

Bottom Friction in a Baroclinic Ocean: Influences on Sea Level and Slope Currents

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ABSTRACT: In the flat-bottomed open ocean, the seawater density distribution yields alone the existence of geostrophic baroclinic currents and steric sea-level spatial changes. However, at basin margins, the bathymetry exerts a considerable control on both these quantities. Indeed, the steric sea level vanishes at the coast where depth is zero. Also, continental slopes are vorticity barriers hindering convergence (divergence) of baroclinic transport toward the coast and accumulation (removal) of water there. In the limit of no temporal development, how the coastal sea level is impacted by open-ocean density changes is hence nontrivial and must involve ageostrophic mechanisms. Here, we focus on bottom friction as one such process, provide derivations extending the arrested topographic wave theories to a fully baroclinic ocean, and discuss an application for an eastern boundary margin (representative of the Rockall Slope Current region, for example). We demonstrate that open-ocean density changes yield important joint effect of baroclinicity and relief (JEBAR) along-slope currents, which generate cross-slope Ekman currents due to friction with the seabed. The latter are associated with divergence and convergence leading to sea-level changes above the slope, mediated to the coast via coastally trapped waves. Through this process, not only the coastal sea level is modified but also the along-slope currents are slowed down to well-known asymptotical solutions. Hence, our results link modern theoretical developments in sea-level research and past analytical studies of slope currents. Our effort describes the fundamental notions, and we anticipate it paves the way for more sophisticated works.

KEYWORDS: Sea level; Boundary currents; Continental shelf/slope; Ageostrophic circulations; Baroclinic flows; Barotropic flows

1. Introduction

Due to the ongoing and anthropologically driven global warming, the global mean sea level is rising at a rate of $\sim 3.2 \text{ mm yr}^{-1}$ since the early 1990s and accelerating [Intergovernmental Panel on Climate Change (IPCC); Fox-Kemper et al. 2021]. It is projected to be 0.4–1.0 m higher by 2100 than it was during the period 1995–2014, depending on the exact emission scenario (IPCC; Fox-Kemper et al. 2021), and will continue to rise an additional 2–7 m over the subsequent centuries (Strauss et al. 2015). However, although the ongoing and future landward shoreline retreat and increased inundation vulnerability impact most of Earth's seaboards, a myriad of mechanisms drive spatiotemporal differences in coastal sea-level changes. These include geoid effects, vertical land motions, inverse barometer effects, wave setup, and many more processes (Woodworth et al. 2019). Sea level is further influenced by ocean circulation, which is set in motion by the wind and seawater density gradients. The present contribution focuses on the role of the latter in driving the circulation along oceanic margins and the behavior of the sea-level distribution in such conditions.

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In recent years, some efforts have been undertaken to comprehend the effects of the large-scale ocean circulation on the spatiotemporal changes in coastal sea level at low frequency. They have highlighted the role of bottom friction processes, which allow the penetration of open-ocean sea-level signal to the shelf and eventually to the coast (Wise et al. 2018; Hughes et al. 2019; Wu 2021; Lin et al. 2022). Wise et al. (2018) reintroduced the arrested topographic wave (ATW) framework [the term was originally popularized by Csanady (1978)], a vorticity conservation equation which for a homogeneous ocean with arbitrary bathymetry and coastline reads

$$J\left(\frac{gh}{f}, \eta\right) + \nabla \cdot \left(\frac{gr}{f^2} \nabla \eta\right) = 0, \quad (1)$$

where $J(a, q) = \nabla a \times \nabla q = \nabla \times (a \nabla q)$ is the Jacobian operator, η is the sea level, h is the water depth (strictly positive quantity), f is the Coriolis parameter, g is the acceleration of gravity, and r represents a linear bottom friction coefficient. Equation (1) is a depth-integrated mass conservation equation. In this homogeneous ocean context, the sea level η is equivalent to ζ , the manometric (or residual) sea level related to bottom pressure ($\eta \equiv \zeta$). In Eq. (1), the wind stress curl was omitted. This term is not of importance for the present study.

The first term in Eq. (1) is a pressure torque, related to barotropic geostrophic convergence. If friction is altogether neglected, $J(gh/f, \eta) = 0$, the sea level is a function of h/f only, and currents are exactly parallel to h/f contours. When changes in f are negligible, the sea level is a function of water depth only and flow is along isobaths. The effect of bottom friction appears through the second term in Eq. (1). This term represents the bottom Ekman transport convergence. This convergence is, in

the homogeneous ATW framework, the sole driver of changes in η along h/f contours. Without bottom drag, open-ocean sea-level signals do not influence sea elevation at the coastline, where h/f is constant and equals zero. The ATW formulation is analogous to an “advection–diffusion” equation (Wise et al. 2018; Wu 2021; Lin et al. 2022 and references therein). The sea level η is “advected” cyclonically along isolines of h/f , but for the action of friction “diffusing” the signal en route on either side of h/f contours. On an f plane, with a bathymetry profile invariant in the along-slope direction (along y), and neglecting cross-slope friction, Eq. (1) is simply

$$\frac{\partial h}{\partial x} \frac{\partial \eta}{\partial y} + \frac{r}{f_0} \frac{\partial^2 \eta}{\partial x^2} = 0, \quad (2)$$

the original formulation of Csanady (1978).

For the most part, the ATW developments have neglected the role of baroclinicity in the ocean. This is despite formulations broadly of the form of Eq. (1) but including density gradient effects date back to Sarkisyan and Ivanov (1971). For example, Lin et al. (2022) and Csanady (1978, section 7) investigated the role played by longshore winds in generating a sea-level signal growing from the coastline. Csanady (1978, section 8), Wang (1982), and Wu (2021) investigated the effects of the sea level η prescribed at the upstream boundary of the domain considered when solving for Eq. (1) (or a simplified version of this equation). Note “upstream” is meant in a coastally wave-propagating sense. In all these efforts, the effects of the seawater density distribution were neglected. Further efforts yielded some limited attention to the role of baroclinicity. Csanady (1978, section 9) included considerations of density changes, although solely caused by freshwater influx at the shore. Csanady (1978, section 10), Middleton (1987), Wise et al. (2018), Wu (2021), and Lin et al. (2022) considered a boundary region where the homogeneous framework of Eq. (1) applied, but under the influence of an open-ocean sea level η prescribed at the offshore boundary. Through this imposed boundary setup, the forcing of the baroclinic open ocean was included to some extent. For example, Wise et al. (2018) and Lin et al. (2022) set the open-ocean sea level η equal to that expected from reduced-gravity gyre dynamics. This works rather well at western boundaries; it is, however, unlikely to hold elsewhere. Besides, the method leads to a discontinuity in bottom stress at the boundary between the ocean interior and the ocean margin, which is difficult to treat (Wise et al. 2018). Csanady and Shaw (1983) considered cross-slope gradients of density but neglected along-slope gradients. We will see the latter play a considerable role.

In parallel, studies have emphasized the importance of the joint effect of baroclinicity and relief (JEBAR; after Sarkisyan and Ivanov 1971) and friction for circulation above continental slopes (Shaw and Csanady 1983; Huthnance 1984, 1986; Pingree and Le Cann 1990; Marsh et al. 2017; Pringle 2018; Huthnance et al. 2020). In simplified terms, depth-integrated baroclinic geostrophic currents do not cross f/h contours. As a consequence and to maintain nondivergence, strong along-slope currents lie above slopes in the presence of along-slope gradients of density. This allows to “flush” the geostrophic flow incoming toward the slope (Marsh et al. 2017), which is the typical situation at eastern

boundaries. The opposite is true on the western margins of basins, where along-slope currents “feed” the geostrophic transport flowing away from the slope. This is the meaning of JEBAR, and this is in stark contrast with the flat-bottomed open ocean where density gradients yield geostrophic currents solely in the perpendicular direction. With oceanic density generally increasing poleward from the equator, JEBAR slope currents are nonnegligible at eastern and western boundaries (Huthnance 1984; Myers et al. 1996). At the former, JEBAR may be the dominant contribution to the currents over the continental slopes (Huthnance 1984). Note that, throughout this contribution, we use the wording JEBAR to signify its literal meaning (the combined effect of baroclinicity and changing bathymetry), not the more narrowed notion of the JEBAR torque, which appears only in geostrophic contour vorticity balance formulations (Waldman and Giordani 2023). Joint effects of baroclinicity and relief impact currents regardless of the choice of depth-integrated vorticity balance, through the presence of the potential energy anomaly gradient in the momentum equations. We introduce this quantity formally below. It can be thought of as the combination of the steric sea level and the twice depth-integrated horizontal gradients of seawater density.

Huthnance (1984) further highlighted how bottom drag acts to reduce the growth of JEBAR slope currents, with velocities evolving along slope toward an arrested state known as the equilibrated form. His equation for slope currents (the first unnumbered equation in his section 5b),

$$\bar{v} = \frac{gh_0 - h\partial\rho}{r \frac{2\rho_r}{\partial y}}, \quad (3)$$

is indicative that the depth-averaged meridional flow $\bar{v}$ is directly proportional to along-slope density gradients $\partial_y \rho$. It helped diffusing the idea that eastern boundary slope currents are directly related to density changes in the ocean. Here, h_0 is the depth of the distant open ocean and ρ_r is a constant reference density; other variables are as above. This idea was pursued in Huthnance et al. (2020), where expressions for the equilibrated slope current were given for arbitrary density distributions.

While both the JEBAR and ATW theories consider a geostrophic–frictional balance in an ocean with realistic bathymetry, the former is more complete than the latter. The ATW formalism is, for the most part, limited to barotropic oceans. The JEBAR theory includes in addition baroclinicity. However, most JEBAR research has focused on ocean circulation, with limited emphasis on the distribution of sea level implied by such circulation. For example, Huthnance (1984) investigated the role of bottom friction in a fully baroclinic ocean, but his formulation was expressed in terms of the mass transport streamfunction rather than with sea level [see his Eq. (4.1)]. The exact same comment can be made about Eq. (22) of Mertz and Wright (1992) or Eq. (1) of Myers et al. (1996). In these studies, the formalism adopted is a geostrophic contour vorticity balance (Waldman and Giordani 2023). Alternatively, Rattray (1982) relied on a barotropic vorticity balance (Waldman and Giordani 2023), where the transport streamfunction is swapped for bottom pressure. The formalism we propose below is strongly related to this effort because the manometric

sea level has a strong tie to bottom pressure. Nonetheless, Rat-tray's goal was to understand the circulation, not the sea level. Finally, a number of JEBAR equational frameworks were formulated in terms of sea level (e.g., Sarkisyan and Ivanov 1971; Sarkisyan 2006; Pringle 2018). However, the focus was never on the precise way the open-ocean sea level is mediated to the shelf and coast; modeling the sea-level distribution was an intermediary step to assess the circulation. On the other hand, ATW theories have, from the initial work of Csanady (1978), always featured a strong interest for the coastal and shelf sea level. They only lack a proper consideration of the spatial variations of density in the ocean.

There are a few studies which elude to the simplistic classification we have presented thus far of 1) ATW studies missing considerations of baroclinicity and 2) JEBAR studies lacking interest in sea level. Sheng and Thompson (1996a,b) derived equations to diagnose ocean circulation and sea level in the context of a baroclinic ocean with bottom friction. Their developments were expressed using the manometric sea level and are reminiscent of the bottom pressure framework of Rat-tray (1982). Recently, Renkl et al. (2025) developed diagnostic equations for the evolution of the coastal sea level bordering a baroclinic ocean. They form a geostrophic contour vorticity balance (Waldman and Giordani 2023) and relate coastal sea-level alongshore changes to vorticity dynamics integrated over the adjacent ocean, including JEBAR and bottom stress. Our analysis can be seen as a continuation of these different contributions. Finally, Huthnance (1987, 2004) showed great interest in the spatiotemporal variations of sea level in a baroclinic ocean with bottom friction. In particular, they demonstrated that only large-scale baroclinic features impact the coastal sea level. These contributions go beyond the simple work proposed here, as they retain considerations for the temporal developments of currents and sea level. As such, these papers are more complex than the present effort.

In a context where the understanding of ocean circulation influence on coastal sea level is of paramount importance, inclusion of (true) baroclinicity is lacking in the most modern developments in shelf and coastal sea-level research, despite density distribution being of prime importance for boundary currents. This is also despite considerations of the joint effect of baroclinicity and relief have been an integral part of oceanography research in the last 60 years and allow for such inclusion. In the present contribution, we fill this gap for an arbitrary ocean region and then explore implications for currents and sea level at an eastern boundary. This eastern boundary is here representative of the Scottish-Irish margins and of the Rockall Slope Current, but similar contexts exist elsewhere. The framework we propose in section 2 is a generalization of the arrested topographic wave theory to nonhomogeneous oceans. A simplified slope geometry is introduced in section 3 to present solutions for sea level and currents. We discuss our results in section 4 and conclude in section 5.

2. Theoretical development

The depth-averaged, linear, steady-state momentum conservation equations are expressed as follows:

$$-\frac{f}{h} \int_{-h}^{\eta} \rho_r v dz + \frac{\tau_b^x}{h} = -\frac{1}{h} \int_{-h}^{\eta} \frac{\partial p}{\partial x} dz, \quad (4)$$

$$+\frac{f}{h} \int_{-h}^{\eta} \rho_r u dz + \frac{\tau_b^y}{h} = -\frac{1}{h} \int_{-h}^{\eta} \frac{\partial p}{\partial y} dz, \quad (5)$$

where $u(x, y, z)$ and $v(x, y, z)$ are the zonal and meridional velocities, $\eta(x, y)$ is the sea-level anomaly, $\rho_r(z)$ is a reference density profile, $h(x, y)$ is the seafloor depth, $f(y)$ is the Coriolis frequency, and $\tau_b(x, y)$ is the frictional bottom stress. The hydrostatic pressure is $p(x, y, z) = \int_z^{\eta} \rho g dz$, where $\rho = \rho_r(z) + \rho'(x, y, z)$ is the (spatially evolving) in situ density, which is throughout this contribution regarded as prescribed. Here, the effects of lateral stress, surface stress, and nonlinear advection have been neglected. These additional forces are typically not included in arrested topographic wave theories. They are outside of the scope of the fundamental notions covered in the present contribution. Note our reference density $\rho_r(z)$ is vertically varying, meaning the context is anelastic. This framework—also known as weak Boussinesq—is superior to the more classical strong Boussinesq hypothesis where ρ_r is constant and more suited for JEBAR studies. We write the surface reference density $\rho_{r_s} = \rho_r(z=0)$. Furthermore, in the remainder of this paper, the following notations for the depth-averaged mass fluxes (of reference density) are used:

$$\overline{\rho_r v} = \frac{1}{h} \int_{-h}^{\eta} \rho_r v dz, \quad (6)$$

$$\overline{\rho_r u} = \frac{1}{h} \int_{-h}^{\eta} \rho_r u dz.$$

After using the hydrostatic relationship, considering that $\eta \ll h$ and making the assumption that $\rho(x, y, z \geq 0) \approx \rho_{r_s}$, the momentum balance is reformulated as

$$-f \overline{\rho_r v} + \frac{\tau_b^x}{h} \approx -g \rho_{r_s} \frac{\partial \eta}{\partial x} - \frac{g}{h} \int_{-h}^0 \int_{-h}^0 \frac{\partial \rho}{\partial x} dz' dz, \quad (7)$$

$$+f \overline{\rho_r u} + \frac{\tau_b^y}{h} \approx -g \rho_{r_s} \frac{\partial \eta}{\partial y} - \frac{g}{h} \int_{-h}^0 \int_{-h}^0 \frac{\partial \rho}{\partial y} dz' dz. \quad (8)$$

Note the horizontal gradients (∂_x, ∂_y) in Eqs. (7) and (8) can be freely moved in and out of the inner integral because z is an independent variable.

Neglecting any inverse barometer effects, the total sea level η is the sum of manometric ζ and steric η_s contributions, i.e.,

$$\eta = \zeta + \eta_s. \quad (9)$$

We define the latter as

$$\eta_s = -\frac{1}{\rho_{r_s}} \int_{-h}^0 (\rho - \rho_r) dz = -\frac{1}{\rho_{r_s}} \int_{-h}^0 \rho' dz, \quad (10)$$

which is similar to the strong Boussinesq formulation for steric sea level:

$$\eta_s = - \int_{-h}^0 \frac{\rho - \rho_r}{\rho_r} dz = - \int_{-h}^0 \frac{\rho'}{\rho_r} dz \quad \text{with } \rho_r \text{ constant.}$$

Equation (10) implies a simple relationship between bottom pressure p_b and manometric sea level ζ :

$$\begin{aligned} p_b &= g \int_{-h}^0 \rho_r dz + g \rho_{r_s} (\eta - \eta_s) \\ &= g \int_{-h}^0 \rho_r dz + g \rho_{r_s} \zeta. \end{aligned} \quad (11)$$

This relation is formally demonstrated in [appendix A](#). It indicates that ζ is a bottom pressure head anomaly. “Manometric” sea level is the modern terminology to refer to bottom pressure changes expressed in terms of sea level ([Gregory et al. 2019](#)). Noting that

$$\nabla \int_{-h}^0 \rho_r dz = \rho_{r_b} \nabla(h), \quad (12)$$

gradients of manometric sea level $\nabla \zeta$ and that of bottom pressure are simply related:

$$\nabla p_b = g \rho_{r_b} \nabla h + g \rho_{r_s} \nabla \zeta, \quad (13)$$

where the bottom reference density is $\rho_{r_b} = \rho_r(-h)$.

The pressure terms in Eqs. (7) and (8) can hence be reformulated with the help of Eqs. (9) and (10). The right-hand side of Eqs. (7) and (8) becomes

$$-g \rho_{r_s} \nabla \zeta + g \nabla \left(\int_{-h}^0 \rho' dz \right) - \frac{g}{h} \int_{-h}^0 \int_z^0 \nabla \rho' dz' dz, \quad (14)$$

where we used $\nabla \rho = \nabla \rho'$. The pressure-gradient force is hence associated with a barotropic part, related to ζ , and two baroclinic contributions, related to density changes in the water column. There is some compensation between the two baroclinic terms, and they can be further simplified. The second term in Eq. (14), related to the steric sea level, is from the Leibniz integral rule:

$$g \nabla \left(\int_{-h}^0 \rho' dz \right) = g \int_{-h}^0 \nabla \rho' dz + g \rho'_b \nabla h, \quad (15)$$

where $\rho'_b = \rho_b - \rho_r(-h)$ is the anomaly of the bottom density $\rho_b = \rho(x, y, -h)$.

Through integration by parts, the double integral term in Eq. (14) is expressed as follows:

$$\begin{aligned} \frac{g}{h} \int_{-h}^0 \int_z^0 \nabla \rho' dz' dz \\ = g \int_{-h}^0 \nabla \rho' dz + \frac{g}{h} \int_{-h}^0 \nabla (z \rho') dz. \end{aligned} \quad (16)$$

Subtracting Eq. (16) from Eq. (15) yields

$$\begin{aligned} g \nabla \left(\int_{-h}^0 \rho' dz \right) - \frac{g}{h} \int_{-h}^0 \int_z^0 \nabla \rho' dz' dz \\ = -\frac{g}{h} \left[\int_{-h}^0 \nabla (z \rho') dz - h \rho'_b \nabla h \right] \\ = -\frac{1}{h} \nabla \chi', \end{aligned} \quad (17)$$

where the scalar χ' is the potential energy anomaly (e.g., [Rattray 1982](#)):

$$\chi' = g \int_{-h}^0 z \rho' dz. \quad (18)$$

The last equality in Eq. (17) was obtained using the Leibniz integral rule. The potential energy anomaly is also known more simply as the potential energy ([Mellor et al. 1982](#)). It is an integrated quantity which combines the energy of the steric sea level and that of density changes at depth.

Substituting Eqs. (14) and (17) in Eqs. (7) and (8), one obtains

$$-f \overline{\rho_r v} + \frac{\tau_b^x}{h} = -g \rho_{r_s} \frac{\partial \zeta}{\partial x} - \frac{1}{h} \frac{\partial \chi'}{\partial x}, \quad (19)$$

$$+f \overline{\rho_r u} + \frac{\tau_b^y}{h} = -g \rho_{r_s} \frac{\partial \zeta}{\partial y} - \frac{1}{h} \frac{\partial \chi'}{\partial y}. \quad (20)$$

We will refer to terms associated with ζ as barotropic and terms associated with χ' as baroclinic. Critically, the depth-integrated geostrophic baroclinic mass flow is solely related to the potential energy anomaly χ' and the Coriolis parameter f :

$$h \overline{\rho_r \mathbf{v}_{bi}} = \frac{\mathbf{k} \times \nabla \chi'}{f}. \quad (21)$$

Following the classical methodology of ATW theories, an assumption that bottom stress is proportional to the bottom geostrophic flow is introduced: $\tau_b = \rho_{r_b} r \mathbf{v}_b^g$, with r a friction parameter in units of velocity and $\rho_{r_b} = \rho_r(-h)$ the reference density at the seafloor. The bottom geostrophic flow is

$$u_b^g = -\frac{1}{f \rho_{r_b}} \frac{\partial p}{\partial y} \bigg|_{-h} \quad \text{and} \quad v_b^g = +\frac{1}{f \rho_{r_b}} \frac{\partial p}{\partial x} \bigg|_{-h}, \quad (22)$$

where the subscript $-h$ denotes that the horizontal gradients of pressure should be evaluated at the seafloor, whose depth varies with x and y . Integrating the hydrostatic equation from the bottom allows to write $p(x, y, z) = p_b - \int_{-h}^z \rho g dz'$ and yields, after substituting in Eq. (22),

$$\mathbf{v}_b^g = \frac{1}{f \rho_{r_b}} \mathbf{k} \times \left| \nabla p_b - \nabla \int_{-h}^z \rho g dz' \right|_{-h}, \quad (23)$$

where $\mathbf{k}$ is the unit vertical vector. The Leibniz integral rule permits to obtain

$$\mathbf{v}_b^g = \frac{1}{f\rho_b} \mathbf{k} \times \left[\nabla p_b - \int_{-h}^z g \nabla \rho dz' - g \rho_b \nabla h \right]_{-h}. \quad (24)$$

In Eq. (24), the integral term cancels upon evaluation at $z = -h$, leaving

$$\mathbf{v}_b^g = \frac{1}{f\rho_b} \mathbf{k} \times (\nabla p_b - g \rho_b \nabla h). \quad (25)$$

This is exactly the equation for the bottom geostrophic flow provided by Schlichtholz (2004). We reformulate this equation so that it is expressed in terms of manometric sea level with the use of Eq. (13) and obtain the bottom stress:

$$\boldsymbol{\tau}_b = \rho_{r_b} r \mathbf{v}_b^g = \frac{gr}{f} \mathbf{k} \times (\rho_{r_s} \nabla \zeta - \rho'_b \nabla h). \quad (26)$$

Substituting in Eqs. (19) and (20) leads to expressions for the depth-averaged mass flux:

$$\overline{\rho_r u} = -\frac{g \rho_{r_s}}{f} \frac{\partial \zeta}{\partial y} - \frac{1}{fh} \frac{\partial \chi'}{\partial y} - \frac{gr \rho_{r_s}}{f^2 h} \frac{\partial \zeta}{\partial x} + \frac{gr \rho'_b}{f^2 h} \frac{\partial h}{\partial x}, \quad (27)$$

$$\overline{\rho_r v} = +\frac{g \rho_{r_s}}{f} \frac{\partial \zeta}{\partial x} + \frac{1}{fh} \frac{\partial \chi'}{\partial x} - \frac{gr \rho_{r_s}}{f^2 h} \frac{\partial \zeta}{\partial y} + \frac{gr \rho'_b}{f^2 h} \frac{\partial h}{\partial y}. \quad (28)$$

Using Eqs. (27) and (28) in the depth-integrated mass conservation equation, which is in an anelastic context,

$$\nabla \cdot \int_{-h}^{\eta} \rho_r \mathbf{u} dz = \frac{\partial h \overline{\rho_r u}}{\partial x} + \frac{\partial h \overline{\rho_r v}}{\partial y} = 0, \quad (29)$$

allows to obtain a simple vorticity conservation law:

$$\begin{aligned} J\left(\frac{gh}{f}, \zeta\right) + \nabla \cdot \left(\frac{gr}{f^2} \nabla \zeta\right) \\ = -J\left(\frac{1}{f}, \frac{\chi'}{\rho_{r_s}}\right) + \nabla \cdot \left(\frac{gr}{f^2} \frac{\rho'_b}{\rho_{r_s}} \nabla h\right). \end{aligned} \quad (30)$$

This expression is a partial differential equation (PDE) for the manometric sea level ζ , which can be solved, provided the density field is fully known. On an f plane, only the bottom density needs to be known as the first right-hand side (rhs) term of Eq. (30) cancels. Once ζ is determined, the total sea level η can be readily obtained using $\eta = \zeta + \eta_b$. Similarly, depth-averaged mass fluxes $\overline{\rho_r u}$ and $\overline{\rho_r v}$ can be derived from Eqs. (27) and (28). If depth-averaged (or, similarly, depth integrated) currents are needed instead, they can be obtained using integration by parts:

$$\bar{\mathbf{u}} = \frac{\overline{\rho_r \mathbf{u}}}{\rho_{r_b}} - \frac{1}{h \rho_{r_b}} \int_{-h}^0 \frac{\partial \rho_r}{\partial z} \int_z^0 \mathbf{u} dz' dz, \quad (31)$$

where the modest transport in between $z = 0$ and $z = \eta$ was neglected. In principle, the obtention of depth-averaged velocity through Eq. (31) demands the vertical distribution of currents throughout the water column is known. This requires, among other things, the knowledge of the stress $\boldsymbol{\tau}(z)$ in the

bottom boundary layer, which is an adverse consequence of using the anelastic approximation over the strong Boussinesq hypothesis. For many applications, however,

$$\bar{\mathbf{u}} \approx \frac{\overline{\rho_r \mathbf{u}}}{\rho_{r_b}}, \quad (32)$$

which is a sufficient approximation.

In an ocean without horizontal density gradient, $\rho = \rho_r$, and the terms on the right-hand side of Eq. (30) cancel. In that case, $\eta = \zeta$ and our formulation is consistent with the homogeneous ocean ATW equation of Wise et al. (2018) and Wu (2021) [Eq. (1)]. There are no flow and no sea-level changes unless they originate from the coast or from open-ocean boundaries (Wang 1982; Wu 2021). In the more general case where variations in density cannot be neglected, the two right-hand-side terms of Eq. (30) control the way the manometric sea-level solution will deviate from the simpler homogeneous ocean case. The analogy with an advection–diffusion equation remains. The manometric sea level ζ radiates “fast” along h/f contours while diffusing “slowly” across these isolines. However, ζ has now sources and sinks associated with the rhs terms of Eq. (30), which arise due to density differences in the ocean but are of different relative importance. The baroclinic Jacobian (first term on the rhs) is the divergence of the baroclinic geostrophic transport and one formulation for the baroclinic β effect (BARBE; after Sarkisyan 2006). Simply put, meridional variations in the Coriolis frequency ($f \neq f_0$) cause a fraction of the geostrophic baroclinic transport to cross isobath. The second rhs term reflects the divergence of the bottom Ekman currents generated by the bottom geostrophic baroclinic currents. In all generality, both these two terms can be sources or sinks and will vary spatially. In effect, variations in bathymetry h dominate variations in f/h at oceanic margins and the effect of bottom drag is larger than the baroclinic β effect there [for details, see appendix C of Huthnance et al. (2020)].

3. Analysis

a. Setup for a simplified Rockall Slope Current

Up to now, the derivations have been consistent with bathymetry, seawater density distribution, and coastline, all of which are arbitrary. Hence, the equations so far hold for a quite general ocean. The problem can at present be simplified for an application to the Rockall Slope Current (or any other similar eastern boundary slope circulation). The Rockall Slope Current is a slope circulation above the continental slope to the west and north of Ireland and Scotland. Typical velocities are 15–20 cm s^{−1} (Fraser et al. 2022; Souza et al. 2001). The framework we opt for shares many similarities with that of Huthnance et al. (2020) but does not require that the slope current has reached an equilibrium without along-slope flow changes ($\partial_y \bar{v}$ is not required to be zero), allowing for a greater investigation of the spatial distribution of η and of currents. We also give a greater emphasis on the behavior of the sea level. A schematic of the modeled region is presented in Fig. 1a. The

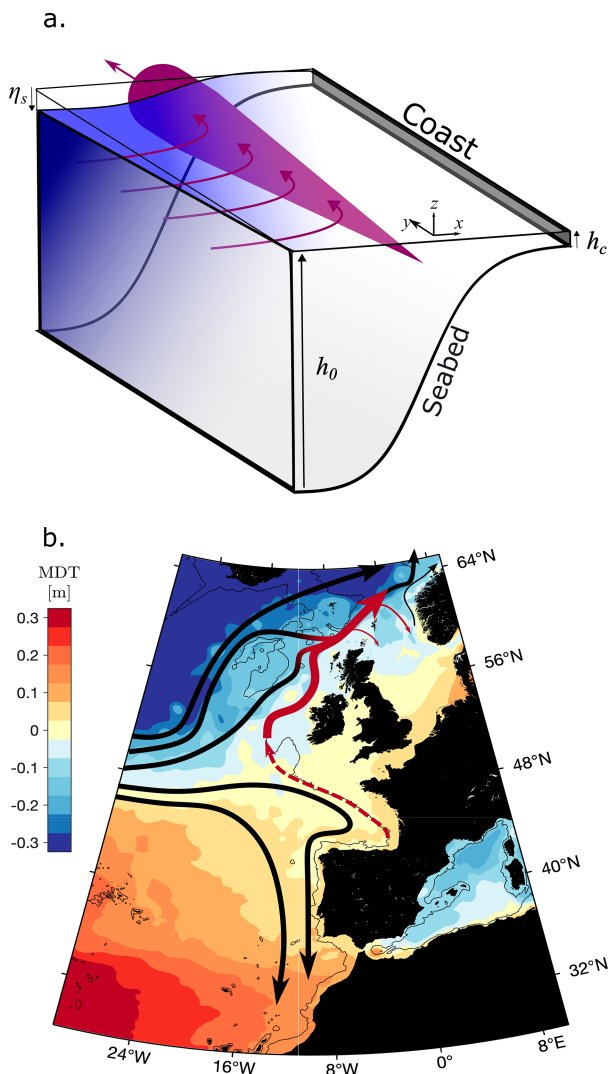


FIG. 1. Slope current schematic and northeast Atlantic context. (a) Schematic illustrating the simplified slope geometry, as well as the physical processes at play. Zonal baroclinic currents (small purple arrows) arise from seawater differences in the ocean, depicted as shadings with blue colors indicating denser waters. They drive an important “JEBAR” slope current (large purple cone). This poleward current is balancing the cross-slope gradient of steric sea level η_s . It is intensified over the mid-to-upper slope and through friction against the seafloor generates bottom return flow toward the ocean as Ekman currents (not shown in the schematic). The manometric sea level ζ must adjust to the flow divergence associated with these frictional currents. In the schematic, h_0 represents the open-ocean depth, whereas h_c is the depth at the coastal boundary. (b) Color shadings represent the mean dynamic topography of the sea surface for the 1993–2012 era (Mulet et al. 2021). Overlaid is the general pattern of flow in the region, including the different branches of the North Atlantic Current (thick black arrows). The thick dark red arrow shows the Rockall Slope Current. The 1000-m isobath is shown as thin black contours.

sloping oceanic margin geometry is similar to Huthnance et al. (2020). The 1993–2012 mean dynamic topography (Mulet et al. 2021) around the European margin is shown in Fig. 1b for context, with the thick red arrow showing the Rockall Slope Current. Its upstream origin is relatively contested (e.g., Moritz et al. 2021). The Rockall Slope Current may emerge from the slope circulation farther to the south in the Bay of Biscay, as is shown with the dashed red arrow. The authors who consider so tend to label the whole poleward slope circulation the “European Slope Current.” The mean dynamic topography was obtained from the Copernicus Marine Environment Monitoring Service (CMEMS) Information (<https://doi.org/10.48670/moi-00150>) (accessed on 15 April 2024).

We consider a simplified topography, where depth $h(x)$ is invariant in the along-slope (y) direction ($\partial_y h = 0$). The continental slope is parallel to the y direction, which is oriented from south to north for simplicity. The bathymetry is represented by a sigmoid function:

$$h(x) = h_0 + \frac{h_c - h_0}{1 + \exp(kx)}, \quad (33)$$

with $h_c = 100$ m being the depth at the coast, $h_0 = 2300$ m being the depth at the open-ocean boundary, and $k = -7 \times 10^{-5} \text{ m}^{-1}$ being a growth parameter. This definition means the slope is well represented, but the shelf is flat and bounded by a vertical wall at the coast. This is a purposeful decision to focus on slope processes. The $x = 0$ mark is at the center of the slope, where $\partial_x(-h)$ is maximum. The coast is 150 km eastward of this position at $x = x_c$. The growth rate $|k|$ is large enough that it can be reasonably said $\partial_x h$ is zero at the coast, which simplifies the formulation of the coastal boundary condition. The approximate slope width is 100 km. Maxima of $|\partial_x^2 h|$ are found ± 19 km off the slope center.

We define the density forcing in terms of the potential density $\sigma^\theta(x, y, z)$, which is set to increase linearly in the poleward direction in the upper thermal layer: $\sigma^\theta = \sigma_s^\theta = \sigma_{y_0}^\theta + \kappa y$, for $z > -h_T + \epsilon$, with κ being the rate of density poleward increase (Fig. 2a). To obtain a realistic κ , we retrieved Jones et al. (2023) climatology of temperature and salinity along the 1000-m isobath of the subpolar North Atlantic. We derived the along-isobath gradient of the potential density averaged in the top 800 m (see Fig. 1 in the online supplemental material) and found setting $\kappa = 1.5 \times 10^{-7} \text{ kg m}^{-4}$ was appropriate to represent the Rockall Trough. At the upstream boundary, we set $\sigma_{y_0}^\theta$ to 27 kg m^{-3} , a value which is obtained from an Absolute Salinity of 35.45 g kg^{-1} and a Conservative Temperature of 11°C . At depth, an abyssal layer is kept homogeneous ($\sigma^\theta = \sigma_b^\theta$ for $z \leq -h_T - \epsilon$). It is set at 28 kg m^{-3} , from an Absolute Salinity of 35.45 g kg^{-1} and a Conservative Temperature of 4°C . A transitional layer of 2ϵ thickness lies between the two, allowing for σ^θ to be continuous. A sinusoidal function is used, i.e.,

$$\sigma^\theta = \sigma_b^\theta + \frac{\sigma_s^\theta - \sigma_b^\theta}{2} \left[\sin \frac{\pi(z + h_T)}{2\epsilon} + 1 \right],$$

for $-h_T - \epsilon < z \leq -h_T + \epsilon$. We set the depth of the middle of this stratification layer $h_T = 1000$ m and use a stratification

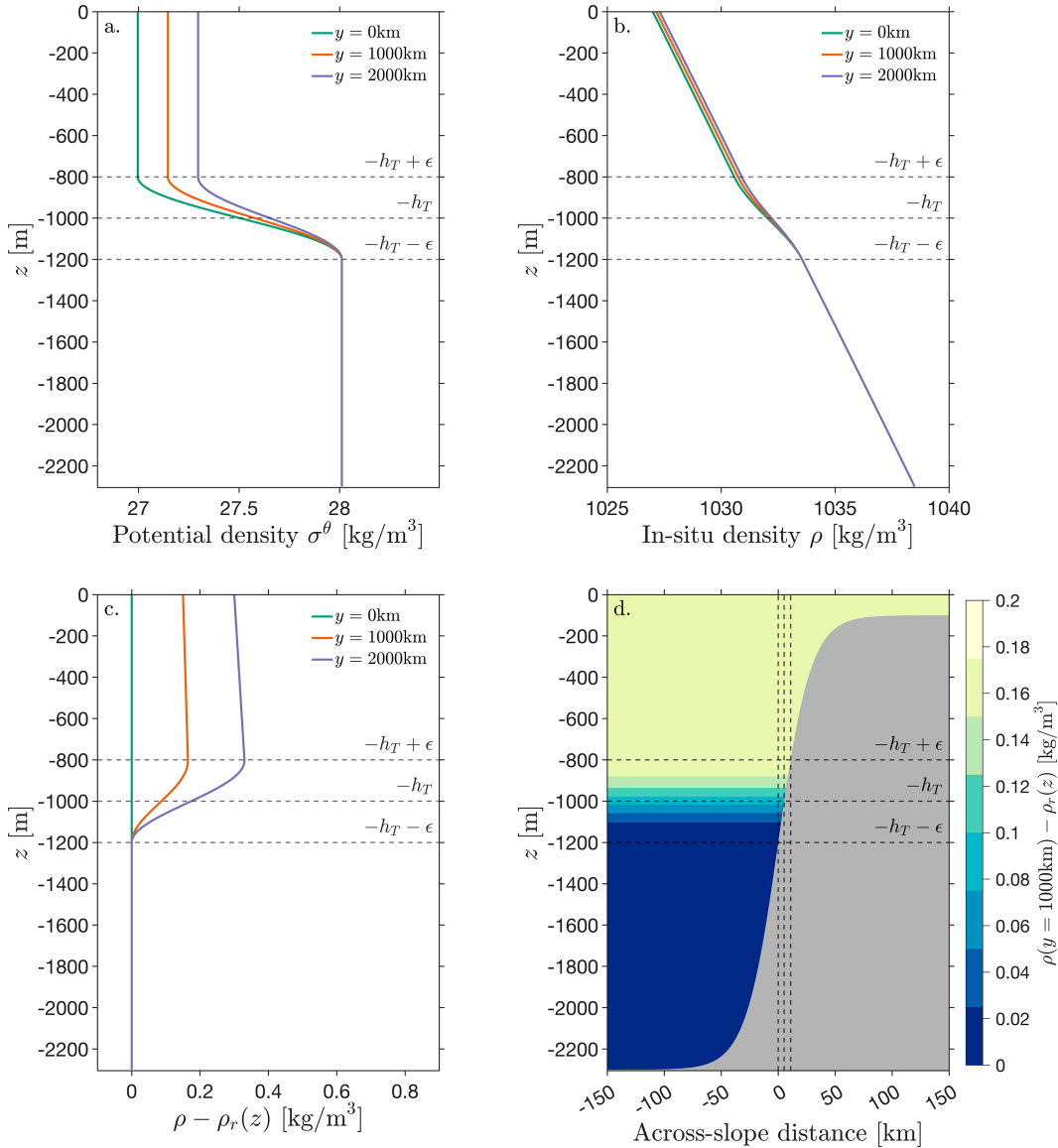


FIG. 2. The density and bathymetry definition. (a) Potential density σ^θ for three different along-slope distances from $y = 0$. (b) The associated in situ density ρ . (c) The difference $\rho - \rho_r(z)$ at $y = 1000$ km (shadings). The gray patch indicates the seafloor, as a function of x . Horizontal dashed lines in all plots indicate the lower, middle, and upper limits of the stratification layer. Vertical dashed lines in (d) represent where these lines meet the slope (respectively, at $x = 0$ km, $x \approx +5$ km, and $x \approx +11$ km). The difference $\rho - \rho_r(z)$ is slightly greater at the upper limit of the stratification layer ($z = -h_T + \epsilon$) than at the surface because of thermobaricity.

layer half-thickness of $\epsilon = 200$ m. With this choice, the active layer does not make contact with the lower slope. The lower limit of the stratification layer intersects the slope at $x(-h_T - \epsilon) = 0$ km. The middle and upper limit of the stratification layer respectively intersect the seafloor at $x(-h_T) = x_T \approx +5$ km and $x(-h_T + \epsilon) \approx 11$ km.

Altogether, this formulation for potential density is obviously a coarse approximation, but it is enough to mimic the poleward decline of dynamic height observed in the thermal layer of

the subpolar northeast Atlantic. In situ density ρ , the needed variable in the equations of motion is derived from this piecewise potential density definition using $\rho = \rho(\sigma^\theta, -\rho_s g z)$ (Fig. 2b). The value of ρ_s is unimportant, and we set it to $\sigma_{y_0}^\theta + 1000 \text{ kg m}^{-3}$. We define the reference density profile $\rho_r(z)$ equal to $\rho(z, y_0)$, the density profile at the upstream boundary. The density forcing, associated with the density anomaly $\rho' = \rho(y, z) - \rho_r(z)$, is hence that represented in Fig. 2c. Although ρ' does not vary with x , χ' and ρ'_b do because they

implicitly depend on $h(x)$. We represent in Fig. 2d the cross-slope geometry, including the density distribution and the bathymetry profile.

We assume an f plane and set $f = f_0 = 1.22 \times 10^{-4} \text{ s}^{-1}$. This approximation is appropriate at the length scales of the Rockall Slope Current (Huthnance et al. 2020), whose characteristic lengths are of order $\mathcal{O}(10^1)$ km across slope and $\mathcal{O}(10^3)$ km along slope. The action of across-slope bottom friction τ_b^x is considered negligible in comparison with the Coriolis force of alongshore flow and neglected. This implies the meridional flow is purely geostrophic. The friction parameter r is set equal to $4 \times 10^{-3} \text{ m s}^{-1}$, a value which was found to yield appropriate results for currents and sea level with the bathymetry and density distribution used here. It is on the higher end of commonly found values in the literature: often r is set at $5 \times 10^{-4} \text{ m s}^{-1}$ (e.g., Wu 2021; Wright 1986) or at $1 \times 10^{-3} \text{ m s}^{-1}$ (Wise et al. 2018; Csanady 1978). Lin et al. (2022) use $1.5 \times 10^{-3} \text{ m s}^{-1}$, and r values as high as $1 \times 10^{-2} \text{ m s}^{-1}$ have been used (Huthnance et al. 2020).

In the context of no cross-slope density variations, the cross-slope gradient of potential energy χ' simplifies to $\partial_x \chi' = gh\rho_s \partial_x \eta_s$ through Eq. (17). This leads to a simple expression for depth-averaged along-slope mass flux:

$$\overline{\rho_r v} = + \frac{g\rho_s}{f_0} \frac{\partial(\zeta + \eta_s)}{\partial x} = \frac{g\rho_s}{f_0} \left(\frac{\partial \zeta}{\partial x} - \frac{\rho'_b}{\rho_s} \frac{\partial h}{\partial x} \right), \quad (34)$$

where the second equality is obtained through the use of the Leibniz integral rule. The depth-averaged across-slope mass flux is unchanged with respect to section 2:

$$\overline{\rho_r u} = - \frac{g\rho_s}{f_0} \frac{\partial \zeta}{\partial y} - \frac{1}{f_0 h} \frac{\partial \chi'}{\partial y} - \frac{g\rho_s r}{f_0^2 h} \left(\frac{\partial \zeta}{\partial x} - \frac{\rho'_b}{\rho_s} \frac{\partial h}{\partial x} \right), \quad (35)$$

but we have grouped the frictional currents so that it is more immediately apparent they are simply proportional, in the present context, to the along-slope flow scaled by h [Eq. (34)]. Finally, the ATW equation becomes

$$\frac{\partial h}{\partial x} \frac{\partial \zeta}{\partial y} + \frac{r}{f_0} \frac{\partial^2 \zeta}{\partial x^2} = \frac{r}{f_0} \frac{\partial}{\partial x} \left(\frac{\rho'_b}{\rho_s} \frac{\partial h}{\partial x} \right). \quad (36)$$

The first term on the left-hand side (lhs) represents the convergence of the barotropic geostrophic transport. The second term on the lhs is the convergence of the bottom Ekman transport generated by the barotropic geostrophic current. The term on the rhs is the convergence of the bottom Ekman transport generated by the baroclinic geostrophic current. Altogether, this formulation for ζ does not depend on the density distribution $\rho(x, y, z)$ but simply on the bottom density anomaly $\rho'_b(x, y)$. It is in fact valid for any other depth profiles invariant in y and for arbitrary density distributions. This expression is very similar to the original framework of Csanady (1978) [see Eq. (2) above] but differs with the presence of a right-hand side reflecting the convergence–divergence of Ekman currents generated at the bottom of the ocean by the

large meridional slope currents induced by density changes in the ocean.

The condition of no-normal flow at the coast is obtained from Eq. (35):

$$\frac{r}{f_0} \frac{\partial \zeta}{\partial x} + h_c \frac{\partial \zeta}{\partial y} = \frac{1}{g\rho_s} \frac{\partial \chi'}{\partial y} + \frac{r}{f_0} \frac{\rho'_b}{\rho_s} \frac{\partial h}{\partial x}, \quad (37)$$

where the last term cancels because $\partial_x h$ at the coast is zero with our depth profile choice. The upstream (i.e., southern) boundary is kept clamped [$\zeta(x=0)=0$] so that the initial geostrophic along-slope transport is zero. Equation (36) being parabolic in nature, it is in principle not necessary to specify a downstream boundary condition (and certainly not desired). However, this is not possible with the solver we opted for and a Neumann boundary condition was used. The model along-slope length was set at 8000 km so that numerical issues associated with this treatment of the downstream boundary condition are not seen in the first part of the domain, with our focus being on the evolution of sea level and currents for $y < 2000$ km. The open-ocean (western) boundary is similarly problematic. It is maintained clamped but pushed well away at $x = 3000$ km so that it has a minimum influence. Finally, and for readability, we use Eq. (32) and report the modeled flow in terms of depth-averaged velocity rather than as a depth-averaged mass flux [we divide Eqs. (35) and (34) by ρ_r and neglect the second term on the rhs of Eq. (31)].

b. Results

Figure 3 presents the baroclinic “forcing.” Figure 3a presents the steric sea-level anomaly η_s implied by the poleward density increase in the ocean. Larger changes in η_s occur over the open ocean than over the shelf, simply because steric effects are small in shallow water. A similar comment can be made about the potential energy anomaly χ' (see supplemental Fig. 2). The associated eastward geostrophic flow (Fig. 3b) is hence greater in the open ocean than on the shelf. Note the depth-averaged velocities admit a maximum around the slope center, but in effect, the depth-integrated transport is strictly diminishing from west to east. This leads to convergence over the slope, which is exactly balanced by along-slope growth in poleward baroclinic currents as can be seen in Fig. 3c. We refer to the latter currents as JEBAR slope currents. These currents are large [here $\mathcal{O}(0.1\text{--}1) \text{ m s}^{-1}$, and note the color bar axes are not identical on the different panels]. They do not stop increasing poleward for they are “fed from the west” by the maintained geostrophic baroclinic zonal currents (Marsh et al. 2017). These meridional currents emerge where the active layer meets the slope, immediately shoreward of the $h = h_T + \epsilon$ bathymetry contour. From an equatorial perspective, they result from the combination of strong across-slope topographic variations and bottom density anomalies in Eq. (34) (or equally, from the cross-slope gradient of steric sea level η_s).

Through friction against the bottom of the ocean floor, the JEBAR slope currents (Fig. 3c) generate an offshore Ekman transport (Fig. 3d). With density being prescribed, the divergence associated with this ageostrophic transport can only be balanced through an adjustment of the manometric sea-level

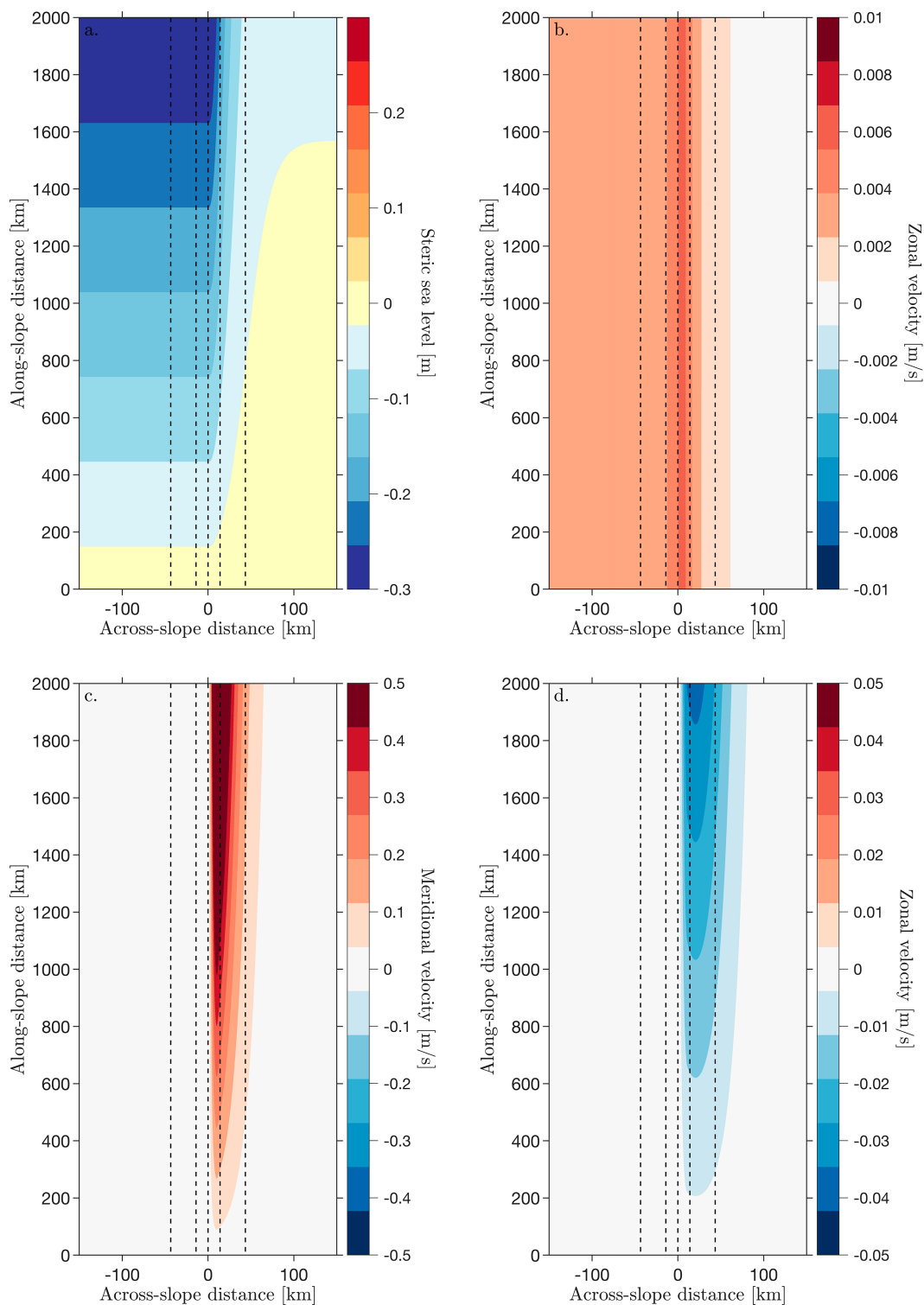


FIG. 3. The baroclinic forcing. (a) The steric sea-level anomaly η_s . (b),(c) The associated zonal and meridional baroclinic geostrophic currents $[\bar{\mathbf{v}} = \mathbf{k} \times \nabla \chi' / (\rho_b f_0 h)]$. (d) The ageostrophic zonal currents arising from the friction of the meridional baroclinic geostrophic flow on the seafloor $[\bar{u} = g r \rho'_b \partial_x h / (f_0^2 h \rho_b)]$; see Eq. (35). All currents are depth averaged. Note the different color bar axis limits on all panels. Dashed black lines indicate the 2200-, 1700-, 1200-, 700-, and 200-m isobaths.

anomaly ζ . Specifically, ζ must adjust such that the barotropic geostrophic currents and the bottom Ekman currents of barotropic origin compensate for the divergence of the bottom Ekman transport of baroclinic origin. This divergence is presented in Fig. 4. Equation (36) is solved numerically, and the obtained manometric sea level ζ is shown in Fig. 5a. Negative ζ emerges from around the shelf edge (Fig. 5a), where the bottom Ekman transport of baroclinic origin (Fig. 4) diverges over the outer shelf and upper slope. Positive ζ anomalies, which have a lesser magnitude than negative ones, develop immediately shoreward of the slope center, where the ageostrophic transport of baroclinic origin largely converges (for it completely disappears for $x \leq 0$). This forms the source of along-slope growth in ζ over the lower slope and in the open ocean. With water depth being large at the midslope, small changes in ζ are enough to counterbalance this large midslope convergence.

The ζ anomalies are then advected and diffused away according to the classic ATW theory. This leads to decreased ζ over the shelf and upper slope. Similarly, weak positive ζ anomalies progress toward the lower slope and eventually the open ocean, but in all generalities, anomalies (of any sign) progress along slope with different across-slope penetration rates due to the changing bathymetry. The ratio $|(f_0 \partial_x h)/r|$ expresses the importance of the geostrophic term relative to the frictional term on the lhs of Eq. (36). It is greater over the slope than over the shelf. This indicates that, for a given along-slope distance, the cross-slope penetration of a ζ isoline is greater over the shelf than over the slope. This has three immediate consequences. 1) The shelf and eventually the coastal sea levels are promptly and greatly affected. Here, “promptly” is meant in the sense of short along-slope distance. 2) A large geostrophic equatorward-flowing current is generated at the intermediate and upper slope (Fig. 5c). This current opposes the JEBAR slope current of Fig. 3c. Note, however, that the transport divergence balance is in between the cross-slope currents of Figs. 3d, 5b, and 5d. 3) A relatively weaker poleward geostrophic current is generated at the lower slope (Figs. 5c). The barotropic along-slope slope currents drive bottom Ekman currents (Figs. 5d).

Figure 6 presents the total sea-level anomaly η , which is the sum $\eta_s + \zeta$, as well as the total currents. Through the action of friction, the open-ocean steric height anomaly η_s has been mediated to the coastline as the manometric sea-level anomaly ζ (Figs. 3a, 5a, and 6a). There is, however, a displacement in the direction of coastally trapped wave propagation (here, poleward) as well as some attenuation. Figures 6c and 6d present, respectively, the depth-averaged and depth-integrated meridional currents. A poleward slope current is well developed with core velocities greater than 10 cm s^{-1} poleward of $y = 1050 \text{ km}$. Initially, along-slope growth in this slope current is large [greater than 1 cm s^{-1} per hundred kilometers for $y < 300 \text{ km}$]. This development is gradually slowed down to 0.1 cm s^{-1} per hundred kilometers at $y \sim 2000 \text{ km}$. As we will at present demonstrate, the flow tends, with increasing y , toward the equilibrated regime described in Huthnance et al. (2020), where $\partial_y(h\bar{v})$ is equal to zero and there is no along-slope development.

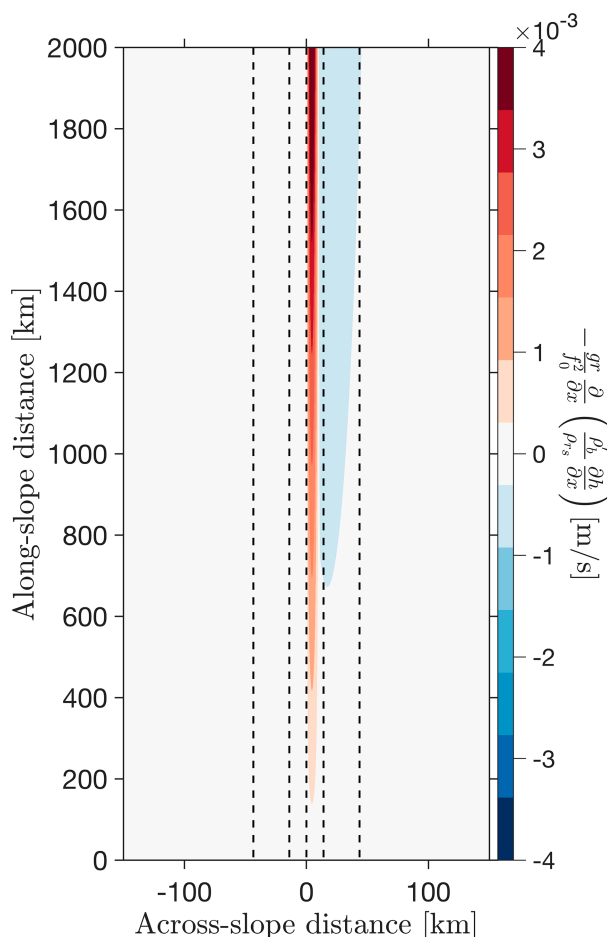


FIG. 4. Divergence of the bottom Ekman currents generated by the along-slope geostrophic baroclinic bottom flow $-gr\partial_x(\rho'_b\partial_x h)/(f_0^2\rho_s)$. This definition is consistent with $-(g/f_0)$ times Eq. (36).

c. Comparison with Huthnance's equilibrated slope current

Huthnance et al. (2020) focused on the particular situation where the total along-slope current has reached an equilibrium without along-slope development, continuing an earlier effort undertaken by Huthnance (1984). This equilibrium solution is of interest because of its historical significance and lasting impact on the community.

Consider a column of water flowing poleward along the slope of an eastern boundary, with forces at play as above (i.e., inertial, friction, and pressure-gradient forces). This situation is depicted in Fig. 7. We continue assuming no cross-slope density changes, which means along-slope velocities do not change with depth. Where the balance of the forces acting upon it sums up to zero, the column moves in a uniform rectilinear motion (=does not accelerate nor decelerate) as is illustrated in Fig. 7. The Coriolis and drag forces are unchanged in this equilibrated regime as the column moves poleward. Indeed, they both depend only on the steady along-slope speed. Therefore, the pressure-gradient force, associated with the horizontal distribution of both χ' and ζ , must also remain unchanged for

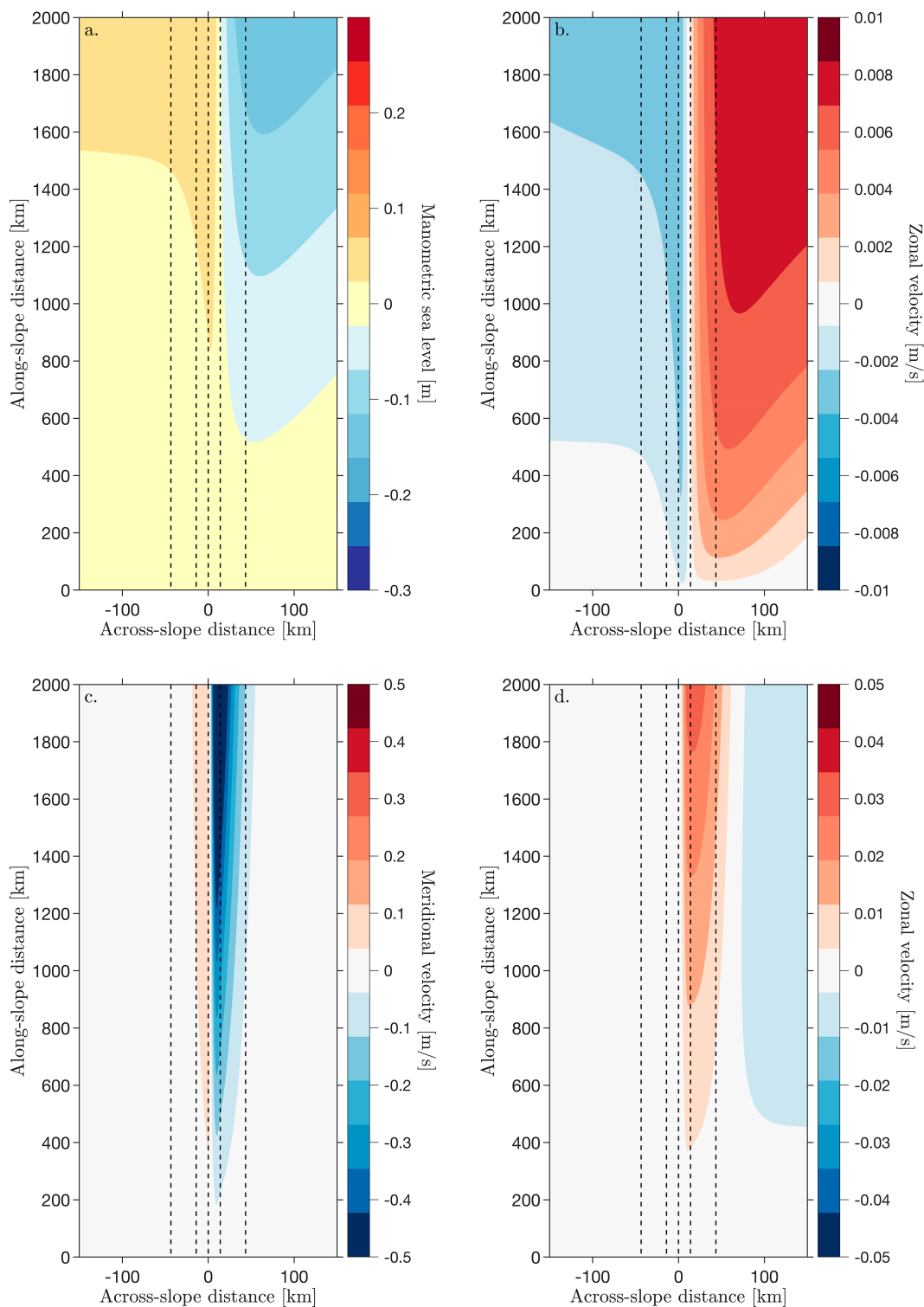


FIG. 5. The barotropic response. (a) The manometric sea-level anomaly ζ . (b),(c) The associated zonal and meridional barotropic geostrophic currents [$\bar{\mathbf{v}} = g\rho_s \mathbf{k} \times \nabla\zeta/(f_0\rho_b)$]. (d) The ageostrophic zonal currents arising from the friction of the meridional barotropic geostrophic flow on the seafloor [$\bar{u} = -g\rho_s \partial_x \zeta/(f_0^2 h \rho_b)$]. All currents are depth averaged. Note the different color bar axis limits on all panels. Dashed black lines indicate the 2200-, 1700-, 1200-, 700-, and 200-m isobaths.

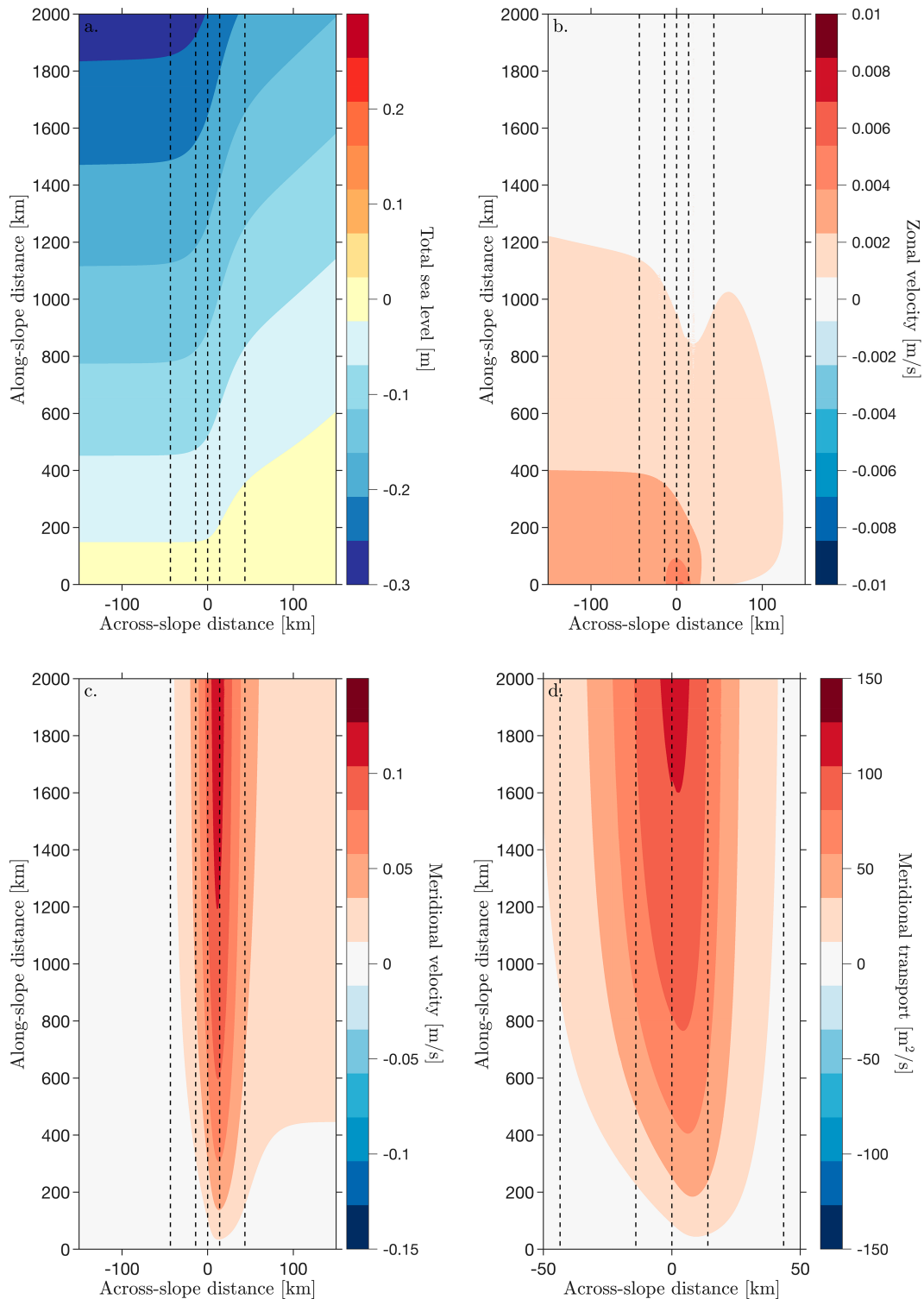


FIG. 6. Resulting total sea level and currents. (a) The total sea-level anomaly $\eta = \eta_s + \zeta$. (b),(c) The total depth-averaged zonal and meridional currents. Note currents in (b) include the ageostrophic circulation. (d) The meridional transport $(h\bar{v})$ in square meters per second, with a zoom-in on the slope region. Note the different color bar axis limits on all panels and the different across-slope distance range in (d). Dashed black lines indicate the 2200-, 1700-, 1200-, 700-, and 200-m isobaths.

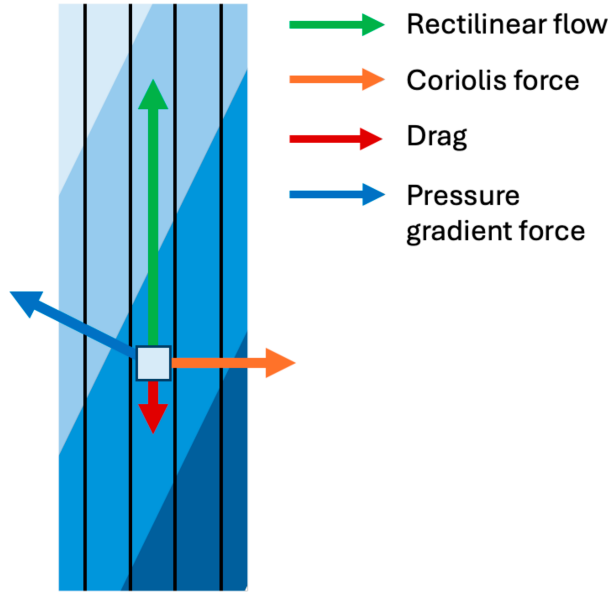


FIG. 7. Equilibrated slope current schematic. This representation presents the forces acting on a column of water (square marker) in a uniform rectilinear poleward motion ($\sim$ no acceleration) along the slope of a Northern Hemisphere eastern boundary. The coast is to the right of the schematic, and the open ocean is to the left. Both may be at some distance in which case the schematic represents only a portion of the eastern boundary margin where the equilibrated balance applies. The equator and north pole are, respectively, toward the bottom and top of the schematic. Isobaths are indicated by the vertical thick black lines. The pressure field is represented as shadings, with darker (lighter) colors associated with relatively higher (lower) pressure. Note all isobars are parallel to each other and perpendicular to the blue arrow representing the pressure-gradient force.

the sum of the forces acting upon the parcel to equate zero. The baroclinic pressure-gradient force (associated with χ') is not conserved along slope because of the poleward increase in seawater density. Hence, these changes must exactly be matched by changes in opposing sign in the barotropic pressure-gradient force (associated with ζ), yielding

$$x \text{ momentum: } \frac{\partial^2 \zeta}{\partial y \partial x} = -\frac{\partial^2 \eta_s}{\partial y \partial x}, \quad (38)$$

$$y \text{ momentum: } \frac{\partial^2 \zeta}{\partial y^2} = -\frac{1}{\rho_{r_b} g h} \frac{\partial^2 \chi'}{\partial y^2} (= 0). \quad (39)$$

In effect, changes in ζ are governed by friction as we have demonstrated throughout the present work. Nonetheless, the equilibrated slope current may be reached following an initial along-slope adjustment (Huthnance 1984) after which the requirements of Eqs. (38) and (39) are met.

The column is a conceptual simplification, used here for illustrative purposes. The different transport contributions are found at different depths—the emphasis being on the cross-slope Ekman transports, which are in close proximity to the seafloor. The cross-slope flow is hence depth varying,

and there is some overturning occurring in the across-slope-vertical plane. There are nonetheless great insights gained by studying the balance of the depth-integrated forces, and Eqs. (38) and (39) are simply obtained by enforcing no along-slope changes in along-slope velocity ($\partial_y \bar{p}_r \bar{v} = 0$) in Eq. (34) and thereafter in Eq. (35).

In the equilibrated regime, the velocity can be simply related to the density distribution. Huthnance's (1984) expression [Eq. (3)] is only valid for density profiles that do not vary across slope or with depth. Huthnance et al. (2020) derived a new formulation for more general density distributions:

$$\bar{v} = \frac{gh}{\rho_{r_b} r} \left(\frac{1}{h_0} \int_{-h_0}^0 \int_z^0 \frac{\partial \rho}{\partial y} dz' dz - \frac{1}{h} \int_{-h}^0 \int_z^0 \frac{\partial \rho}{\partial y} dz' dz \right). \quad (40)$$

Equation (40) is Eq. (10) of Huthnance et al. (2020) with winds and cross-slope changes in density neglected. The first integral is evaluated at the offshore ocean boundary, which has a depth of h_0 . We formally demonstrate in appendix B how Eqs. (3) and (40) can be obtained from our Eqs. (34) and (35).

In the real ocean, density changes along y are not linear, and equilibrated slope currents do not occur exactly. Nonetheless, Eq. (40) can be used as a first approximation for the flow structure at a given cross-slope section where frictional-JEBAR slope currents are known to exist. Huthnance et al. (2020) applied the equation, retaining the density variations in x , to the NEMO Atlantic Margin Model (AMM15) density field and replicated the AMM15 Rockall Slope Current at 56°N with great success. In our setup, $\partial_y \rho$ would be exactly constant throughout y but for thermobaricity. In practice, the effect of thermobaricity is small enough that we anticipate our modeled currents converge toward Huthnance's equilibrated solution.

In Fig. 8, the total meridional currents we obtained (Fig. 6c) are represented for $y = 500$ km, $y = 1000$ km, $y = 2000$ km, and $y = 3000$ km alongside Huthnance et al. (2020) diagnosis expression [Eq. (40), although effectively we use Eq. (B8)/ ρ_{r_b} : The two are equivalent]. The velocities obtained from solving the partial differential Eq. (36) and thereafter Eq. (34) were slightly smoothed to correct for errors associated with the numerical solving. As previously noted, along-slope growth in our along-slope velocity is initially large but gradually reduced with increasing y . We furthermore compare our currents at $y = 3000$ km with those obtained from the analytic expression of Huthnance et al. (2020) estimated at $y = 3000$ km. Discrepancies of order $\mathcal{O}(1) \text{ mm s}^{-1}$ or inferior exist, particularly in the open ocean. These small differences can be traced back to our treatment of the offshore boundary condition, which hinders that ζ isolines become exactly parallel to the x axis toward the offshore limit (for largely negative x). In addition, small along-slope growth inferior to $0.02 \text{ cm s}^{-1} (100 \text{ km})^{-1}$ in our modeled along-slope currents exists past $y = 3000$ km (not shown). We verified it is solely due to thermobaricity (not shown). To the very least, these different discrepancies are extremely minor and the agreement is excellent. We conclude that our currents have reached the equilibrated state.

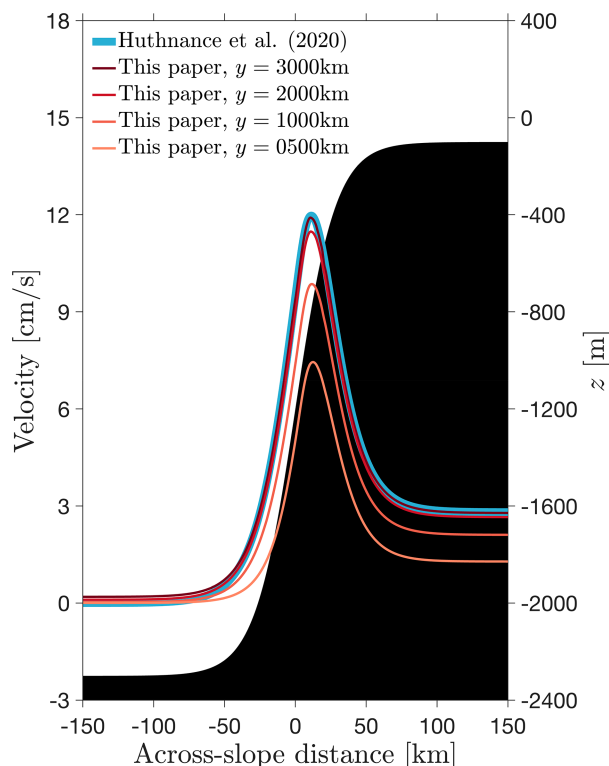


FIG. 8. Depth-averaged meridional velocities in the equilibrated regime. Huthnance et al. (2020) analytic expression (left y axis) [Eq. (40); thick light blue line] alongside our along-slope meridional currents (Fig. 6c) at several along-slope distances y . Positive velocities indicate poleward flow. The seafloor elevation, shown for reference (right y axis) (black patch).

The equilibrated slope current concept is rather different from the view that the European slope circulation may be fed from the west (Marsh et al. 2017; Clark et al. 2022). In the equilibrated regime, the baroclinic cross-slope inflow is fully recirculated toward the open ocean via the geostrophic barotropic currents arising from the changes in manometric sea level induced by bottom Ekman currents. Therefore, once they are established, equilibrated slope currents do not grow along slope to flush the upper-level baroclinic cross-slope transport but rather return exactly the incoming transport offshoreward. The constraint of no cross-slope transport was desired by Huthnance (1984), as it hinders the development of anomalously large gradient of sea level across the slope. It is not without cons, however. In the open ocean, where friction effects become negligible because meridional currents are negligible, the barotropic geostrophic transport must oppose the baroclinic geostrophic transport:

$$\lim_{x \rightarrow -\infty} \frac{\partial \zeta}{\partial y} = -\frac{1}{\rho_r g h} \frac{\partial \chi'}{\partial y}.$$

This is to meet the constraint of no cross-slope flow. Therefore, the equilibrated form can only occur if there is along-slope growth of positive ζ anomalies in the open ocean. The open-ocean growth of positive ζ anomalies is visible in Fig. 5a

for large y . This buildup weakens the zonal inflow to the slope region, which is a disputable requisite of the equilibrated regime.

4. Discussion

a. Generalities

In section 2, we derived a powerful formulation for the manometric sea-level distribution in nonhomogeneous oceans [Eq. (30)]. Expressed in terms of bottom pressure p_b , it is

$$\begin{aligned} J\left(\frac{h}{f}, p_b\right) + \nabla \cdot \left(\frac{r}{f^2} \nabla p_b\right) \\ = -J\left(\frac{1}{f}, \chi\right) + \nabla \cdot \left(\frac{g r \rho_b}{f^2} \nabla h\right), \end{aligned} \quad (41)$$

where the baroclinic potential energy is defined (Waldman and Giordani 2023) as follows:

$$\chi = g \int_{-h}^0 \rho z dz. \quad (42)$$

Equations (30) and (41) are equivalent, with the former arguably more appealing to scientists familiar with the ATW framework. Indeed, using the manometric sea level ζ allows for the comparison with the homogeneous ocean case of Eqs. (1) and (2). Besides this point, the velocity formulations with ζ and χ' for velocities are more meaningful than the expressions with p_b and χ . Indeed, there is considerable compensation between the geostrophic currents associated with ∇p_b and those associated with $h^{-1} \nabla \chi$. This can be shown from the depth-averaged pressure-gradient force in Eqs. (4) and (5), which is alternatively expressed as

$$-\frac{1}{h} \int_{-h}^{\eta} \nabla p dz = -\nabla p_b - \frac{1}{h} \nabla \chi. \quad (43)$$

Using the bottom pressure relation to the manometric sea level in gradient form [Eq. (13)] and making use of the Leibniz integral rule to re-express $h^{-1} \nabla \chi$, the decomposition with p_b and χ yields

$$-\nabla p_b = -g \rho_r \nabla \zeta - g \rho_b \nabla h, \quad (44)$$

$$-\frac{1}{h} \nabla \chi = -\frac{g}{h} \int_{-h}^0 z \nabla \rho dz + g \rho_b \nabla h. \quad (45)$$

A straightforward scale analysis for a simple slope region indicates the second terms on the right-hand side of these equations are approximately three orders of magnitude greater than the first terms on the right-hand sides. They hence dominate the bottom pressure gradient ∇p_b and the (h^{-1}) -scaled gradient of baroclinic potential energy $h^{-1} \nabla \chi$. However, $+g \rho_b \nabla h$ and $-g \rho_b \nabla h$ largely oppose one another, but for a difference term proportional to $\rho'_b = \rho_b - \rho_r$. The sum in Eq. (43) is hence a small difference in large quantities and does not appear as the most clever decomposition available. Subtracting $g \rho_r \nabla h$ from Eq. (44) and adding it to Eq. (45), the pressure-gradient force decomposition can be obtained

with ζ and χ' instead of p_b and χ . It further demands to use $\nabla\rho = \nabla\rho'$ in Eq. (45) and group all terms associated with ρ' and ρ'_b with the help of the Leibniz integral rule. This decomposition is the one we used throughout:

$$-\frac{1}{h} \int_{-h}^{\eta} \nabla p dz = -g\rho_s \nabla \zeta - \frac{1}{h} \nabla \chi'. \quad (46)$$

The advantage of this formulation is that the two pressure terms do not awkwardly compensate for one another. The geostrophic currents related to $\nabla\zeta$ and $h^{-1}\nabla\chi'$ are therefore more meaningful than those expressed in terms of ∇p_b and $h^{-1}\nabla\chi$. In the end, the two formulations are similar and the choice is a matter of personal preference. We further note that Rattray (1982) and Mellor et al. (1982) used a perturbation bottom pressure $\nabla p'_b = \nabla p_b - g\rho_{r_s} \nabla h$. This is akin to our method, as $\nabla p'_b = g\rho_{r_s} \nabla \zeta$, and similarly allows to circumnavigate the issue of compensation between ∇p_b and $h^{-1}\nabla\chi$.

Expressions of the kind of Eqs. (30) and Eq. (41) [see Eq. (20) of Hughes 2008 and Eq. (16) of Waldman and Giordani 2023], termed “transport divergence vorticity balance” by Waldman and Giordani (2023), are relatively rare among modeling studies despite being rather intuitive. The two Jacobians in Eq. (30) [or, equally, in Eq. (41)] indicate the convergence–divergence of the geostrophic transport (Waldman and Giordani 2023). In the complete absence of friction ($r = 0$), convergence of barotropic geostrophic transport [associated with the Jacobian on the lhs of Eq. (30)] must be balanced by divergence of baroclinic geostrophic transport (associated with the Jacobian on the rhs). When considering domains of relatively small extent, the latter tends toward zero because the variations in f are small. This indicates that, on an f plane, the baroclinic geostrophic transport does not cross isobaths and cannot lead to mass convergence or divergence. In this case, the baroclinic Jacobian cancels in Eq. (30), which is an advantage of the transport divergence vorticity balance over other vorticity expressions (Waldman and Giordani 2023). Hence, unless domains of substantial horizontal length scale are considered, convergence–divergence of the barotropic geostrophic transport is mostly caused by bottom friction, represented as the r terms in Eq. (30). In section 3, we made full use of this fact by assuming an f -plane ocean. This approximation, besides being suited for the Rockall Slope Current (Huthnance et al. 2020), simplifies Eq. (30) considerably.

Alternatively, it is possible to construct a vorticity equation “from the top” rather than “from the bottom,” leading to an expression in terms of the total sea level η . After bottom geostrophic velocities are expressed with η ,

$$\mathbf{v}_b^g = \frac{g}{f\rho_{r_b}} \mathbf{k} \times \left(\rho_{r_s} \nabla \eta + \int_{-h}^{\eta} \nabla \rho' dz \right), \quad (47)$$

the bottom stress is obtained and Eqs. (7) and (8) lead to, with the use of mass conservation,

$$\begin{aligned} J\left(\frac{gh}{f}, \eta\right) + \nabla \cdot \left(\frac{gr}{f^2} \nabla \eta \right) \\ = -\nabla \times \left(\frac{g}{f} \int_{-h}^{\eta} \nabla \rho' dz' \right) - \nabla \cdot \left(\frac{gr}{f^2} \int_{-h}^{\eta} \frac{\nabla \rho'}{\rho_{r_s}} dz \right). \end{aligned} \quad (48)$$

Equation (48) is exactly Eq. (9) of Pringle (2018), with wind effects neglected. The framework of Huthnance et al. (2020) is formulated in a similar form from the top perspective, although the authors do not provide a vorticity conservation law. Oftentimes, the left-hand side of Eq. (48) is described as the barotropic component (this is not the terminology we adopted in the present contribution), but it should be noted the steric sea level η_s is “hidden” within η . Reformulating Eq. (48) with $\eta = \zeta + \eta_s$ and moving η_s terms to the rhs lead to our vorticity conservation [Eq. (30)]. Multiplying Eq. (17) by h/f and taking the curl shows that

$$J\left(\frac{gh}{f}, \eta_s\right) + \nabla \times \left(\frac{g}{f} \int_{-h}^{\eta} \nabla \rho dz' \right)$$

is simply

$$J\left(\frac{1}{f}, \frac{\chi'}{\rho_{r_s}}\right)$$

and plays a little role unless the meridional variations of the Coriolis parameter dominate the problem. Similarly,

$$\nabla \cdot \left(\frac{gr}{f^2} \nabla \eta_s \right) + \nabla \cdot \left(\frac{gr}{f^2} \int_{-h}^{\eta} \frac{\nabla \rho}{\rho_{r_s}} dz \right)$$

is simply

$$-\nabla \cdot \left(\frac{gr}{f^2} \frac{\rho'_b}{\rho_{r_s}} \nabla h \right),$$

which is exactly zero where the seabed is flat and generally small if ρ'_b is small (e.g., in the open ocean where changes in bottom density are subtle). Therefore, and quite similarly to Eq. (41), there is a large cancellation in between terms of Eq. (48), which is fully due to the choice of pressure-gradient force variables rather than to the physics of the problem. Equation (48) is also a transport divergence vorticity balance, but because of this choice, it is a bit harder to interpret than Eq. (30).

Besides the use of a transport divergence vorticity balance formulation, the strength of our derivations stems in part from our use of the ATW bottom drag parameterization ($\tau_b = \rho_{r_b} r \mathbf{v}_b^g$). Such form of bed stress has over the years enabled considerable advances in the research of sea-level distribution and circulation over shelves and slopes of homogeneous oceans (Csanady 1978, 1982; Wang 1982; Wise et al. 2018; Pringle 2018; Wu 2021; Lin et al. 2022). Our derivations build upon the state-of-the-art ATW theories (Wise et al. 2018; Wu 2021; Lin et al. 2022) but in addition allow for the diagnosis of shelf and coastal sea level in an inhomogeneous ocean (on an f plane, only bottom density needs to be known to determine the bottom pressure head anomaly ζ). Further, they allow the investigation of the spatial evolution of slope currents of Huthnance (1984) kind. That is, JEBAR slope currents slowed down by bottom friction.

Finally, our framework makes use of the anelastic approximation, also known as the weak Boussinesq approximation. It is superior to the strong Boussinesq approximation within

which ρ_r is constant but is also more complex to manipulate. We opted for the anelastic framework because it allows for an appropriate definition of geostrophic currents, something which is difficult to achieve in the strong Boussinesq case when bathymetry is sloping. Indeed, imagine an ocean with density horizontally homogeneous but varying vertically. Such ocean should not, in principle, have any baroclinic geostrophic currents. Under the strong Boussinesq approximation ($\rho_r = \text{constant}$), both

$$\chi' = g \int_{-h}^0 z(\rho - \rho_r) dz \quad \text{and} \\ \eta_s = - \int_{-h}^0 \frac{\rho - \rho_r}{\rho_r} dz$$

will vary greatly where h is changing. This translates into fictitious and incorrect geostrophic baroclinic currents. The issue is particularly problematic at open-ocean boundaries. For example, if ρ_r is kept constant in our setup for a simple eastern boundary margin (section 3), the steric sea level η_s cannot be zero all along the domain southern boundary if $\rho(x, 0, z)$ varies with x , z , or both. With the steric sea level being non-zero at this upstream boundary, the analysis of section 3 is largely complexified. Furthermore, if η_s is nonconstant across the slope of the upstream boundary—a likely situation with ρ_r fixed—along-slope baroclinic currents are not equal to zero either. In that scenario, one may ask whether along-slope currents upstream of the domain are responsible for bottom Ekman transport and should result in a modified distribution of ζ at the domain boundary. Clearly, the strong Boussinesq approximation hampers the demonstration of section 3 because it incorrectly informs on the distribution of η_s , χ' , and baroclinic currents.

b. Simplified eastern oceanic margin

In line with previous studies (e.g., Huthnance 1984, 1986; Pingree and Le Cann 1989; Pingree and Le Cann 1990; Simpson and Sharples 2012; Marsh et al. 2017; Huthnance et al. 2020), the results presented in section 3 highlight that a large-scale poleward decline in dynamic height in the open ocean leads to accelerated poleward currents at the eastern boundary subpolar slope. These currents arise through the JEBAR, which—in a transport divergence vorticity formulation—reflects that baroclinic geostrophic transport is nondivergent on an f plane (Waldman and Giordani 2023). That baroclinic geostrophic transport does not create mass convergence implies that the zonal baroclinic geostrophic transport in the open ocean is balanced by meridional baroclinic geostrophic transport over the slope (see Marsh et al. 2017; Simpson and Sharples 2012, 311–313).

The strong poleward baroclinic currents generate bottom Ekman currents because of friction with the seafloor. This ageostrophic transport is everywhere directed offshoreward, but its magnitude is larger immediately offshore of the shelf edge than immediately shoreward of the shelf edge, leaving a “void” centered roughly at the shelf edge, which can only be managed by a rearrangement of the manometric sea level ζ

(in this case, a depletion of the shelf manometric sea level). Similarly, positive ζ anomalies are created where the active layer meets the slope, again because of the convergence of the Ekman currents of baroclinic origin. The poleward JEBAR slope current is reduced by the opposing barotropic geostrophic flow arising from the distribution of ζ , which respects the general rules of the ATW theory [i.e., analogous to an advection–diffusion propagation; see Wise et al. (2018); Wu (2021); Lin et al. (2022), for recent findings]. After some along-slope distance, the sum of the two contributions eventually reaches a “balanced” state without along-slope development. This regime is the equilibrated form discussed by Huthnance et al. (2020) (but see also Huthnance 1984, 1986; Pingree and Le Cann 1990; Souza et al. 2001). Finally, creation at the slope foot of positive ζ anomalies through the convergence of ageostrophic transport is in theory possible. However, this is not seen in our framework because ρ'_b was set to tend toward zero at depth, yielding a close-to-motionless abyssal flow in the open ocean and lower slope. We believe this is a realistic setup.

The physical mechanisms detailed here are sufficient to explain qualitatively the presence of the Rockall Slope Current over the slope of the eastern Rockall Trough (Huthnance 1986; Souza et al. 2001; Hill and Mitchelson-Jacob 1993; Fraser et al. 2022). The slope region between 48° and 50°N (from the Whitard Canyon to the Goban Spur) can be thought to represent very roughly where $y = 0$ in our model. Indeed, cross-slope gradients of sea level are small there (Fig. 1b), yielding close to zero along-slope geostrophic currents. In line with this proposition, we disagree with the idea that the slope circulation does not grow with along-slope distance, most especially immediately downstream of this initial region (say, between 50° and 54°N). Farther downstream, the Rockall Slope Current flows at a speed of approximately 10–20 cm s^{−1} in the northeast Rockall Trough (Fraser et al. 2022; Souza et al. 2001), with less evidence of along-slope growth. This suggests an equilibrated regime is achieved. The northeast Rockall Trough is located 1000–1500 km downstream of the 48°–50°N region. In this y -distance range, the equilibrated form is not yet exactly reached in our framework. Similarly, the maximum along-slope total velocity we obtained for $y \leq 2000$ km is 11.5 cm s^{−1}, which lies on the lower end of maximum along-slope velocities observed on the Rockall Slope. To start, these results are strongly dependent on the (somewhat subjective) choices of friction parameter r and bathymetry profile definition. Therefore, results closer to the truth can be tailored by modifying the problem parameters and geometries. This is, however, not the aim of the present contribution. We further note that, beside the mechanisms described here, there are numerous additional drivers to the velocity on the Rockall Slope. While they may be of second order, they play a role in increasing the total currents. They include not only cross-slope variations in density (Huthnance et al. 2020; Fraser et al. 2022) but also wind forcing (Huthnance et al. 2020; Pingree and Le Cann 1989) and tidal rectification (Stashchuk et al. 2017).

The assumption made in section 3 that along-slope geostrophic currents are zero at the upstream boundary is embedded in the different slope current publications of Huthnance (Huthnance 1984, 1986; Huthnance et al. 2020). For example,

Huthnance (1986) most clearly states that they assume the sea surface to be level across the continental slope at the equator, which serves as the upstream limit. In contrast, in the previous paragraph, we considered the 48°–50°N region as the upstream reference. This question demands further work. Clearly, if both the along-slope geostrophic transport at the upstream boundary is neglected and the density is set to increase poleward from a constant value ρ_r [or a reference profile $\rho_r(z)$] representative of the upstream region, the JEBAR slope currents are artificially constrained to flow poleward. This simplification was done by us and in the different works of Huthnance. However, in principle, the JEBAR slope currents may initially be equatorward. This depends on the distribution of η_s (or rather, χ') at the upstream boundary. This matter is tied to thorny questions, beyond the scope of the present work. Where does the Rockall Slope Current originate? Is it a part of a continuous European Slope Current or is it a branch of the North Atlantic Current inflow which becomes trapped at the slope?

While the transmission of ζ along and across the slope from the shelf edge controls the barotropic geostrophic slope current opposing the JEBAR flow, ζ is also transmitted on the shelf toward the coastline. To keep the present contribution streamlined, we have preferred to model in section 3 a flat shelf bounded by a vertical wall at the coastline. A sloping shelf is more realistic and will modify the shelf and coastal sea-level distribution. There are two processes of relevance which are at play. First, because $\partial_x h$ is non-zero over the shelf in this scenario, the negative ζ anomalies emerging from the shelf edge penetrate the shelf at a smaller rate than in the case presented in section 3 and reach the coastline at greater along-slope distances from $y = 0$. However, and second, negative ζ anomalies emerge from the coastline, which becomes a greater source due to the presence of the second rhs term in Eq. (37). This growth of coastal origin is analogous to the one discussed in the study of arrested topographic wave generation by the wind (Csanady 1978; Lin et al. 2022). Although it depends on the exact shelf slope and the friction parameter r , one outcome is that ζ isolines may be bowed, with, for a given y distance larger negative ζ anomalies at the shelf edge and coastline than above the midshelf. In section 2 of the supplemental material, we have included reproductions of Figs. 2–6 for a bathymetry including a sloping shelf. Altogether, the conclusion that the open-ocean steric sea-level signal is transmitted coastward as manometric sea level holds in the sloping shelf case. This is in agreement with Huthnance (2004, 1987), who showed that the transmission across the slope and shelf of the surface elevation η is possible provided bottom friction is large enough and the oceanic level is of large enough along-slope scale.

Finally, we briefly considered the more realistic case where the linear bottom drag parameterization is swapped for a quadratic definition. Derivations are provided in the supplemental material (section 3), but note they were not investigated numerically. We do not expect that this change would bring major improvements but leave it for others to explore.

c. Implications for more realistic oceans

In the real ocean, zonal baroclinic currents flowing toward the slope are not maintained throughout oceanic basins.

Section 3's assumption of a steady poleward decrease in dynamic height is maintained over 2000 km or more is too simplistic. In the eastern North Atlantic, it would be more correct to consider that along-slope density changes are large where the branches of the North Atlantic Current approach the slope and smaller elsewhere. These approach areas are in the southern Rockall Trough (52°N), west of the Hebrides (58°N), and in the Wyville–Thomson ridge (60°N). We anticipate the attachment points onto the slope of these branches may determine the regions of larger growth in along-slope current. Of course, these considerations are not solely important to understand the flow above continental slopes. They also open the door to investigate the role played by open-ocean baroclinic currents in controlling shelf and coastal sea level. These currents are often associated with density gradients maintained over distances of order of only $\mathcal{O}(10^1\text{--}10^2)$ km. Considering the nature of such baroclinic flow beyond the simple steady rate of density increase examined here may help explain why coastal sea-level changes along many coastlines of the world covary with remote open-ocean temperature anomalies in open-ocean currents [see, among others, Dangendorf et al. (2021); Diabaté et al. (2021); Sasaki et al. (2014)].

Our effort focused on bottom friction as one specific ageostrophic mechanism allowing for transport divergence to occur in the ocean and drive manometric sea-level changes. Of course, additional ageostrophic mechanisms are potential sources of the divergence that is necessary to equilibrate coastal manometric sea level with open-ocean steric sea level. They include temporal development, nonlinear momentum advection, and horizontal dissipation [Eq. (16) of Waldman and Giordani (2023)]. The role of these ageostrophic mechanisms in controlling coastal sea-level changes remains to be explored, especially in the baroclinic ocean case. Equation (16) of Waldman and Giordani (2023) and Eq. (20) of Hughes (2008) provide some hindsights on how these ageostrophic terms appear in a transport divergence vorticity balance formulation. In the temporal mean state, there are clear indications that the horizontally integrated barotropic vorticity balance in the subpolar North Atlantic is dominated by the equilibrium in between the bottom drag curl and the bottom pressure torque [Fig. 9 of Le Corre et al. (2020)]. This is most especially the case along the basin margins and justifies our choice to focus on the associated forces in the present contribution. However, the nonlinear momentum advection has locally a greater control of the bottom pressure distribution than the bottom drag [Fig. 7 of Le Corre et al. (2020)]. We can therefore expect that the manometric sea level is locally changed by nonlinear dynamics, a contribution which is possibly important for the coastal sea level. Regarding fast motions, temporal development plays a role through the oscillatory displacement of isopycnals and subsequent behavior of baroclinic currents (Gill and Clarke 1974) and the adaptive response of shelf waves (Huthnance 1987, 2004).

5. Conclusions

We extended the arrested topographic wave theory to non-homogeneous oceans [Eq. (30)]. This modified framework is extremely malleable to different contexts of bathymetry and seawater density distribution, and the scope of applications is

very large. Here, we investigated the spatial evolution of sea level and currents at an eastern subpolar margin under large-scale density forcing. In the northeast Atlantic, this is the context of the Rockall Slope Current. Our results highlight that bottom friction is responsible for both the deceleration of baroclinic slope currents and the mediation of gradient of sea-level anomaly, initially steric and of open-ocean origin, to the shelf and the coast (as manometric changes). This new arrested topographic wave framework yields interesting perspectives. For example, it could be used to investigate the temporal changes in sea level along coastlines bordering subtropical western boundary currents. Indeed, in some of these regions, there are numerous evidences of a link between remote open-ocean temperatures and coastal sea-level changes (see Dangendorf et al. 2021; Diabaté et al. 2021; Sasaki et al. 2014; Ezer 2019). In principle, inverse methods would allow for the reconstruction of past behavior of the western boundary currents (position, transport) from tide gauge measurements. Furthermore, our developments are very suited to investigate coastal sea-level changes in the coming centuries in the North Atlantic Ocean, where large changes in density distribution are expected due to a plausible shutdown of the Atlantic meridional overturning circulation. Of course, our framework can only be used where the limit of small temporal development ($\partial_t \sim 0$) remains valid, as demonstrated by Wright (1986) for the barotropic case.

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Data availability statement. No datasets were generated during the current study. The only analyzed data are the MDT map shown, which was obtained from the Copernicus Marine Service Information (CMEMS), <https://doi.org/10.48670/moi-00150> (accessed on 15 April 2024). The PDE solving was done using the MATLAB Partial Differential Equation Toolbox, and scripts are provided alongside this paper (see the supplemental material). The provided scripts form a “working example” are extremely portable, run in less than 5 mn on a modern computer, and require as few toolboxes as possible.

As provided, they effectively give poor solutions because of the extremely coarse grid resolution, which ensures the short running time and portability of the produced MATLAB .mat files. Users are left free to uncomment the code excerpts generating fine grid and mesh and allowing for solutions of the quality shown in the paper to be obtained. Computing time is greatly affected when seeking high-resolution solutions. The required toolboxes are the MATLAB Partial Differential Equation Toolbox and Symbolic Math Toolbox, as well as the TEOS-10 Gibbs-Seawater (GSW) Oceanographic Toolbox (<https://www.teos-10.org/software.htm>).

APPENDIX A

Bottom Pressure and Manometric Sea Level

We here provide some insights on the link between bottom pressure and manometric sea level. Bottom pressure is

$$p_b = g \int_{-h}^{\eta} \rho dz. \quad (\text{A1})$$

This depth integral can be split at $z = 0$:

$$p_b = g \int_{-h}^0 \rho dz + g \int_0^{\eta} \rho dz, \quad (\text{A2})$$

and the anelastic formulation for density ($\rho = \rho_r + \rho'$) may be introduced in the first of these integrals, yielding

$$p_b = g \int_{-h}^0 \rho_r dz + g \int_{-h}^0 \rho' dz + g \int_0^{\eta} \rho dz. \quad (\text{A3})$$

The second term in Eq. (A3) is easily identified as $-g\rho_r\eta_s$ with the help of Eq. (10). The last term in Eq. (A3) simplifies to $g\rho_r\eta$ with the approximation that $\rho(x, y, z \geq 0) \approx \rho_r$. Hence, bottom pressure is simply

$$\begin{aligned} p_b &= g \int_{-h}^0 \rho_r dz + g\rho_r(\eta - \eta_s), \\ &= g \int_{-h}^0 \rho_r dz + g\rho_r\zeta. \end{aligned} \quad (\text{A4})$$

Therefore, there is a straightforward relationship between bottom pressure and manometric sea level. Manometric sea level is the “head” of bottom pressure, expressed in fluid height. Furthermore, because bottom pressure is primarily a function of the seafloor depth [through the integral term in Eq. (A4)], a variable of greater interest for ocean dynamics is the “perturbation” bottom pressure (Rattray 1982). Perturbation bottom pressure is directly proportional to the manometric sea level ζ , and the two are equivalent: $p'_b = g\rho_r\zeta$.

APPENDIX B

Derivations of Diagnostic Expressions for Huthnance's Equilibrated Slope Current

Huthnance (1984) and Huthnance et al. (2020) considered a slope current having reached an equilibrium without along-slope

flow development ($\partial_y \bar{v} = 0$) on a similar eastern boundary bathymetry invariant alongshore. We walk through their derivations but using our expressions for velocities [Eqs. (35) and (34)]. Since neither h nor $\bar{v}$ varies with y , anelastic mass conservation is simply $\partial_x(h\bar{\rho}_r\bar{u}) = \partial_y(h\bar{\rho}_r\bar{v}) = 0$, with the immediate consequence that the cross-slope transport is nil everywhere given that it is zero at the coast:

$$h\bar{\rho}_r\bar{u} = 0. \quad (\text{B1})$$

From Eqs. (35) and (B1),

$$h\left(\bar{\rho}_r\frac{\partial\zeta}{\partial y} + \frac{1}{gh}\frac{\partial\chi'}{\partial y}\right) = -\frac{r\rho_{r_s}}{f_0}\left(\frac{\partial\zeta}{\partial x} - \frac{\rho'_b}{\rho_{r_s}}\frac{\partial h}{\partial x}\right). \quad (\text{B2})$$

Otherwise said, the zonal geostrophic transport (lhs) must exactly oppose the zonal Ekman transport generated by meridional geostrophic currents (rhs) at any given x . An expression for $\partial_x\zeta$ can be derived from Eq. (B2), and after substituting in Eq. (34), we are left with

$$\bar{\rho}_r\bar{v} = -\frac{g\rho_{r_s}h}{r}\left(\frac{\partial\zeta}{\partial y} + \frac{1}{\rho_{r_s}gh}\frac{\partial\chi'}{\partial y}\right). \quad (\text{B3})$$

This is Csanady's (1978) asymptotic solution [their Eq. (33)], corrected for the effect of baroclinicity in the domain [second term within the brackets of Eq. (B3)] as is appropriate when the whole boundary, including the slope region, is considered [Csanady (1978) only focused on the shelf].

Again invoking that a flow solution steady along slope is sought ($\partial_y \bar{v} = 0$), a simple expression for the manometric sea-level anomaly is obtained from Eq. (34):

$$\frac{\partial^2\zeta}{\partial y\partial x} = -\frac{\partial^2\eta_s}{\partial y\partial x}, \quad (\text{B4})$$

meaning the along-slope evolution of the cross-slope gradient of manometric sea level exactly opposes the along-slope evolution of the cross-slope gradient of steric sea level, so that the cross-slope gradient of total sea level $\partial_x\eta$ does not evolve along slope. Integrating this equation from a distant offshore location x_d yields

$$\frac{\partial\zeta}{\partial y} - \frac{\partial\zeta}{\partial y}\bigg|_{x_d} = -\left(\frac{\partial\eta_s}{\partial y} - \frac{\partial\eta_s}{\partial y}\bigg|_{x_d}\right). \quad (\text{B5})$$

An expression can be obtained for the currents using Eqs. (B3) and (B5):

$$\bar{\rho}_r\bar{v} = \frac{g\rho_{r_s}h}{r}\left(\frac{\partial\eta_s}{\partial y} - \frac{\partial\eta_s}{\partial y}\bigg|_{x_d} - \frac{\partial\zeta}{\partial y}\bigg|_{x_d} - \frac{1}{\rho_{r_s}gh}\frac{\partial\chi'}{\partial y}\right). \quad (\text{B6})$$

Equation (B6) enables diagnostic computation of the currents from the density field only, at the condition that $\partial_y\zeta$ is known at x_d . Huthnance et al. (2020) assume bottom friction vanishes well away in the open ocean ($x_d = -\infty$), letting the cross-slope barotropic geostrophic transport exactly oppose the cross-slope baroclinic geostrophic transport at

this offshore limit. Indeed, with this assumption, the rhs of Eq. (B2) is zero and $\partial_y\zeta$ at x_d is obtained:

$$\frac{\partial\zeta}{\partial y}\bigg|_{-\infty} = -\frac{1}{\rho_{r_s}gh_0}\frac{\partial\chi'}{\partial y}\bigg|_{-\infty}. \quad (\text{B7})$$

From there,

$$\frac{\bar{\rho}_r\bar{v}}{r} = \frac{g\rho_{r_s}h}{r}\left[\left(\frac{1}{\rho_{r_s}gh_0}\frac{\partial\chi'}{\partial y}\bigg|_{-\infty} - \frac{\partial\eta_s}{\partial y}\bigg|_{-\infty}\right) - \left(\frac{1}{\rho_{r_s}gh}\frac{\partial\chi'}{\partial y} - \frac{\partial\eta_s}{\partial y}\right)\right], \quad (\text{B8})$$

where h_0 is the depth of the abyssal ocean. From Eqs. (10) and (17) relating χ' and η_s , the anelastic version of Eq. (10) of Huthnance et al. (2020) is obtained (winds and cross-slope density changes are neglected):

$$\bar{\rho}_r\bar{v} = \frac{gh}{r}\left(\frac{1}{h_0}\int_{-h_0}^0\int_z^0\frac{\partial\rho'}{\partial y}\bigg|_{-\infty}dz'dz - \frac{1}{h}\int_{-h}^0\int_z^0\frac{\partial\rho'}{\partial y}dz'dz\right). \quad (\text{B9})$$

Further using Eq. (32), we obtain Eq. (40). In the simplified scenario where ρ is only a function of y , Eq. (40) [or Eq. (B8)] simplifies to Eq. (3), the original formulation of Huthnance (1984).

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