})$, $\boldsymbol{g}(\boldsymbol{x})$, and $h(\boldsymbol{x})$, being differentiable with respect to $\boldsymbol{x}$ and absolutely continuous functions of time, $\boldsymbol{d}(\boldsymbol{x},t)$ being bounded external disturbances and/or model uncertainty, and ν being measurement noise.

Let the relative degree of $h(\boldsymbol{x})$, with respect to the control action, u, be one, and assume it is possible to transform (1) to a normal form using $z_1 = h(\boldsymbol{x})$, i.e.:

$$\Sigma_s : \begin{cases} \dot{z}_1 = \underbrace{L_{\boldsymbol{f}}(h(\boldsymbol{x}))}_{\alpha(\boldsymbol{z})} + \underbrace{L_{\boldsymbol{g}}(h(\boldsymbol{x}))}_{\beta(\boldsymbol{z})} u + \underbrace{L_{\boldsymbol{d}}(h(\boldsymbol{x}))}_{\delta(\boldsymbol{z},t)}, & \text{(2a)} \\ \dot{\bar{\boldsymbol{z}}} = \boldsymbol{q}(\boldsymbol{z}), & \text{(2b)} \\ y = z_1 + \nu, & \text{(2c)} \end{cases}$$

where $\boldsymbol{z}^\intercal = [z_1 \ \bar{\boldsymbol{z}}^\intercal]$, $\alpha(\boldsymbol{z})$, $\beta(\boldsymbol{z})$ and $\delta(\boldsymbol{z},t)$ are unknown smooth functions, and $\boldsymbol{q} \in \mathcal{Q} \subset \mathbb{R}^{n-1}$ are the states of the zero dynamics in (2). Additionally, assume:

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(H1) $|\gamma(\boldsymbol{z},t)| = |\alpha(\boldsymbol{z}) + \delta(\boldsymbol{z},t)| < \infty$, and $|\dot{\gamma}(\boldsymbol{z},t)| < \Delta < \infty$ for some $\Delta \in \mathbb{R}_+$, $\forall t$ and all $\boldsymbol{z} \in \mathcal{Z} \subset \mathbb{R}^n$.

(H2) In (2), $0 < \beta(\boldsymbol{z}) < \infty$, and $|\dot{\beta}(\boldsymbol{z})| < B < \infty$ for some $B \in \mathbb{R}_+$, $\forall \boldsymbol{z} \in \mathcal{Z} \subset \mathbb{R}^n$.

(H3) The zero dynamics of system (2) are asymptotically stable, hence, with $\delta(\boldsymbol{z},t) = 0$, $\alpha(\boldsymbol{z}) \to 0$, as $z_1 \to 0$.

Assuming (H1)–(H3) hold, given a smooth user-defined reference, y_r, the control objective is to steer $\sigma(z_1,t) = z_1 - y_r(t)$ to zero in finite time, exactly in the absence of measuring noise ($\nu = 0$), and robustly using a measured $\sigma_m(z_1,t)$:

$$\sigma_m(z_1,t) = \underbrace{z_1 - y_r}_{\sigma(z_1,t)} + \nu = \sigma(z_1,t) + \nu. \tag{3}$$

Additionally, to achieve $\sigma = 0$, two different control scenarios are separately considered: First, solely employing a SM control, and second, considering an SM control combined with a feedforward control action.

B. Revisiting ST convergence

In this section, using concepts of homogeneous systems, the convergence of the classic ST algorithm is revisited.

First, consider a surface:

$$\mathcal{S} = \{\boldsymbol{z} \in \mathcal{Z} \subset \mathbb{R}^n : \sigma(z_1,t) = \dot{\sigma}(z_1,t) = 0\}, \tag{4}$$

An ideal second-order sliding mode is said to take place on (4) if $\boldsymbol{z}(t)$ evolves such that $\sigma(z_1(t_r),t_r) = \dot{\sigma}(z_1(t_r),t_r) = 0$ for some finite $t_r \in \mathbb{R}_+$, and $\sigma(z_1(t),t) = \dot{\sigma}(z_1(t),t) = 0$, $\forall t > t_r$. To achieve finite time convergence to (4), the ST control:

$$u_{sm} = -k_1\lceil\sigma\rfloor^{1/2} + u_1, \tag{5a}$$

$$\dot{u}_1 = -k_2\lceil\sigma\rfloor^{0}, \tag{5b}$$

can be applied [1]. In (5), the notation $\lceil\cdot\rfloor^n = |\cdot|^n\mathrm{sign}(\cdot)$, is used. To analyse ST convergence, compute the derivative $\dot{\sigma}$ and substitute (5) to obtain the control dynamics:

$$\dot{\sigma} = \gamma(\boldsymbol{z},t) + \beta(\boldsymbol{z})(-k_1\lceil\sigma\rfloor^{1/2} + u_1) - \dot{y}_r, \tag{6a}$$

$$\dot{u}_1 = -k_2\lceil\sigma\rfloor^{0}. \tag{6b}$$

Note that (6) possesses a discontinuous right-hand side. In this paper, solutions to differential equations with a discontinuous right-hand side are understood in the Filippov sense. As a result, the dynamics (6) are replaced by an equivalent differential inclusion (DI):

$$\dot{\sigma} = \sigma_1 - k_1\beta(\boldsymbol{z})\lceil\sigma\rfloor^{1/2}, \tag{7a}$$

$$\dot{\sigma}_1 \in [-\Gamma;\Gamma] - k_2\beta(\boldsymbol{z})\lceil\sigma\rfloor^{0}, \tag{7b}$$

obtained by defining $\sigma_1 = \gamma(\boldsymbol{z},t) + \beta(\boldsymbol{z})u_1 - \dot{y}_r$, and assuming $|\Delta + Bu_1 - \ddot{y}_r| < \Gamma \in \mathbb{R}_+$. For conciseness, rewrite (7) as:

$$\dot{\boldsymbol{s}} \in \boldsymbol{\Phi}(\boldsymbol{s}), \quad \boldsymbol{\Phi}(\boldsymbol{s}) \subset T_{\boldsymbol{s}}\mathbb{R}^2, \tag{8}$$

with $\boldsymbol{s} = \begin{bmatrix}\sigma & \sigma_1\end{bmatrix}^\intercal \in \mathcal{S} \subset \mathbb{R}^2$, and $T_{\boldsymbol{s}}\mathbb{R}^2$ being the tangent space to $\mathbb{R}^2$, where the vector set, $\boldsymbol{\Phi}$, is nonempty, closed, convex, locally bounded, and upper-semicontinuous [7]. It is simple to prove that, assuming $0 \leqslant \Gamma < \infty$, the DI (8) is homogeneous with an associated triple $(\mathbf{r}, q, \boldsymbol{\Phi})$ where $\mathbf{r} \in \mathbb{R}^2$, with $[r_1, r_2]^\intercal = \mathbf{r}$, $r_1 = 2$, $r_2 = 1$ are the weights, and $q = -1$ is the HD. Homogeneity of (8) implies that, $\forall \kappa \in \mathbb{R}_+$:

$$\boldsymbol{\Phi}(\boldsymbol{s}) = \kappa^{-q}(d^{\mathbf{r}}_{\kappa_2})^{-1}\left(\boldsymbol{\Phi}(d^{\mathbf{r}}_{\kappa_2}\boldsymbol{s})\right), \tag{9}$$

with $d^{\mathbf{r}}_{\kappa_2}$ being a coordinate dilation in $\mathbb{R}^2$, with weight $\mathbf{r}$, defined as:

$$d^{\mathbf{r}}_{\kappa_2} : (x_1, x_2) \mapsto (\kappa^{r_1}x_1,\ \kappa^{r_2}x_2). \tag{10}$$

If the origin of a locally homogeneous DI with negative HD is globally uniformly asymptotically stable, then it is also globally uniformly finite-time stable [8, Theorem 4.4], and the DI is contractive [9][10]. In particular, for system (7), it has been proven [3][6] that, with an appropriate selection of gains k_1 and k_2, an ideal SM takes place after a finite time t_r, which implies $\sigma = \dot{\sigma} = \sigma_1 = 0$.

III. Main results

In this section, the FST control structure is presented in Subsection III-A, and the extension of the results with state-dependent perturbations is analysed in Subsection III-B. First, the class of filterable functions is presented in definitions 1 and 2. For a broader discussion of these definitions, the reader is referred to [6][11].

Definition 1: A function $\nu(t)$, $\nu : [0, \infty) \to \mathbb{R}$ is called a signal of *global filtering* order k, $k \geqslant 0$ if ν is a locally integrable Lebesgue measurable function, and there exists a solution ξ for the differential equation $\xi^{(k)} = \nu$. Then, $|\xi(t)|$ is the k-th global order integral magnitude of ν.

Definition 2: Any signal $\nu(t)$, $\nu : [0, \infty) \to \mathbb{R}$ is termed *locally filterable* if it can be represented as $\nu(t) = \varepsilon_0 + \varepsilon_1 + ... + \varepsilon_k$, where each ε_i, with $i = 0, 1, ..., k$, are signals of global filtering order $0, 1, ..., k$, respectively.

A. Filtering ST control

Consider the measured sliding variable σ_m in (3), with y_r being a desired reference. To drive $\sigma = \dot{\sigma} = 0$, the FST controller is designed as follows:

$$\Sigma_c : \begin{cases} \dot{w}_1 = w_2 - k_{f1}\lceil w_1\rfloor^{\frac{1+n_f}{2+n_f}}, & \text{(11a)} \\ \dot{w}_2 = w_3 - k_{f2}\lceil w_1\rfloor^{\frac{n_f}{2+n_f}}, & \text{(11b)} \\ \quad\vdots & \\ \dot{w}_{n_f-1} = w_{nf} - k_{f(n_f-1)}\lceil w_1\rfloor^{\frac{3}{2+n_f}}, & \text{(11c)} \\ \dot{w}_{n_f} = \sigma_m - k_{fn_f}\lceil w_1\rfloor^{\frac{2}{2+n_f}}, & \text{(11d)} \\ u_{sm} = -k_1\lceil w_1\rfloor^{\frac{1}{2+n_f}} + u_1, & \text{(11e)} \\ \dot{u}_1 = -k_2\lceil w_1\rfloor^{0}, & \text{(11f)} \end{cases}$$

where k_{fi}, with $i = 1, ..., n_f$ are the filtering gains (associated with the filtering states w_1–w_{nf}), k_1 and k_2 are user-defined variables, adjusted to guarantee controller convergence, and u_{sm} the FST control action.

Theorem 1: Consider dynamics (1) with conditions (H1)–(H3), and surface (4). Then, with appropriate selection of the gains k_{fi}, with $i = 1, ..., n_f$, k_1 and k_2, FST controller (11) provides, in the absence of measurement noise ($\nu = 0$), exact finite-time convergence to $y - y_r = \sigma = \dot{\sigma} = 0$, and robust finite-time convergence, in the presence of noise of a filtering order not exceeding n_f.

Proof: First, the error dynamics are computed, by substituting the FST dynamics (11) to (2) and redefining $\sigma_1 = \gamma + \beta u_1 - \dot{y}_r$ (for simplicity, the $\boldsymbol{z}$ and t arguments are dropped):

$$\boldsymbol{\Phi}(\boldsymbol{e}): \begin{cases} \dot{w}_1 = w_2 - k_{f1}\lceil w_1 \rfloor^{\frac{1+n_f}{2+n_f}}, & \text{(12a)} \\ \dot{w}_2 = w_3 - k_{f2}\lceil w_1 \rfloor^{\frac{n_f}{2+n_f}}, & \text{(12b)} \\ \vdots \\ \dot{w}_{n_f-1} = w_{nf} - k_{f(n_f-1)}\lceil w_1 \rfloor^{\frac{3}{2+n_f}}, & \text{(12c)} \\ \dot{w}_{n_f} = \sigma + \nu - k_{fn_f}\lceil w_1 \rfloor^{\frac{2}{2+n_f}}, & \text{(12d)} \\ \dot{\sigma} = \sigma_1 - k_1\beta\lceil w_1 \rfloor^{\frac{1}{2+n_f}}, & \text{(12e)} \\ \dot{\sigma}_1 \in [-\Gamma;\Gamma] - k_2\beta\lceil w_1 \rfloor^{0}, & \text{(12f)} \end{cases}$$

where, $\boldsymbol{e}^\intercal = [\boldsymbol{w}^\intercal\ \boldsymbol{s}^\intercal]$, and assuming $|\Delta + Bu_1 - \ddot{y}_r| < \Gamma < \infty$. Note that (12) coincide with the error dynamics of the SM differentiator proposed in [6], whose convergence has already been proven and extensively studied using Lyapunov functions in [4]. Specifically, (12) is homogeneous with associated triple $(-1, \mathbf{r}, \boldsymbol{\Phi})$, with $\mathbf{r}_i = n_f + 3 - i$, $i = 1, ..., n_f + 2$. Hence, first, with $\nu = 0$, (12) provides exact finite-time convergence to $\sigma = \dot{\sigma} = \sigma_1 = 0$, and $\boldsymbol{w} = \boldsymbol{0}$. With $\nu \neq 0$ of global filtering order n_f, then $\xi_{n_f}^{(n_f)} = \nu$, $|\xi_{n_f}| < \varepsilon_{n_f}$ (the following result is easily extrapolated for locally filterable ν). Define $\omega_i = w_i - \xi_{n_f}^{(i-1)}$, for $i = 1, ..., n_f$, and rewrite (12) as:

$$\boldsymbol{\Phi}(\cdot): \begin{cases} \dot{\omega}_1 = \omega_2 - k_{f1}\lceil \omega_1 + \xi_{n_f} \rfloor^{\frac{1+n_f}{2+n_f}}, & \text{(13a)} \\ \dot{\omega}_2 = \omega_3 - k_{f2}\lceil \omega_1 + \xi_{n_f} \rfloor^{\frac{n_f}{2+n_f}}, & \text{(13b)} \\ \vdots \\ \dot{\omega}_{n_f-1} = \omega_{nf} - k_{f(n_f-1)}\lceil \omega_1 + \xi_{n_f} \rfloor^{\frac{3}{2+n_f}}, & \text{(13c)} \\ \dot{\omega}_{n_f} = \sigma - k_{fn_f}\lceil \omega_1 + \xi_{n_f} \rfloor^{\frac{2}{2+n_f}}, & \text{(13d)} \\ \dot{\sigma} = \sigma_1 - k_1\beta\lceil \omega_1 + \xi_{n_f} \rfloor^{\frac{1}{2+n_f}}, & \text{(13e)} \\ \dot{\sigma}_1 \in [-\Gamma;\Gamma] - k_2\beta\lceil \omega_1 + \xi_{n_f} \rfloor^{0}, & \text{(13f)} \end{cases}$$

which coincide with the error dynamics of a perturbed SM differentiator. It follows, from [6][12], that (13) is robust with respect to perturbations causing locally small changes in the DI around the origin [13], and with $\nu \neq 0$ of the filtering order n_f, any trajectory starting at the origin at $t = t_0$, satisfies $\sup|\sigma| < \mu_0 \left(\frac{\xi_{n_f}}{\varepsilon_{nf}}\right)^{\frac{2}{n_f+2}}$, $\sup|\sigma_1| < \mu_1 \left(\frac{\xi_{n_f}}{\varepsilon_{nf}}\right)^{\frac{1}{n_f+2}}$ for some $\mu_0 > 0$, $\mu_1 > 0$, and $\forall t > t_0$, and the control structure is locally finite-time stable. ■

Remark 1: The requirements for Γ are the same in both the ST and FTS algorithms. As a result, the class of perturbations that FST is capable of rejecting are the same as in the ST case. However, FST permits robust operation in the presence of measurement noise, which cannot be guaranteed using the ST structure.

Remark 2: Two mature procedures to select the FST gains can be directly applied: A numerical approach, as in [6], or via Lyapunov analysis, as in [4]. Using either approach guarantees local homogeneity and finite time convergence of the FST.

B. Results with state-dependent perturbations

Assume the control action is composed of two terms, a feedforward control term, u^*, designed to provide, *on average*, the control action that drives the system close to $\sigma = 0$, and a feedback SM control term, u_{sm}. That is:

$$u = u^* + u_{sm}, \tag{14}$$

where $u^* = -\frac{\alpha(\boldsymbol{z}^*)}{\beta(\boldsymbol{z}^*)}$. Using $\beta(\boldsymbol{z}) = \beta(\boldsymbol{z}^*) + \Delta\beta(\boldsymbol{z})$, (2) becomes:

$$\begin{aligned} \dot{\sigma} &= \alpha(\boldsymbol{z}, t) + \beta(\boldsymbol{z}, t)(u^* + u_{sm}) + \delta(\boldsymbol{z}, t) - \dot{y}_r, \qquad (15)\\ &= \alpha(\boldsymbol{z}, t) - \alpha(\boldsymbol{z}^*, t) + \delta(\boldsymbol{z}, t) - \dot{y}_r \\ &\quad - \alpha(\boldsymbol{z}^*, t)\frac{\Delta\beta(\boldsymbol{z}, t)}{\beta(\boldsymbol{z}^*, t)} + \beta(\boldsymbol{z}, t)u_{sm}. \end{aligned}$$

Assume it is possible, via (15), to rewrite:

$$\dot{\sigma} = \alpha(\sigma, t) + \rho(\boldsymbol{z}, \boldsymbol{z}^*, t) + \beta(\boldsymbol{z}, t)u_{sm}, \tag{16}$$

where $\rho(\boldsymbol{z}, \boldsymbol{z}^*, t)$, represents bounded perturbations satisfying (H1) for some $\Delta^* \in \mathbb{R}_+$. In (16), it is not possible to bound $\dot{\alpha}(\sigma, t)$, since this is the output of the zero-dynamics of system (2), using σ as input. However, it is possible to state convergence conditions for a class of homogeneous perturbations, as formalised in Theorem 2.

Theorem 2: Let a control for system (1), satisfying assumptions (H1)–(H3), be designed as in (14), using the FST controller (11). If α is a homogeneous function with HD $r_\alpha > 1/2$, then FST guarantees, with $\nu = 0$, exact finite-time convergence to $\sigma = \dot{\sigma} = 0$, and robust finite time convergence with $\nu \neq 0$ of a filtering order not exceeding n_f.

Proof: First, evaluate the error dynamics by substituting (16) to (11), and using $\sigma_1 = \rho + \beta u_1 - \dot{y}_r$:

$$\boldsymbol{\Phi}(\boldsymbol{e}): \begin{cases} \dot{w}_1 = w_2 - k_{f1}\lceil w_1 \rfloor^{\frac{1+n_f}{2+n_f}}, & \text{(17a)} \\ \dot{w}_2 = w_3 - k_{f2}\lceil w_1 \rfloor^{\frac{n_f}{2+n_f}}, & \text{(17b)} \\ \vdots \\ \dot{w}_{n_f-1} = w_{nf} - k_{f(n_f-1)}\lceil w_1 \rfloor^{\frac{3}{2+n_f}}, & \text{(17c)} \\ \dot{w}_{n_f} = \sigma(z_1, t) - k_{fn_f}\lceil w_1 \rfloor^{\frac{2}{2+n_f}}, & \text{(17d)} \\ \dot{\sigma} = \alpha(\sigma, t) + \sigma_1 - k_1\beta(\boldsymbol{z}, t)\lceil w_1 \rfloor^{\frac{1}{2+n_f}}, & \text{(17e)} \\ \dot{\sigma}_1 \in [-\Gamma^*;\Gamma^*] - k_2\beta\lceil w_1 \rfloor^{0}, & \text{(17f)} \end{cases}$$

where $\boldsymbol{e}^\intercal = [\boldsymbol{w}^\intercal\ \boldsymbol{s}^\intercal]$, and $|\Delta^* + Bu_1 - \ddot{y}_r| < \Gamma^* < \infty$. Apply the dilation $d^{\mathbf{r}}_{\kappa_{n_f+2}}$, and transformation (9), to (17), with weights $\mathbf{r}_i = n_f + 3 - i$, $i = 1, ..., n_f + 2$, and $q = -1$. Analyse, in particular, (17e):

$$\dot{\sigma} = \sigma_1 - k_1\beta(\boldsymbol{z}, t)\lceil w_1 \rfloor^{\frac{1}{2+n_f}} + \kappa^{-1}\alpha(\kappa^2\sigma, \kappa^{-1}t), \tag{18}$$

$$= \sigma_1 - k_1\beta(\boldsymbol{z},t)\lceil w_1 \rfloor^{\frac{1}{2+n_f}} + \kappa^{-1+2r_\alpha}\left(\alpha(\sigma,\kappa^{-1}t)\right).$$

Then, for $\alpha(\cdot)$ with a HD $r_\alpha > 1/2$, it follows from the 0-limit approximation (see Appendix) that there exists some $\kappa_0, \varepsilon_0 \in \mathbb{R}_+$, that defines an homogeneous ball $B_{\kappa_0} \subset \Omega_{\kappa_0}$:

$$\Omega_{\kappa_0} := \{\boldsymbol{e} \in \mathbb{R}^{n_f+2} \,|\, \kappa < \kappa_0 \wedge \kappa^{-1+2r_\alpha}\left(\alpha(\sigma,\kappa^{-1}t)\right) < \varepsilon_0\},$$

where $\alpha(\cdot)$ is *dominated* by the FST correction terms. Hence, for $\boldsymbol{e} \in B_{\kappa_0} \subset \Omega_{\kappa_0}$, (17) is locally homogeneous with HD $= -1$ and, with appropriate selection of gains (k_{fi}, with $i = 1, ..., n_f$, k_1 and k_2), locally strongly finite-time convergent [4][12].

Analogously, for $\varepsilon_\infty \in \mathbb{R}_+$, define $\bar{\Omega}_{\kappa_0}$:

$$\bar{\Omega}_{\kappa_0} := \{\boldsymbol{e} \in \mathbb{R}^{n_f+2} \,|\, \kappa > \kappa_0 \wedge \kappa^{-2r_\alpha}(\sigma_1 - k_1\beta(\boldsymbol{z},t)\lceil w_1 \rfloor^{\frac{1}{2+n_f}}) < \varepsilon_\infty\},$$

where $\alpha(\cdot)$ *dominates* the FST correction term, and hence:

$$\dot{\sigma} \approx \alpha(\sigma, t). \tag{19}$$

Thus, $\forall \boldsymbol{e} \in \bar{B}_{\kappa_0} \subset \bar{\Omega}_{\kappa_0}$, the dynamics of (17) are approximated as the cascade interconnection of the stable systems:

$$\dot{\boldsymbol{w}} = \boldsymbol{h}(\boldsymbol{w}, \sigma), \tag{20a}$$
$$\dot{\sigma} = \alpha(\sigma, t), \tag{20b}$$
$$\dot{\sigma}_1 \in [-\Gamma; \Gamma] - k_2\beta\lceil w_1 \rfloor^0, \tag{20c}$$

where (20a) are the error dynamics of a truncated SM differentiator (17a)–(17d). Now, Lemma 1 (see Appendix) is applied. Using (H3), (19) is asymptotically stable, and $\alpha(\sigma, t) \to 0$ as $\sigma \to 0$. Since $\dot{\boldsymbol{w}} = \boldsymbol{h}(\boldsymbol{w}, 0)$ is asymptotically stable, $\forall \boldsymbol{e} \in \bar{B}_\kappa \subset \bar{\Omega}_{\kappa_0}$, as $\sigma \to 0$, $w_i \to 0$, and (20a)-(20b) is asymptotically stable. Observe that (20c) plays no role. Extending the solutions of (20) forward in time, σ decreases until $\boldsymbol{e} \in B_{\kappa_0} \subset \Omega_{\kappa_0}$. Then, inside Ω_{κ_0}, (17) is locally homogeneous with HD $= -1$, strongly finite-time convergent, and $\sigma \to 0$ (and $\alpha(\sigma, t) \to 0$) in finite time. Thus, robustness around the origin and noise rejection properties follow from Theorem 1. ■

Remark 3: The homogeneity degree of $\alpha(\sigma, t)$ plays a fundamental role. If $\alpha(\cdot)$ is not homogeneous, it is possible to define $\alpha(\cdot)$ inside a class of larger homogeneous perturbations as in [12]. Importantly, if $\alpha(\cdot)$ satisfies (H3), is locally homogeneous, and there exists a 0-limit approximation of $\alpha(\cdot)$, $\alpha_0(\cdot)$ with HD $r_{\alpha,0} > 1/2$, then (17) is also locally homogeneous. Two important cases are: (i) $r_\alpha = 1/2$, which is extensively studied in [5], and (ii) $\alpha(\cdot)$ is a linear function of σ, i.e., $r_{\alpha,0} = 1$, in which case Theorem 2 is directly applicable.

Remark 4: Due to the inclusion of u^*, as $\alpha \to \alpha^*$, and $\beta \to \beta^*$:

$$|\rho(\boldsymbol{z}, \boldsymbol{z}^*, t)| \approx |\delta(\boldsymbol{z}, t) - \dot{y}_r| \leqslant |\underbrace{\delta(\boldsymbol{z}, t) + \alpha(\boldsymbol{z}, t)}_{\gamma} - \dot{y}_r|,$$

hence, typically, $\Gamma^* \leqslant \Gamma$, resulting in reduced values for the FST (alternatively ST) gains [14].

IV. Case study: Wave energy converter

In this section, in-silico evaluations are conducted on a one-degree-of-freedom wave energy converter (WEC). The goal of this section is to evaluate FST performance around $\sigma = \dot{\sigma} = 0$, and compare it with ST performance in noisy environments. Since u^* does not affect the local homogeneity of the controllers, only the case $u^* = 0$ is considered. The simulation parameters are presented in Subsection IV-A. Then, the FST controller velocity-tracking results are presented and compared in Subsection IV-B.

A. Wave energy system and controller tuning

Consider the one-degree-of-freedom WEC dynamics:

$$\Sigma_w : \begin{cases} \dot{v} = \mathcal{M}\left(-\mathbf{H}\bar{\boldsymbol{z}} - k_z x + d(t) + u\right), & (21a) \\ \dot{x} = v, & (21b) \\ \dot{\bar{\boldsymbol{z}}} = \mathbf{F}\bar{\boldsymbol{z}} + \mathbf{G}v, & (21c) \\ y = v + \nu, & (21d) \end{cases}$$

where x is displacement, v is velocity, $\bar{\boldsymbol{z}} \in \mathbb{R}^7$ are the radiation states, and $d(t)$ is the wave excitation force, which can be modelled as an external bounded disturbance, satisfying $|\dot{d}(t)| < L_d$. To generate a realistic wave profile, $d(t)$ is generated using the Bretschneider spectrum [15], considering a peak period $T_p = 8s$, and significant wave height $H_s = 1m$. Also, $\mathcal{M} = 6.8 \times 10^{-6}$ and $k_z = 5.57 \times 10^5$.

For evaluation of the FST, a controller is designed with $n_f = 2$, and compared with the ST. To tune both controllers, the gains are as follows: $L = 6m/s^2$, $k_{1_{\text{ST}}} = 9.97 \times 10^6$, $k_{2_{\text{ST}}} = 8.95 \times 10^6$, $k_{f_{1\text{FST}}} = 13.7$, $k_{f_{2\text{FST}}} = 53.95$, $k_{1_{\text{FST}}} = 9 \times 10^7$, and $k_{2_{\text{FST}}} = 8.95 \times 10^6$. The simulations are conducted with 20% uncertainty in the parameters of (21), and initial conditions outside the sliding surface. As detailed in the following, robustness to model uncertainty and perturbations is observed for both the ST and the FST.

B. FST and ST comparison

This section analyses the performance of the ST and FST algorithms for $\nu = \nu_1 + \nu_2$, with $\nu_1 \sim \mathcal{N}(0, 10^{-6})$, and $\nu_2 = 0.5\sin(100\pi t)$, emulating 50Hz grid noise. The signal-to-noise ratio of the measured velocity (see Figure 1), with the simulated noise, is SNR $= 5.3$dB.

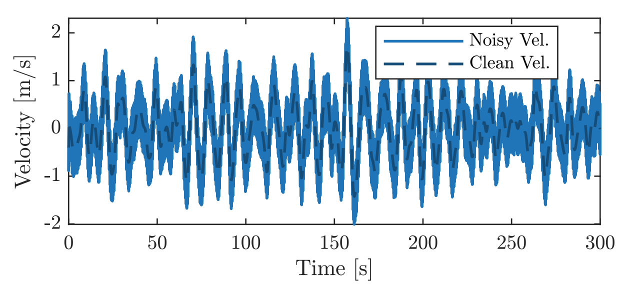

Figure 1. Velocity measurement (solid line) and noiseless velocity (dashed line).

In the following, the convergence of the ST and FST controllers with $\nu = 0$ is analysed. First, observe the σ-$\dot{\sigma}$ plane in Figure 2. As expected, both algorithms converge to

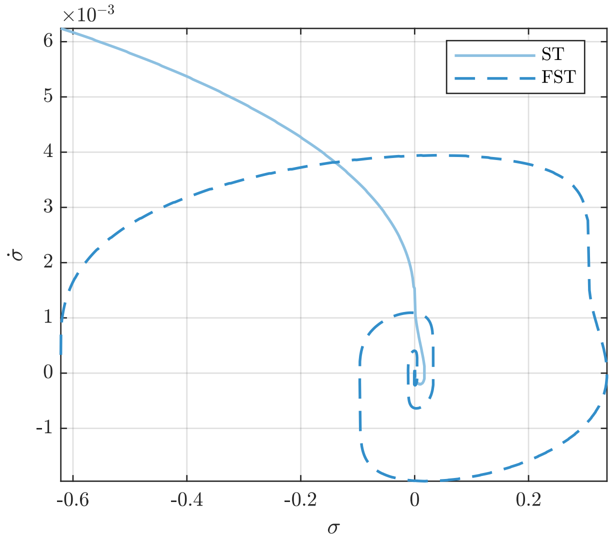

Figure 2. Phase plane convergence of ST and FST to the sliding surface when the measurement is noiseless.

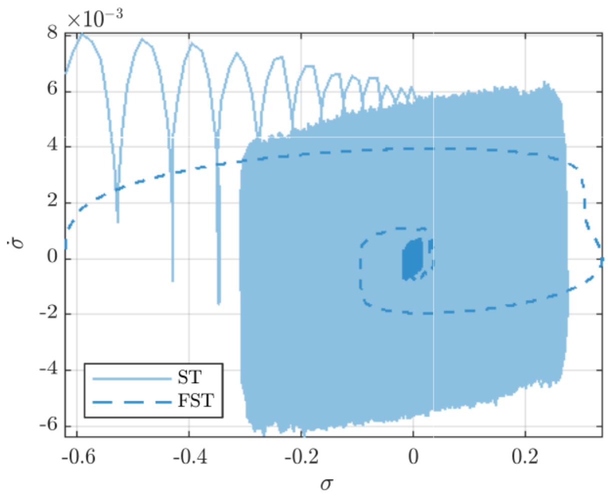

Figure 3. Phase plane convergence of ST and FST to the sliding surface when the measurement is noisy.

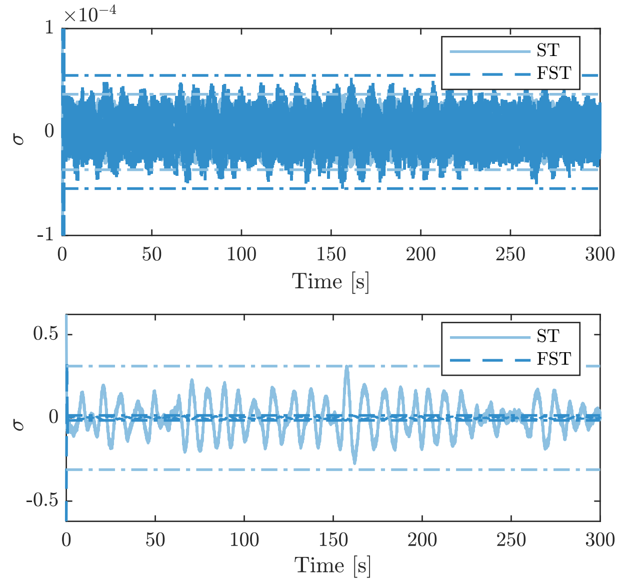

Figure 4. Sliding variable evolution for ST and FST when the measurement is noiseless (top) and noisy (bottom). The dot-dashed lines delimit the maximum values for ST and FST errors.

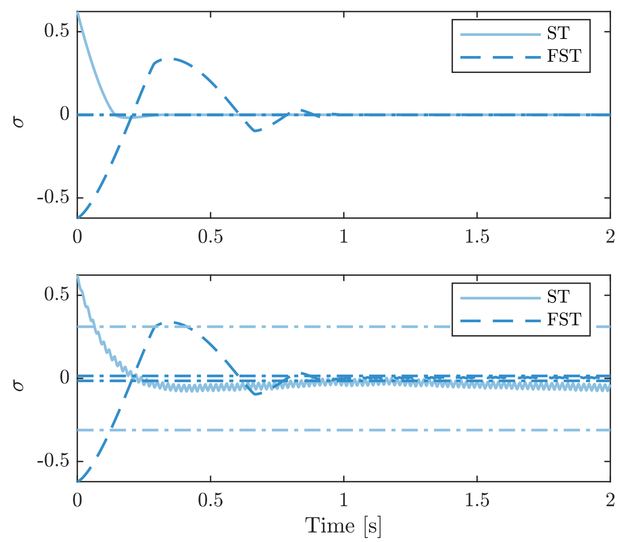

Figure 5. Sliding variable evolution for ST and FST when the measurement is noiseless (top) and noisy (bottom). The dot-dashed lines delimit the maximum values for ST and FST errors.

$\sigma = \dot{\sigma} = 0$, in spite of model uncertainty. However, with $\nu \neq 0$, the performance of both algorithms is affected, as shown in Figure 3. Notably, when compared with the case with $\nu = 0$, the FST presents a similar reaching phase, while the ST initial convergence is deteriorated. Around the origin, FST exhibits a clear advantage in achieving convergence to a smaller region of the phase plane compared to that of the ST. Complementarily, $\sigma(t)$ is depicted in the time domain in Figure 4. In Figure 4 (top), it is shown that, when $\nu = 0$, both controllers exhibit a similar performance. However, when $\nu \neq 0$ (see Figure 4 (bottom)), ST convergence is considerably degraded, with σ showing increased oscillations around the convergence region. In contrast, FST robustly retains the sliding variable around zero, considerably reducing the impact of measurement noise.

Also, the initial convergence of the ST and FST algorithms is depicted, in the time domain, in Figure 5. In Figure 5 (top), with $\nu = 0$, it is observed that FST exhibits a longer convergence time, approximated by the dot-dashed line. Complementary, Figure 5 (bottom), representing the noisy case, reveals that the reaching phase for the FST algorithm remains similar to the noise-free scenario. However, the evolution of σ in the ST algorithm begins to exhibit noticeable deviations, consistently with the result in Figure 3.

Finally, Figure 6 illustrates the influence of measurement noise on the control action. In Figure 6 (top), where the measured signal is noiseless, ST and FST exhibit similar control actions and hence, performance. However, in Figure 6 (bottom), where $\nu \neq 0$, a significant degradation of the ST control action can be observed. This is due to the higher sensitivity of the ST to noise. In contrast, the

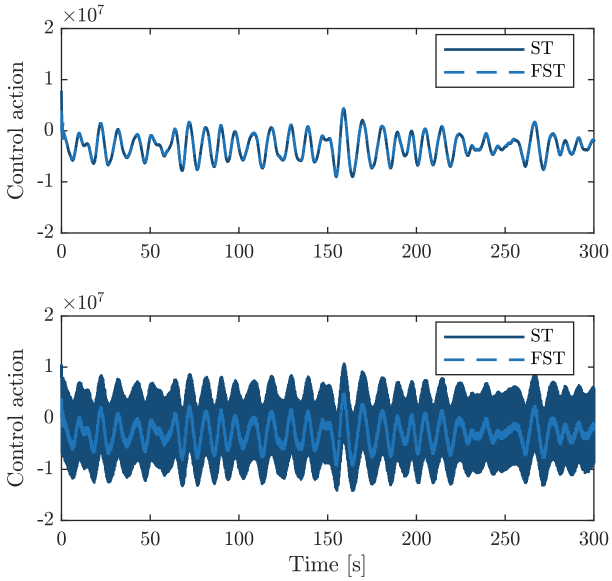

Figure 6. ST (solid line) and FST (dashed line) control action. Top: Control action when the measured signal is noiseless. Bottom: Control action when the measured signal is noisy.

FST demonstrates improved mitigation of the impact of noise, resulting in a control signal with reduced amplitude variations. Importantly, this latter feature proves essential for real-time application of FST, especially to prevent control action saturation and/or slew-rate effects.

V. Conclusions

The proposed filtering super-twisting (FST) controller fulfils several key requirements for improving conventional super-twisting (ST) control performance in noisy environments. First, FST effectively mitigates the impact of noise in measured signals, enhancing overall performance without compromising the inherent ST robustness to unmodelled dynamics and perturbations. Second, FST reduces control effort in noisy environments, addressing critical issues such as wind-up and slew-rate saturation. Third, by integrating a sliding-mode filtering structure directly within the control design, FST eliminates phase delays in the tracked variable. These enhancements collectively lead to a more efficient and reliable control strategy when the noise power is significant. Overall, FST represents an appealing solution for applications demanding both high robustness and strong noise resilience. Importantly, the preliminary results presented in this paper can be extended to (i) an analysis of discretised FST, (ii) address FST fixed-time convergence, and (iii) sensitivity analysis on the FST gains, among other areas of interest.

Appendix

Definition 3 (0-limit homogeneous approximation [16]): A vector field $\mathbf{\Phi}$ is said homogeneous in the 0-limit with associated tripe $(\mathbf{r}_0, q_0, \mathbf{\Phi}_0)$, where $\mathbf{r}_0 \in \mathbb{R}^n$ is the weight, $q \in \mathbb{R}$ is the HD, and $\mathbf{\Phi}_0$ is an approximating vector field, if $\mathbf{\Phi}$, $\mathbf{\Phi}_0$, are continuous and not identically zero, and, for each compact set $C \in \mathbb{R}^n \backslash \{0\}$, and each $\varepsilon > 0$, there exists a $\kappa_0 > 0$, such that for each $\mathbf{\Phi}_{0,i}$, $\mathbf{\Phi}_i$, is satisfied:

$$\max_{x \in C} \left| \frac{\mathbf{\Phi}_i(d_\kappa^{\mathbf{r}_0} \boldsymbol{x})}{\kappa^{q_0 + \mathbf{r}_{0,i}}} - \mathbf{\Phi}_{0,i}(\boldsymbol{x}) \right| \leqslant \varepsilon, \quad \forall \kappa \in (0, \kappa_0]. \tag{22}$$

In words, definition 3 states that there must exist a sufficiently small κ_0, such that the 0-limit approximation $\mathbf{\Phi}_0$, approximates $\mathbf{\Phi}$ with a small error ε.

Lemma 1 ([17, Appendix B]): Consider the system:

$$\dot{\boldsymbol{z}} = \boldsymbol{q}(\boldsymbol{z}, \boldsymbol{y}), \tag{23a}$$

$$\dot{\boldsymbol{y}} = \boldsymbol{g}(\boldsymbol{y}). \tag{23b}$$

Suppose $(\boldsymbol{z}, \boldsymbol{y}) = (0, 0)$ is an equilibrium of (23), the equilibrium $\boldsymbol{z} = 0$ of $\dot{\boldsymbol{z}} = \boldsymbol{q}(\boldsymbol{z}, 0)$ is asymptotically stable, and the equilibrium $\boldsymbol{y} = 0$ of (23b) is asymptotically stable. Then the equilibrium $(\boldsymbol{z}, \boldsymbol{y}) = (0, 0)$ of (23) is asymptotically stable.

References

[1] A. Levant, “Sliding order and sliding accuracy in sliding mode control,” *International Journal of Control*, vol. 58, no. 6, pp. 1247–1263, Dec. 1993.

[2] R. Seeber and M. Horn, “Stability proof for a well-established super-twisting parameter setting,” *Automatica*, vol. 84, pp. 241–243, 2017.

[3] R. Seeber, M. Horn, and L. Fridman, “A novel method to estimate the reaching time of the super-twisting algorithm,” *IEEE Transactions on Automatic Control*, vol. 63, no. 12, pp. 4301–4308, 2018.

[4] E. Cruz-Zavala and J. A. Moreno, “Levant’s Arbitrary-Order Exact Differentiator: A Lyapunov Approach,” *IEEE Trans. Automat. Cont.*, vol. 64, no. 7, pp. 3034–3039, 2019.

[5] R. Seeber and M. Horn, “Guaranteeing disturbance rejection and control signal continuity for the saturated super-twisting algorithm,” *IEEE Control Systems Letters*, vol. 3, no. 3, pp. 715–720, 2019.

[6] A. Levant and M. Livne, “Robust exact filtering differentiators,” *Eur. J. Control*, vol. 55, pp. 33–44, Sep. 2019.

[7] A. F. Filippov, *Differential Equations with Discontinuous Righthand Sides*, F. M. Arscott, Ed. Springer Netherlands, 1988.

[8] Y. V. Orlov, Ed., *Discontinuous Systems*, ser. SpringerLink. London: Springer London, 2009.

[9] S. P. Bhat and D. S. Bernstein, “Finite-time stability of homogeneous systems,” in *Proceedings of the 1997 American Control Conference (Cat. No.97CH36041)*. IEEE, 1997.

[10] A. Levant, “Homogeneity approach to high-order sliding mode design,” *Automatica*, vol. 41, no. 5, pp. 823–830, may 2005.

[11] J.-P. Barbot, A. Levant, M. Livne, and D. Lunz, “Discrete differentiators based on sliding modes,” *Automatica*, vol. 112, Feb. 2020.

[12] A. Levant and M. Livne, “Weighted homogeneity and robustness of sliding mode control,” *Automatica*, vol. 72, pp. 186–193, 2016.

[13] A. Levant, “Homogeneity approach to high-order sliding mode design,” *Automatica*, vol. 41, pp. 823–830, 2005.

[14] N. Tietze, K. Wulff, and J. Reger, “Local stabilisation of nonlinear systems with time- and state-dependent perturbations using sliding-mode model-following control,” in *2024 IEEE 63rd Conference on Decision and Control (CDC)*. IEEE, Dec. 2024, pp. 6620–6627.

[15] C. L. Bretschneider, *Wave variability and wave spectra for wind-generated gravity waves*. USA: The Board, 1959, no. 118.

[16] V. Andrieu, L. Praly, and A. Astolfi, “Homogeneous approximation, recursive observer design, and output feedback,” *SIAM J. Cont. Opt.*, vol. 47, no. 4, pp. 1814–1850, 2008.

[17] A. Isidori, *Nonlinear Control Systems*. Springer London, Limited, 2013.