OPTIMAL ALGORITHMS FOR BLIND SOURCE SEPARATION

--- APPLICATION TO ACOUSTIC ECHO CANCELLATION

A Dissertation Submitted
For the Degree of
M.Eng.Sci

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Abstract

We are all familiar with the sound which can be viewed as a wave motion in air or other elastic media. In this case, sound is a stimulus. Sound can also be viewed as an excitation of the hearing mechanism that results in the perception of sound. The interaction between the physical properties of sound, and our perception of them, poses delicate and complex issues. It is this complexity in audio and acoustics that creates such interesting problems.

Acoustic echo is inevitable whenever a speaker is placed near to a microphone in a general full-duplex communication application. The most common communication scenario is the hands-free mobile communication kits for a car. For example, the voice from the loudspeaker is unavoidably picked up by the microphone and transmitted back to the remote speaker. This makes the remote speaker hear his/her own voice distorted and delayed by the communication channel or called end to end delay, which is known as echo. Obviously, the longer the channel delay, the more annoying the echo resulting a decrease in the perceived quality of the communication service such as VoIP conference call.

In the thesis, we propose to use different approaches to perform acoustic echo cancellation. In addition, we exploit the idea of blind source separation (BSS) which can estimate source signals using only information about their mixtures observed in each input signal. In addition, we provide a wide theoretical analysis of models and algorithmic aspects of the widely used adaptive algorithm Least Mean Square (LMS). We compare these with Non-negative Matrix Factorization (NMF), and their various extensions and modifications, especially for the purpose of performing AEC by employing techniques developed for monaural sound source separation.

Acknowledgement

This thesis would not be what it is without the help, encouragement and friendship of many people, so now is the time to give credit where credit is due.

I would like to thank my principal supervisor, Dr. Bob Lawlor, for all his patience, guidance, support and encouragement, especially for all the times when I'm sure it seemed like this research was going nowhere. He was always there to meet and talk about my ideas, and to ask me good questions to help me think through my project. Without his encouragement and constant guidance, I could not have finished my project. He helped me all the time through the project and also provided me with constructive feedback on my project report.

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Acronyms

Symbols	**Description**
ADRess	Azimuth Discrimination and Resynthesis
AEC	Acoustic Echo Cancellation
BD	Beta Divergence
BSS	Blind Source Separation
CASA	Computational auditory scene analysis
CNMF	Convolutive Non-negative Matrix Factorization
DCT	Discrete Cosine Transform
DFT	Discrete Fourier Transform
DUET	Degenerate Unmixing Estimation Technique
EEG	Electroencephalography
ERLE	Echo Reduction Loss Enhancement
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FLMS	Fast Least Mean Square
ICA	Independent Component Analysis
IID	Interaural Intensity Difference
IFFT	Inverse Fast Fourier Transform
ISD	Itakura-Saito Divergence
LEM	Loudspeaker-enclosure-microphone coupling
LMS	Least Mean Squares
LNMF	Local Non-negative Matrix Factorization
KLD	Kullback-Leibler Divergence
ML	Maximum Likelihood
MSSS	Monaural Sound Source Separation
NFLMS	Normalized Fast Least Mean Square
NLMS	Normalized Least Mean Square

NMF	Non-negative Matrix Factorization
PCA	Principal Component Analysis
RIR	Relative Incremental Reactivity
SAR	Signal to Artifacts Ratio
SED	Squared Euclidean Distance
SDR	Signal to Distortion Ratio
SIR	Signal to Interference Ratio
SNMF	Sparse Non-negative Matrix Factorization
STFT	Short Time Fourier Transform
TI	Texas Instruments
VoIP	Voice over Internet Protocol

1. Introduction

This thesis will address some of the aims of signal processing and machine learning techniques, including extracting an interesting knowledge from experimental raw datasets. In particular, we focus on the techniques related to blind source separation (BSS) to solve one of its applications: Acoustic echo cancellation (AEC). The purpose of this project focuses on finding a high quality and efficient technique to perform AEC. Furthermore, to address the issue of sound dataset structure, we explore a recent iterative technique called Non negative Matrix Factorization (NMF) **[Daniel 01]**, also we place particular emphasis on the initialization of current NMF algorithms for efficiently computing NMF.

An aforementioned research area is blind source separation method. The sources separation problems arise when a number of sources emit signals that mix and propagate to one or more sensors. The objective is to identify the underlying source signals based on measurements of the mixed sources. We have studied the feasibility of various source separation techniques such as Independent Component Analysis (ICA), Principal Component Analysis (PCA), and Degenerate Unmixing Estimation Technique (DUET). In this thesis, we use both different types of LMS algorithms and Non-negative Matrix Factorization (NMF) model to derive and implement in MATLAB, using efficient and relatively simple iterative algorithms that work well in practice for real-world data. Finally, we present an echo effect and echo control experiment on real-time DSP board Texas Instruments Develop Start Kits (TMS320C6713 DSK) in order to demonstrate a simple AEC solution.

1.1 Research problem description

This project aims to use different conventional mathematical techniques to perform Acoustic Echo Cancellation. We will review the adaptive algorithms which are discussed in later chapters and introduce a new optimal computational algorithm called NMF to find the best suitable solution for AEC problem.

As the theory and applications of NMF is still being developed. In this project we choose NMF algorithm to perform AEC using various divergence as a general cost function of NMF, and find the optimal method that can give the best performance of AEC problem.

In addition, the workhorse in this project related NMF include initialization problem and morphological constraints. These constrains include nonnegativity, sparsity, orthogonality and smoothness. This research we also implement and optimize algorithm for NMF and provide psedu-source code and efficient source code in MATLAB.

1.2 Thesis organization and overview

The focus of this thesis is the Acoustic Echo Cancellation using widely used adaptive algorithm LMS and sound separation technique – NMF. Special emphasis is provided coverage of the models and algorithms for nonnegative matrix factorizations both from a theoretical and practical point of view. The main objective is to derive and implement in MATLAB simulation. Actually, almost all of the experiments presented in this thesis have been implemented in MATLAB and extensively tested. The layout of the thesis is as follows.

In chapter two we provide the necessary background information and theory in sound source separation and includes the different BSS generative mixing model. . In addition, we also discuss the general principle of acoustic echo cancellation. It is main application we have it involved in this project. And, we introduce the optimum solution for the conversional acoustic echo canceller limitation at the end of this chapter.

In chapter three we discuss the blind source separation (BSS) and related methods which present various optimization techniques and statistical methods to derive efficient and robust learning or update rules. We present the conventional optimize algorithms (i.e. ICA, PCA, DUET ADRess). This section discussed using different mathematical techniques to perform sound source separation.

In chapter four we introduce the learning algorithms for Nonnegative Matrix Factorization (NMF) and its properties of a large family of generalized and flexible divergences between two nonnegative sequences or matrices. This chapter puts particular emphasis on discussing NMF numerical approaches and various useful cost functions and regulations of NMF, including those based on generalized Kullback-Leibler, Pearson and Neyman Chi-squared divergences etc. Many of these measures belong to the class of Alpha-divergences and Beta-divergences. In addition, we give novel experiments on acoustic echo cancellation using extended NMF algorithms.

In chapter five, two MATLAB simulation experiments present the requirements for implementing the algorithms discussed in chapter three and four, and the measurements that used to examine the output speech quality. We focus on Non-negative Matrix Factorization algorithm implementation. Also the main contribution of this work is the development a version of the NMF algorithm that combined the BSS principle, represented the best route for tacking the AEC problem.

In chapter six, we extended the AEC problem on real-time implementation, and demonstrated a simple straightforward echo control experiment based on TI C6713 DSP start kits.

Chapter seven then contains conclusion on the work done and also highlights areas for the future research in the area of NMF algorithm for blind source separation.

2. Acoustic Blind Source Separation background and theory

What is the blind source separation? The technique for estimation of individual source components from their mixtures at multiple sensors is known as blind source separation (BSS). In a real room environment, one well known BSS application is the separation of audio sources which have been mixed and then captured by multiple sensors or microphones. These sources could be different output signals from speakers in the same room. Therefore, each sensor acquires a slightly different mixture of the original source signals. One of the examples is solving the cocktail party problem **[Bronkhorst 00]**; we will discuss it in chapter two. The term "blind" stresses the fact that the original source signals and the generic mixing system are assumed to be unknown. Additionally, the estimation is performed blindly, in other words, if the sources are to be separated blindly, they should have some distinct characteristics, such as nonstationarity, non- Gaussianity. One optimal learning algorithm: Independent component analysis (ICA) can calculate the separation matrix, which is sometimes regarded as synonymous with BSS, relies on non-Gaussianity **[Lee 98]**[**Haykin 00**][**Hyvärinen 01**].

Furthermore, the fundamental assumption necessary for applying blind source separation methods is that the original source signals are mutually statistically independent. The fundamental problem of BSS refers to finding a demixing system whose outputs are statistically independent. We will explain in detail the different mixture and separation models for which most early BSS algorithms were designed in this chapter.

2.1 BSS Generative Model

One of the difficulties of the blind source separation task more particularly rely on the way in which the signals are mixed within the physical environment. The simplest mixing scenario deals with an instantaneous mixing model, for where no delayed versions of the sources signals appear. This is the ideal case for which most early BSS algorithms were designed, but such algorithms have limited practical applicability in real time speech separation problems. In real world acoustical paths lead to convolutive mixing of the sources when measured at acoustic sensors. It is an extension of the instantaneous mixing model by considering also delayed versions of the source signals leading to a mixing system. The system generally can be modelled by finite impulse response (FIR) filters.

When measuring the convolutive mixing of the sources, the degree of mixing is significant since the reverberation time of the room space is large.

2.1.1 Instantaneous mixture model

In instantaneous mixing, they can be described as a set of *m* unknown source signals $\{s_i(k)\}$, where $1 \le i \le m$ are combined to yield the *n* measured sensor signals $\{x_j(k)\}$, where $1 \le j \le n$ as:

$$x_j(k) = \sum_{i=1}^{m} a_{ji} s_i(k) + v_j(k) \tag{2.1}$$

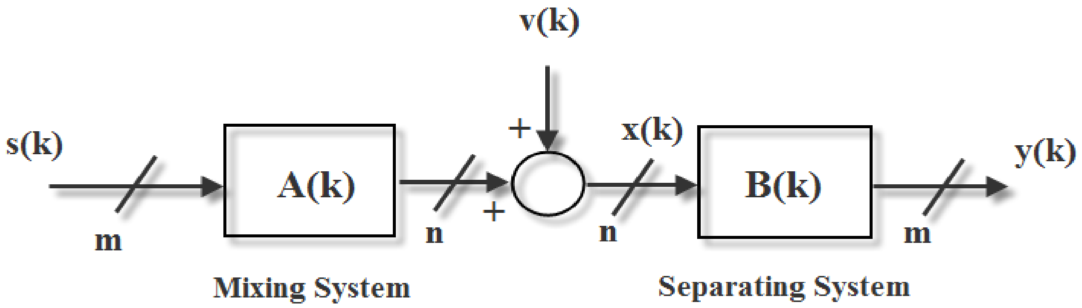


FIGURE 2.1: BLOCK DIAGRAM OF THE INSTANTANEOUS BSS TASK

From Eq. 2.1 where $\{a_{ji}\}$ are the coefficients of the linear time-invariant mixing system represented by the $(n \times m)$ matrix $\mathbf{A}$ and $v_j(k)$ is additive noise signal at the *j*th sensor. The goal of BSS for instantaneous mixtures is to adjust the coefficients of a $m \times n$ separation or demixing matrix $\mathbf{B}$, which recover estimates $y_i(k)$, of the original sources $x_j(k)$ from

$$y_i(k) = \sum_{j=1}^{n} b_{ij}(k) x_j(k) \tag{2.2}$$

The block diagram of this task is shown in Fig. 2.1.

There are several applications where the instantaneous mixture model is applicable. For example, in brain science BSS helps to identify underlying components of brain activity from recordings of brain activity as given by an electroencephalogram (EEG**) [Cichocki 02].** In other fields like image processing applications, which are the extraction of independent features in image and improving the image quality. A comprehensive treatment of the instantaneous BSS case and related algorithms can be found in **[Hyvärinen 01].** However, the practical algorithm for speech separation must take the convolutive mixing of the acoustic paths into account. In this thesis we deal with BSS for acoustic environments and thus the instantaneous mixture model is not appropriate as no delayed versions of the source signals are considered. Therefore, in the next section we extend this model and show how the convolutive mixture model works in practical acoustic scenario.

2.1.2 Convolutive mixture model

In acoustic scenario, we extend the instantaneous mixture model by considering the time delays resulting from sound propagation over space and probably the multipath generated by reflections of sound off different objects, particularly in large rooms and other enclosed settings. Normally, the convolutive mixing system consists of finite impulse response filters. As a result, the *m* sources are mixed by a time-dispersive multichannel system , described by

$$x_j(k)=\sum_{l=-\infty}^{\infty}\sum_{i=1}^{m}a_{jil}s_i(k-l)+v_j(k) \qquad (2.3)$$

where $\{x_j(k)\}, 1\le j\le n$ are the *n* sensor signals. The parameter *m* also denotes the FIR filter length of the demixing filter a_{jil} or we call the coefficients of the discrete-time linear time-invariant mixing system $\{\mathbf{A}_l\}_{l=-\infty}^{\infty}$, where each matrix $\mathbf{A}_l$ is of dimension $(n\times m)$.

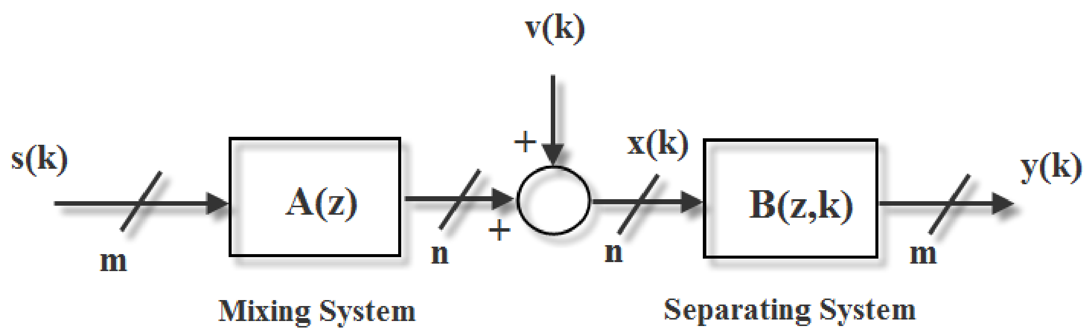


FIGURE 2.2: BLOCK DIAGRAM OF THE CONVOLUTIVE BSS TASK

In the above diagram, $\mathbf{A}(z)=\sum_{l=-\infty}^{\infty}\mathbf{A}_l z^{-l}$ and $\mathbf{B}(z,k)=\sum_{l=-\infty}^{\infty}\mathbf{B}_l(k)z^{-l}$ represent the *z* transform of the sequences of the system $\{\mathbf{A}_l\}$ and $\{\mathbf{B}_l(k)\}$.

Most commonly, BSS algorithms are developed under the assumption that the number *m* of simultaneously active source signal $s_i(k)$ equals the number *n* of the sensor signals $x_j(k)$. The number of unknown source signals m plays an important role in BSS algorithms in that, under reasonable constraints on the mixing system, the separation problem remains linear if the number of mixture signals n is greater than or equal to $m\,(n \geq m)$. This case that the sensors outnumber the sources is termed overdetermined BSS. The main approach to simplify the separation problem in this case is to apply principal component analysis (PCA) **[Hyvärinen 01].** In order to perform matrix dimension reduction by extracting the first m components and then use a standard BSS algorithm. A situation is called underdetermined BSS or BSS with overcomplete bases, which means that the sources outnumber the sensors $(n \leq m)$. This is the significantly more difficult case. Mostly the sparseness of the sources in the time-frequency domain is used to determine clusters which correspond to the separated sources (e.g. **[Zibulevsky 01] [Bofill 03]**. Currently, many researchers proposed methods to estimate the sparseness of the sources based on modelling the human auditory system and then subsequently apply time-frequency masking to separate the sources.

2.2 Speech source signal characteristics and BSS criteria

In this section we are going to discuss the signal properties of acoustic source signals such as speech signals and their relevant utilization for BSS algorithms.

As we know, speech signals are feature-rich and possess certain characteristics that enable BSS algorithm to be applied.

2.2.1 Basic signal properties of acoustic signals

Statistical properties: a good statistical model of a signal in the time domain is a zero-mean Gaussian process $\mathbb{N}(\mu_N, \sigma_N)$ with a given variation σ_N^2, mean $\mu_N = 0$ and normal probability density function (PDF) given by:

$$p(x/\mu_N, \sigma_N) = \frac{1}{\sigma_N \sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_N^2}\right) \tag{2.4}$$

In the discrete time domain this simple model means that every sample has a random value with a Gaussian PDF, also called Gaussian noise or Gaussian distribution.

Temporal properties: one of the widely used temporal properties of a noise signal is the assumption that the noise is a stationary signal. In most cases in this thesis this is a human speech signal. In other words, it is called "temporal dependencies" which means audio signals are in general showing temporal dependencies, for example, the speech signals by the vocal tract. Speech can also be separated using second-order statistics alone if the source signals have unique temporal structures with distinct autocorrelation functions. In other words, if the temporal sample of a signal is uncorrelated, then the signal exhibits strict-sense whiteness.

Stationarity: speech is also a highly non-stationary signal due to the amplitude modulations inherent in the voiced portions of speech and to the intermingling of voiced and unvoiced speech patterns in most dialects **[Scott 07].** The non-stationary characteristics of individual talks (sources) are not likely to be similar. The majority of audio signals are considered in literature as non-stationary signals, but strict-sense stationarity is only assumed.

2.2.2 Criteria for BSS in Speech Separation

- Nonstationarity. BSS algorithms can be designed to exploit the statistical independence of different talkers in an acoustic environment. It is known that the statistic of jointly-Gaussian random processes can be completely specified by their first or second order statistic; hence, the higher and lower order statistical features do not carry any additional information about Gaussian signals. Therefore, in most acoustic BSS applications nonstationarity of the source signals can be exploited by simultaneous diagonalization of short-term output correlation matrices at different time instants **[Weinstein 93].**
- Non-Gaussianity, in such case, statistical independence of the individual talker's signals need not be assumed, and the non-Gaussian nature of the speech signals are not very important when these statistics are used. Additionally, the non-gaussianity can be exploited by using higher-order statistics yielding a statistical decoupling of higher-order joint moment of the BSS output signals. BSS algorithms utilizing higher-order statistics are also termed independent component analysis (ICA) algorithm **[Cardoso 89][Jutten 91][Comon 91].**
- Non-whiteness. As audio signals exhibit temporal dependencies this can be exploited by the BSS criterion. Therefore, it can be assumed that samples of each source signal are not independent along the time axis however; the signal

samples from different sources are mutually independent. Based on the assumption of mutual statistical independence for non-white sources several algorithms can be found in the literature. Mainly the non-whiteness is exploited using second-order statistics by simultaneous diagonalization of output correlation matrices over multiple time-lags. It notes that convolution based BSS algorithm which is based on the mutual statistical independence for temporally white signals.

2.3 Acoustic echo cancellation

2.3.1 General Principle

The effect of sound reflection from objects is called "reverberation." Echoes are distinct copies of the reflected sound. Humans can hear echoes when the difference between arrival times of the direct signal and the reflection is more than 100ms, but even with differences of 50ms the audio still sounds echoic. Most acoustic echo reduction applications do not supress the echoes in the room environment, however, it actually supresses the effect when the local sound source is captured by the receive device such as microphone, transmitted through the communication line, reproduced by the loudspeaker in the receiving room, captured by the microphone there, returned back through the communication line, reproduced from the local loudspeaker, and so on. That is the simple entire system converts to a signal generator, reproducing an annoying constant one.

In addition, acoustic echo is inevitable whenever a speaker is placed near to a microphone in a general full-duplex communication application. The most common communication scenario is the hands-free mobile communication kits for the cars. For example, the voice from the loudspeaker is unavoidable to be picked up by the microphone and transmitted back to the remote speaker. This makes the remote speaker hear his/her own voice distorted and delayed by the communication channel or called end to end delay, which is known as echo. Obviously, the longer the channel delay, the more annoying the echo and the worse is the perceived quality of the communication service such as VoIP conference call.

There are some properties of acoustic echo:

- It is not stationary, and is varies based on a multitude of external factors – intensity and position of the sound source.
- It is a non-linear signal; the non-linearity might be created by the analogue circuitry.
- It is more dispersive, with dispersion times up to 100ms.

2.3.2 Joint Blind Source Separation and Echo Cancellation

2.3.2.1 Cause of Echo in digital network

In most situations, background noise is generated through the network when we use digital phones operated in hands-free mode. In the real-time environment, the additional sounds are directly and indirectly transmitted to the microphone, so the multipath audio is created and transmitted back to the talker. These additional sounds pass through the digital cellular vocoder and cause distortion of speech. Meanwhile, the digital processing delays and speech-compression applied further contribution of the echo generation and degraded voice quality.

Under this circumstance, the echo-control systems are required in today's digital wireless networks. Because of the speech process delays ranging from 80ms to 100ms are introduced, and then resulting in total end-to-end delay of approximately 160ms to 200ms. At this stage, the echo cancellation devices are required within the wireless network.

There are two main echo cancellation types: line echo cancelation and acoustic cancellation. General speaking, line echo is created by a telephone hybrid which transforms a 4 wire line to a 2 wire line. Usually there are two hybrids in the telephone line. One corresponds to the near end terminal and the other one corresponds to the far end (remote) terminal. See the figure 2.3 for the line echo flow diagram.

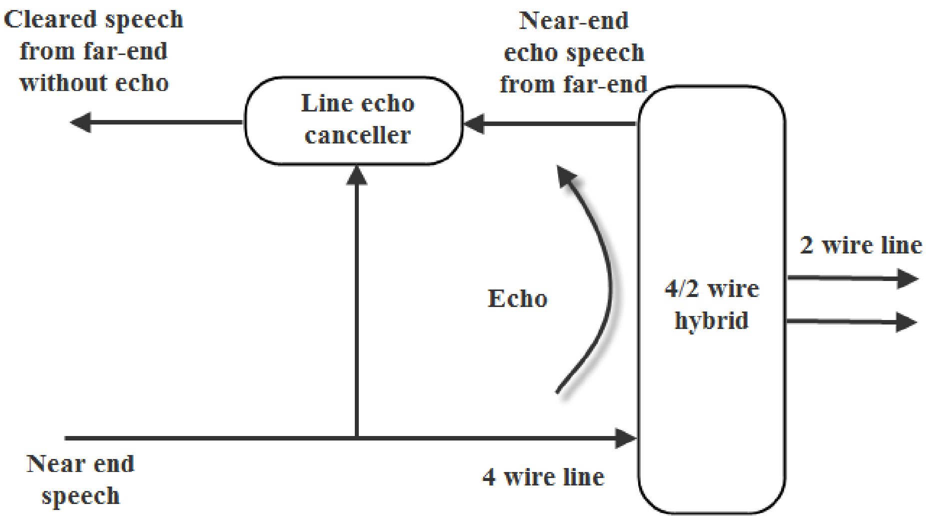


FIGURE 2.3: LINE ECHO CANCELLER INTEGRATION FLOW DIAGRAM

Line echo canceller features include: fast convergence, fast re-convergence after echo path change, robustness in respect to background noise and non-linear distortion, maximal echo path up to 256ms, reliable work in networks with VoIP segments.

Additionally, acoustic echo cancellation compares with line echo cancellation, both of them address the similar problems, and are often based on the same technology. However, a line echo canceller generally cannot replace an acoustic echo canceller; due to acoustic echo cancellation is a more difficult problem. With line echo cancellation there are generally less than two reflections from telephone hybrids or impedance mismatches in the telephone line. These echoes are usually delayed by less than 32 ms, and do not change very frequently. As mentioned before, with acoustic echo cancellation, the echo path is complex and also varies continuously as the speaker moves around the room.

2.3.2.2 The Process of Echo Cancellation and performance measurement

Today's digital cellular network technologies require significantly more processing power to transmit signals through the channels.

Simply said, the process of cancelling echo involves two steps.

- Calling set up: the echo canceller employs a digital adaptive filter to set up a model of voice signal and echo passing through the echo canceller. As a voice path passes back through the cancellation system, the echo canceller compared the original signal and "modelled" signal to cancel existing echo dynamically.

- The second process utilizes a non-linear processor to eliminate the remaining residual echo by attenuating the signal to achieve the lower noise level.

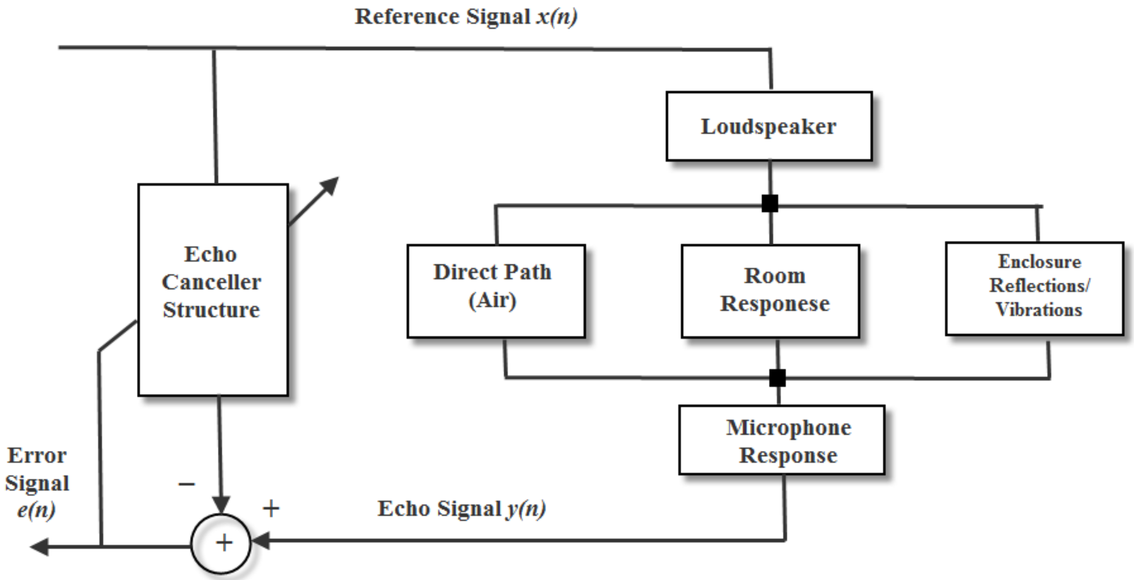


FIGURE 2.4: STRUCTURE OF ACOUSTIC ECHO CANCELLER IN THE ROOM ENVIRONMENT

In Figure 2.3 the acoustic echo canceller estimates the transfer path loudspeaker microphone and subtracts the estimated portion of the loudspeaker signal from the microphone signal. One important evaluation parameter is called the "Echo Return Loss Enhancement (*ERLE*). It is used in evaluating the residual energy or echo residual. We suppose the signal captured from the loudspeaker will be completely suppressed. Owing to the near-end noise, shorter filters than the actual reverberation, and estimation errors, a portion of the captured loudspeaker signal will remain. This portion is called the echo residual.

A measure of the AEC performance is the Echo Return Loss Enhancement (*ERLE*) which is defined as follows:

$$ERLE(dB) = 10\log_{10}\left(\frac{E\left\{y^2(t)\right\}}{E\left\{e^2(t)\right\}}\right) \tag{2.5}$$

where $y(t)$ is the echo signal and $e(t)$ is the echo left after processing. In next chapter a simulation experiment will plot an example output from the two optimal algorithms - NMF and LMS.

2.3.2.3 AEC applications with BSS algorithm

In some applications such as teleconferencing and voiced-controlled machinery, AEC has been widely used in this kind of real applications. However, this straightforward approach would be to use multichannel AEC which has two important drawbacks:

- The AEC can only operate reliably when one of the speakers are talking; it means it will not work properly when there is double talk. As louder speakers to microphone fast adaptation is required which cannot be obtained in the presence of double talk.
- The BSS algorithm is obstructed by contributions of the loud speaker signals that remain present in the microphone signal despite the AEC. Because BSS can only be applied on independent signals, otherwise the overall system performance deteriorates accordingly **[Kwong 92]**.

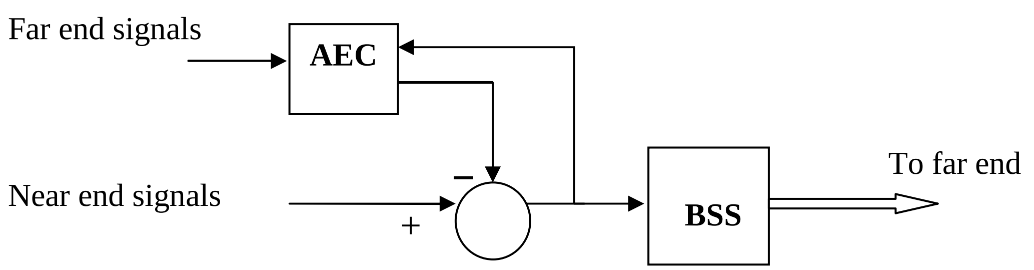


FIGURE 2.5: ECHO CANCELLATION FOLLOWED BY BLIND SIGNAL SEPARATION

An alternative way is that applying BSS to both the microphone signals (near end signals) and the far- end signals would overcome these drawbacks but it will cause the higher computational complexity.

In the real-time scenario, the problem of recovering source signals from mixtures of them which are contaminated by acoustic echo. We assumed that the original sources (near-end sources) to be independent of each other, but far-end signals that are reproduced in the same room and they are generally not independent. Therefore, Kwong introduced a correlation estimator which measures the cross-correlations among all microphone (modelled) signals include known input signals. Thus, this will be resulting updated outputs which are passed by multichannel filters. More algorithm detail processing can be found in **[Kwong 92]**.

The above example is taking advantage of BSS algorithm over conventional echo cancellation is that can operate in many suitable applications such as teleconferencing and hands free telephony.

2.3.3 Limitation of conventional Acoustic Echo Canceller

Much work has be carried out aimed at **[Kwong 92][Makino 93][Mathews 93]** improving the convergence speed of LMS type algorithm. Ideally, an acoustic echo canceller is to completely remove any signal emanating from a loudspeaker from the signal picked up by a closely coupled microphone. In short conclusion of limitations of echo cancellers for speakerphones includes:

- Acoustic, thermal and DSP related noise
- Inaccurate modelling of the room impulse response
- Slow convergence and dynamic tracking
- Nonlinearities in the transfer function caused mainly due to the loudspeaker
- Resonances and vibration in the plastic enclosure.

To be commercially viable the AEC needs to be developed in products for a self-contained handsfree device in a typical room environment. An important part of the acoustic each canceller evaluation is the convergence time and it is necessary to be set on the order of 100ms with Echo Return Loss Enhancement (ERLE) on the order of 30dB.

2.3.4 Conclusions

Acoustic echo cancellation is useful in any hands-free or other telecommunications situation involving two or more locations. Acoustic echo is most noticeable and annoying when delay is present in the transmission path. This would happen primarily in long distance circuits, or systems utilizing speech compression such as VoIP application. However the echo might not be as annoying when there is no delay (e.g. with short links between conference rooms in the same building or distance learning over high speed fibre-optic cable connection. As the existence of imperfection of speech quality in the modern telecommunication, acoustic echo cancellation techniques will have large commercial potential in the future.

3 Optimum Algorithms for Blind Source Separation

3.1 Independent Component Analysis (ICA)

3.1.1 Background Theory of Independent Component Analysis

Blind source separation (BSS) is the problem of recovering signals from several observed linear mixtures. These signals could be from different directions or they could have different pitch levels along the same directions. When we deal with the BSS, there is no need for information on the source signals or mixing system (location or room acoustics) **[Makino 07a].** Here, we should point out that the characteristics of the source signals are statistically independent, as well as independent from the noise components. Therefore the goal of BSS is to separate an instantaneous linear even-determined mixture of non-Gaussian independent sources **[Paul 05]**.

As we mix independent components (random independent variables) the resulting mix tends towards having a Gaussian distribution, making the Independent Components Analysis (ICA) method impossible. ICA is the classical blind source separation method to deal with problems that are closely related to the cocktail-party problem. The following simple model shows what the Blind Source Separation is:

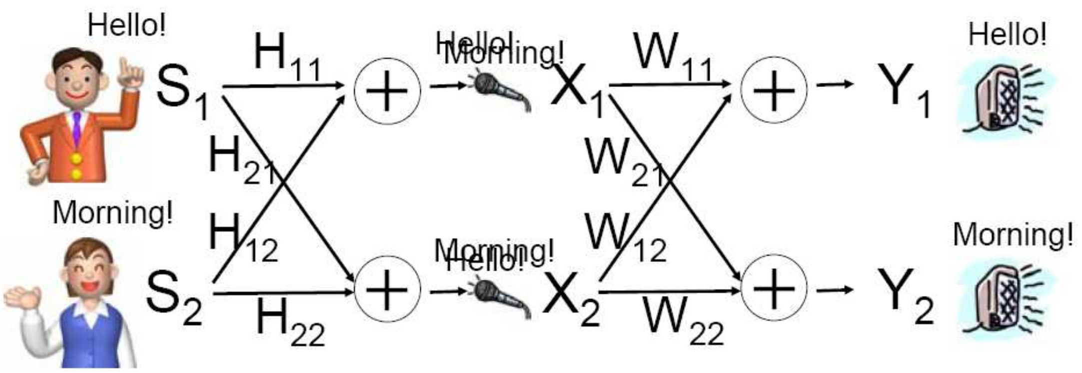


FIGURE 3.1: MODEL OF BLIND SOURCE SEPARATION

In detail, this model has five main parts: Source signals $\mathbf{S}_1$, $\mathbf{S}_2$, mixing system $\mathbf{H}$, observed signals $\mathbf{X}_1, \mathbf{X}_2$, separation system W and separated signals $\mathbf{Y}_1$, $\mathbf{Y}_2$. Initially, the source signals $\mathbf{S}_1$ $\mathbf{S}_2$ are independent, and then in the mixing system $\mathbf{H}$, it delays, attenuates and reverberations the source signals. During the separation processing, the separation system **W** only uses the observed signals $\mathbf{X_1}$, $\mathbf{X_2}$ to estimate $\mathbf{S_1}$, $\mathbf{S_2}$. The separated signals $\mathbf{Y_1}$, $\mathbf{Y_2}$ should become mutually independent.

Ideally, the aim of the source separation is not necessarily to recover the originally source signal. Instead, the aim is to recover the model sources without interferences from the other source. Therefore, each model source signal can be a filtered version of the original source signals.

3.1.2 Notation of Blind Source Separation

In the Blind Source Separation problem, for example, *m* mixed signals are linear combinations of *n* unknown mutually statistically, independent, zero-mean source signal, and are noise-contaminated source signals. So this is can be written as:

$$x_i(t) = \sum_{j=1}^{n} h_{ij} s_j(t) + n_i(t) \quad i = 1...m \tag{3.1}$$

Its matrix notation:

$$\mathbf{X(t) = HS(t) + N(t)} \tag{3.2}$$

Where $\mathbf{X(t) = [x_1(t), x_2(t), ..., x_m(t)]^T}$, is a vector of sensor signals, $\mathbf{N(k)}$ is the vector of additive noise. **H** is the unknown full rank $n \times m$ mixing matrix. The block diagram as shown below:

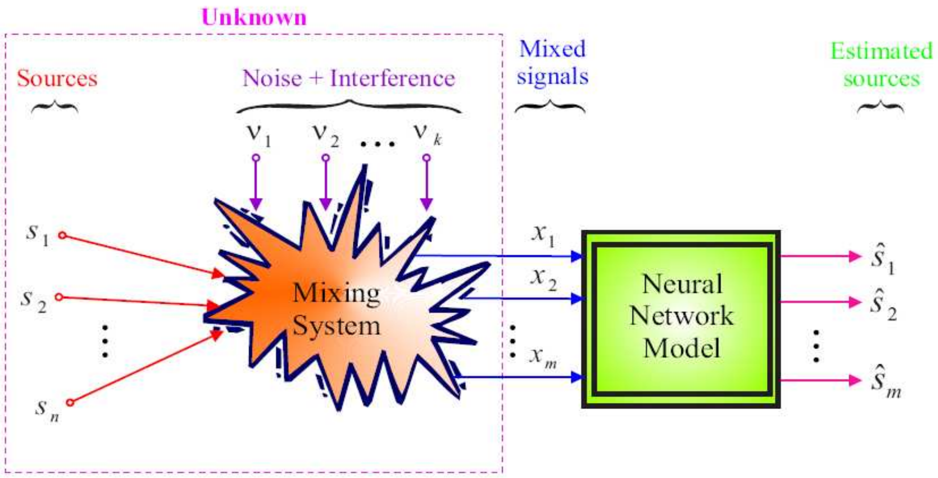


FIGURE 3.2: BLOCK DIAGRAMS ILLUSTRATING BLIND SIGNAL PROCESSING PROBLEM

We consider equation (3.1) as a linear function in most cases, and every component $x_i(t)$ is expressed as a linear combination of the observed variables $s_j(t)$.

3.1.3 Definition of ICA

There are several definitions of ICA and all include the above linear mixing model. In the literature, we will review the different three basic definitions of linear ICA as follows.

1) Temporal ICA: it is the first general definition of ICA. The mathematical model can be expressed as:

$$\mathbf{y_i} = \mathbf{Wx_i} \tag{3.3}$$

It is the ICA of a noisy random vector $x(k)$ is obtained by finding the output of a linear transform y_i with the full rank separating matrix $\mathbf{W}(n \times m)$. And such that the output signal vector $y_i = [y_1, y_2, ..., y_n]^T$ contains the estimated source component s_i which are as independent as possible, because we try to maximize some function $F(s_1, ..., s_m)$ of source independence. **[Hyvarinen 99][Cichocki 02]**.

2) Random noisy model ICA is defined by:

$$\mathrm{x_i} = \mathbf{H}\,\mathrm{s_i} + \mathrm{n_i} \tag{3.4}$$

Where $\mathbf{H}$ is a $(n \times m)$ mixing matrix, $s_i = [s_1, s_2, ..., s_n]^T$ is a source vector of statistically independent signals, $n_i = [n_1, n_2, ..., n_m]^T$ is a vector of uncorrelated noise terms. ICA is obtained by estimating both the mixing matrix **H** and the independent source (vectors) components.

3) Noise-free ICA model: it is a simplified definition in which the noise vectors (components) are omitted.

And it is can be expressed as:

$$\mathrm{x_i} = \mathbf{H}\,\mathrm{s_j} \tag{3.5}$$

The matrix form is: $\mathbf{X} = \mathbf{HS}$. In many applications, especially when a large number of Independent Components (ICs) occur and they have sparse distribution. It is more convenient to use this noisy-free ICA model (the equivalent form: $\mathbf{X^T} = \mathbf{S^T H^T}$)**[Hyvarinen 99][Cichocki 02]**.

Note: The temporal ICA and Noise-free ICA. They are asymptotically equivalent. Generally, the natural relation $\mathbf{W} = \mathbf{H^{-1}}$ is used with **n**=**m** which is the unique matrix.

From the definition 3, the basic noisy-free ICA model is a generative model **[Hyvarinen 99b]**, which means that it describes how the observed data are generated by a process of mixing the components s_j (sources), and these components are latent variables, meaning that they cannot be directly observed. All we observe are the random variables x_i, and we must estimate both the mixing coefficients $\mathbf{H}$, and the ICs s_i (estimated sources) using x_i. Here we have dropped the time index t and this is because in the basic ICA model, we assume that each mixture x_i as well as each independent component s_j (sources) is a random variable, instead of a proper time signal or time series. We also neglect any time delays during the mixing. So this is often called the *instantaneous* mixing model.

3.1.4 Restrictions in ICA

There are three certain assumptions and restrictions to make sure the basic ICA model can be estimated.

1) The independent components are assumed statistically independent.

   The random variables are said to be independent if the source component s_i does not give any information on the value of another source component s_j for $i \neq j$. Technically, the independence can be defined by the probability densities.
   (Note: more details relate joint pdf and marginal pdf, see section 2.3 on ICA **[Hyvarinen 99c]**)

2) The independent components must have Non-Gaussian distributions.

   The Gaussian components mix the independent components and cannot be separated from each other. In other words, some of the estimated components will be arbitrary linear combinations of the Gaussian components and in the Non-Gaussian distributions we can find the independent components. Thus, ICA is essentially impossible if the observed mixtures x_i (variables) have Gaussian distributions.

3) We can assume that the unknown, mixing matrix is square.

   This assumption means, the number of independent components s_i is equal to the number of observed mixture x_i. This simplifies the estimation (from original source) very much.

3.1.5 Background theory of ICA

There are three basic and intuitive principles for estimating the model of independent component analysis.

1) ICA by minimization of mutual information.

There is a basic definition of information-theoretic concepts explained in this section.

The differential entropy $\mathbf{H}$ of a random vector *y* with density *p(y)* is defined as **[Hyvarinen 99c]**:

$$H(y) = -\int p(y)\log p(y)dy \tag{3.6}$$

The entropy is closely related to the code length of the random vector. Basically, the mutual information $\mathbf{I}$ between *m (scalar)* random variables $y_i, i = 1....m$ is defined as follows:

$$\mathbf{I}(y_1, y_2, ..., y_m) = \sum_{i=1}^{m} \mathbf{H}(y_i) - \mathbf{H}(y) \tag{3.7}$$

Here is the simple diagram to illustrate what is mutual information between two random variables:

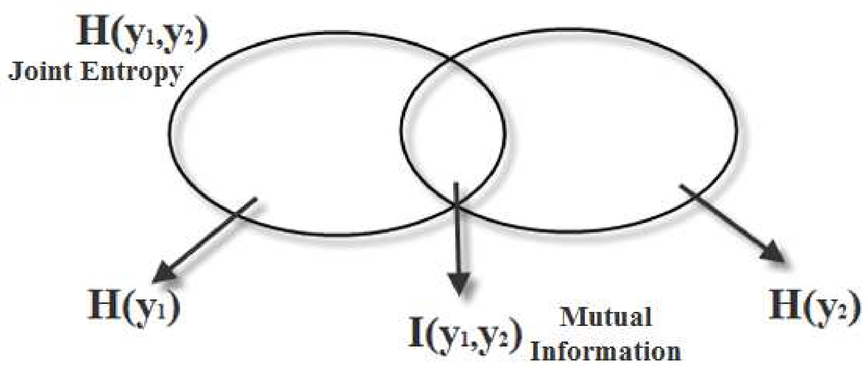


FIGURE 3.3: MUTUAL INFORMATION BETWEEN TWO RANDOM VARIABLES

The mutual information is: $\mathbf{I}(y_1, y_2) = \sum_{i=1}^{2} \mathbf{H}(y_i) - \mathbf{H}(y_1, y_2)$, where $\sum_{i=1}^{2} \mathbf{H}(y_i)$ is marginal entropy and $\mathbf{H}(y_1, y_2)$ is joint entropy. The mutual information is a natural measure of the dependence between random variables. It is always nonnegative, and zero if and only if the variables are statistically independent. Therefore, we can use mutual information as the criterion for finding the ICA representation, i.e. to make the output "decorrelated". In any case, minimization of mutual information can be interpreted as giving the maximally independent components **[Hyvarinen 99c]**.

2) ICA by maximization of Non-Gaussianity.

Non-Gaussianity is actually most important in ICA estimation. In classic statistical theory, random variables are assumed to have Gaussian distributions. So we start by motivating the maximization of Non-Gaussianity by the central limit theorem. It has important consequences in independent component analysis and blind source separation. As mentioned in the first section, a typical mixture of the random data vector $\mathbf{x}$, is of the form $\mathbf{x}_i = \sum_{j=1}^{m} a_{ij}\mathbf{s}_j$, where a_{ij}, $j = 1,....,m$, are constant mixing coefficients and $\mathbf{s}_j$, $j = 1,...,m$, are the m unknown source signals. Even for a small number of sources the distribution of the mixture is usually close to Gaussian.

Simply explained as follows:

Let us assume that the data vector x is distributed according to the ICA data model: $x = \mathbf{H}s$, it is a mixture of independent components. Estimating the independent components can be accomplished by finding the right linear combinations of the mixture variables. We can invert the mixing model as: $\mathbf{s = H^{-1}x}$, so the linear combination is $\mathbf{x_i}$. In other words, we can denote this by $\mathbf{y = b^T x = \sum_{i=1} b_i x_i}$. We could take $\mathbf{b}$ as a vector that maximizes the Non-Gaussianity of $\mathbf{b^T x}$. This means that $\mathbf{y = b^T x}$ equals one of the independent components. Therefore, maximizing the Non-Gaussianity of $\mathbf{b^T x}$ gives us one of the independent components. **[Hyvarinen 99c]** To find several independent components, we need to find all these local maxima. This is not difficult, because the different independent components are uncorrelated: We can always constrain the search to the space that gives estimates uncorrelated with the previous ones. **[Hyvarrinen 04]**

3) ICA by maximization of likelihood.

Maximization of likelihood is one of the popular approaches to estimate the independent components analysis model. Maximum likelihood (ML) estimator assumes that the unknown parameters are constants if there is no prior information available on them. It usually applies to large numbers of samples. One interpretation of ML estimation is calculating parameter values as estimates that give the highest probability for the observations.

There are two algorithms to perform the maximum likelihood estimation:

- Gradient algorithm: this is the algorithms for maximizing likelihood obtained by the gradient method. (Further Ref. See **[Hyvarinen 99d])**

- Fast fixed-point algorithm **[Ella 00]**: the basic principle is to maximize the measures of Non-Gaussianity used for ICA estimation. Actually, the FastICA algorithm (gradient-based algorithm but converge very fast and reliably) can be directly applied to maximization of the likelihood.

3.2 Principal Component Analysis (PCA)

3.2.1 Introduction

Principal Component Analysis is one of the simplest and better known data analysis techniques. The main purpose of PCA analytic techniques are: a) to reduce the number of variables. b) to detect structure in the relationships between variables, that is to classify variables. In other words, PCA is combining two or more variables into a single factor where these variables might be highly correlated with each other.

1) Scatter plot for PCA

The results of PCA can be summarized in a scatter plot (diagram). A regression line can be fitted that represents the "best" summary of the linear relationship between the variables. Essentially, we have reduced the two variables to one factor and the new factor is actually a linear combination of the two variables. The scatter plot can show various kinds of relationships, including positive (rising), negative (falling), and no relationship (independent)**[Utts 05]**. If we extend the two variables to multiple variables, then the computations become more involved, but the basic principle of expressing two or more variables by a single factor remain the same. When we have three variables, we could plot a three dimensional scatter plot and we could fit a plane through the data.

2) PCA Factor Analysis

The computational aspect of PCA is the extraction of principal components which amounts to a variance maximizing rotation of the original variable spaces. In PCA, the criterion for the rotation is:

- Maximize the variance of the “new” variables (factor).

- Minimizing the variance around the new variable.

After the first regression line has been found through the data, we iteratively continue to define other lines that maximize the remaining variability. In this manner, consecutive factors are extracted and these factors are independent of each other. In other words, consecutive factors are uncorrelated or orthogonal to each other **[Dinov 04]**. Note that the decision of when to stop extracting factors basically depends on when there is only very little random variability left. Also the variances extracted by the factor are called the *eigenvalues*. As expected, the sum of the eigenvalues is equal to the number of variables. We will discuss more about eigenvalues in the next section.

3.2.2 Mathematics Background

- Eigenvalue and Eigenvector

Calculating Eigenvalues and Eigenvectors is the key point in PCA. PCA involves determining of these two parameters of the covariance matrix. We will talk in more detail about the covariance matrix in the next section.
Eigenvalues are a special set of scalars associated with a linear system of equations that are sometimes also known as characteristic roots. Each eigenvalue is paired with a corresponding so-called eigenvector. The determination of the eigenvalues and eigenvectors of a system is very important in engineering, where it is equivalent to matrix diagonalization**.**

Matrix diagonalization is the process of taking a square matrix and converting it into a so-called diagonal matrix that shares the same fundamental properties of the underlying matrix. The relationship between a diagonalized matrix, eigenvalues, and eigenvectors follows from the great mathematical identity. For example, a square matrix $\mathbf{A}$ can be decomposed into the very special form: $\mathbf{A} = \mathbf{PDP}^{-1}$**,** where $\mathbf{P}$ is a matrix composed of the eigenvectors of $\mathbf{A}$; $\mathbf{D}$ is the diagonal matrix constructed from the corresponding eigenvalues, and the $\mathbf{P}^{-1}$ is the inverse matrix of $\mathbf{P}$ **[George 97]**.

- Covariance

Firstly we need to understand what covariance is. The covariance of two datasets $C_{x,y}$ (x and y) can be defined as their tendency to vary together. We usually define these two datasets as a two dimensional dataset. In statistics, the variability of the data set around its mean is called the data standard deviation. In the same way, covariance can describe variability—as the product of the averages of the deviation of the data points from the mean value. There are three possible results which can indicate the relationship between the two datasets.

$C_{x,y}$ value will be larger than 0 (positive) if x and y tend to increase together.

$C_{x,y}$ value will be less than 0 (negative) if x and y tend to decrease together.

$C_{x,y}$ value will equal 0 if x and y are independent.

Since the covariance value can be calculated between any 2 dimensions in the data set, this technique is often used to find relationships between dimensions in high-dimensional data sets where visualisation is difficult.

Also measuring the covariance between *x* and *y* would give us the variance of the *x*, *y* dimensions respectively. The formula for covariance is:

$$\mathrm{cov}(x,y)=\frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{n-1} \tag{3.8}$$

For each item, multiply the difference between the *x* value and the mean of *x*, by the difference between the *y* value and the mean of *y* and add all these up, and divide by *n-1*.

- Covariance matrix

In fact, for an *n*-dimensional data, there are $\frac{n!}{(n-2)!*2}$ different covariance values. Generally, a useful way to get all the possible covariance values between all the different dimensions is to calculated them all and put them in a matrix. For example, for 2D data the covariance matrix has two dimensions, and the values are this:

$$\mathbf{C}=\begin{pmatrix}\mathrm{cov}(x,x) & \mathrm{cov}(x,y)\\ \mathrm{cov}(y,x) & \mathrm{cov}(y,y)\end{pmatrix} \tag{3.9}$$

Basically, if we have an *n*-dimensional data set, then the matrix has n rows and n columns (must be square) and each entry in the matrix is the result of calculating the covariance between two separate dimensions.

3.2.3 PCA Methodology

The dimension of the data is the number of variables that are measured on each observation. A high dimensional dataset contains more information compared with a low dimension counterpart. To reduce the dimensionality of the data while retaining as much as possible of the variation present in the original dataset is the goal of PCA. In mathematical terms, we can state this as follows:

Given the *p*-dimensional random variable $\mathbf{x} = (x_1, ..., x_p)^T$, find a lower dimensional representation of it, $\mathbf{s} = (s_1, ..., s_k)^T$ with $k \le p$, that captures the content in the original data. But dimensionality reduction implies information loss; our task is to preserve as much information as possible and determine the best lower dimensional space. Technically, the best low-dimensional space can be determined by the "best" eigenvectors of the covariance matrix of **x** (i.e. the "best" eigenvectors corresponding to the "largest" eigenvalues – also called "principak components") **[Simth 02]**.

3.2.4 Procedure of PCA

Step1: Collect and prepare a set of data and obtain the mean value

Suppose $x_1, x_2, ..., x_m$ are $m \times 1$ vector, and mean is $\overline{x} = \frac{1}{m}\sum_{i=1}^{m} x_i$

Step2: Subtract the mean value from each data element $(x - \overline{x})(y - \overline{y})$

The mean subtracted is the average across each dimension and it produces a data set whose mean is zero. So, all the x values have $\overline{x}$ subtracted, and y values have $\overline{y}$ subtracted from them.

Step3: Calculate the covariance matrix

This is done in the same way as was discussed in the previous section.

Step4: Determine the eigenvalues and eigenvectors of the covariance matrix

Since the covariance matrix is square, we can calculate the eigenvalues and eigenvectors for this matrix. It is important to tell us useful relationship information about the data – increase, decrease together or independent. Each eigenvalue is a measure of how much variance each successive factor extracts, and associated the eigenvector shows us how these dataset are related along a regression line. The process of taking the eigenvector of the covariance matrix, we have been able to extract lines that characterise the scatter of the data.

Step5: Choosing components and forming a feature vector

It is import to choose the components in terms of the eigenvalues which are determined by the covariance matrix. In general, we order the eigenvectors by eigenvalue from highest to lowest. This gives us the components in order of significance. If there are a large number of components, we could ignore the components of much lesser significance. However, this means we will lose some information and the final data set will have fewer dimensions than the original.

Here, the feature vector is constructed by taking the eigenvectors that we want to keep from the list of original eigenvectors, and forming a matrix with these eigenvectors in the columns.

FeatureVector = (eig_vec$_1$ eig_vec$_2$ eig_vec$_3$ ... eig_vec$_n$)

Step6: Deriving the final new data set

The final step of PCA is generating the new final data set. It is also an easy way to calculate. We simply take the transpose of the feature vector and multiply it on the left of the original data set transposed.

FinalData = FeatureVector (Transposed) x MeanAdjustData (Transposed)

Where the mean-adjust-data vector is the original data vector with the mean subtracted from each dimension. Here, what will we get? It will give us the original data solely in terms of the vectors we choose. In the case of when the new data set has reduced dimensionality, the new data is only in terms of the vectors that we choose. For example, we could take only the eigenvector with the largest eigenvalue. As expected, it only has a single dimension compared with the one resulting from using more eigenvectors; we will notice that this data set is exactly the first column of the other. But the single-eigenvector decomposition has removed the contribution due to the smaller eigenvectors. The contribution means the combination of contributions from each of the lines (patterns) which most closely describe the relationships between the data **[Smith 02]**.

Step7: Reconstruction of the original data

If we want the original data back, we just reverse the steps that we took above and we will get the original data set back. Note that if we discarded some eigenvectors in steps, we will lose that information in the retrieved data.

TransDataAdjust = TransFeatureVector^{-1}×FinalData

After calculating the adjusted data set, we need to add the mean to each dimension of the data set to retrieve the original data set.

TransOriginalData = TransDataAdjust + OriginalMean

The following figure shows the essential procedure of PCA.

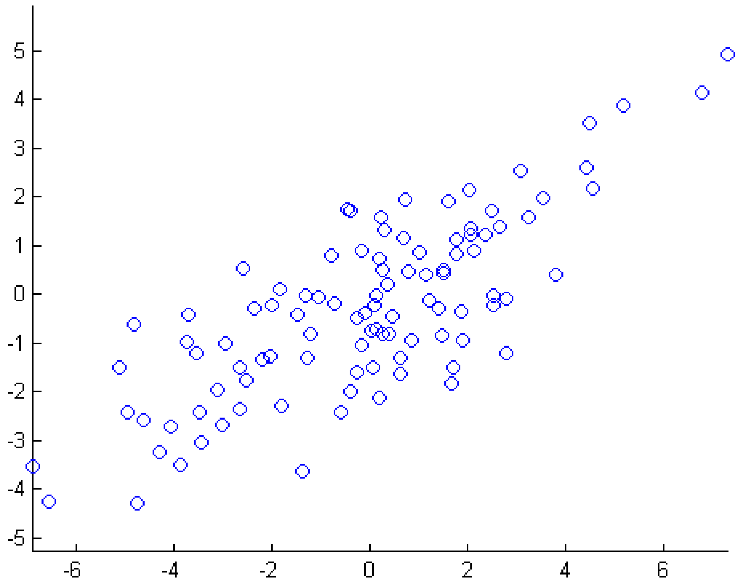


FIGURE 3.4A: ORIGINAL TWO DIMENSIONAL DATA

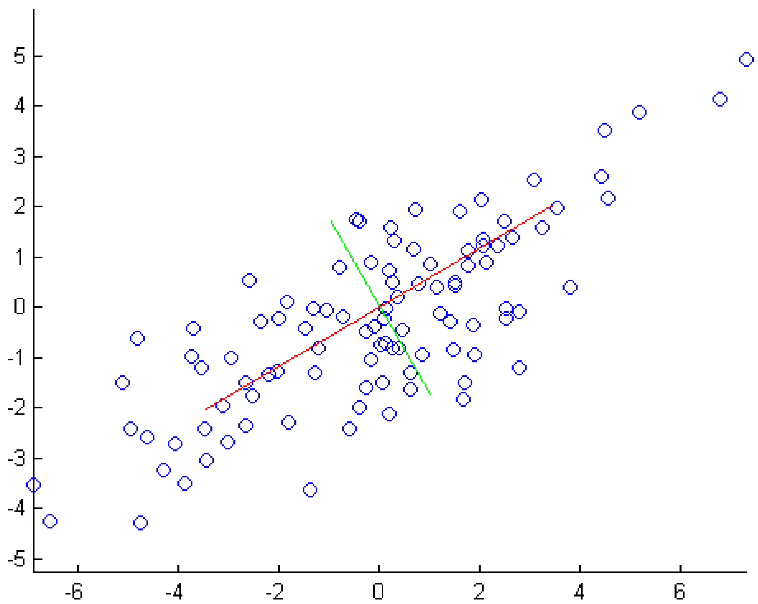


FIGURE 3.4B: NORMALIZED TWO DIMENSIONAL DATA

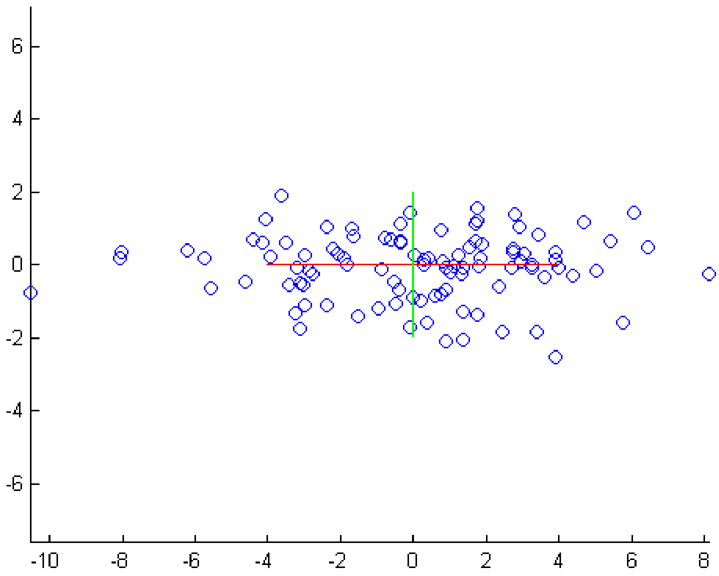


FIGURE 3.4C: DATA BY APPLYING THE PCA ANALYSIS USING BOTH EIGENVECTORS

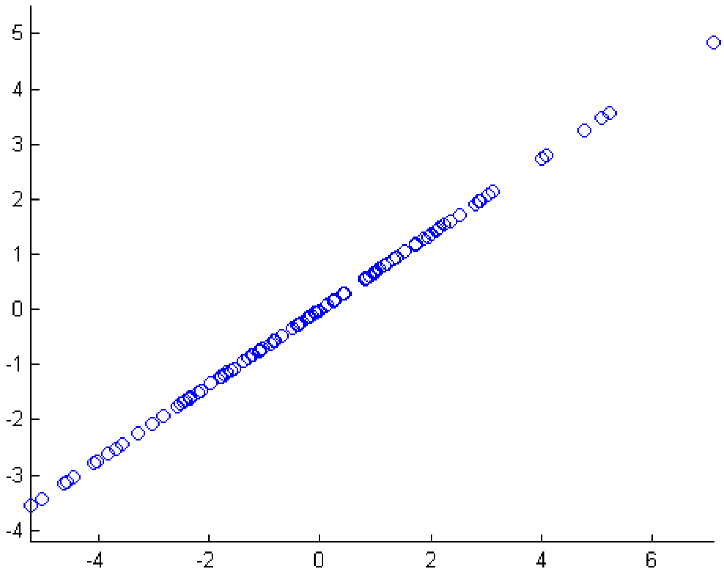


FIGURE 3.4D: THE RECONSTRUCTION FROM THE DATA THAT WAS DERIVED USING ONLY A SINGLE EIGENVECTOR

3.3 Degenerate unmixing estimation technique (DUET)

3.3.1 Introduction to DUET

Degenerate Unmixing estimation technique (DUET) is one of the demixing algorithms in the fields of blind source separation (BSS). It can separate any number of sources using only two mixtures **[Scott 01][Makino 07]**. This method is based on the sources being w-disjoint orthogonal. Common assumptions about the statistical properties of the sources are statistically independent **[Bell 05][Cardoso 97]**, are statistically orthogonal **[Weinstein 93],** are nonstationary **[Parra 00]**, or can be generated by finite dimensional model spaces **[Broman 99].** Moreover, the DUET algorithm is efficient for sources having a property of sparseness in the time-frequency domain, such as speech signal, that is, the target speech signal in a noisy environment can be effectively recognised using the DUET algorithm for Blind Source Separation.

However, in many cases there are more sources than mixtures so we refer to such a case as degenerate. In degenerate Blind Source Separation poses a challenge because the mixing matrix is not invertible. Basically, the traditional method such as Independent Component Analysis (ICA) of demixing by estimating the inverse mixing matrix does not work. Therefore, most blind source separation research has focussed on the square or non-degenerate case **[Scott 01][Makino 07]**. Despite the difficulties, there are several approaches for dealing with degenerate mixtures. We will review these approaches in the next few sections.

Generally, DUET solves the degenerate demixing problem in an efficient and robust manner. We can summarized in one sentence as a definition: DUET makes it possible to blindly separate an arbitrary number of sources given just two anechoic mixtures provide the time-frequency representations of the sources do not overlap too much, which is ideal for speech **[Makino 07]**.

3.3.2 Sources assumptions and mathematics background

- Anechoic Mixing

Consider the mixture of N source signals, $\mathbf{s_j}(t),\ j=1,...,N$, being received at a pair of microphones on a direct path. Suppose we can absorb the attenuation and delay parameters of the first mixture $\mathbf{x_1}(t)$ into the definition of the sources without loss of generality. Then the two anechoic mixtures can be expressed as:

$$\mathbf{x_1}(t)=\sum_{j=1}^{N}\mathbf{s_j}(t) \qquad (3.10)$$

$$\mathbf{x_2}(t)=\sum_{j=1}^{N}a_j\mathbf{s_j}(t-\delta_j) \qquad (3.11)$$

Where a_j is a relative attenuation factor corresponding to the ratio of the attenuations of the paths between sources and sensors, δ_j is the arrival delay between the sensors.

Actually the DUET method, which is based on the anechoic model is quite robust even when applied to echoic mixtures.

- W-Disjoint Orthogonality

In mathematics, disjoint means if two or more sets are disjoint they have no element in common, or say their intersection is the empty set.

W-disjoint orthogonality is crucial to DUET because it allows for the separation of a mixture into its component sources using a binary mask. (Note: a binary mask is used to change specific bits in the original value in the time-frequency plane to the desired setting(s) or to create a specific output value).

We can call two functions $s_j(t)$ and $s_k(t)$ W-disjoint orthogonal. For a given windowing function $W(t)$, the supports of the windowed Fourier transform of $s_j(t)$ and $s_k(t)$ are disjoint. The windowed Fourier transform of $s_j(t)$ is defined as:

$$\hat{s}_j(\tau,\omega) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W(t-\tau) s_j(t) e^{-i\omega t} dt \qquad (3.11)$$

We can state the W-disjoint orthogonality assumption concisely as the following expression:

$$\hat{s}_j(\tau,\omega)\hat{s}_k(\tau,\omega) = 0, \quad \forall \tau,\omega, \quad \forall j \neq k. \qquad (3.12)$$

This assumption is a mathematical idealization of the condition (Note: Idealization is the over-estimation of the desirable qualities and underestimation of the limitations of a desired thing **[Changing 00]** .) In other words, it is likely that every time-frequency point in the mixture with significant energy is dominated by the contribution of one source. In this case, W-disjoint orthogonality can be expressed as,

$$\hat{s}_j(\omega)\hat{s}_k(\omega) = 0, \quad \forall \omega, \quad \forall j \neq k \qquad (3.13)$$

As mentioned before, the binary mask can be used to separate the mixture. So consider the mask function for the support of $\hat{s}_j$,

$$M_j(\tau,\omega) := \begin{cases} 0 & \hat{s}_j(\tau,\omega) \neq 0 \\ 1 & \text{otherwise} \end{cases} \qquad (3.14)$$

M_j separates $\hat{s}_j$ from the mixture via

$$\hat{s}_j(\tau,\omega) = M_j(\tau,\omega)\hat{x}_1(\tau,\omega), \quad \forall \tau,\omega \qquad (3.15)$$

We must determine the masks which are the indicator functions $M_j(\tau,\omega)$ for each source and separate the sources by partitioning. The question is: how do we determine the masks? We will review and discuss it shortly.

3.3.3 Local stationarity and Microphones close together

Local stationarity can be viewed as a form of narrowband assumption. It is necessary for DUET that for all arrival delay time δ, $|\delta| \le \Delta$, where Δ is the maximum time difference possible in the mixing model (Maximum distance of two microphones divided by the speed of signal propagation), even when the window function $W(t)$ has finite support. Additionally, in the common array processing literature **[Krim 96]**, the physical separation of the sensors is small such that the relative delay between the sensors can be expressed as a phase shift of the signal.

We can utilize the local stationarity assumption to turn the delay in time into a multiplicative factor in time-frequency. Basically, this multiplicative factor $e^{-i\omega\delta}$ only uniquely specifies δ if $|\omega\delta| < \pi$ as otherwise we have an ambiguity due to phase-wrap **[Makino 07b].** So we require, $|\omega\delta_j| < \pi, \forall\omega, \forall j$, avoiding phase ambiguity. Therefore, this is guaranteed when two microphones are separated by less than $\pi c / \omega_{\max}$ where $\omega_{\max}$ is the maximum frequency present in the sources and c is the speed of sound.

3.3.4 DUET demixing model and parameters

The assumptions of anechoic mixing and local stationarity allow us to rewrite the mixing equations (1) and (2) in the time- frequency domain as,

$$\begin{bmatrix} \hat{x}_1(\tau,\omega) \\ \hat{x}_2(\tau,\omega) \end{bmatrix} = \begin{bmatrix} 1 & \dots & 1 \\ a_1 e^{-i\omega\delta_1} & \dots & a_N e^{-i\omega\delta_N} \end{bmatrix} \begin{bmatrix} \hat{s}_1(\tau,\omega) \\ \vdots \\ \hat{s}_N(\tau,\omega) \end{bmatrix} \tag{3.16}$$

This is the mixing model for two sources and if the number of sources is equal to the number of mixtures, the non-degenerate case or the standard demixing method is to invert the mixing matrix from the above equation. When the number of sources is greater than the number of mixtures, we can demix by partitioning the time-frequency plane using one of the mixtures based on estimates of the mixing parameters between mixture **[Jourjing 00]**.

With the further assumption of W-disjoint orthogonality, at most one source is active at every (τ,ω), and the mixing process can be described as,

$$\begin{bmatrix} \hat{x}_1(\tau,\omega) \\ \hat{x}_2(\tau,\omega) \end{bmatrix} = \begin{bmatrix} 1 \\ a_j e^{-i\omega\delta_j} \end{bmatrix} \hat{s}_j(\tau,\omega), \text{ for some } j \tag{3.17}$$

In the above equation, j is the index of the source active at (τ,ω). The main DUET observation which is the ratio of the time-frequency representations of the mixtures does not depend on the source components but only on the mixing parameters associated with the active source component.

The mixing parameters associated with each time-frequency point can be calculated as,

$$\tilde{a}(\tau,\omega) := \left| \hat{x}_2(\tau,\omega) / \hat{x}_1(\tau,\omega) \right| \tag{3.18}$$

$$\tilde{\delta}(\tau,\omega) := (-1/\omega)\angle(\hat{x}_2(\tau,\omega) / \hat{x}_1(\tau,\omega)) \tag{3.19}$$

Under the assumption that if the two sensors are sufficiently close then the delay estimation can be ignored, the local attenuation estimator $\tilde{a}(\tau,\omega)$ and the local delay estimator $\tilde{\delta}(\tau,\omega)$ can only take on the values of the actual mixing parameters. As we saw in equation (7), we can demix via binary masking by determining the indicator function of each source. So the indicator functions are determined via,

$$M_j(\tau,\omega) := \begin{cases} 0 & (\tilde{a}(\tau,\omega), \tilde{\delta}(\tau,\omega)) = (a_j, \delta_j) \\ 1 & \text{otherwise} \end{cases} \tag{3.20}$$

And then demix using the masks. Where $(\tilde{a}(\tau,\omega), \tilde{\delta}(\tau,\omega)) = (a_j, \delta_j)$ is the mixing parameter pairs which take over all the time-frequency plane (τ,ω).

3.3.5 Construction of the 2D weighted histogram

Histogram is the key structure used for localization and separation. By using $(\tilde{a}(\tau,\omega), \tilde{\delta}(\tau,\omega))$ pairs to indicate the indices into the histogram, clusters of weight will emerge centred on the actual mixing parameter pairs **[Makino 07b]**. Figure 3.5 shows the two-dimensional weighted histogram.

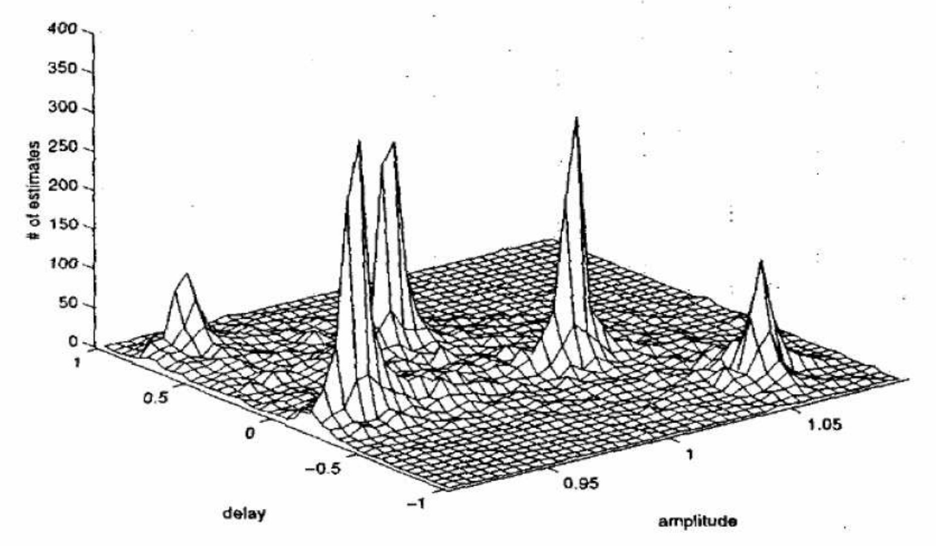


FIGURE 3.5: DUET TWO-DIMENSIONAL CROSS POWER WEIGHTED HISTOGRAM OF SYMMETRIC ATTENUATION $(a_j - 1/a_j)$ AND DELAY ESTIMATE PAIRS FROM TWO MIXTURES OF FIVE SOURCES. EACH PEAK CORRESPONDS TO ONE SOURCE AND THE PEAK LOCATIONS REVEAL THE SOURCE MIXING PARAMETERS.

We can formally define that the weighted histogram separates and clusters the parameter estimates of each source. The number of peaks corresponding to the number of sources, and the peak locations reveal the associated source's anechoic mixing parameters.

There are several different automatic peak identification methods including weighted *k*-means, model-based peak removal, and peak tracking **[Rickard 01]**. Once the peaks have been identified, our goal is to determine the time-frequency masks which will separate each source from the mixtures.

3.3.6 Maximum-likelihood (ML) estimators

Our assumptions made previously will not be satisfied in real-time (real signals with noise) cases, we need a mechanism for clustering the relative attenuation- delay estimates. Thus, we considered the ‘’maximum likelihood (ML) estimators’’ for the a_j attenuation factor and the δ_j delay factor in the following mixing model:

$$\begin{bmatrix} \hat{x}_1(\tau,\omega) \\ \hat{x}_2(\tau,\omega) \end{bmatrix} = \begin{bmatrix} 1 \\ a_j e^{-i\omega\delta_j} \end{bmatrix} \hat{s}_j(\tau,\omega) + \begin{bmatrix} \hat{n}_1(\tau,\omega) \\ \hat{n}_2(\tau,\omega) \end{bmatrix} \tag{3.21}$$

Where $\hat{n}_1$ and $\hat{n}_2$ are noise terms which represent the assumption inaccuracies. One thing we need to point out is: rather than estimating a_j, we estimate $a_j := a_j - \dfrac{1}{a_j}$ which we call the "symmetric attenuation". That is, the attenuation is reflected symmetrically about a centre point (a_j=0) because it has the property that the two microphone (sensor) signals can be swapped **[Makino 07b]**. We can define the local symmetric attenuation estimate,

$$\tilde{a}(\tau,\omega) := \left|\frac{\hat{x}_2(\tau,\omega)}{\hat{x}_1(\tau,\omega)}\right| - \left|\frac{\hat{x}_1(\tau,\omega)}{\hat{x}_2(\tau,\omega)}\right| \tag{3.22}$$

It is motivated by the form of the ML estimators.

However, the difficulty with the estimators is that they require knowledge of time-frequency supports of each source. On the other hand, the local symmetric attenuation and delay observation estimates will cluster around the actual symmetric attenuation and delay mixing parameters of the original sources, so we need a mechanism for determining these clusters.

The estimators suggest the construction of a two-dimensional weighted histogram to determine the clusters and the estimated mixing parameters (a_j, δ_j). Thus, the mixing parameters can be extracted by locating the peaks in the histogram. In this review, we won't go over much mathematics involved in the mixing model, but well explained the basic DUET BSS algorithm theory

3.3.7 Summary of DUET Algorithm

1) Construct time-frequency representations $\hat{x}_1(\tau,\omega)$ and $\hat{x}_2(\tau,\omega)$ from anechoic matrix $x_1(t)$ and $x_2(t)$.

2) Calculate the mixing parameters $(\tilde{a}(\tau,\omega), \tilde{\delta}(\tau,\omega)) = \left(\left|\frac{\hat{x}_2(\tau,\omega)}{\hat{x}_1(\tau,\omega)}\right| - \left|\frac{\hat{x}_1(\tau,\omega)}{\hat{x}_2(\tau,\omega)}\right|, \frac{-1}{\omega}\angle\left(\frac{\hat{x}_2(\tau,\omega)}{\hat{x}_1(\tau,\omega)}\right)\right)$.

3) Construct a 2D smoothed weighted histogram for all weights associated with time-frequency plant.

4) Locate peaks and find peak centres which determine the mixing parameter estimates

5) Construct time-frequency binary masks for each peak centre $(\tilde{a}_j, \tilde{\delta}_j)$ via indicator functions $M_j(\tau,\omega)$ for each source and separate the sources by partitioning.

6) Apply each mask to the appropriately alighted mixtures.

7) Convert each estimated source time-frequency representation back into the time domain.

3.4 Azimuth Discrimination and Resynthesis

3.4.1 Background and Introduction

The Azimuth Discrimination and Resynthesis is a novel sound source separation algorithm which was presented in **[Barry 04a]** to separate stereo musical recordings into independent constituent sources that comprise the mixture. So a typical example is recording stereo music, this process involves recording N sources (each instrument source) individually and then summing and distributing between the right and left channels by using a panoramic potentiometer (pan pot).

The pan pot is a device which usually increases intensity of one source in one channel relative to the other by scaling the gain of source appropriately. By virtue of this, a single source may be virtually positioned at any point between the speakers. Therefore in this case is achieved by creating an interaural intensity difference (IID) **[Rayleigh 76]**. What is the IID? It is better called interaural level differences (ILD), are differences of the sound pressure level arriving at the two ears or sensors; and are the important cues that human use to localise higher frequency sounds.

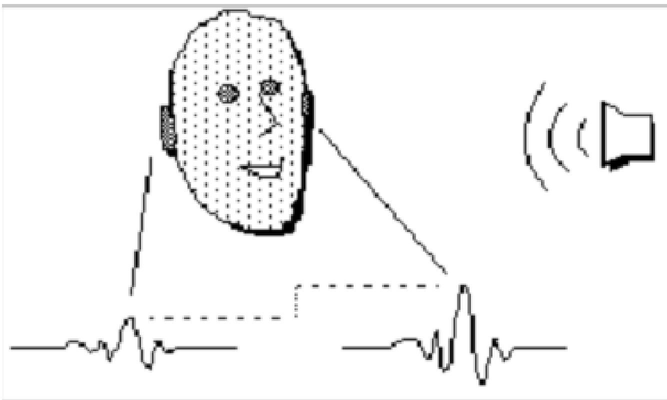


FIGURE 3.6: ILLUSTRATION OF INTERAURAL INTENSITY DIFFERENCE

See above figure, there is a difference in the volume of the sound reaching either ear. Listeners perceive IID as the apparent location of the sources along a horizontal stereo field from left to right. The pan pot was devised to simulate IID's by attenuating the source signal fed to one reproduction channel, causing it to be localised more in the opposite channel **[Barry 04b]**.

ADRess uses gain scaling subtraction and phase cancellation in the time-frequency domain to spatially discriminate between the time-frequency points of a stereo mixture **[Cahill 06]**. The purpose of developing the ADRess algorithm is to perform the noise reduction in mobile or other communication applications. Like other sound source separation algorithms, it has a mathematics model. So we will discuss more detailed in ADRess methodology based on its discrete time mixing model in the next chapter.

3.4.2 ADRess Methodology

ADRess can be described as the mixing model for a channel audio and the following discrete time mixing model defined as:

$$l(n) = \sum_{i=0}^{j-1} pl_i s_i(n), \quad for \quad n = 1, ..., N-1 \tag{3.23}$$

$$r(n) = \sum_{i=0}^{j-1} pr_i s_i(n), \quad for \quad n = 1, ..., N-1 \tag{3.24}$$

Where $l(n)$ and $r(n)$ are the left and right mixed stereo signals, pl_i and pr_i are the panning coefficients for the i^{th} independent source $s_i(n)$ (note: these two coefficients defined the amount we want to scale the volume (or pan) of the source in the left and right channels), j is the number of sources and N is the length of the mixtures in the audio samples.

In fact, we can look at these signals in the frequency domain by performing a short-time Fourier transform (STFT) on one sample frame of the time signal. It means the algorithm takes these two signals as its initial input data and then divides them into short overlapping frames. These frames are transformed into the frequency domain using the Fourier Transform **[Cahill 06]** using the following equations:

$$l_f(\tau,\omega) = \sum_{n=0}^{N-1} w(n-\tau) l(n) e^{\frac{-j\omega n}{N}} \tag{3.25}$$

$$r_f(\tau,\omega) = \sum_{n=0}^{N-1} w(n-\tau) r(n) e^{\frac{-j\omega n}{N}} \tag{3.26}$$

where $\omega = 2\pi f$ is sample rate, N is the frequency sampling factor and $2\pi / N$ is the frequency sampling interval. w is usually a Hamming window and τ is the frame number.

From the equation (3.23) and (3.24), the ratio of the left and right panning coefficients pl and pr of the i^{th} source can be expressed as:

$$g(i) = pl_i / pr_i \tag{3.27}$$

Similarly,

$$pl_l = g(i).pr_i \tag{3.28}$$

Where $g(i)$ is also called the intensity ratio. Adjust the intensity ratio to control the volume (or pan) between the right and left channel. Equation 3.28 implies we can scale the right channel to the same volume as the left channel for a given source $s_i(n)$. In fact, if we can expect to subtract the two audio channels after performing the scaling, then the source $s_i(n)$ can be cancelled out. (i.e. $l - g(i).r = 0$) Similarly as scaling the left channel which when subtracted from the right channel (i.e. $r - g(i).l = 0$) will be cancelled out as well. The question is how to define the gain scales factor when the panning coefficients are unknown as is the case of a stereo recording. The gain scale factors are defined as follows:

$$g(i) = i.(1/\beta) \tag{3.29}$$

for all i and for $0 \le i \le \beta$ where i and β are integer values.

From equations (3.25) (3.26) l_f and r_f are short time frequency domain representations of the left and right channel respectively and these equations also indicate to create a frequency-azimuth plane for the left and right channel individually. In equation (3.29) the azimuth resolution β refers to how many equally spaced gain scaling values of g we will use to construct the frequency azimuth plane. Thus, right and left channel azimuth-frequency planes are created according to the following equation:

$$Azl(\tau,\omega,i) = \left| r_f(\tau,\omega) - g(i).l_f(\tau,\omega) \right| \tag{3.30}$$

$$Azr(\tau,\omega,i) = \left| l_f(\tau,\omega) - g(i).r_f(\tau,\omega) \right| \tag{3.31}$$

for the integer values of i such that $0 \le i \le \beta$. Depending on the choice of β, the algorithm can create different resolution azimuth planes. Also large value of β will achieve more accurate azimuth discrimination but will increase the Fourier computational load because the frequency –azimuth plane will be an $N \times \beta$ array for each channel. In equation (3.30) (3.31), combining Azl and Azr creates the azimuth frequency plane of the mixture, here the “azimuth” we mentioned is purely a function of the intensity ration, created by the pan pot.

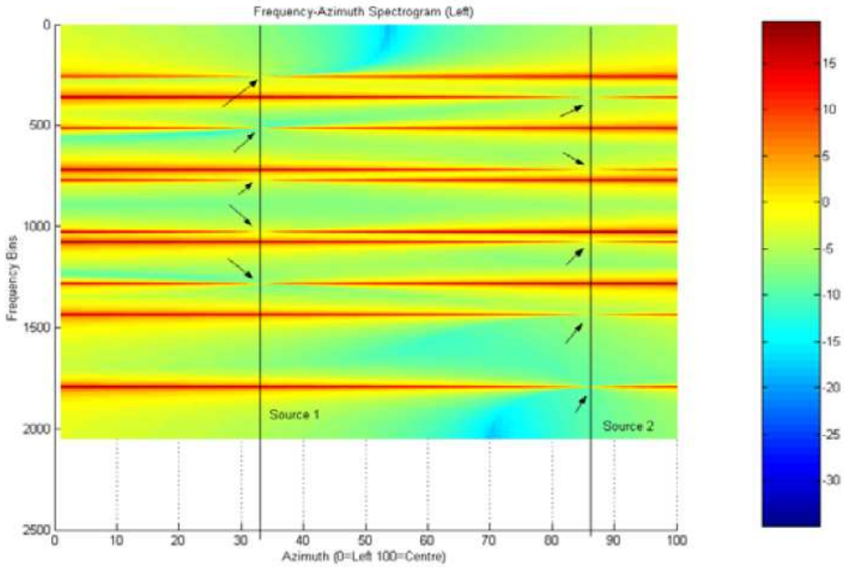


FIGURE 3.7: FREQUENCY-AZIMUTH PLANE. PHASE CANCELLATION HAS OCCURRED WHERE THE NULLS APPEAR AS SHOWN.

It can be seen that the arrows point out the cancellation points along the azimuth axis. For each frequency, there exist peaks (see figure 3.7) of varying magnitude resulting from the phase cancellations or the gain scale subtraction process. These peaks converge to a minimum value or even null (see figure 3.8), which corresponds to the location of that frequency within the azimuth plane. In the ADRess algorithm, for the purpose of resynthesis and so we need to invert these nulls, since the amount of energy lost through cancellation is proportional to the actual energy contributed by the source **[Coyle 07].**

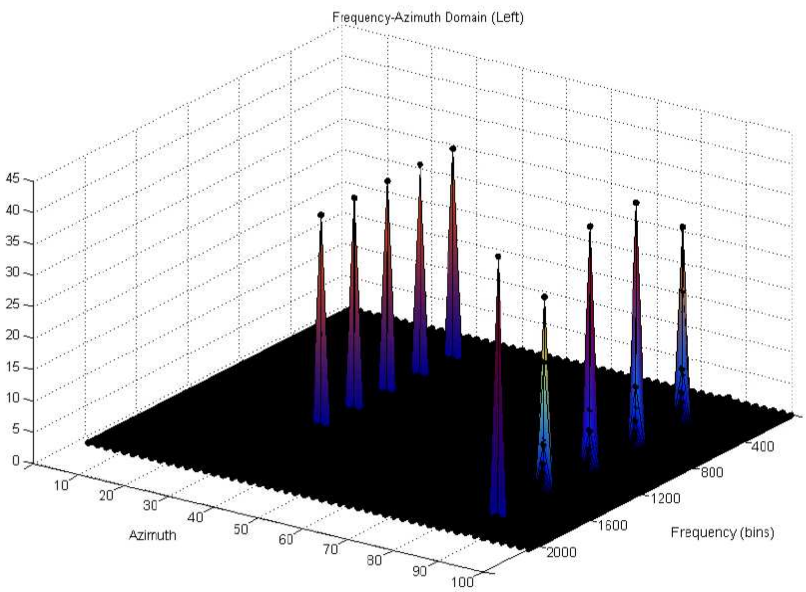


FIGURE 3.8: BY INVERTING THE NULLS OF THE FREQUENCY AZIMUTH COMPOSITION THE FREQUENCY COMPOSITION OF EACH SCORE CAN BE CLEARLY SEEN

The frequency azimuth spectrogram is assigned to the location of the null or minimum value having a magnitude equal to the difference between the value of the null and maximum value of the azimuth plane at the frequency. All other points in the azimuth plane are zeroed, also the plot in figure 3.8 and 3.9 represent the decomposition on a single frame basis.

To estimate the magnitude of frequency azimuth spectrogram we define:

$$Azl(\tau,\omega,i)=\begin{cases} Azl(\tau,\omega)_{\max}-Azl(\tau,\omega)_{\min}, & \text{if } Azl(\tau,\omega,i)=Azl(\tau,\omega)_{\min} \\ 0, & Otherwise. \end{cases} \tag{3.32}$$

$$Azr(\tau,\omega,i)=\begin{cases} Azr(\tau,\omega)_{\max}-Azr(\tau,\omega)_{\min}, & \text{if } Azr(\tau,\omega,i)=Azr(\tau,\omega)_{\min} \\ 0, & Otherwise. \end{cases} \tag{3.33}$$

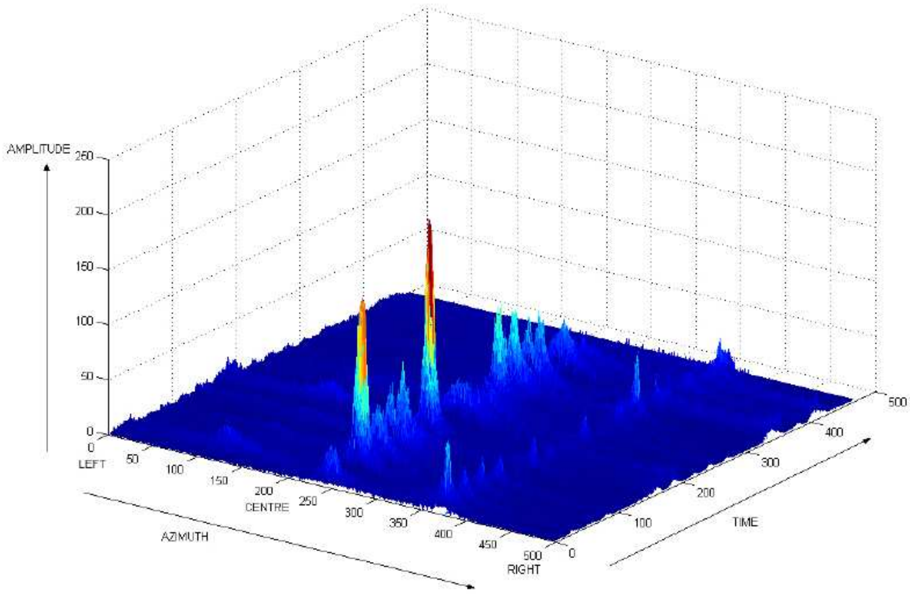


FIGURE 3.9: THE PLOT DISPLAYS THE ENERGY DISTRIBUTION OF SOURCE ACROSS THE STEREO FILED WITH RESPECT TO TIME. (A source in the centre can clearly be seen as well as several others less prominent sources in the left and right regions of the stereo field.) **[Coyle 07]**

From figure 3.9, by summing energy at all frequencies located at different points along the azimuth axis an energy distribution plot emerges. These peaks are used with the original bin phases to synthesise the source present at that azimuth. On the other hand, the plot shown in figure 3.9 is the ideal case that is no harmonic overlap between two sources.

3.4.3 Problem with ADRess

In practice, a single frequency bin may contain energy from multiple sources and also each source in a mixture is not strictly orthogonal with every other source. Then the peaks of these frequencies drift away from a source position and lead to locate at an erroneous azimuth where there may or may not be a source. In other words: there are two or more sources contributing to one frequency bin of the STFT and this results in sources not grouping perfectly on the azimuth planes. This is called "azimuth-smearing phenomenon" which results in frequencies being excluded from the resynthesis of the target source. Therefore, an "azimuth subspace width" H is defined, such that $1 \le H \le \beta$. This permits including peaks that have drifted away from the target azimuth in the resynthesis of the source. Two types of "azimuth subspace width" H are:

- A wide azimuth subspace will result in worse rejection of nearby sources.
- A narrow azimuth subspace will lead to poor resynthesis and missing harmonics (peak).

Meanwhile, an extra term the "discrimination index" d is also introduced at this point, where $0 \le d \le \beta$. This index, d, along with the azimuth subspace width, H, will define what portion of the frequency-azimuth plane is extracted for resynthesis.

3.4.4 Resynthesis

Collectively d and H will define what portion of the azimuth frequency plane will be used for resynthesis. In practice, we set the azimuth subspace to span [**Barry 04b] [Barry 04c]** from $d-(H/2)$ to $d+(H/2)$. The peaks for resynthesis are extracted using,

$$Y(\tau,\omega) = \sum_{i=d-H/2}^{i=d+H/2} Az(\tau,\omega,i) \tag{3.34}$$

Where Az is the combined $Azl - Azr$ inverse azimuth frequency plane and Y is the output time frequency points. The resultant Y must be left and right channel, each channel containing only the bin magnitudes pertaining to a particular azimuth subspace as defined by d and H. The bin phases from the original FFT are used to resynthesis the extracted source. Thus, the magnitude and phase component of each bin are combined and converted from polar to complex form. The azimuth subspace is then resynthesisd using the Inverse Short Time Fourier Transform (ISTFT), see equation 3.35.

$$X(n) = \frac{1}{\tau}\sum_{k=1}^{N} Y(\tau,\omega) e^{\frac{+j\omega n}{N}} \tag{3.35}$$

Where X is the output signal rendition. The resynthesisd time frames are then recombined using a simple overlap and add scheme **[Barry 04c]**.

In practice, the resynthesis is not perfect due to the fact of the power spectrum for each frame and source is an estimate. The windowing function (hamming window) is not preserved and therefore the frames at the output do not overlap perfectly. At the frame boundaries, there may be some distortion. Ideally, we need smoother frame transitions, so it can be resolved by multiplying the output frame by a suitable windowing function **[Barry 04c]**. In another words, by controlling the parameter d and H be set subjectively until the required separation is achieved.

3.5 Conclusions

ICA is a very general-purpose statistical technique that is used to find underlying factors by analyzing a set of observed random data. These observed random data are linearly transformed into components that are maximally independent of each other. ICA was originally developed to deal with sound source separation for audio processing, but now has been widely used in many different areas such as biomedical signal processing, image processing, telecommunications, and econometrics. In addition, ICA can be estimated as a latent variable model. There are two approaches that can be used to estimate ICA: optimization of the maximum of non-gaussianity can be used for the estimation of the ICA model; alternatively, maximum likelihood estimation or minimization of mutual information can also be used to estimate ICA.

PCA is a widely used statistical technique in many applications. It can be used to perform data compression while it can also be used to analyse data sets. However, PCA is not commonly associated with sound source separation. The fact that all the eigenvectors are orthogonal makes this technique useless for most mixtures, except artificial constructions where the columns of the mixing matrix are orthogonal. Even though this method is of little use for the separation of audio signals, this discussion gives a geometrical interpretation of the separation problem that can be useful in the following discussion of other techniques.

DUET is another technique which can be used for sound source separation. Theoretically it can separate any number of sources using just two mixed records if the sources are W-Disjoint orthogonal with each other. This technique is based on the fact that all frequencies coming from one source should have the same attenuation and time delay relative to the microphones. DUET is well suited to human speech separation; however, due to its assumption that all sources are W-Disjoint orthogonal with each other, its performance of musical signal separation is not as good as human speech separation. Nevertheless, using DUET to separate anechoically mixed and stereophonic music streams is an interesting research topic.

The ADRess algorithm is a new technique that can perform sound source separation by using the idea that sources occupy unique azimuth positions in the frequency-azimuth plane. This algorithm breaks down the sound mixture into frequency-azimuth subspaces, these subspaces can then be resynthesised according to different sources, resulting in source separation. In addition, the ADRess algorithm is able to separate multiple sources from only two mixtures. This feature makes it capable of enhancing sound quality in many areas. One of the possible applications is that by adding a second microphone in the mobile phone, the algorithm can perform noise reduction and sound quality enhancement in the mobile communication.

4. NMF algorithm

4.1 Introduction

In real-world many data or signals are non-negative and the corresponding hidden components have a physical meaning only when nonnegative. However, the data or variables with constrains such as sparsity and non-negativity is in order to seek a trade-off between the two goals of interpretability and statistical fidelity. In other words, we should make sure the estimated data components have physical sense and meaning; also need explain these data components are consistent and avoiding impurities (external noise).

Why non-negative and sparsity constrains? In general, compositional data are natural representations of the variables (features) of some whole or we call it is a sample space. For example, in image processing, involved variables and parameters are corresponded to pixels, and non-negative sparse decomposition is related to extraction of relevant parts from the targeted image **[Lee 99]**. Furthermore, it is note that non-negative matrix factorization (NMF) is an additive model which does not allow subtraction; therefore it often quantitatively describes that parts that comprise the whole object. In other words, NMF is usually to be considered as a parts-based representation.

The basic NMF problem can be stated as follows:

Give a nonnegative data matrix $\mathbf{V} \in \mathbb{R}_{+}^{M \times N}$ and a reduced rank R, find two nonnegative matrices $\mathbf{W} \in \mathbb{R}_{+}^{M \times R}$ and $\mathbf{H} \in \mathbb{R}_{+}^{R \times N}$ which factorize $\mathbf{V}$ as well as possible.

$$\mathbf{V} \approx \mathbf{WH}\,, \mathbf{V} = \mathbf{WH} + \mathbf{E}\,, \qquad v_{ik} \approx \sum_{j=1}^{R} w_{ij} h_{jk} \tag{4.1}$$

where $R < \min\{M, N\}$ is positive integer. The matrix $\mathbf{E} \in \mathbb{R}^{M \times N}$ represents approximation error.

4.2 Cost function

It is interesting to note that the NMF problem can be considered as natural extension of Nonnegative Least Squares (NLS) problem formulated as the following optimization problem.

In the NMF algorithm, Lee and Seung **[Lee 99]** suggested an approach similar to that used in Expectation- Maximization algorithms to iteratively update the factorization based on a given objective function. Two conventional NMF algorithms were introduced by them, each seeking to minimize a different object function or distance measure with a particular iterative update strategy chosen for its implementation ease and each optimizing its own measure of reconstruction quality: first measure is the Euclidean distance,

$$D_{ED}(\mathbf{V},\mathbf{W},\mathbf{H})=\frac{1}{2}\left\|\mathbf{V}-\mathbf{WH}\right\|^2 \tag{4.2}$$

In computing an NMF using the Euclidean Distance Algorithm, we wish find factors, $\mathbf{W}$ and $\mathbf{H}$, that minimize the objective function. In order to balance algorithm complexity and convergence speed and we use the following multiplicative update rules:

$$w_{ij} \leftarrow w_{ij}\frac{[\mathbf{VH}^T]_{ij}}{[\mathbf{WHH}^T]_{ij}}, \qquad h_{ij} \leftarrow h_{ij}\frac{[\mathbf{W}^T\mathbf{V}]_{jk}}{[\mathbf{W}^T\mathbf{WH}]_{jk}} \tag{4.3}$$

Where $[\cdot]_{ij}$ indicates that the noted divisions and multiplications are computed element-by element.

The second objective function commonly used in practical measure is the divergence; we called a generalized version of the Kullback-Leibler divergence, (also called the I-divergence) **[Sajda 03]**

$$D_{KL}(\mathbf{V}\,\|\,\mathbf{W},\mathbf{H})=\sum_{ik}\left(v_{ik}\log\frac{v_{ik}}{[\mathbf{WH}]_{ik}}-v_{ik}+[\mathbf{WH}]_{ik}\right) \tag{4.4}$$

The above objective function D_{KL} is not a distance measure due to it is not symmetric in $\mathbf{V}$ and approximation $\mathbf{WH}$. In this case, D_{KL} reduces to the Kullback-Leibler information measure used in statistics that quantifies in bits how close a probability distribution $\mathbf{V}$ is to a model distribution $\mathbf{WH}$, zero if the distributions match exactly and can potentially equal infinity. In addition, this object function is related to likelihood of generating the columns in $\mathbf{V}$ from the basis $\mathbf{W}$ and coefficients $\mathbf{H}$. Same again, in order to balance complexity and convergence speed, the following update rules are commonly used:

$$w_{ij} \leftarrow w_{ij}\frac{\sum_{k=1}^{N}(v_{ik}/[\mathbf{WH}]_{ik})h_{jk}}{\sum_{k=1}^{N}h_{jk}}, \quad h_{jk} \leftarrow h_{jk}\frac{\sum_{i=1}^{M}w_{ij}(v_{ik}/[\mathbf{WH}]_{ik})}{\sum_{i=1}^{M}w_{ij}} \tag{4.5}$$

where the subscripts again indicate element by element division or multiplications.

Currently most existing approaches minimize only one kind of cost function by alternately switching between sets of parameters. In this thesis we use a more general approach (algorithm) in which instead of one cost function we use called multi-layer NMF using alternating minimization of two cost functions; one of them is minimized with respect to $\mathbf{W}$ and the other one with respect to $\mathbf{H}$. The following pseudo code represents most NMF algorithm to AEC application discussed in next two chapters.

Algorithm 4.1: Multi-layer NMF two cost function minimization

Input: $\mathbf{V} \in \mathbb{R}_+^{M\times N}$; input matrix data. R: rank of factorization

Output: $\mathbf{W} \in \mathbb{R}_+^{M\times R}$ and $\mathbf{H} \in \mathbb{R}_+^{R\times N}$; the given cost functions are minimized.

1 Begin

2 $\quad \mathbf{H} = \mathbf{V}, \mathbf{W} = \mathbf{I}$

3 $\quad$ **for** l = 1 to L **do**

4 $\qquad$ Initialize randomly $\mathbf{W}_{(l)}$ and $\mathbf{H}_{(l)}$

5 $\qquad$ **repeat**

6 $\qquad\quad \mathbf{W}_{(l)} = \arg \min\limits_{\mathbf{W}_{(l)} \geq 0} \left\{ D_1 \left(\mathbf{H} \,\|\, \mathbf{W}_{(l)} \mathbf{H}_{(l)} \right) \right\}$ for fixed $\mathbf{H}_{(l)}$

7 $\qquad\quad \mathbf{H}_{(l)} = \arg \min\limits_{\mathbf{H}_{(l)} \geq 0} \left\{ D_2 \left(\mathbf{H} \,\|\, \mathbf{W}_{(l)} \mathbf{H}_{(l)} \right) \right\}$ for fixed $\mathbf{W}_{(l)}$

8 $\qquad$ **until** a convergence condition is met

9 $\qquad \mathbf{H} = \mathbf{H}_{(l)}$

10 $\qquad \mathbf{W} \leftarrow \mathbf{W}\mathbf{W}_{(l)}$

11 $\quad$ **end**

12 End

Table 4.1: Multi-layer NMF using alternating minimization of two cost function

Here is the MATLAB function to perform basic NMF algorithm which is mainly used in the rest of the thesis:

```
function [H,W] = NMF(spec,R,num_iter);
```

```
V = abs(spec(:,1:513))';

index = size(V);   % must be nonnegative
M = index(1,1);
N = index(1,2);

W = rand(M,R);     % random initialization
H = rand(R,N);

num_iter = 100;    % can be adjusted

for i = 1:1:num_iter
    W = W.*((V./(W*H+1e-9))*H')./(ones(M,N)*H');
    H = H.*(W'*(V./(W*H+1e-9)))./(W'*ones(M,N));
end
```

Table 4.2: Standard NMF Algrithm in MATLAB Form

4.3 Initialization of NMF

The motivation behind NMF is that besides the dimensionality reduction sought in many image or signal processing applications. As defined, the NMF problem is a more general instance of the case where the two nonnegative matrices whose product exactly equals the original matrix. In common sense, there is no guarantee that an exact nonnegative factorization exists for arbitrary R which is rank of approximation. It is known, however, that if $\mathbf{V} \geq 0$, then the nonnegative rank and nonnegative $\mathbf{W}$ and $\mathbf{H}$ having that number as rank so that $\mathbf{V} = \mathbf{WH}$ holds exactly **[Gregory 83].** Furthermore, NMF is a part of nonconvex optimization problem with inequality constraints and iterative methods become necessary for its solution **[Bertsekas 99][Salakhutdinov 03].** However, the current NMF algorithms typically converge comparative slowly and then at local minima. Most algorithms for NMF are iterative and required initial values of $\mathbf{W}$ and $\mathbf{H}$, and many authors prescribe initializing $\mathbf{W}$ and $\mathbf{H}$ with random non-negative numbers. A suitable chosen initialization, can lead to faster convergence, and since the solution of most NMF algorithm problems is not unique, different initializations can lead to different solutions.

4.3.1 Optimization problem

The solution and convergence provided by the NMF algorithm usually highly depend on initial conditions, typically starting guess values, especially in a multivariate context. Therefore, it is important to have efficient and consistent ways for initialization matrices $\mathbf{W}$ and $\mathbf{H}$. Due to the iterative nature of NMF algorithms, most of them in the literature use random nonnegative initialization for ($\mathbf{W}$, $\mathbf{H}$). Iterates converge to a local minimum and poor initializations also often result in slow convergence, and in certain instances may lead even to an incorrect or irrelevant solution which we aim to. The problem of selecting an appropriate starting point or starting initialization matrices becomes even more complicated for large-scale NMF problems **[Dhillon 01]** and when certain structures or constraints are imposed on the factorized matrices involved. In the real time case, initialization in NMF plays a key role since the objective function to be minimized may have local minima, and the intrinsic alternating minimization in NMF is nonconvex, even though the objective function is strictly convex with respect to one set of variables. The issues of initialization in NMF have been widely discussed in the literature **[Baeza 92] [Carmona 06] [Ruspini 69].**

4.3.2 Basic initialization for NMF algorithm

As a rule of thumb, we can obtain a robust initialization using the following three steps which the main idea is to find better initial estimates with the multi-start initialization algorithm:

- First, we can generate *S* (number of restarts) by a search method to initial matrices $\mathbf{W}$ and $\mathbf{H}$. This could be based on random starts or the output from a simple conventional NMF algorithm. The parameter *S* depends on the number of required iterations. We typically set *S* between 15 and 20.
- Run a specific NMF algorithm for each set of initial matrices and with a fixed but small number of iterations (15-20). As a result, the NMF algorithm provides *S* initial estimates of the matrices $\mathbf{W}^{(s)}$ and $\mathbf{H}^{(s)}$.
- Select the estimates ("candidates"), we denoted that $\mathbf{W}^{(s_{\min})}$ and $\mathbf{H}^{(s_{\min})}$ correspond to the lowest value of the cost function (the best likelihood) among the *R* trials as initial values for the final factorization.

The following pseudo code represents above steps:

Algorithm 4.2: Multi-start initialization

Input: $\mathbf{V} \in \mathbb{R}_+^{M \times N}$: input matrix data,

R: rank of factorization, S: number of restarts,

K_{init}, K_{fin}: number of alternating steps for initialization and completion

Output: $\mathbf{W} \in \mathbb{R}_+^{M \times R}$ and $\mathbf{H} \in \mathbb{R}_+^{R \times N}$; the given cost functions are minimized.

1 Begin

2 **parfor** s = 1 to S **do** % process in parallel mode

3 Initialize randomly $\mathbf{W}^{(0)}$ or $\mathbf{H}^{(0)}$

4 $\left\{\mathbf{W}^{(s)}, \mathbf{H}^{(s)}\right\} \leftarrow \text{nmf\_algorithm}\left(\mathbf{V}, \mathbf{W}^{(s)}, \mathbf{H}^{(s)}, K_{init}\right)$

5 $d_s = D\left(\mathbf{V} \,||\, \mathbf{W}^{(s)}\mathbf{H}^{(s)}\right)$

6 **endfor**

7 $s_{\min} = \arg\min_{1 \le s \le S} d_s$

8 $\left\{\mathbf{W}, \mathbf{H}\right\} \leftarrow \text{nmf\_algorithm}\left(\mathbf{V}, \mathbf{W}^{(s_{\min})}, \mathbf{H}^{(s_{\min})}, K_{fin}\right)$

9 End

Table 4.3: Multi-start initialization to initial NMF alogorithm

Thus, the multi-start initialization selects the initial estimates for $\mathbf{W}$ and $\mathbf{H}$ which give the steepest decrease in the assumed objective (cost) function $D(\mathbf{V} \,||\, \mathbf{WH})$ via alternating steps.

4.3.3 Termination condition

In many practical situations, the iterations usually continue until some combinations of termination conditions or stopping criteria are satisfied. There are several possible stopping criteria for the iteration algorithm used in NMF:

- The cost function achieves a zero-value or a value just below a given threshold ε, also during the NMF divergence updating, the stopping criterion can be adjusted, for example: Frobenius norm of cost function,

$$D_F^{(k)}\left(\mathbf{V} \,||\, \hat{\mathbf{V}}^{(k)}\right) = \left\|\mathbf{V} - \hat{\mathbf{V}}^{(k)}\right\|_F^2 \le \varepsilon, \quad \hat{\mathbf{V}}^{(k)} = \mathbf{W}^{(k)}\mathbf{H}^{(k)} \tag{4.6}$$

  $\hat{\mathbf{V}}$ is estimated value.

- There is little or no improvement between successive iterations in the minimization of a cost function, for example: Frobenius norm of the estimated matrices,

$$D_F^{(k+1)}\left(\hat{\mathbf{V}}^{(k+1)} \,||\, \hat{\mathbf{V}}^{(k)}\right) = \left\|\hat{\mathbf{V}}^{(k)} - \hat{\mathbf{V}}^{(k+1)}\right\|_F^2 \leq \varepsilon \tag{4.7}$$

or Ratio of the distance

$$\frac{\left|D_F^{(k)} - D_F^{(k-1)}\right|}{D_F^{(k)}} \leq \varepsilon \tag{4.8}$$

- There is little or no change in the updates for factor matrices $\mathbf{W}$ and $\mathbf{H}$.
- The number of iterations achieves or exceeds a predefined maximum number of iterations and the maximum number of iterations also can be adjusted.

4.4 Convolutive NMF

The Convolutive NMF (CNMF) is a natural extension and generalization of the standard NMF. The standard NMF represents regularly repeating patterns which span multiple columns of the $\mathbf{V}$ matrix using a number of different bases to describe the entire sequence. CNMF uses a single basis function that spans the pattern length. This kind of situation can be very frequently found when analysing audio signals. In the Convolutive NMF, we process a set of nonnegative matrices or patterns which are horizontally shifted (or time delayed) versions of the primary matrix $\mathbf{W}$ **[Zass 05]**. In the simplest form the CNMF can be defined as (see Figure: 4.1)

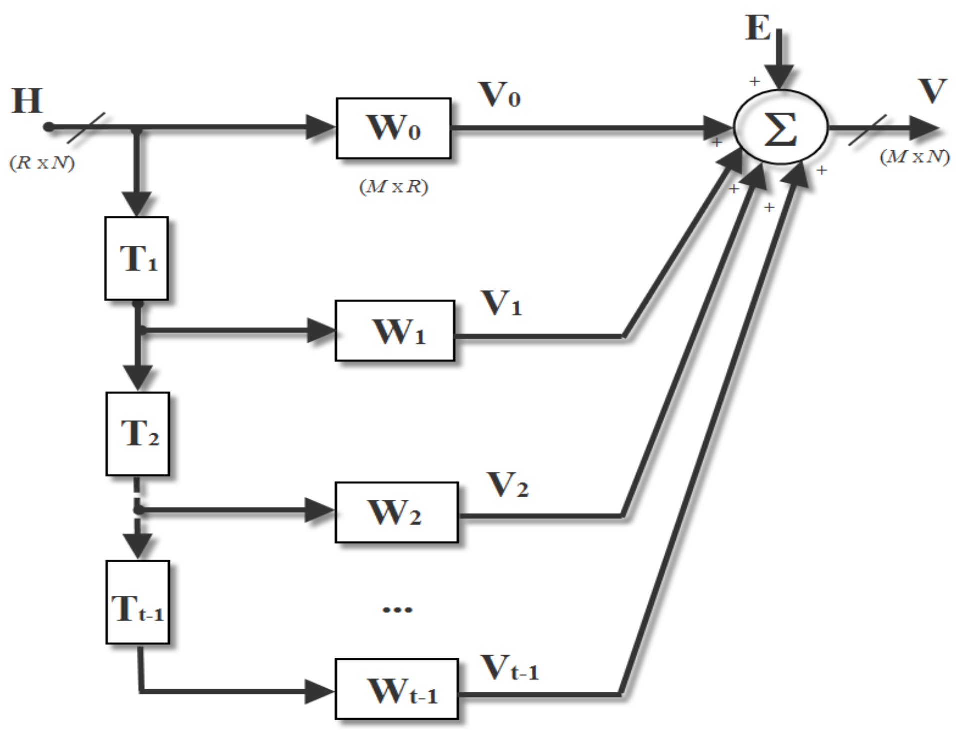


FIGURE 4.1: ILLUSTRATION OF CONVOLUTION NMF

In the previous section, we saw the NMF uses a matrix product $\mathbf{V} \approx \mathbf{WH}$ to reconstruct the estimated data matrix $\mathbf{V}$, in the convolutive Non-Negative Matrix Factorization they extend this expression to:

$$\mathbf{V} \approx \sum_{t=0}^{T-1} \mathbf{W}_t \cdot \overset{t\rightarrow}{\mathbf{H}} + \mathbf{E} \tag{4.9}$$

where $\mathbf{V} \in \mathbb{R}_+^{M\times N}$ is a given input data matrix to be decomposed, $\mathbf{W}_t \in \mathbb{R}_+^{M\times R}$ is a set of unknown nonnegative matrices, $\mathbf{H} = \overset{0\rightarrow}{\mathbf{H}} \in \mathbb{R}_+^{R\times N}$ is the matrix representing coding information of the source (such as position of activation and it's amplitude). Here $\overset{t\rightarrow}{\mathbf{H}}$ is a t column shifted version of $\mathbf{H}$. In other words, $\overset{t\rightarrow}{\mathbf{H}}$ denotes the t positions (columns) shifting operator to the right, with the columns shifted in from outside the matrix set to zero. This shift (sample-delay) is performed by a basic operator denoted as $\overset{i\rightarrow}{(\bullet)}$. $\overset{\leftarrow i}{(\bullet)}$ performs the reverse. The matrix $\mathbf{E} \in \mathbb{R}^{M\times N}$ represents approximation error.

The i^{th} column of $\mathbf{W}_t$ describes the spectrum of the i object t time steps after the object has begun.

Equation 4.12 is a summation of convolution operations between corresponding elements from a set of two-dimensional bases $\mathbf{W}$ and a set of weights $\mathbf{H}$.

The set of i^{th} columns of $\mathbf{W}(t)$ defines a two-dimensional structure. This matrix will be shifted and scaled by convolution across the axis of t with the i^{th} row of $\mathbf{H}$. The resulting reconstruction will be a summation of all the basis convolution results for each of the R bases.

The estimation of the appropriate set of matrices $\mathbf{W}(t)$ and $\mathbf{H}$ to approximate V is based on the framework of NMF that Lee and Seung used in **[Lee 99]**. In accordance to the NMF cost function, they defined the Convolutive NMF cost function as:

$$D = \| \mathbf{V} \bullet In\left(\frac{\mathbf{V}}{\hat{\mathbf{V}}}\right) | - \mathbf{V} + \hat{\mathbf{V}} \|_F \tag{4.10}$$

Where $\hat{V}$ is the approximation of V defined as:

$$\hat{\mathbf{V}} = \sum_{t=0}^{T-1} \mathbf{W}(t) \cdot \overset{t\rightarrow}{\mathbf{H}} \tag{4.11}$$

They decomposed the above cost function to a series of simultaneous NMF approximations according to the linearity property, one for each value of *t*. Then they optimized the above cost function by optimizing this set of *T* NMF approximations. For each NMF approximation they updated the equivalent *W(t)* and the appropriately shifted *H*. This gives the convolutive NMF updates equations which are:

$$\mathbf{H} = \mathbf{H} \bullet \frac{\mathbf{W}(t)^T \cdot \left[\frac{\mathbf{V}}{\hat{\mathbf{V}}}\right]}{\mathbf{W}(t)^T \cdot 1}, \mathbf{W}(t) = \mathbf{W}(t) \bullet \frac{\left[\frac{\mathbf{V}}{\hat{\mathbf{V}}}\right] \cdot \overset{t\rightarrow^T}{\mathbf{H}}}{1 \cdot \overset{t\rightarrow^T}{\mathbf{H}}} \tag{4.12}$$

They updated *H* and *W(t)* in every updating iteration and each *t*. Actually for each *t*, *W(t)* is updated by the corresponding NMF, but *H* is shared and shifted across all *t*'s in an iteration. Update *W(t)* and *H* for each *t* may result in a mistaken estimate of *H* with the update for *t = T −1* dominating over others. Therefore it is best to update all *W(t)* first and then assign to *H* the average of all the NMF sub-problems:

$$\mathbf{H} = \left(\mathbf{H} \bullet \frac{\mathbf{W}(t)^T \cdot \overset{\leftarrow t}{\left[\frac{\mathbf{V}}{\hat{\mathbf{V}}}\right]}}{\mathbf{W}(t)^T \cdot 1} \right), \forall t \tag{4.13}$$

In terms of computational complexity this technique depends mostly on *T*. If *T = 1* then it reduces to standard NMF, otherwise it is burdened with extra matrix updates equivalent to one NMF per unit of *T* **[Smaragdis 07]**.

In addition, we utilize this idea, realize it in the MATLAB simulation environment and implement it to perform the specific application which is Acoustic Echo Cancellation. Experimental results are presented in chapter 6 and we will show both NMF and CNMF approached to acoustic echo cancellation.

4.5 Conclusions

In this chapter we have presented two different models (NMF and CNMF), graphical and mathematical representations for NMF and the related matrix factorizations and decompositions. Our emphasis has been on the formulation of the problems and establishing relationships and links among different models. Each model usually provides a different interpretation of the data and may have different applications. Various equivalent representations have been presented which will serve as a basis for the development of learning algorithms in next two chapters.

5. Acoustic echo cancellation MATLAB experiment

This chapter is organized as follows: in section 5.1 we present a detailed description of numerical aspects of the Least Mean Square (LMS) algorithm. The second section is focused on the different versions of the LMS algorithm simulation in MATLAB and an experiment result will be presented. In section 6.3, we use two set of NMF experiments (standard NMF and convolution NMF) to perform AEC. The convolution NMF experiment is based on the process of using standard NMF. The purpose of these experiments is finding a better solution for AEC and comparing the results with LMS counterparts.

5.1 Least Mean Square Solution for Acoustic Echo Cancellation

5.1.1 Steepest Decent Algorithm

The Steepest Decent algorithm is a method of gradient decent minimization or an "adaptive" approach. We can find a single global minimum corresponding to the optimum weights based on the quadratic cost function.

Formally the gradient is defined as:

$$\mathbf{g} = \nabla_{\mathrm{w}} J = \frac{\partial J}{\partial \mathbf{w}} \tag{5.1}$$

Since $\mathbf{g}$ is the direction of steepest ascent $-\mathbf{g}$ gives us the direction of steepest descent.

The iterative procedure of the steepest or gradient descent method as follows:

Start with an arbitrary initial weights vectors $\mathbf{w}_k, k = 0$

Calculate the gradient $\mathbf{g}_k = \dfrac{\partial J}{\partial \mathbf{w}_k} = 2[\mathbf{R}\mathbf{w}_k - \mathbf{p}]$

Update the weights vector in the direction of steepest descent using the rule:

$$\mathbf{w}_{k+1} = \mathbf{w}_k - \mu \mathbf{g}_k \tag{5.2}$$

Where μ is a positive constant known as the step size or learning rate.

Set $k = k+1$ and repeat until the algorithm converges.

5.1.2 LMS Derivation

It is simple to derive the Least-Mean-Square based on the steepest decent algorithm. We have Mean Square Error (MSE) cost function $J(\mathbf{w}) = E[d_k^2] - 2\mathbf{w}^T\mathbf{p} + \mathbf{w}^T\mathbf{R}\mathbf{w}$, both d_k and $\mathbf{x}_k$ are jointly wide-sense stationary. Also we have the Wiener Solution (Eq. 5.3) $\mathbf{w}^* = \mathbf{R}^{-1}\mathbf{p}$.

Therefore, a steepest-decent-based algorithm can be used to search the Wiener solution as follows:

$$\begin{aligned}\mathbf{w}_{k+1} &= \mathbf{w}_k - \mu\mathbf{g}_{\mathbf{w}}^k \\ &= \mathbf{w}_k - \mu[-2\mathbf{p}_k + 2\mathbf{R}_k\mathbf{w}_k] \\ &= \mathbf{w}_k + 2\mu[d_k\mathbf{x}_k - \mathbf{x}_k^T\mathbf{x}_k\mathbf{w}_k] \\ &= \mathbf{w}_k + 2\mu e_k\mathbf{x}_k\end{aligned} \tag{5.3}$$

This is the Least-mean-square algorithm that was proposed by Bernard Widrow in the late 1960s **[Widrow 60].**

5.1.3 Gradient behaviour

The ideal search direction is on the MSE surface for the optimum coefficient vector solution (Eq. 5.8). In the LMS algorithm, instantaneous estimates of $\mathbf{R}$ and $\mathbf{p}$ are used to determine the search direction:

$$\hat{\mathbf{g}}_{\mathbf{w}}^k = 2[\mathbf{x}_k\mathbf{x}_k^T\mathbf{w}_k - d_k\mathbf{x}_k] \tag{5.4}$$

In general, the LMS gradient direction has the tendency to approach the ideal gradient direction since for a fixed coefficient vector (filter weight factor) $\mathbf{w}$ and its convergence behaviour is different from the steepest-decent algorithm counterpart. Hence,

$$\begin{aligned}E(\hat{\mathbf{g}}_{\mathbf{w}}^k) &= 2\left\{E[\mathbf{x}_k\mathbf{x}_k^T]\mathbf{w} - E[d_k\mathbf{x}_k]\right\} \\ &= \mathbf{g}_{\mathbf{w}}\end{aligned} \tag{5.5}$$

Under an ergodic condition, the average direction tends to $\mathbf{g}_{\mathbf{w}}$ with a fixed $\mathbf{w}$ vector when calculated for a large number of inputs and reference signals.

5.1.4 Condition for the LMS convergence

Determine the range of convergence factor μ of the LMS algorithm. Firstly, we should know the error in the filter coefficients as related to the ideal coefficient vector $\mathbf{w}^*$, then gives:

$$\Delta\mathbf{w}_k = \mathbf{w}_k - \mathbf{w}^*. \tag{5.6}$$

Using Eq.5.6 the gradient $\mathbf{g}_k$ is given by:

$$\mathbf{g}_k = 2\mathbf{R}\Delta\mathbf{w}_k \tag{5.7}$$

and the steepest-decent update rule, $\mathbf{w}_{k+1} = \mathbf{w}_k - \mu\mathbf{R}\Delta\mathbf{w}_k$, we have:

$$\mathbf{w}_{k+1} = \mathbf{w}_k - 2\mu\mathbf{R}\Delta\mathbf{w}_k \tag{5.8}$$

Subtracting $\mathbf{w}*$ from both sides and colleting terms gives:

$$\begin{aligned} &\Rightarrow \mathbf{w}_{k+1} - \mathbf{w}* = \mathbf{w}_k - \mathbf{w}* - 2\mu\mathbf{R}\Delta\mathbf{w}_k \\ &\Rightarrow \Delta\mathbf{w}_{k+1} = \Delta\mathbf{w}_k - 2\mu\mathbf{R}\Delta\mathbf{w}_k \end{aligned} \tag{5.9}$$

Finally we obtain:

$$\Delta\mathbf{w}_{k+1} = [I - 2\mu\mathbf{R}]\Delta\mathbf{w}_k \tag{5.10}$$

If it is assumed that the elements of $\mathbf{x}_k$ are statistically independent of the element of $\Delta\mathbf{w}_k$ and e_k; the expected error in the coefficient vector from Eq. 5.10 is simplified as follows:

$$E[\Delta\mathbf{w}_{k+1}] = (I - 2\mu\mathbf{R})E[\Delta\mathbf{w}_k] \tag{5.11}$$

Starting with an initial weight deviation $\Delta\mathbf{w}_o = \mathbf{w}_o - \mathbf{w}*$ and it is in order to guarantee convergence, so the condition we require is $\lim\limits_{k\to\infty} \Delta\mathbf{w}_k = 0$ and hence:

$$\lim_{k\to\infty} [I - 2\mu\mathbf{R}]^k = 0 \tag{5.12}$$

To find acceptable values for μ, we can use the eigenvalue/eigenvector decomposition of $\mathbf{R}$.

So $\mathbf{R}$ can be written as $\mathbf{Q}\Lambda\mathbf{Q}^T$ where Λ is the diagonal eigenvalue matrix of $\mathbf{R}$ and $\mathbf{Q}$ is the corresponding orthonormal eigenvector matrix. Thus Eq.5.12 becomes:

$$\lim_{k\to\infty} [\mathbf{I} - 2\mu(\mathbf{Q}\Lambda\mathbf{Q}^T)]^k = 0 \tag{5.13}$$

We can rewrite Eq.5.13 using the matrix calculation fact that $\mathbf{Q}\mathbf{Q}^T = \mathbf{Q}^T\mathbf{Q} = \mathbf{I}$ and $[\mathbf{Q}\mathbf{R}\mathbf{Q}^T]^k = \mathbf{M}\mathbf{R}^k\mathbf{M}^T$:

$$\begin{aligned} &\lim_{k\to\infty} [\mathbf{Q}\mathbf{Q}^T - 2\mu(\mathbf{Q}\Lambda\mathbf{Q}^T)]^k = 0 \\ &\Rightarrow \lim_{k\to\infty} [\mathbf{Q}[\mathbf{I} - 2\alpha\mu\Lambda]\mathbf{Q}^T]^k = 0 \\ &\Rightarrow \lim_{k\to\infty} [\mathbf{Q}[\mathbf{I} - 2\mu\Lambda]^k\mathbf{Q}^T] = 0 \end{aligned} \tag{5.14}$$

Since is the constant eigenvector matrix, we can simplify Eq.5.14 and gives:

$$\lim_{k\to\infty}[I-2\mu\Lambda]^k=0$$

$$\Rightarrow \lim_{k\to\infty}\begin{bmatrix}[1-2\mu\lambda_1]^k & 0 & \cdots & 0\\ 0 & [1-2\mu\lambda_2]^k & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & [1-2\mu\lambda_N]^k\end{bmatrix}=0 \qquad (5.15)$$

Therefore, for the convergence we require:

$$\lim_{k\to\infty}[1-2\mu\lambda_i]^k=0 \quad \text{for all } \lambda_i \qquad (5.16)$$

Thus, it provides: $|1-2\mu\lambda_{\max}|<1$.

The condition for convergence (stability) of the mean of the weight vector is:

$$0<\mu<\frac{1}{\lambda_{\max}} \qquad (5.17)$$

Here $\lambda_{\max}$ is the largest eigenvalue of the input correlation matrix $\mathbf{R}=\mathbf{x}_k\mathbf{x}_k^T$ .The value of μ in this range guarantees that all elements of the diagonal matrix in the Eq.5.15 tend to zero as $k\to\infty$. The critically damped point is given by $1-2\mu\lambda_i=0$ which we get a step size: $\mu=\frac{1}{2\lambda_i}$.

5.1.5 Rate of convergence of LMS algorithm

The rate of convergence of LMS algorithm is identical to the Steepest-Decent algorithm. A useful way of quantifying rate of convergence is to measure it in terms of equivalent weight error exponential decay time constant along each of the principal axes, gives:

$$\exp\left(-\frac{k}{\tau_i}\right)=[1-2\mu\lambda_i]^k \qquad (5.18)$$

Then solving for the time constant in terms of $\tau_{\max}$ gives,

$$\tau_{\max}=\frac{-1}{\ln[1-2\mu\lambda_{\min}]} \qquad (5.19)$$

The largest time constant corresponds to the smallest eigenvalue and it can determine the rate of convergence of the overall algorithm. By defining a normalized step size (convergence factor) $\mu=\frac{\mu_\lambda}{\lambda_{\max}}$ where $0<\mu_\lambda<1$ for stability. Eq.5.19 can be rewritten as:

$$\tau_{\max} = \frac{-1}{\ln\left[1 - 2\mu_\lambda \frac{\lambda_{\min}}{\lambda_{\max}}\right]} = \frac{-1}{\ln\left[1 - \frac{2\mu_\lambda}{C(\mathbf{R})}\right]} \tag{5.20}$$

Here $C(\mathbf{R}) = \frac{\lambda_{\min}}{\lambda_{\max}}$ is called condition number of $\mathbf{R}$. By approximation $\ln(1-x) = -x$ for small x, Eq.5.20 gives: $\tau_{\max} \cong \frac{C(\mathbf{R})}{2\mu_\lambda}$ for poorly conditioned problem. W of the can see that the rate of convergence of the LMS algorithm is directly proportional to the condition number of the input correlation matrix and inversely proportional to the normalized step size.

We can rewrite Eq. 5.19 as:

$$\tau_{\max} = \frac{1}{2\mu\lambda_{\min}} \tag{5.21}$$

The constant above is for the convergence of the weights to their optimum values. In addition, the corresponding learning curve time constant is defined as:

$$\tau_{MSE} = \frac{\tau_{\max}}{2} = \frac{1}{4\mu\lambda_{\min}} \tag{5.22}$$

5.1.6 Steps associated with the NLMS algorithm

An alternative formulation of LMS-based algorithm known as the Normalized Least Mean Square algorithm (NLMS).The convergence factor is chosen with the objective of achieving a faster convergence. The weight update rule is defined as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + 2\mu \frac{e_k \mathbf{x}_k}{\mathbf{x}_k^T \mathbf{x}_k} \tag{5.23}$$

and guarantees the convergence when $0 < \mu < 1$. The normalized LMS algorithm usually converges faster than the conventional LMS algorithm, since it utilizes a variable convergence factor to obtain the reduction of instantaneous output error. The major advantage of NLMS is that the learning rate is independent of $\mathbf{x}_k^T \mathbf{x}_k$. However, the convergence factor μ is usually chosen as fixed value in the NLMS in order to control the misadjustment (see eq. 5.29) since all the derivations are based on instantaneous values of the squared errors and not on the MSE. Additional, a parameter γ should be added in order to avoid large steps when $\mathbf{x}_k^T \mathbf{x}_k$ becomes small. The parameter γ also means it can overcome potential numerical instability in the update of the weights. In practise, a small positive adaption constant ε (usually far smaller than 1) multiplies the step size to achieve a proper compromise between the convergence rate and the misadjustment **[Haykin 02]**. The updated coefficient of NLMS is given by:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + 2\mu \frac{e_k \mathbf{x}_k}{\varepsilon\gamma + \mathbf{x}_k^T \mathbf{x}_k} \tag{5.24}$$

In summary, there are similar processes to the LMS algorithm as follows:

1) Initial condition: input signal $\mathbf{x}$ and weight vector $\mathbf{w}_k = [0, \cdots, 0]^T$, $k = 0,1,2,....$

2) The convergence factor is $0 < \mu < 1$.

3) γ is a small constant.

Calculate the output for the current training input: $y_k = \mathbf{w}^T \mathbf{x}_k$

Estimate the error: $e_k = d_k - y_k$

Update the weight vector: $\mathbf{w}_{k+1} = \mathbf{w}_k + 2\mu \frac{e_k \mathbf{x}_k}{\varepsilon\gamma + \mathbf{x}_k^T \mathbf{x}_k}$ with positive constant step size μ .

5.1.7 Excess Mean-Square Error and Misadjustment

The LMS algorithm uses a noisy estimate of the gradient the Mean Square error (MSE). Thus, misadjustment is defined as the ratio of the excess MSE to the minimum MSE and is a measure the performance of the adaptive process tracks the true Wiener solution – i.e. it is a measure of the "cost of adaptability".

The excess in the Mean Square Error is given by:

$$\text{Excess MSE} = E[J_{\text{w}} - J_{\min}] = E[\Delta\mathbf{w}_k^T \mathbf{R} \Delta\mathbf{w}_k] = E[\mathbf{m}_k^T \Lambda \mathbf{m}_k] \tag{5.25}$$

Since $\Delta\mathbf{w}_k = \mathbf{w}_k - \mathbf{w}^*$ and Λ is diagonal as mentioned before, this can be written as a sum (non-matrix form):

$$J_{MSE} = \sum_{i=1}^{N+1} \lambda_i E[m_{ik}^2] \tag{5.26}$$

If the LMS has converged the only variation in the weights will be due to gradient noise causing the weights to the wander around the minimum value. Therefore,

$$E[m_{ik}^2] \approx \mu J_{\min} \tag{5.27}$$

And the excess MSE formula becomes:

$$J_{MSE} = \mu J_{\min} \sum_{i=1}^{N+1} \lambda_i = \mu J_{\min} \mathbf{tr}[\mathbf{R}] \qquad (5.28)$$

Finally, we obtain the NLMS misadjustment M ,is defined as

$$M = \frac{\text{Excess MSE}}{J_{\min}} = \frac{\mu J_{\min} \mathbf{tr}[\mathbf{R}]}{J_{\min}} = \mu \mathbf{tr}[\mathbf{R}] \qquad (5.29)$$

The trade-off analysis among the rate of convergence, the amount of excess mean-square error, and the ability of the adaption to track the signal is important. Thus misadjustment is directly proportional to step-size. We therefore have to trade rate of adaption with accuracy as measured by misadjustment.

5.2 Using different LMS Algorithms to Perform AEC

5.2.1 Experiment principles and procedure

The experiment is about the normalized least mean square (LMS) algorithm. The application in this experiment is echo cancellation in real-time VoIP scenario. Actually, we recorded the speech data into MATLAB as testing data. Here we need to point out, is that echo can't origin from a VoIP network. But delay time due to codec and buffering quickly makes even the slightest echo received very annoying. Echo is generated by digital with 4 wire to analogue with 2 wire conversions either in the public switched telephone network (PSTN). As aforementioned chapter five, there are a couple of mechanisms to prevent echo that is *ERLE* (Echo Return Loss Enhance).

ERLE is often named echo canceller. *ERLE* is expressed in *dB*. The higher the value, the better the echo canceller. Furthermore, *ERLE* as a function of the discrete-time index *n* provides information about the convergence behaviours of the canceller. The input signal of an echo canceller system is often a speech signal. Speech signals are non-stationary, which makes the choice of step size rather difficult. One advantage of the NLMS algorithm is the choice of its step size. In the previous section we have detailed discussed LMS and NLMS in math form. In this section, we write a series function via MATLAB simulation to perform echo cancellation. Meanwhile, we will compare the *ERLE*-curves of LMS and NLMS to tell why the NLMS do work better than the stand LMS for input speech signal with strong varying amplitude. To compare LMS and NLMS, we also introduced another more efficient LMS algorithm named FastLMS.

In the experiment we used the setup shown in figure 6.1.

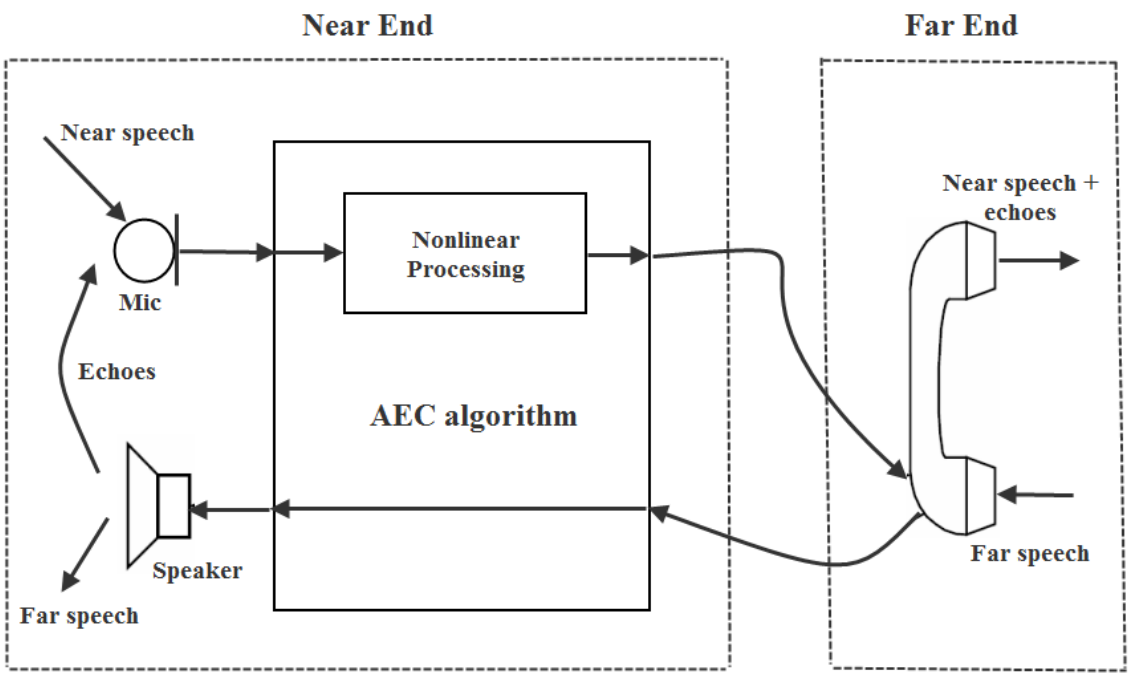


FIGURE 5.1: AEC OPERATION IN THE ROOMACOUSTIC ENVIRONEMENT

In the simulation, we need to create three functions which are *erle.m*, *lms.m, nlms.m* and *flms.m* to perform echo cancelation and compare results.

1) Create a function that calculates *ERLE* given the residual error *e(n)* and the output signal of the speech voice *d(n).*
2) Create a function, which takes an input vector *u* and a reference or desired signal *d(n),* both of length *N*, and calculates the error *e(n)* for all time instants. Furthermore, the input signal vector *u* is required to be a column vector.
3) For the NLMS function, in order to lower the influence of the input signal amplitude on the gradient noise, the step size is scaled where it is divided by the variance of the input signal *u(n)*. In case the input signal is zero, a positive constant in the denominator prevents the step size from being infinite. This modification of the standard LMS is referred to as normalized LMS.
4) Lastly, for the FastLMS simulation, its algorithm is an alternative frequency domain implementation of the standard LMS which designed to avoid circular convolution effects **[Ferrara 80]**. We will plot the speech signal's spectrum over time, which shows the frequency representation of the first 10k samples in

the time-frequency plane. There are two versions of FastLMS which are FastLMS without normalization and FastLMS with normalization.

Additionally, in both LMS and NLMS testing function we apply to the echo canceller with M =128 filter length and a value for step size that guarantees convergence and allows the fastest adaptation possible. The input signal and the desired signal are *u* and *d* respectively. Again, in order to investigate the convergence behaviour, use the function *erle.*

5.2.2 LMS and NLMS Simulation Results

LMS vs. NLMS (full MATLAB script, see Appendix A):

The following are LMS, NLMS, *ERLE* MATLAB function call script

```
function [e,w]=lms(mu,M,u,d)
```

```
Call:

Input arguments:
  mu  = step size, dim 1x1
  M   = filter length, dim 1x1
  u   = input signal, dim Nx1
  d   = desired signal, dim Nx1

 Output arguments:
   e  = estimation error, dim Nx1
   w  = final filter coefficients, dim Mx1

 % LMS
  for n = M:N
     uvec = u(n:-1:n-M+1);
     e(n) = d(n)-w'*uvec;
     w = w+mu*uvec*conj(e(n));
  end
```

Table 5.1: Least Mean Square function call

```
function [e,w]=nlms(mu,M,u,d,a)
```

```
Call:

Input arguments:
  mu  = step size, dim 1x1
  M   = filter length, dim 1x1
  u   = input signal, dim Nx1
  a   = constant, dim 1x1

 Output arguments:
   e  = estimation error, dim Nx1
   w  = final filter coefficients, dim Mx1

  % NLMS
for n = M:N
   uvec = u(n:-1:n-M+1);
   e(n) = d(n)-w'*uvec;
   w = w+mu/(a+uvec'*uvec)*uvec*conj(e(n));
end
```

Table 5.2: Normalized Least Mean Square function call

```
function [erle]=erle(e,d);
```

```
Call:

Input arguments:
  e   = residual echo, dim Nx1
  d   = desired signal, dim Nx1

Output arguments:
  r  = ERLE curve in dB

% ERLE
  erle = 10*log10(d./e);
```

Table 5.3: *ERLE* function call

In the simulation experiment, we showed the performance of acoustic echo cancellation by using LMS and NLMS. The speech data is collected by TI C6713 DSK real-time. Figure 5.2 is comparison of two algorithms and figure 5.3 is *ERLE* value in *dB*.

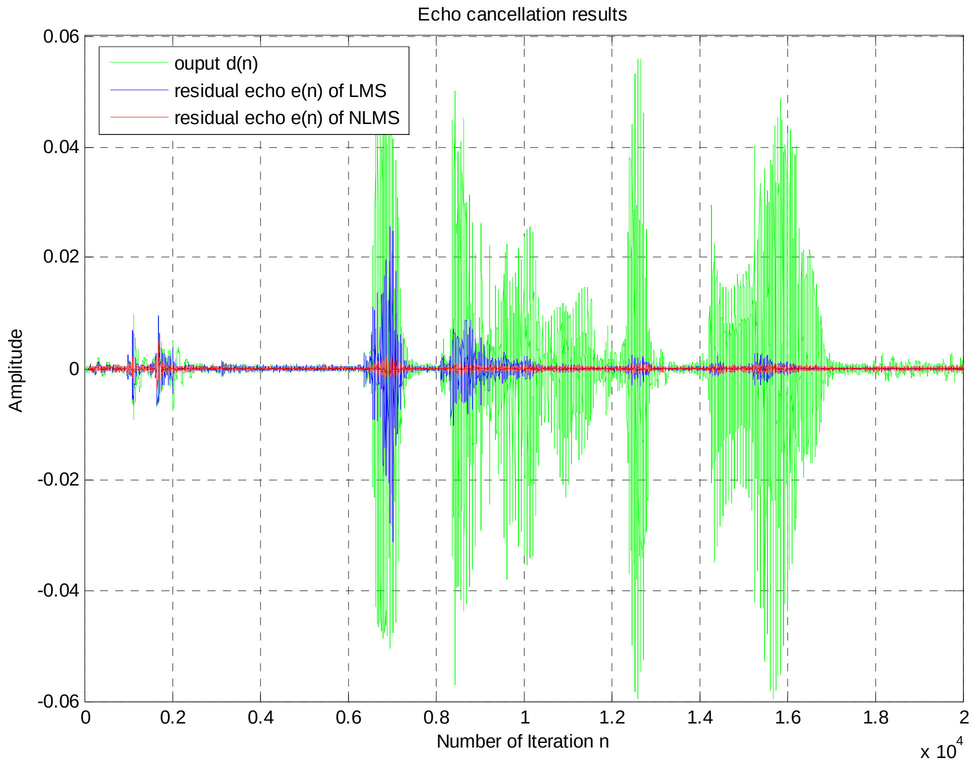


FIGURE 5.2: ECHO CANCELLATION RESULTS PERFORMED BY LMS AND NLMS

Normalized LMS usually converges much more quickly and efficiently than standard LMS at very little extra cost; NLMS is very commonly used in adaptive applications such as AEC. Furthermore, in the LMS function algorithm step size must be nonnegative scalar, we use *max_step_size* to determine a reasonable range of step size values for the speech signals being processed, and in the NLMS function algorithm, the step size must be a scalar between 0 and 2. Setting this step value to 1 provides the fastest convergence.

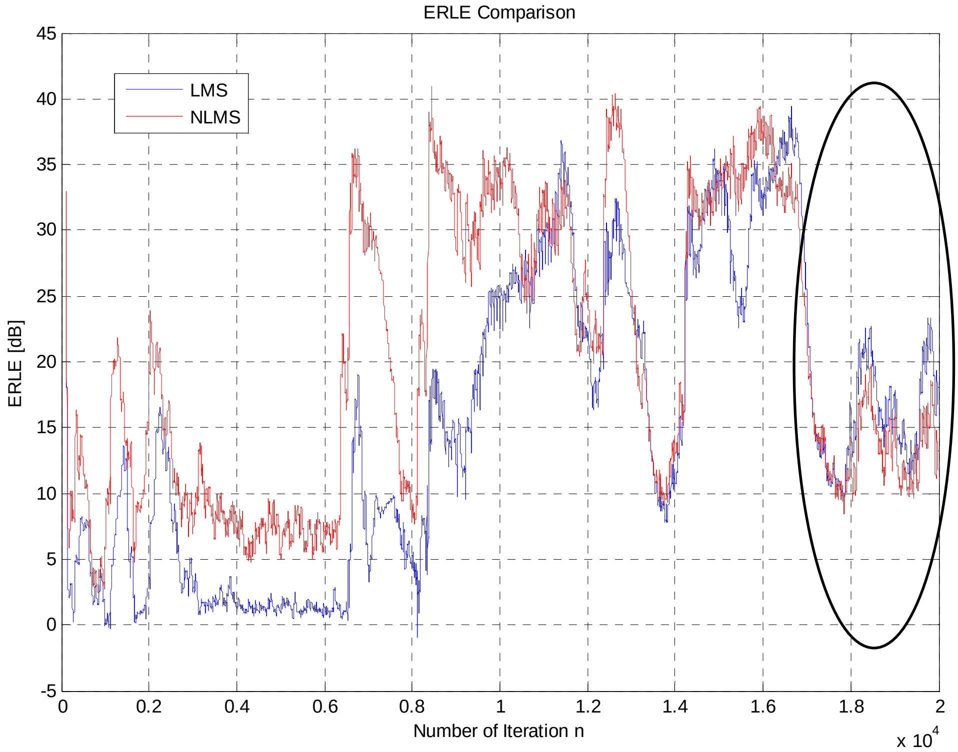


FIGURE 5.3: *ERLE* VALUE COMPARISON (LMS VS. NLMS)

Iteration no. (*10^4)	0.1	0.4	0.6	0.8	1.2	1.4	1.6	1.8	2.0
ERLE(*dB*) for LMS	1.412	1.322	1.318	4.121	21.881	11.97	32.88	**16.84**	**17.01**
ERLE(*dB*) for NLMS	5.012	7.84	7.788	8.243	22.54	12.54	37.12	**12.36**	**12.21**

Table 5.4: *ERLE* value comparison (LMS vs. NLMS)

5.2.3 FastLMS and NFastLMS Simulation Results

FastLMS vs. NFastLMS (full MATLAB script, see Appendix A):

```
function [e,w] = fastlms(st,M,u,d,gamma,P)
```

```
Call:

Input arguments:
st = step size, dim 1x1
M = filter length, dim 1x1
u = input signal, dim Nx1
d = desired signal, dim Nx1
P = initial value, energy, dim 2Mx1

Output arguments:
e  = estimation error, dim Nx1
w  = final filter coefficients, dim Mx1
```

Table 6.5: Fast Least Mean Square function call

The Fast LMS algorithm involves three diagonal matrices of dimension 2*M* by 2*M* (see Table 5.5 function call), which hence contain only information in their 2*M* diagonal elements. A so called element-wise multiplication of the vectors operations are denoted with a dot in MATLAB. In addition, the speech signals are transformed from time domain to frequency domain and backwards using the FFT and the IFFT, respectively. Hence all vectors in MATLAB are complex valued, even though they are real valued in time domain. Here it is a problem when plotting the vectors. Therefore, we have a possible solution is to extract only the real part in MATLAB. (See the MATLAB script in Appendix A). We simply repeat part 1 experiment to see how Fast LMS performance over the LMS algorithm. And we still use the same speech signal as before.

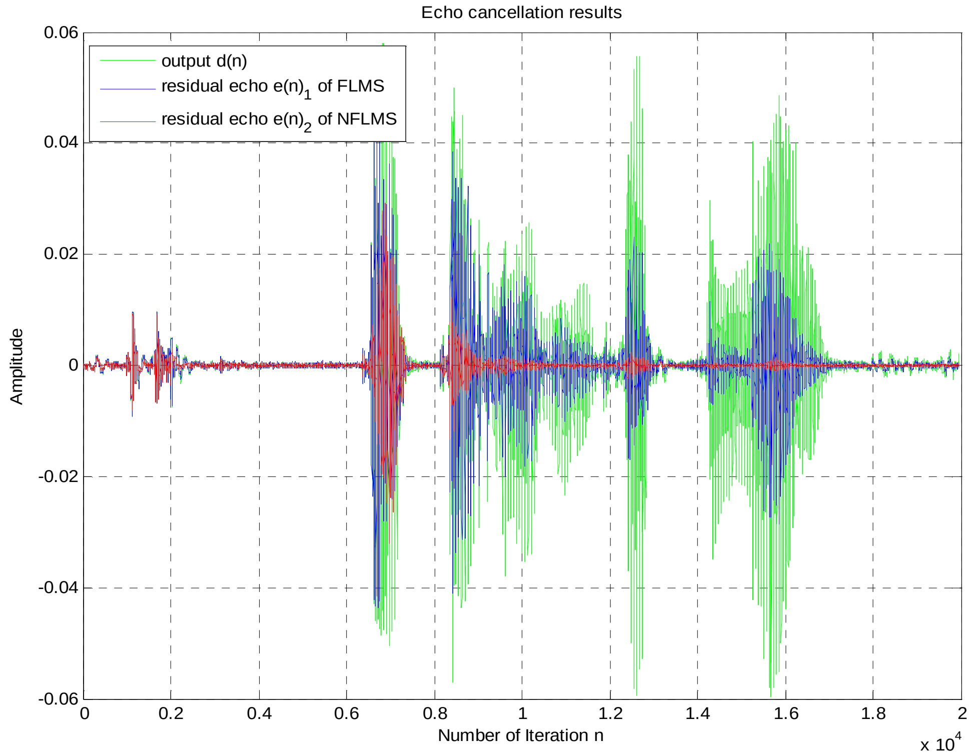


FIGURE 5.4: ECHO CANCELLATION RESULTS PERFORMED BY FAST LMS

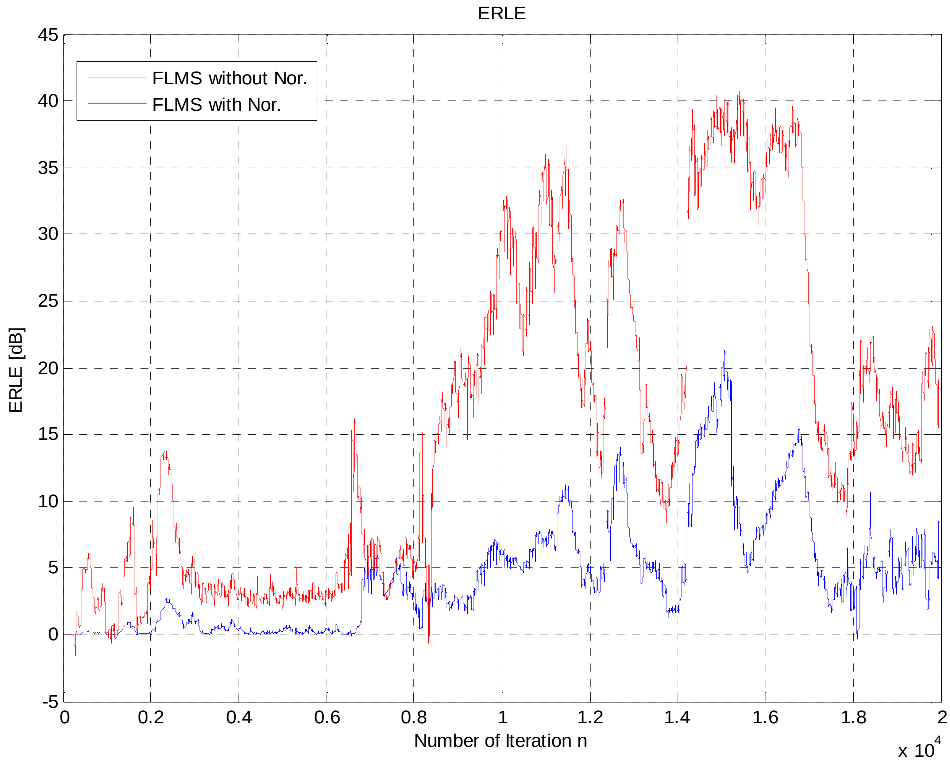


FIGURE 5.5: *ERLE* VALUE COMPARISON (FLMS VS. NFLMS)

The above figures are the investigation of *ERLE* for the different versions of the fast LMS algorithm.

Iteration no. (*10^4)	0.4	0.6	0.8	1.2	1.4	1.6	1.8	2.0
ERLE(*dB*) for FLMS	0.211	0.198	2.601	4.106	2.499	8.127	3.111	4.67
ERLE(*dB*) for NFLMS	4.31	3.981	5.322	17.995	11.33	33.544	13.241	16.77

Table 5.6: *ERLE* value comparison (FLMS vs. NFLMS)

We also observe the variation of the speech signal's spectrum over time. It can be done using the MATLAB function *spectrum*, which shows the frequency representation of the first 10k samples in the time-frequency plane. Figure 5.6 shows the spectrumgram of residual echo using Fast LMS without normalization and figure 5.7 shows the spectrumgram of residual echo using Fast LMS with normalization, respectively. When we use Fast LMS with normalization and it is clear to see the overall results are better than the performance of the counterpart without normalization

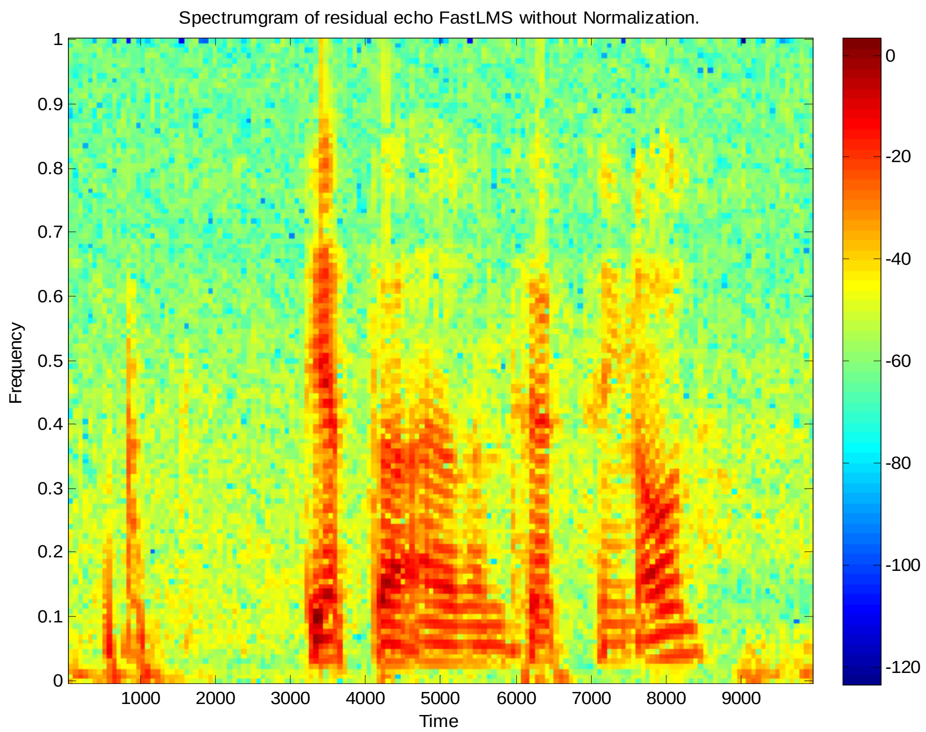


FIGURE 5.6: THE SPECTRUMGRAM OF RESIDUAL ECHO USING FAST LMS WITHOUT NORMALIZATION

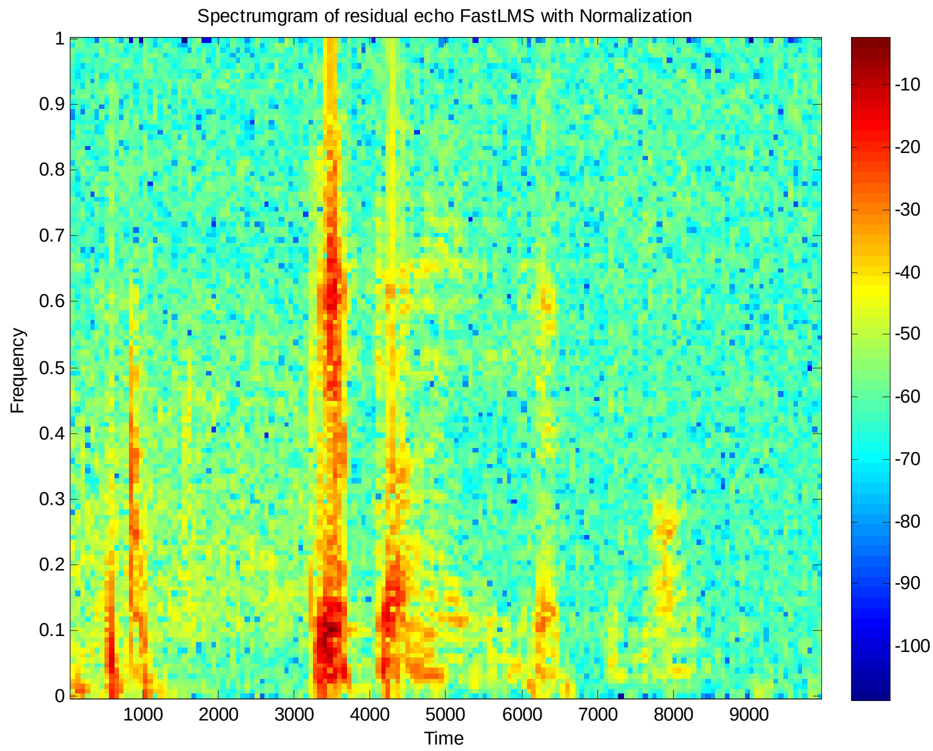


FIGURE 5.7: THE SPECTRUMGRAM OF RESIDUAL ECHO USING FAST LMS WITH NORMALIZATION

5.2.4 Summary of the performance of LMS algorithm

- LMS: is the simplest to implement and is stable when the step size parameter is selected appropriately see equation 5.10. This requires prior knowledge of the input signal. It is not the best choice for the real-time acoustic echo cancellation system.
- Normalized LMS: Simple to implement and computationally efficient. Shows very good attenuation and variable step size allows stable performance with non-stationary signals see equation 5.24. This is the obvious choice for real time implementation.
- Fast LMS: is an alternative frequency domain implementation of the LMS type algorithm designed to avoid circular convolution effects (overlapping output). It provides both faster convergence and simple normalization possibilities. This is also the obvious choice for real time implementation.

5.3 Using NMF to Perform AEC

5.3.1 Experiment Principle and procedure

In this experiment, we choose four different speakers: two male and two female speakers. These pre-recorded voice speeches were chosen from audio databases **TIMIT** by their metadata. IN MATLAB, the database toolbox will save the learning time of the database structure and will enable us to focus on algorithmic aspects of source code. The **TIMIT** database data can take the form of sentences words or phonemes. The MATLAB query or read functions will return a cell array and its waveforms will contain waveforms of entire sentence, words or phonemes, depends whether the query result is sentence, word or phoneme. For more information on **TIMIT** see **[Lingustic 10].**

We used both objective and subjective measurements to analyze the results of the experiments. In the subjective listening tests, a panel of subjects listened to the input and output speech to assess the effect of the algorithm. The objective analysis used three objective ratios based on the input and output speech to analyze the performance of the each value of beta of NMF to perform AEC. Two of the three ratios were taken from a standardized set of energy ratios defined in **[Vincent 05]**.

- Signal to Interference Ratio (SIR), which measures the amount of echo still left in the returning near end speech,

$$SIR = 10\log_{10}\left(\frac{\| s_{target} \|^2}{\| e_{interf} \|^2}\right) \qquad (5.23)$$

- Signal to Distortion Ratio (SDR) which measures the amount of the distortion in the original signal depends on the algorithm applied

$$SDR = 10\log_{10}\left(\frac{\| s_{target} \|^2}{\| e_{interf} + e_{artef} \|^2}\right) \qquad (5.24)$$

Where e_{interf} is the amount of interference energy left in the output, e_{artef} is the energy of processing artifacts left after processing and s_{target} the near end speech.

- Signal to Artifacts energy Ratio is a measure of the level of artifacts, the signal to artifacts ratio (SAR) defined as follows

$$SAR = 10\log_{10}\left(\frac{\| S_{target} + e_{interf} \|^2}{\| e_{artef} \|^2}\right) \tag{5.25}$$

In the convolutive NMF experiments, we want to measure the level of echo reduction during the pauses in speech recording, the energy ratio which is a measure of the level of echo suppression, the echo reduction loss enhancement (*ERLE*) was employed (same measurement in previous LMS experiments). It is defined as follows

$$ERLE = 10\log_{10}\left(\frac{E\{y^2(t)\}}{E\{e^2(t)\}}\right) \tag{5.26}$$

where *y(t)* is the echo signal and *e(t)* is the echo after processing.

Each experimental testing mixture, consists of a nearend speaker contribution and a main farend contribution. Both these contributions were obtained by convolving separate sentences of speech with the respective Room Impulse Responses (*RIRs*). In order to test the echo suppression when there is no nearend speech, also we need create large pause in nearend utterances leaving just the LEM response.

5.3.2 Conventional NMF Simulation results

In MATLAB implementation, we process mixture data frame by frame. For each frame we perform these two steps. In the training step, firstly we train the near-end basis matrix $\mathbf{B_n}$, we define two random matrices $\mathbf{B_n}$ and $\mathbf{H_n}$ of size M x R and R x N, perform update formulae to calculate get the suitable value of $\mathbf{B_n}$. The original NMF uses Kullback-Leibler divergence as the optimized cost function, the update rules to calculate $\mathbf{B_n}$ and $\mathbf{H_n}$ is given as:

$$\mathbf{H} = \mathbf{H} \bullet \frac{\mathbf{W}^T \cdot \left[\frac{\mathbf{V}}{\mathbf{WH}}\right]}{\mathbf{W}^T \cdot 1}, \mathbf{W} = \mathbf{W} \bullet \frac{\left[\frac{\mathbf{V}}{\mathbf{WH}}\right] \cdot \mathbf{H}^T}{1 \cdot \mathbf{H}^T} \tag{5.27}$$

Secondly perform the same procedure to calculate the far-end basis, i.e. the echo basis B_e.

```
for i = 1:1:train_num_iter_v
        Bn = Bn.*((V./( Bn *Hn+1e-9))* Hn ')./(ones(M,N)* Hn '+1e-9);
        Hn = Hn.*( Bn '*(V./( Bn * Hn +1e-9)))./( Bn '*ones(M,N)+1e-9);
end
```

Table 5.7: Update rules of training basis using conventional NMF algorithm

After both near-end and far-end basis are trained, next step is forming the mixture basis. This mixture basis contain both near-end and far-end echo basis and used to remove echo from the input mixture data $\mathbf{V}$.

Next step is matching, matches echo and near-end basis to the correlated parts in the mixture data $\mathbf{V}$. The procedure is as follows:

- Using the mixture basis $\mathbf{B_m}$ and input mixture data $\mathbf{V_m}$ train $\mathbf{H_m}$
- Get the near-end output matrix by multiplying the near-end parts of the mixture basis $\mathbf{B_m}$ with the correlated parts of the contribution matrix $\mathbf{H_m}$.
- Get the far-end echo matrix by multiplying the far-end parts of the mixture basis $\mathbf{B_m}$ with the correlated parts of the contribution matrix $\mathbf{H_m}$.

```
for i = 1:1:match_num_iter
        Hm = Hm.*( Bm'*(Vm./(Bm*Hm+1e-9)))./((Bm'*ones(Mx,1))+1e-9);
        if (i == match_num_iter-1)
          Hnolate = Hm;
        end
        if (i == match_num_iter-1)|(i == match_num_iter)
          Bm  = Bm .*((Vm./(Bm *Hm+1e 9))*Hm')./((ones(Mx,1)*Hm')+1e-9);
        end
 end
    Nearend(NumberOfFrame,:) = Bm(:,NearendFrames)*Hm(NearendFrames,:);
    Echo(NumberOfFrame,:) = Bm(:,EchoFrame)*Hm(EchoFrame,:);
```

Table 5.8: Update rules of matching and removing process with original NMF

Finally we resynthesis, take IFFT translation of the near-end data matrix and resynthesis it for audio. Calculate the objective ratios using the three objective measures described in next section.

```
Nearend = [Nearend,fliplr(Nearend(:,2:512))];
[xf,yf] = pol2cart(angle(mix_frames), Nearend);
resyn = complex(xf,yf);

for i = 1:1:num_frames
    resyn(start:stop) = real(ifft(spec(i,:))) + resyn(start:stop);
```

```
    start = start  + stepsize;
    stop = stop + stepsize;
    if stop > num_samples
        break
    end
end
Output_Nearend = resyn;
```

Table 5.9: Update rules of resynthesis process of output data

We choose male 1 and male 2 as sample speech mixture in the following simulation experiments. The energy ratio measurements results are show the first line of the table in section 5.4.

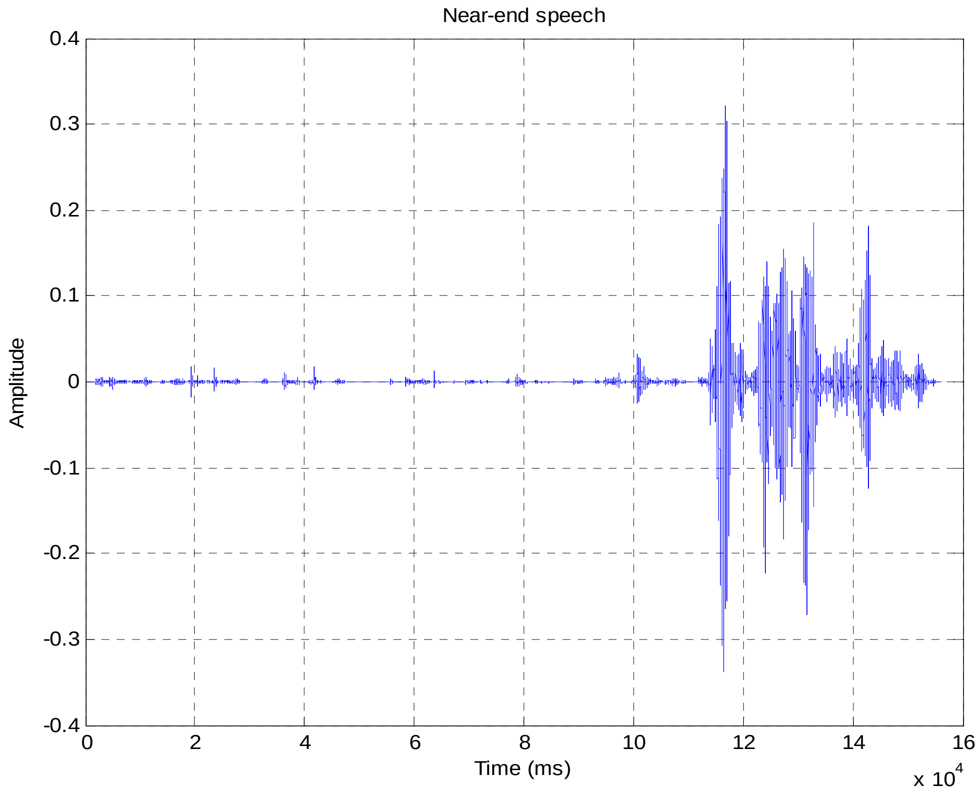


FIGURE 5.8: NEAR-END SPEECH WITH NOISY PAUSE WAVEFORM

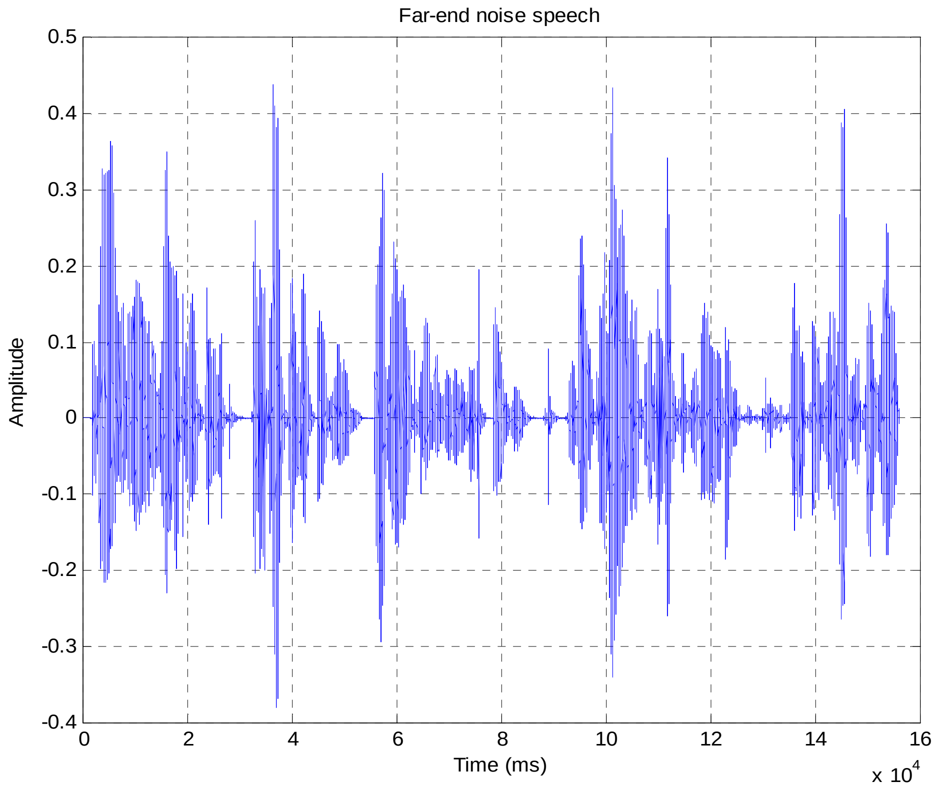


FIGURE 5.9: FAR-END NOISE SPEECH WAVEFORM

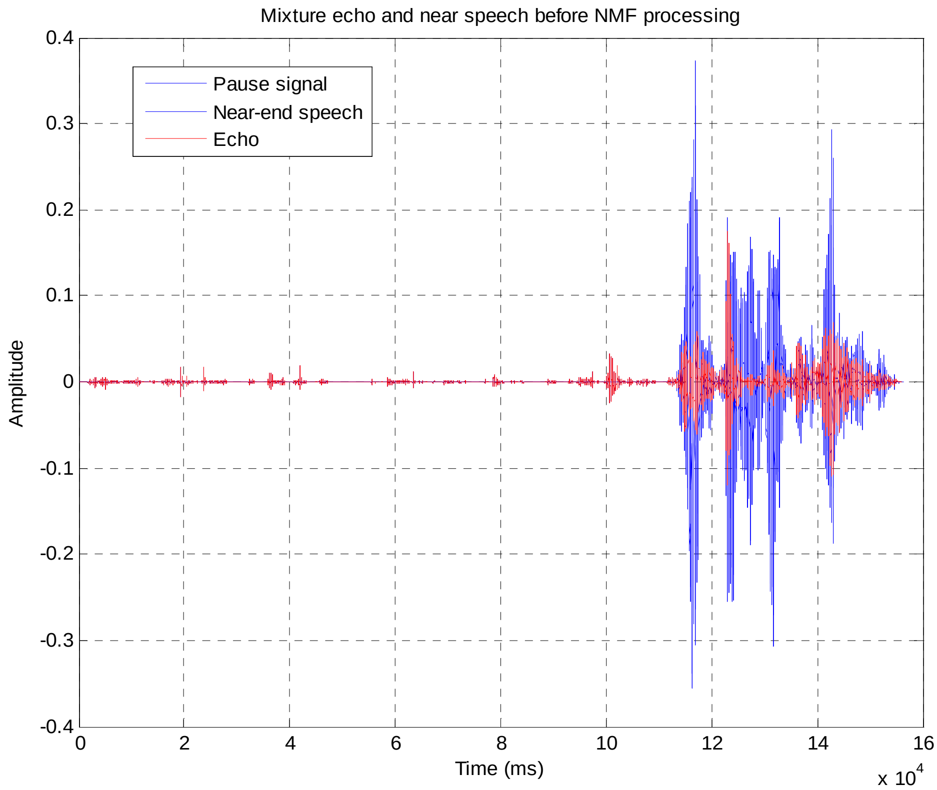


FIGURE 5.10: MIXTURE ECHO AND NEAR-END SPEECH BEFORE NMF PROCESSING

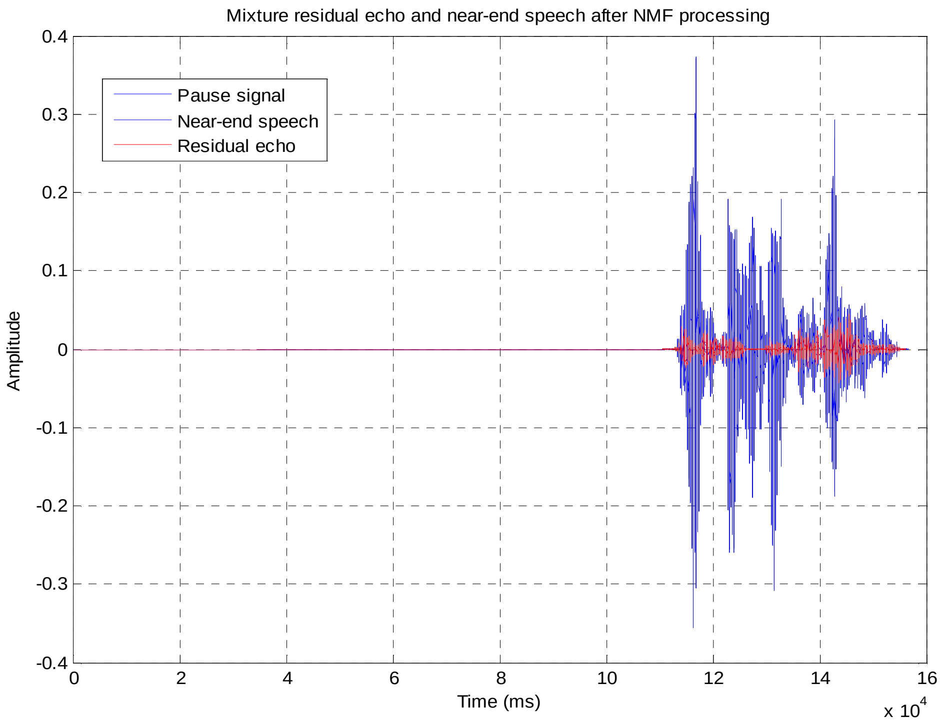


FIGURE 5.11: MIXTURE ECHO AND NEAR-END SPEECH AFTER NMF PROCESSING

5.3.3 Convolutive NMF Simulation Results

The implementation of convolutive NMF to perform Acoustic Echo Cancellation is using the similar frame work as the conventional NMF. The original NMF process the data frame by frame, i.e. each training and matching procedure only process one frame of data. The convolutive NMF uses a single $\mathbf{V}$ matrix which covers t frames of data instead of one. In each updated iteration, first only update $\mathbf{W}(t)$ and shift $\mathbf{H}$ for one frame for t times, then uses the average value of $\mathbf{W}(t)$ to update $\mathbf{H}$. That's because update $\mathbf{W}(t)$ and $\mathbf{H}$ for each t may result in a mistaken estimate of $\mathbf{H}$ with the update for $t = T-1$ dominating over others.

$$\mathbf{H} = \mathbf{H} \bullet \frac{\mathbf{W}(t)^T \cdot \left[\frac{\mathbf{V}}{\hat{\mathbf{V}}}\right]}{\mathbf{W}(t)^T \cdot 1}, \mathbf{W}(t) = \mathbf{W}(t) \bullet \frac{\left[\frac{\mathbf{V}}{\hat{\mathbf{V}}}\right] \cdot \overset{t\rightarrow}{\mathbf{H}}^T}{1 \cdot \overset{t\rightarrow}{\mathbf{H}}^T} \tag{5.28}$$

The process of convolutive NMF becomes:

- Read in a number of frames of mixture.
- Training near-end basis using convolutive NMF update function in Eq. 5.28.
- Training echo basis using convolutive NMF update function.
- Forming the mixture basis using near-end and echo basis.

- Using the mixture basis and input mixture data train **H**.
- Get the near-end output matrix by multiplying the near-end parts of the mixture basis with the correlated parts of the contribution matrix **H**.
- Get the far-end echo matrix by multiplying the far-end parts of the mixture basis with the correlated parts of the contribution matrix **H**.
- Take IFFT translation of the near-end data matrix and resynthesis it for audio
- Start process next frames of data
- After all the frames are processed, calculate the objective ratios of the output speech

```
Hshift = H;

     for t = 1:1:4
        Wt = Wt.*((V./(Wt*Hshift+1e-9))*Hshift')./(ones(M,N)*Hshift'+1e-9);
        W = W + Wt;
        Hshift = circshift(Hshift,[0,1]);
        Hshift(:,1) = 0;
     end
        W = W/t;
        H = H.*(W'*(V./(W*H+1e-9)))./(W'*ones(M,N)+1e-9);
```

Table 5.10: Update rules of convolutive NMF update functions

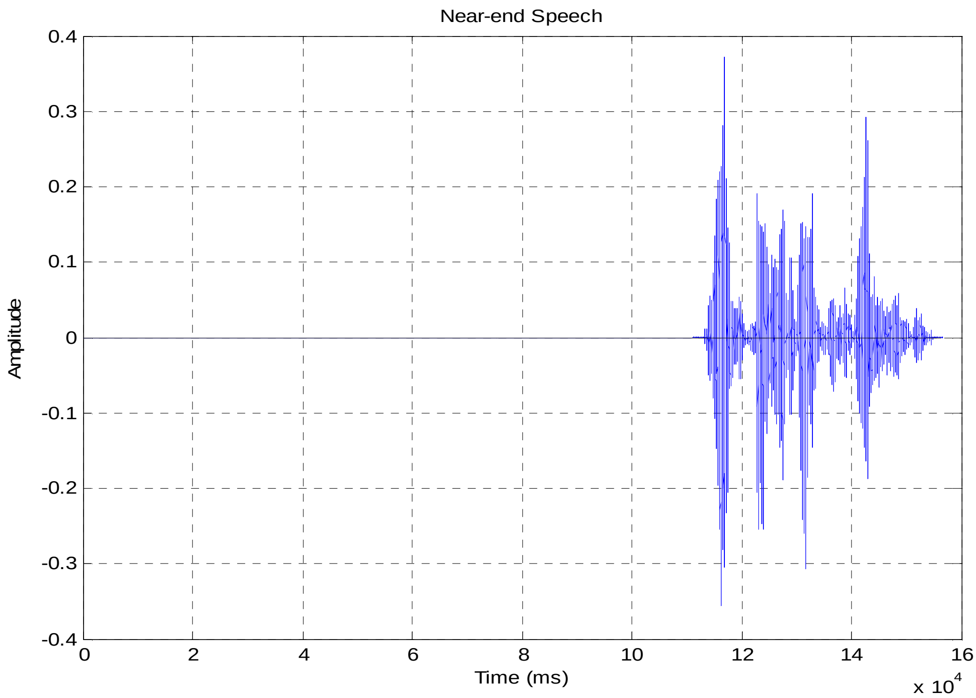


FIGURE 5.12: NEAR-END SPEECH (WITH PAUSE) WAVEFORM

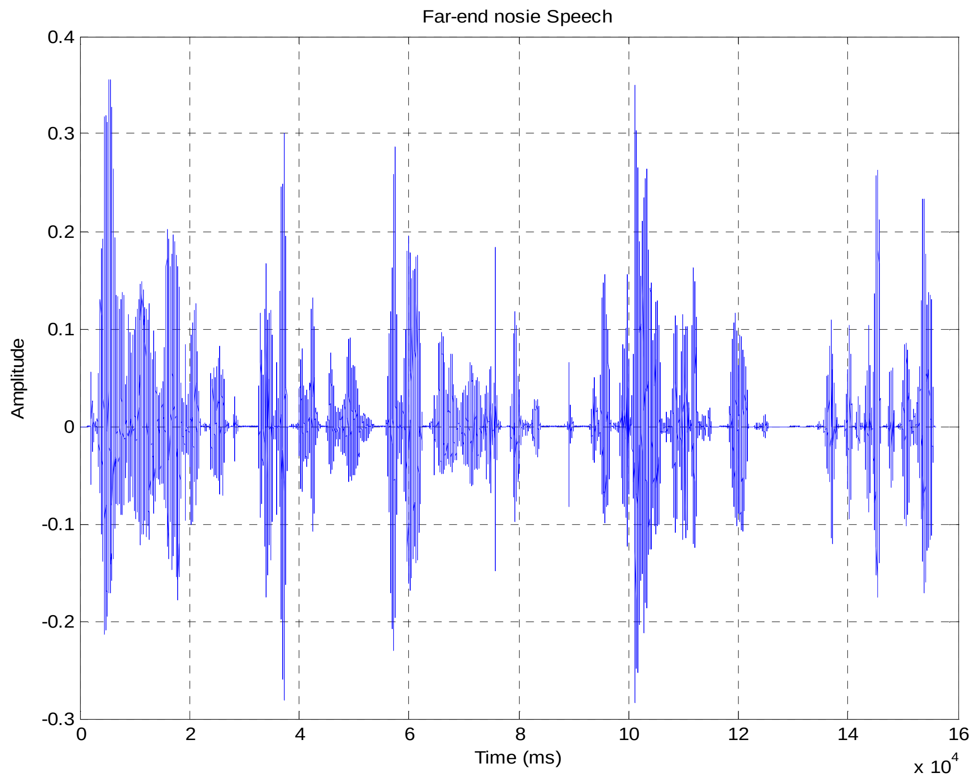


FIGURE 5.13: FAR-END NOISE SPEECH WAVEFORM

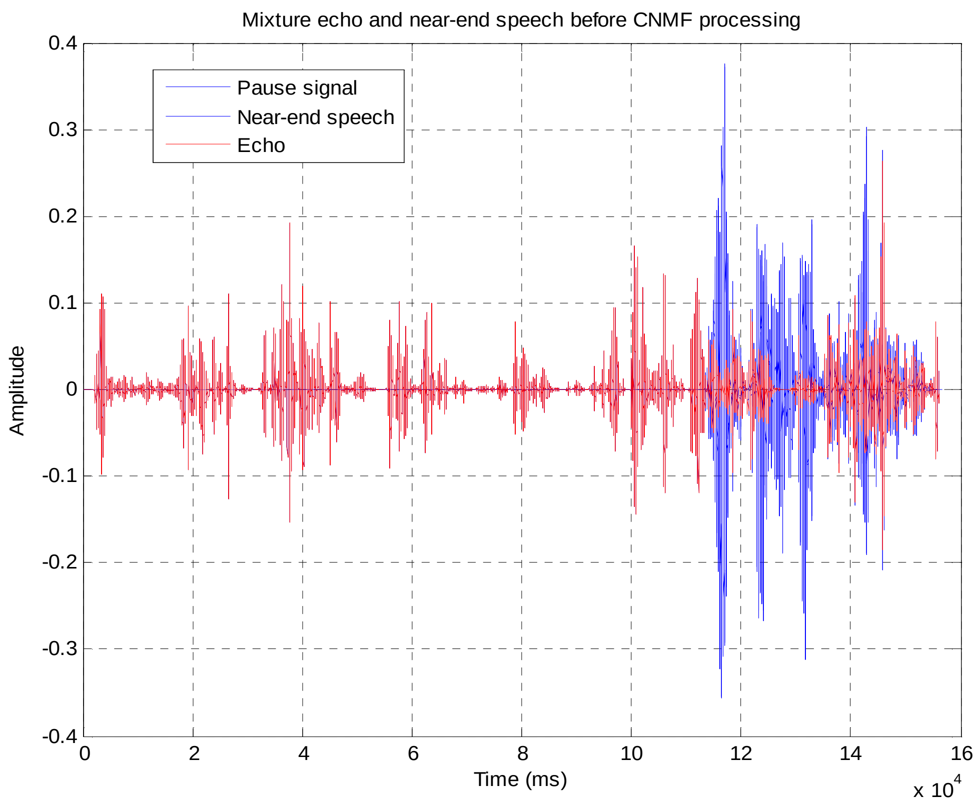


FIGURE 5.14: MIXTURE ECHO AND NEAR-END SPEECH BEFORE CNMF PROCESSING

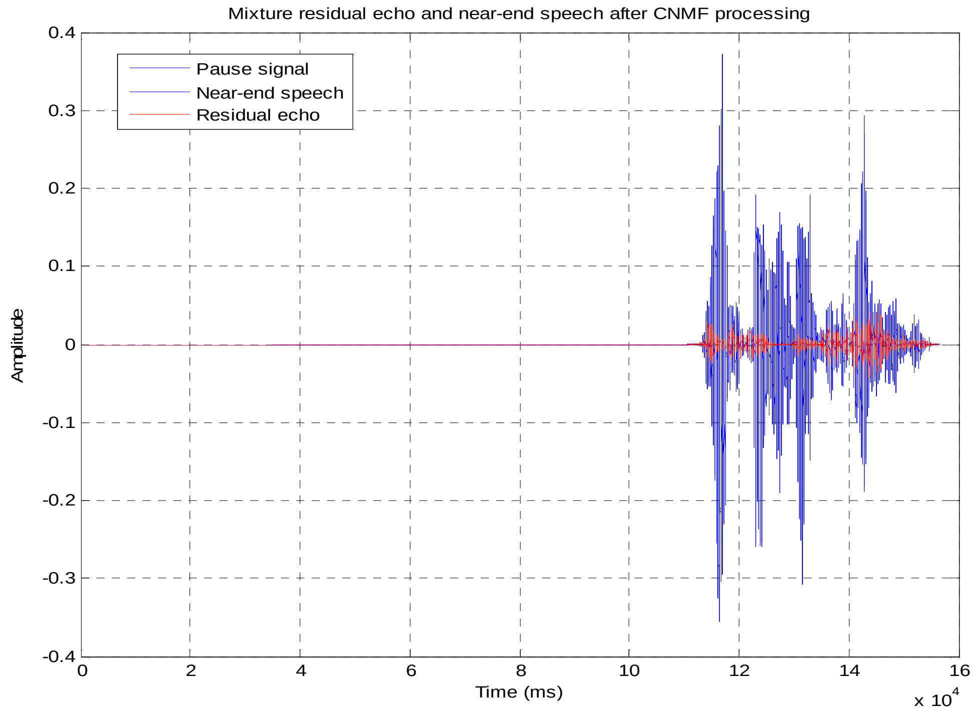


FIGURE 5.15: MIXTURE ECHO AND NEAR-END SPEECH AFTER CNMF PROCESSING

5.4 Measurement results

SIR, SDR were used to measure the performance on mixtures that contained both far-end and near-end speech together. The results of the SIR, SDR ratios are shown in the following Tables. Note that these results are based on the publication **[Zhou 09]** and re-do the experiment on different PC specifications and the results data have been changed. The output 1 SDR and SIR is speaker dependent bases results and output 2 is speaker independent bases.

- Conventional NMF

Near-end	**Far-end (echo)**	**Input SDR dB**	**Input SIR dB**	**Output 1 SDR dB**	**Output 1 SIR dB**	**Output 2 SDR dB**	**Output 2 SIR dB**
Male 1	**Male 2**	2.4238	2.5251	9.2641	32.9231	9.3714	30.8999
Male 2	**Female 1**	1.6504	1.6911	5.5001	25.1911	3.3133	21.4144
Female 1	**Female 2**	3.5942	3.4521	8.0111	23.2422	7.7355	23.8413
Female 2	**Male 1**	4.1011	4.4915	8.2955	27.0112	6.5611	28.1890
Average		2.9424	3.0399	7.7677	27.0919	6.7453	26.0862

Table 5.11: Conventional NMF Energy Ratio Measurements

Near-end	Far-end (echo)	ERLE (dB)
Male 1	**Male 2**	12.1555
Male 2	**Female 1**	14.3672
Female 1	**Female 2**	12.6888
Female 2	**Male 1**	12.0794
Average		12.8227

Table 5.12: *ERLE* for pauses in near end speech (Conventional NMF)

- Convolutive NMF

Near-end	Far-end (echo)	Input SDR dB	Input SIR dB	Output 1 SDR dB	Output 1 SIR dB	Output 2 SDR dB	Output 2 SIR dB
Male 1	**Male 2**	3.0112	2.9385	9.2113	28.2301	9.0012	27.8999
Male 2	**Female 1**	3.2988	3.3111	6.4223	24.1247	6.7781	23.4144
Female 1	**Female 2**	2.6154	2.5908	8.7100	21.4450	7.5644	21.8413
Female 2	**Male 1**	3.0881	2.6557	5.5221	18.0047	5.1229	18.1890
Average		3.0036	2.8740	7.4664	22.9511	7.0067	22.8362

Table 5.13: Convolutive NMF Energy Ratio Measurements

Near-end	Far-end (echo)	ERLE (dB)
Male 1	**Male 2**	10.5442
Male 2	**Female 1**	12.1142
Female 1	**Female 2**	11.0012
Female 2	**Male 1**	9.9912
Average		10.9127

Table 5.14: *ERLE* for pauses in near end speech (Convolutive NMF)

5.5 Discussion and conclusions

In the above both NMF and CNMF simulation experiments, we use the randomly chosen speakers to form the mixtures, two male and two female speeches (Chosen from the TIMIT database). Each experimental mixture had a near end speaker contribution and a far end speaker contribution. From the results of the figures in section 5.3, we can find that both conventional NMF and convolutive NMF can give significant reduction (approximate 8 to10% see figure 5.10, 5.11 and 5.15) in the level of echo. Note that the convolutive NMF approach has trade-off between computational load and the level of echo cancellation. In other words, in CNMF to cover more frames in one mixture gives a more precise result or less residual echo, but it will leads to more computational load.

Therefore, in the experiment we found that processing eight frames can give a best balance between algorithms computational load. The results showed that the convolutive NMF approach gives comparable performance to the conventional NMF but not better. However, as mentioned in chapter 4 if we improve the initialization problem instead of randomly choosing the initial value, then both NMF algorithms can achieve better performance.

The widely used methods are based on different types of Least Mean Squares (LMS) algorithms. And these methods all have limitations in different aspects. Recent research **[Paul 07] [Cahill 08] [Zhou 09]** also revealed that acoustic echo cancellation can also be performed by employing a monaural sound source separation technique based on Non-Negative Matrix Factorization (NMF), and significant echo suppression can be achieved using this method, so using NMF approaches there are a few advantages over the LMS algorithm.

Firstly, consider the effect reverberation has on the $\mathbf{H}$ matrix from NMF decompositions of audio spectrograms. The rows of $\mathbf{H}$ contain a time varying gain for each basis in $\mathbf{W}$ which contains the contribution the basis makes to the mixture over time. The $\mathbf{H}$ matrix is normally a sparse matrix with activations occurring in single spikes for anechoic speech. However in echoic version, if the same $\mathbf{W}$ matrix was used the activations in $\mathbf{H}$ become smeared. This is because the echoes in the speech manifest as repeated and smeared copies of the anechoic spectrogram. The NMF represents these echoes as repeated and scaled copies of the original $\mathbf{W}$ basis over time. This property of the NMF audio spectrogram enables the basis to be trained on anechoic speech and then can be used to separate echoic speech. This applies to AEC as the reference signal first excites a LEM system before reaching the microphone.

Secondly, the effect of misdjustments is reduced. The NMF does not estimate the LEM filter thus it does not require further samples of the reference signal to converge to the new room response like LMS, instead, it continuously adapts to the data present in the speech signal. This also means that the length of the impulse response is insignificant, as NMF will use the best available bases (the reference signal basis) to match the contribution from long impulse responses. In the case of long LEM filters the LMS techniques usually fix the length of the estimation filters.

Lastly using this approach Doubletalk will have less effect on this system, as this approach uses a local speaker basis to match any near end speech **[Cahill 08]**.

6. Real time hardware Implementation

6.1 Introduction

Speech echoes are normally raised from the acoustic coupling between the loudspeaker and microphone. Due to near(far)end acoustic coupling results in a disturbing echo at the far(near)end. Therefore, echo control must be used to insert sufficient echo return loss for comfortable and smooth conversations. There are two challenging aspects of algorithm convergence behaviour namely the large computational complexity and the ability of the filter to track the changes in the acoustic coupling.

The hardware implementation of this project is designed to enable the illustration and demonstration of acoustic echo cancellation (C program) in real-time. The entire experiment is involved MATLAB Simulink^{TM}, Real-time workshop and Embedded Target for TIC6000 DSP toolboxes. They are used to link for CCS (Code composer Studio) which is real-time DSP IDE provided by TI.

6.2 Workstation setup and hardware profile

Most of the work presented in this chapter involves the development and testing of short programs to demonstrate DSP concepts. To perform the experiments described in the chapter, the following tools are used:

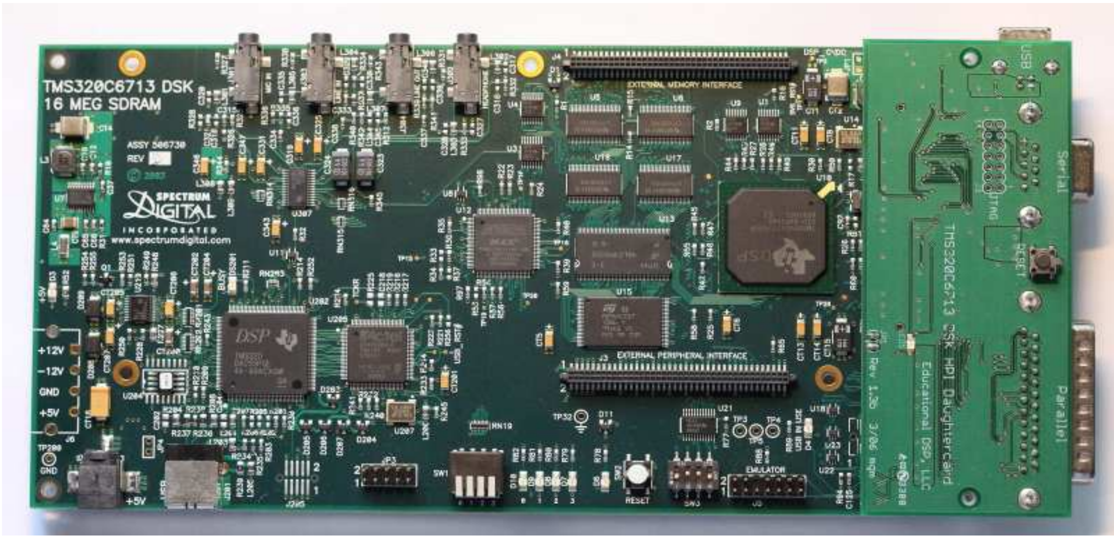
The workstation is equipped with the following items:

1) A Texas Instruments DSP starter kit (DSK) which includes:

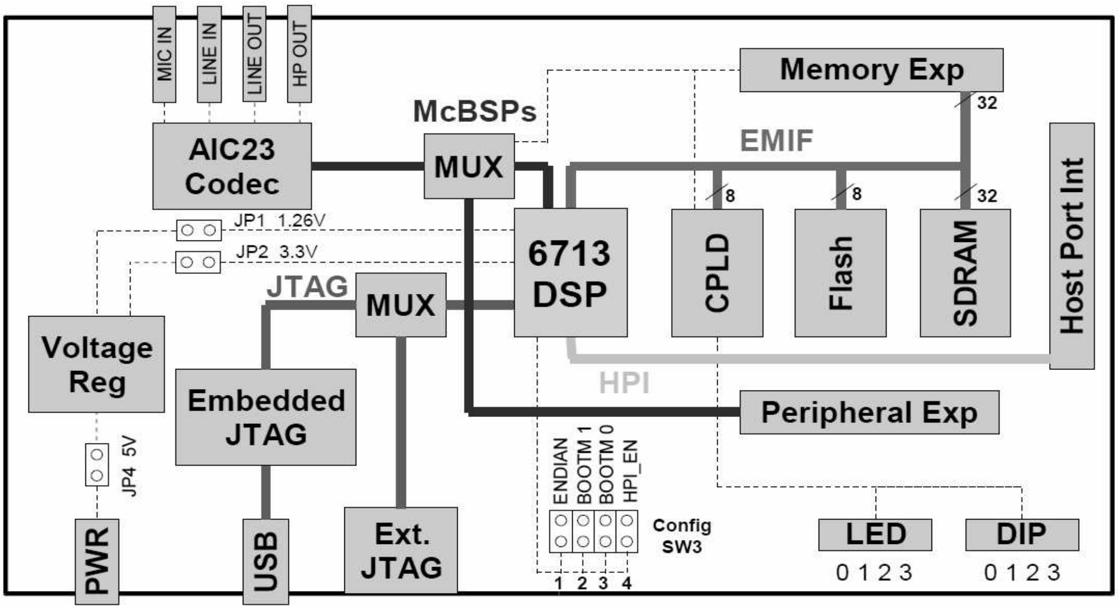
- The DSK package software Code Composer Studio (CCS), which provides the necessary software support tools. CSS provides an integrated development environment (IDE), bringing together the C compiler, assembler, linker, debugger, and so on.
- A circuit board (the TMS320C6713 DSK is shown in Figure 6.1) containing a digital signal processor and a 16-bit stereo codec for analogue signal input and output.
- A universal synchronous bus (USB) cable that connects the DSK board to a PC.

2) A standalone PC. The DSK board connects to the USB port of the PC through the USB cable included with the DSK package
3) An oscilloscope, spectrum analyser (optional) microphones, and speakers

The DSK package are powerful, yet relatively inexpensive, with the necessary hardware and software support tools for real-time signal processing **[TI 01][TI 02a][TI 02b].** The DSK board each include 16MB of synchronous dynamic RAM and 512kB of flash memory. Four connectors on the boards provide analogue input and output: MIC IN for microphone (it is mostly used in the experiment for speech input), LINE IN for line input, LINE OUT for line output, and HEADPHONE for a headphone output (we use this port for catch the output signal or connect to the external loudspeaker).



(A)



(B)

FIGURE 6.1: TMS3206713-BASED DSK BOARD: (A) PHYSICAL BOARD AND (B) BLOCK DIAGRAM

(Courtesy of Texas Instruments)

The DSK C6713 evaluation board installed within a simple enclosure which consists of top and bottom precision machined transparent Plexiglas panels, those are then fastened through the DSK board.

6.3 Real-time application setup

6.3.1 RTDX Technology

Real-Time Data Exchange (RTDX^{TM}) is a technology developed by Texas Instruments that provides effective real-time bi-directional communication between a digital signal processor (DSP) or microcontroller and a host application in other words, it allows system developers to transfer data between a host computer (MATLAB) and targets device (C6713 DSK) without interfering with the target application**[TI 01]**. This bi-directional communication path provides for data collection by the host as well as host interaction with the running target application. RTDX also enables host systems to provide data stimulation to the target application and algorithms **[Dustin 02a].**

6.3.2 RTDX Link to MATLAB

In this experiment, we illustrated the interface between MATLAB and the DSK using RTDX. A buffer of data (i.e. speech wave file) created from MATLAB which running on the host PC is set to the C6713 processor. The C source program running on the DSK increment each data value in the buffer and sends the buffer of data back to MATLAB. In other words, it creates two channels through RTDX: an input channel to transfer data from MATLAB on the PC to the c6713 processor on the DSK and an output channel to transfer data from the target DSK to the PC host. When the input channel is enabled data are read or received as input to the DSK from MATLAB. After each data value in the buffer is incremented by 1, an output channel is enabled to write the data to MATLAB. Note that the input and output designations are from target DSK. There are real-time application literatures **[Dustin 02][Dustin 03][Horst 05][Fu 02]** discussed the RTDX technology throughout TI DSK project.

6.4 Speech recognition Implementation

Speech recognition refers to the concept of recognizing a speaker by his/her voice or speech sample. Simply said speech recognition systems contain two main modules: feature extraction and classification.

1) Feature extraction is a process that extracts a small amount of data from the voice signal that can be used to represent each speaker. Short-time spectral analysis to Short-time Fourier Transform (STFT) is the most common way to characterize a speech signal. In addition, the Mel-frequency cestrum coefficients **[Beth 99]** are used to parametrically represent the speech signal for the speaker recognition task. The implementation steps shown in Figure 6.2.

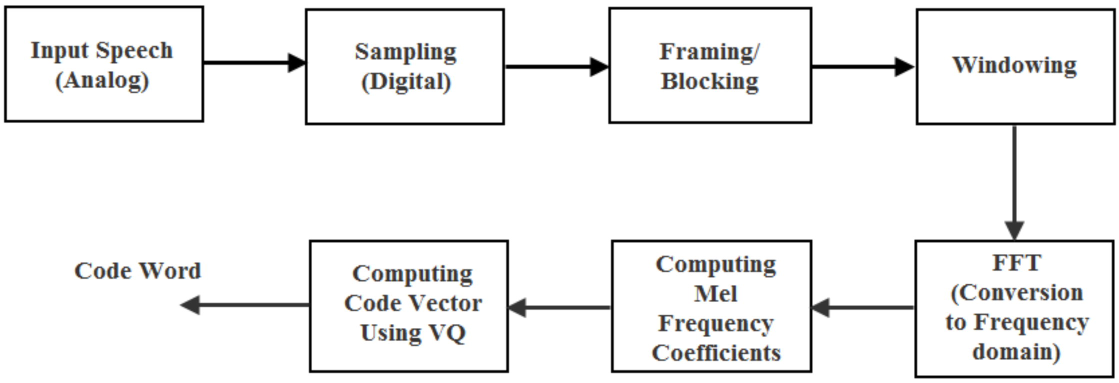


FIGURE 6.2 STEPS FOR SPEECH RECOGNITION IMPLEMENTATION

2) Classification consists of models for classifying extracted featured according to the individual candidate speakers whose voices have been stored. The recorded voice patterns of the speakers are used to derive a classification algorithm such as vector quantization (VQ) **[Allen 01]** is used.

6.5 Echo control Implementation

The following experiment illustrated analogue input and output using the TI DSK. They are included in order to introduce both the DSK hardware and the CCS development environment. The experiment programs demonstrated some important concepts associated with analogue-to-digital conversion, including sampling, aliasing, and reconstruction, additionally, they illustrated the use of interrupts in order to implement real-time applications using the DSK. Many of the concepts and techniques described in the previous section are used again in this chapter.

6.5.1 On board stereo codec for input and output

The experiment testing board C6713 DSK makes use of the AIC23 codec for analogue input and output. The analogue-to-digital converter (ADC), or coder, is part of the codec convert an analogue input signal into a sequence of sample values (16 bit signal integer) to be processed by the digital signal processor (DSP). The digital-to-analogue converter (DAC), or decoder, is part of the codec reconstructs an analogue output signal from a sequence of sample value that have been processed by the DSP as well.

The AIC23 is a stereo audio codec based on sigma-delta technology **[Norsworthy 97][Aziz 96][Candy 92].** Communication with the AIC23 codec for input and output uses two multi-channel buffered serial ports (McBSPs) on the C6713. McBSP0 is used as a unidirectional channel to send 16-bit control word to the AIC23. McBSP1 is used as a bidirectional channel to send and receive audio data. The codec can be configured for data-transfer word lengths of 16 up to 32 bits.

In the experiment, we need to define DSK support files which can initialize the DSK. All the source files are written in C program. The following functions defined in support file and explained for testing purposes:

Main c programme support file 6.1: *c6713dskinit.c*

```
Uint16 inputsource = DSK6713_AIC23_INPUT_MIC;      // select input

void main( )
{
   short sample_data;    // in this case we choose select real-time speech input

   com_poll( );               // initialize DSK, codec, McBSP
   while(1)
   {
      sample_data = input_left_sample( )    // input sample
      output_left_sample(sample_data);      //output sample
   }
}
```

Table 6.1: Loop program using polling

Above C source file for a program, which simply copies input samples read from the AIC23 codec ADC back to the AIC23 codec DAC as output samples is listed in table 6.1. Effectively, the MIC input socket is connected straight through to the HEADPHONE OUT socket on the DSK via the AIC23 codec and the digital signal processor.

6.5.2 Modifying program to create an echo

In the experiment, we create a simple echo speech by feeding back a fraction of the output of the delay line to its input. A fading echo effect can be realized. It showed in Figure 6.3.

Main c programme 6.2: ***echo.c***

```
Uint16 inputsource = DSK6713_AIC23_INPUT_MIC;       // select input

#define gain 0.5                  // fraction of output fed back (value between 0.0 to 1.0)
#define BUF_SIZE 8000       // length of delay (value between 100 to 8000)
short input, output, delayed;
short buffer [BUF_SIZE];
int I;

interrupt void c_int ( )            //interrupt service routine
{
   input = input_left_sample ( );          // read new input sample
   delayed = buffer [i];                        // read output of delay line
   output = input + delayed;               //output sum of new and delayed
   output_left_sample (output);           //buffer index
   buffer [i] = input + delayed*gain;    //store new input and fraction of
                                                          //delayed value
   if (++I >= BUF_SIZE)    I = 0;        //new input sample then increment
   return;
}


void main( )
{
   for (I = 0; I < BUF_SIZE; i++)
      buffer [i] = 0;
   comm_intr( );                                   // initialize DSK, codec, McBSP
   while (1);                                        // infinite loop
}
```

Table 6.2: fading echo program

The value of the constant BUF_SIZE determines the number of samples stored in the array *buffer* and hence the duration of the delay. The value of the constant *gain* determines the fraction of the output that is fed back into the delay line and hence the rate at which the echo effect fades away. In the experiment, we can set the value of *gain* equal to or great than unity would cause instability of the loop. Experiment with different values of *gain* can be set as between 0 and 1 with 0.1 increment and BUF_SIZE can be set as between 100 and 8000 with 1 increment.

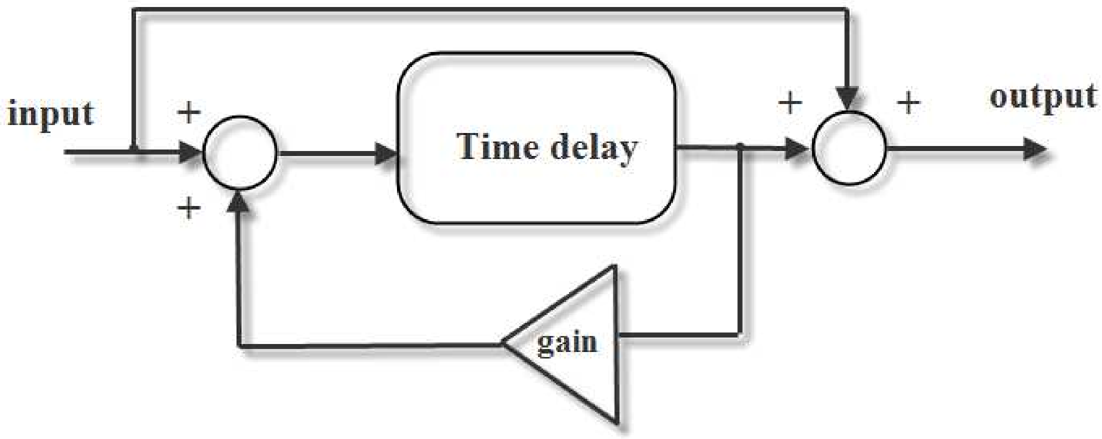


FIGURE 6.3: SIMPLE BLOCK DIAGRAM REPRESENTION OF FADING ECHO PROGRAM

6.5.3 Modifying program to create an echo control

In the experiment we will extend the fading echo program to allow real-time adjustment of gain and delay parameters of the echo effect.

Main c programme 6.3: ***echo_control.c***

```
Uint16 inputsource = DSK6713_AIC23_INPUT_MIC;       // select input

#define MAX_BUF_SIZE 8000       //set maximum length of delay
float gain = 0.5;
short buflength = 1000;
short buffer[MAX_BUF_SIZE];       //storage for previous samples
short input, output, delayed;
int I = 0;

interrupt void c_int ( )          //interrupt service routine
{
   input = input_left_sample ( );              // read new input sample
   delayed = buffer [i];                       // read output of delay line
   output = input + delayed;                   //output sum of new and delayed
   output_left_sample (output);                //buffer index
   buffer [i] = input + delayed*gain ;    //store new input and fraction of
                                              //delayed value
   if (++I    >=    MAX_BUF_SIZE)      //new input sample then increment
      I = MAX_BUF_SIZE – buflength;
   return;
}
```

```
void main( )
{
   short sample_data;    // in this case we choose select real-time speech input

   com_poll( );               // initilize DSK, codec, McBSP
   while(1);                  //infinite loop
}
```

Table 6.3: Echo programme with variable delay and feedback gain for controlling

In above main *echo_control.c* program, array buffer is declared to be the maximum size required, MAX_BUF_SIZE. To achieve a variable delay, integer variable *buflegth* is used to control the length of the circular buffer implemented using array *buffer*. When the value of the index *i*, used to access element of the array *buffer*, is incremented beyond the maximum value allowable (MAX_BUF_SIZE). It is reset not to zero as in previous program see Table 6.2 but to (MAX_BUF_SIZE – *buflength*).

6.6 Notes and Conclusions

There are a few hardware setup information need to be briefly explained in this section.

1) In the hardware implementation, we combined Code Composer Studio (CCStudio) and MATLAB tools to perform the echo control test. The CCStudio IDE provides a graphical interface for using the code generation tools. For example in the *echo_control.prj* project, CCStudio keeps track of all information needed to build a target program or library. A project records:
   - Filenames of source code and object libraries
   - Compiler, assembler, and linker options
   - Include file dependencies

   When we build a project with the CCStudio IDE, the appropriate code generation tools are invoked to compile, assemble, and link out program. For more information, see **[TI 01]** or TI online technical document
2) The experiment shows an echo effect based on the real-time DSK. The length of echo is controlled by changing the buffer size where the samples (real-time speech voice) are stored. A dynamic change of the echo length leads to reverb effect. A fading effect with delaying echo is obtained with a sider. This is the specific way to control the echo in the experiment.

7. Conclusion and future work

An extensive review of optimum algorithms for blind source separation was presented, as well as a review of Non-negative Matrix Factorization (NMF) and Least Mean Square (LMS) based approaches. Based on these reviews it was concluded that using different mathematical techniques to perform Acoustic Echo Cancellation (AEC), by comparing results and considering the trade-off issues we can find the best suitable algorithm for the AEC problem.

This thesis demonstrated two research sub-topics for AEC: the first employs different versions of Least Mean Square algorithms to perform acoustic echo cancellation, we use a dataset from real-time echo speech which is collected TI C6713 DSK. By comparing the *ERLE* value, we can find best suitable version of LMS algorithm which we discussed in the experiment; the second topic presented a new technique called convolutive non-negative matrix factorization to perform acoustic echo cancellation. The two NMF experiments were implemented in MATLAB environment by using the same dataset of input speeches, performing the steps of training near-end and far-end reference bases, forming mixture bases, using the reference bases separate the mixture data, and finally resynthesis the required speech part in the mixture as the output speech. Finally, the output speech is analysed by objective measures included SDR, SIR, *ERIE* and the comparable analytic data shown as a table form. Although the experiments of convolutive NMF showed the new algorithm didn’t give a better performance than conventional version, this can help further research in modifying the algorithm or combine the feature of different version of the algorithm to give better performance.

Also the last part, we present a simple real-time echo control implementation. It is based on TI C6713 development start kit. The real-time scenario let us understand how to create and control echo by modifying the c program function. Also use RTDX Technology to connect MATLAB and DSK is another useful experimental experience.

7.1 Future work

Further work on the topic of Convolutive NMF includes combining features of different versions of NMF such as Local NMF (LNMF) or other mathematical tools, find the optimal or best suitable algorithm for different applications such as AEC, musical separation, etc. Alternatively some of the non-linear post-processing techniques used to improve LMS methods such as component zeroing could be employed to improve performance **[Virtanen 07]**.

The nonnegative matrix factorization has many advantages to alternative techniques for processing such matrices, but it must be initialized and the initialization selected is crucial to getting better solutions. It is an open issue **[Amy 06] [Stefan 04]** for NMF algorithms research.

At present the algorithms described in this thesis are all implemented in MATLAB. A useful area for the future work would be the implementation of these algorithms in C or C++ which would result in a considerable reduction in the time required to run the algorithms. Additionally, to implement the algorithms into real-time is also an important future work. Hardware features such as computational load, delay and floating or fixed point operation of the hardware can affects the performance of algorithms in the real-time environment. Balancing and adjustment of the algorithm parameters are needed, translate the algorithms into C orC++ is also required

In conclusion, the work undertaken has identified a number of possibilities for improvement in acoustic echo cancellation approaches. The technique implementations demonstrated using sound source separation algorithms such as NMF can be further improved by employing more efficient cost functions. It is hoped that future work will further enhance the thrust of this research.

Acknowledgements

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Appendix A:

Least Mean Square MATLAB Script:

```
function [e,w]=lms(mu,M,u,d)
%          Call:
%          [e,w]=lms(mu,M,u,d);
%
%          Input arguments:
%          mu      = step size, dim 1x1
%          M       = filter length, dim 1x1
%          u       = input signal, dim Nx1
%          d       = desired signal, dim Nx1
%
%          Output arguments:
%          e       = estimation error, dim Nx1
%          w       = final filter coefficients, dim Mx1

%initial weights
w=zeros(M,1);

%length of input signal
N=length(u);

%make sure that u and d are column vectors
u=u(:);
d=d(:);

%LMS
for n=M:N
    uvec=u(n:-1:n-M+1);
    e(n)=d(n)-w'*uvec;
    w=w+mu*uvec*conj(e(n));
end
e=e(:)m;
```

Normalized Least Mean Square MATLAB Script:

```
function [e,w]=nlms(mu,M,u,d,a)
%          Normalized LMS
%          Call:
%          [e,w]=nlms(mu,M,u,d,a);
%
%          Input arguments:
%          mu      = step size, dim 1x1
%          M       = filter length, dim 1x1
%          u       = input signal, dim Nx1
%          a       = constant, dim 1x1
%
%          Output arguments:
%          e       = estimation error, dim Nx1
%          w       = final filter coefficients, dim Mx1

%intial value 0
w=zeros(M,1);

%input signal length
N=length(u);

%make sure that u and d are colon vectors
u=u(:);
d=d(:);

%NLMS
for n=M:N
   uvec=u(n:-1:n-M+1);
   e(n)=d(n)-w'*uvec;
   w=w+mu/(a+uvec'*uvec)*uvec*conj(e(n));
end
```

Fast Least Mean Square MATLAB Script:

```
function [e,w]=fastlms(alpha,M,u,d,gamma,P);
%         Call:
%         [e,w]=fastlms(alpha,M,u,d,gamma,P);
%
%         Input arguments:
%         alpha      =step size, dim 1x1
%         M          =filter length, dim 1x1
%         u          =input signal, dim Nx1
%         d          =desired signal, dim Nx1
%         gamma      =forgetting factor, dim 1x1
%         P          =initial value, energy, dim 2Mx1
%
%         Output arguments:
%         e          =estimation error, dim Nx1
%         w          =final filter vector, dim Mx1
%
%         The length N must be chosen such that N/M is integer!
%

% initialization
W=zeros(2*M,1);
N=length(u);

% make sure that d and u are column vectors
d=d(:);
u=u(:);

e=d;

% no.of blocks
Blocks=N/M;

% loop, FastLMS
for k=1:Blocks-1

    % block k-1, k; transformed input signal U(k)
    Uvec=fft([u((k-1)*M+1:(k+1)*M)],2*M);

    % block k, output signal y(k), last M elements
    yvec=ifft(Uvec.*W);
    yvec=yvec(M+1:2*M,1);

    % block k; desired signal
    dvec=d(k*M+1:(k+1)*M);

    % block k, error signal
    e(k*M+1:(k+1)*M,1)=dvec-yvec;

    % transformation of estimation error
    Evec=fft([zeros(M,1);e(k*M+1:(k+1)*M)],2*M);
```

```
    % estimated power
    P=gamma*P+(1-gamma)*abs(Uvec).^2;

    % block k, inverse of power
    Dvec=1./P;

    % estimated gradient
    phivec=ifft(Dvec.*conj(Uvec).*Evec,2*M);
    phivec=phivec(1:M);

    % update of weights
    W=W+alpha*fft([phivec;zeros(M,1)],2*M);
end

% The error vector should have only real values.
% Therefore, extract the real part!
E=real(e(:));

% transform of final weights to time domain.
%
% make sure that w is real-valued
w=ifft(W);
w=real(w(1:length(W)/2));
```

ERLE Function MATLAB Script:

```
function [erle]=erle(e,d)
%         calculation of ERLE
%         Call:
%         [r]=erle(e,d)
%
%         Input arguments:
%         e      = residual echo, dim Nx1
%         d      = hybrid output signal, dim Nx1
%
%         Output arguments:
%         r      = ERLE curve in dB

%make sure that both arguments are column vectors
e=e(:);
d=d(:);

% filtering of squared signals (IIR-filter)
Pd=filter(1,[1, -0.98],d.^2);
Pe=filter(1,[1, -0.98],e.^2);

% ERLE
erle=10*log10(Pd./Pe);
```

Convolutive NMF MATLAB Script:

```
//////////////////////////////////////////////////////////////////////
Function name: Convolutive NMF main function
Description: This is the main function that using convolutive NMF to perform
training and matching process for AEC
//////////////////////////////////////////////////////////////////////

function
[DTD,norm_vhat_energy,echo_measures,nearend_measures,compare_measures,
erleout,output_v] =
NMFAEC_function_subband2(x,y,y_no_v,v,nearendtrain,Thres,num_bases_v,.
..
    train_num_iter_v,num_bases_y,a,b);
%%%%% Variables %%%%%%
%train_num_iter_v = 50;                  %%%% Number of NMF iterations
for I training of the near end basis
train_num_iter_y = 100;                  %%%% Number of NMF iterations
for each trained echo basis
match_num_iter = 250;                    %%%% Number of NMF iterations
for matching/echo nearend separation
for near end speech basis v.
%%%% Number of NMF bases for each echo basis trained from y.
num_prev_bases = 3;                      %%%% Number of previus frames
buffered and used in the calculation of the echo basis
total_num_bases = num_bases_y + num_bases_v;%%%% Total number of NMF basis
vectors
a = 3;                                   %%%% Which speech mixture to
use
Thres = 0.98;                            %%%% Threshold for the
detection of doubletalk
v = nearend(a,:).*Vgain;                    %%%% Clean Near end speech
x = farendclean(a,:);                   %%%% Far end reference signal
y = mixturechange(a,:);                 %%%% Echo + near end mixture
yclean = farendchange(a,:);             %%%% Echo only signal

%%%%%%%%%%%%%% Get near end basis Bn %%%%%%%%%%%
win_length = 1024;       %%% Frame size
stepsize = 512;          %%% Stepsize
[near_train_frames,num_frames] =
STFT(nearendtrain,win_length,stepsize);
%%%% limits of frequency bins
[ref_frames,num_frames] = STFT(x,win_length,stepsize);
[mix_frames,num_frames2,framestart,framestop] =
STFT(y,win_length,stepsize);
[ideal_frames,num_frames2,framestart,framestop] =
STFT(y_no_v,win_length,stepsize);
[ideal_echoframes,num_frames2,framestart,framestop] =
STFT(v,win_length,stepsize);
ref_magframes = abs(ref_frames(:,1:513)).’;
mix_magframes = abs(mix_frames(:,1:513)).’;
mix_magframes2 = abs(mix_frames(:,1:513)).’;
ideal_magframes = abs(ideal_frames(:,a:b));
```

```
ideal_echomagframes = abs(ideal_echoframes(:,a:b));
ideal_magframes2 = abs(ideal_frames(:,1:513));
index = size(ref_magframes);
index2 = size(ideal_magframes2);

M2 = index(1,1);
N2  = index(1,2);
M = index(1,1);
N = index(1,2);

%%%%
num = num_frames*stepsize; %% Only works if stepsize is half window length
DTD = zeros(1,num);
%%%

vhat = zeros(N,M);
yhat = zeros(N,M);
videal = zeros(N,M);
videal_fullband = zeros(N2,M2);
v_energy = zeros(1,N);
y_energy = zeros(1,N);
videal_energy = zeros(1,N);
vhat_energy = zeros(1,N);
yhat_energy = zeros(1,N);
y_no_v_energy = zeros(1,N);
yclean_energy = zeros(1,N);
norm_vhat_energy = zeros(1,N);
ideal_energy = zeros(1,N);
norm_energy =  zeros(1,N);
freq_DTD =  zeros(1,N);
output_v = zeros(1,length(y));
output_v = zeros(1,length(y));

CNMF_start = 1;
CNMF_stop = 1;
time_base = 1;

initialHm = rand(total_num_bases,time_base);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
iter = floor(num_frames/time_base);
for j = 1:1:iter
 V = abs(near_train_frames(CNMF_start:CNMF_stop,1:513)).';
 index = size(V);
 M3 = index(1,1);
 N3 = index(1,2);
 Wt = rand(M3,num_bases_v);
 H = rand(num_bases_v,N3);
 W = 0;
for I = 1:1:train_num_iter_v
Hshift = H;
     for t = 1:1:time_base;
```

```
        Wt = 
Wt.*((V./(Wt*Hshift+1e-9))*Hshift').(ones(M3,N3)*Hshift'+1e-9);
        W = W + Wt;
        Hshift = circshift(Hshift,[0,1]);
        Hshift(:,1) = 0;
     end
        W = W/time_base;
        H = H.*(W'*(V./(W*H+1e-9)))./(W'*ones(M3,N3)+1e-9);
end

Bn = W; %%% Near end basis
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %%%%%%%%%%% Train Echo basis and form mixture basis
    Ve = ref_magframes(:,CNMF_start:CNMF_stop);
    index = size(Ve);
    M4 = index(1,1);
    N4 = index(1,2);
    initialBe = rand(M4,num_bases_y); %%% Echo basis
    initialHe = rand(num_bases_y,N4);
    Be = initialBe;%rand(M,num_bases_y); %%% Echo basis
    He = initialHe;%rand(num_bases_y,N);

    B = 0;
    for I = 1:1:train_num_iter_y
     Hshift2 = He;
     for t = 1:1:time_base
        Be = 
Be.*((Ve./(Be*Hshift2+1e-9))*Hshift2')./(ones(M4,N4)*Hshift2'+1e-9);
        B = B + Be;
        Hshift2 = circshift(Hshift2,[0,1]);
        Hshift2(:,1) = 0;
     end
        B = B/time_base;
        He = He.*(Be'*(Ve./(Be*He+1e-9)))./(Be'*ones(M4,N4)+1e-9);
    end
    Be = B;

    Vm = mix_magframes(a:b,CNMF_start:CNMF_stop);
    Bm = zeros(M,num_bases_y+num_bases_v,time_base);
    Bm = [Bn(a:b,:),Be(a:b,:)];  %%% Mixture basis, Bn nearend basis and 
Be echo basis
    Hm = initialHm;%rand(total_num_bases,1);
    %%%%%% Echo removal stage %%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %%%%%% Match echo to Be and nearend to W using Bm
    Mx = b;
    for I = 1:1:match_num_iter
        Hm = Hm.*(
Bm'*(Vm./(Bm*Hm+1e-9)))./((Bm'*ones(Mx,time_base))+1e-9);
        if (I == match_num_iter-1)
            Hnolate = Hm;
        end
        if (I == match_num_iter-1)|(I == match_num_iter)%|(I == num_iter-2
```

```
            Bm  = Bm .*((Vm./(Bm
*Hm+1e-9))*Hm').*((ones(Mx,time_base)*Hm')+1e-9);
        end

    end
    mat_bot = [Bn(b+1:end,:),Be(b+1:end,:)];
    Bm = [Bm;mat_bot];
    vhat(CNMF_start:CNMF_stop,:) =
(Bm(:,1:total_num_bases-num_bases_y)*Hm(1:total_num_bases-num_bases_y,
:)).';
    yhat(CNMF_start:CNMF_stop,:) =
(Bm(:,num_bases_v+1:end)*Hm(num_bases_v+1:end,:)).';

CNMF_start = CNMF_start + time_base;
CNMF_stop = CNMF_stop + time_base;
end

%%%% Resythesis for audio
vhat = [vhat,fliplr(vhat(:,2:512))];
[xf,yf] = pol2cart(angle(mix_frames),vhat);
resyn = complex(xf,yf);
output_v = resynthesis(resyn,win_length,stepsize);
%%%%%%%%%%%% Objective evaluation %%%%%%%%%%%%%%%%%%
termin = length(output_v);
org_sources = [v(107555:termin);y_no_v(107555:termin)];
index = 1;
[s_target,e_interf,e_artif] = bss_decomp_gain(output_v(107555:termin),
index, org_sources);
[inputSDR,inputSIR,inputSAR] = bss_crit(s_target, e_interf, e_artif);
nearend_measures = [inputSDR,inputSIR,inputSAR];
%%%%%%%% Resythesis no late W updates
yhat = [yhat,fliplr(yhat(:,2:512))];
[xf,yf] = pol2cart(angle(mix_frames),yhat);
resyn = complex(xf,yf);
output_y = resynthesis(resyn,win_length,stepsize);
%%%%%%%%%%%%%%%%
org_sources = [y_no_v(107555:termin);v(107555:termin)];
index = 1;
[s_target,e_interf,e_artif] = bss_decomp_gain(output_y(107555:termin),
index, org_sources);
[inputSDR,inputSIR,inputSAR] = bss_crit(s_target, e_interf, e_artif);
echo_measures = [inputSDR,inputSIR,inputSAR];
%%%% Reconstruct with different phase
[frames,num_frames] = STFT(v,win_length,stepsize);
[framesnear,num_framesnear] = STFT(v,win_length,stepsize);
[xf,yf] = pol2cart(angle(mix_frames),abs(framesnear));
resyn = complex(xf,yf);
diffphasereconstruct = resynthesis(resyn,win_length,stepsize);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
org_sources = [v(107555:termin);y_no_v(107555:termin)];
index = 1;
```

```
[s_target,e_interf,e_artif] =
bss_decomp_gain(diffphasereconstruct(107555:termin), index,
org_sources);
[inputSDR,inputSIR,inputSAR] = bss_crit(s_target, e_interf, e_artif);
compare_measures = [inputSDR,inputSIR,inputSAR];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
erleout = ERLE(y_no_v,output_v,win_length,stepsize);
%%%%%%%%%%

figure
plot(real(output_v))
figure
plot(real(output_y))
figure
plot(v)
grid on;

hold
plot(DTD,'r')
grid on;

figure
plot(vhat_energy)
grid on;
hold

plot(freq_DTD* 0.3,'r')
grid on;
figure

plot(v)
hold
grid on;
plot(real(output_v))
plot(real(output_v) - v(1:length(output_v)),'k')

figure
plot(v)
grid on;

hold
plot(diffphasereconstruct)
plot(diffphasereconstruct - v(1:length(output_v)),'k')
grid on;
```

Objective Measure MATLAB Script:

```
///////////////////////////////////////////////////////////////////
Function Name: Objective measure function
Description: compute evaluation criteria given a decomposition of an
estimated source into target/interference/noise/artifacts of the form
se = s_target + e_interf (+ e_noise) + e_artif

Developers:  - Cedric Fevotte (cf269@cam.ac.uk) – Emmanuel Vincent
( incent@ircam.fr) – Remi Gribonval (remi.gribonval@irisa.fr)
///////////////////////////////////////////////////////////////////
% Usage:
%
% 1) Global mode
%
% [SDR,SIR,(SNR,)SAR]=bss_crit(s_target,e_interf[,e_noise],e_artif)
%
% Input:
%   - s_target: row vector of length T containing the target source(s)
%   contribution,
%   - e_interf: row vector of length T containing the interferences
%   contribution,
%   - e_noise: row vector of length T containing the noise contribution
%   (if any),
%   - e_artif: row vector of length T containing the artifacts
%   contribution.
%
% Output:
%   - SDR: Source to Distortion Ratio,
%   - SIR: Source to Interferences Ratio,
%   - SNR: Signal to Noise Ratio (if e_noise is provided),
%   - SAR: Source to Artifacts Ratio.
%
% 2) Local mode
%
%
[SDR,SIR,(SNR,)SAR]=bss_crit(s_target,e_interf[,e_noise],e_artif,WINDO
W,NOVERLAP)
%
% Additional input:
%   - WINDOW: 1 x W window
%   - NOVERLAP: number of samples of overlap between consecutive windows
%
% Output:
%   - SDR: n_frames x 1 vector containing local Source to Distortion Ratio,
%   - SIR: n_frames x 1 vector containing local Source to Interferences
Ratio,
%   - SNR: n_frames x 1 vector containing local Signal to Noise Ratio,
%   - SAR: n_frames x 1 vector containing local Source to Artifacts Ratio.
%
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% ( incent@ircam.fr) – Remi Gribonval (remi.gribonval@irisa.fr)
```

```
function varargout=bss_crit(varargin)
s_target=varargin{1}; e_interf=varargin{2};

switch nargin
    case 3
        e_noise=[]; e_artif=varargin{3};
        mode='global';
    case 4
        e_noise=varargin{3}; e_artif=varargin{4};
        mode='global';
    case 5
        e_noise=[]; e_artif=varargin{3};
        WINDOW=varargin{4}; NOVERLAP=varargin{5};
        mode='local';
    case 6
        e_noise=varargin{3}; e_artif=varargin{4};
        WINDOW=varargin{5}; NOVERLAP=varargin{6};
        mode='local';
end

T=length(s_target);

switch mode
    case 'global'
        switch isempty(e_noise)
            case 1
                % Computation of the energy ratios

[SDR,SIR,SAR]=bss_energy_ratios(s_target,e_interf,e_artif);
                varargout{1}=10*log10(SDR); varargout{2}=10*log10(SIR);
varargout{3}=10*log10(SAR);
            case 0
                % Computation of the energy ratios

[SDR,SIR,SNR,SAR]=bss_energy_ratios(s_target,e_interf,e_noise,e_artif)
;
                varargout{1}=10*log10(SDR); varargout{2}=10*log10(SIR);
                varargout{3}=10*log10(SNR); varargout{4}=10*log10(SAR);
        end

    case 'local'

        W=length(WINDOW); % Length of window
        n_frames = fix((T-NOVERLAP)/(W-NOVERLAP)); % Number of frames

        switch isempty(e_noise)
            case 1
                F_s_target=bss_make_frames(s_target,WINDOW,NOVERLAP);
                F_e_interf=bss_make_frames(e_interf,WINDOW,NOVERLAP);
                F_e_artif=bss_make_frames(e_artif,WINDOW,NOVERLAP);

[SDR,SIR,SAR]=bss_energy_ratios(F_s_target,F_e_interf,F_e_artif);
```

```
                varargout{1}=10*log10(SDR); varargout{2}=10*log10(SIR);
varargout{3}=10*log10(SAR);
            case 0
                F_s_target=bss_make_frames(s_target,WINDOW,NOVERLAP);
                F_e_interf=bss_make_frames(e_interf,WINDOW,NOVERLAP);
                F_e_noise=bss_make_frames(e_noise,WINDOW,NOVERLAP);
                F_e_artif=bss_make_frames(e_artif,WINDOW,NOVERLAP);

[SDR,SIR,SNR,SAR]=bss_energy_ratios(F_s_target,F_e_interf,F_e_noise,F_
e_artif);
                varargout{1}=10*log10(SDR); varargout{2}=10*log10(SIR);
                varargout{3}=10*log10(SNR); varargout{4}=10*log10(SAR);
        end
end %mode
```

Resynthesis MATLAB Script:

```
//////////////////////////////////////////////////////////////////////
Function name: Resynthesis function
Description: To rebuild the audible output speech from the matched data
//////////////////////////////////////////////////////////////////////
%%%%%%%%%%%%%%%% STFT resynthesis  %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function reconstruct = resynthesis(spec,win_length,stepsize)
dim = size(spec);
num_frames = dim(1,1);
N = dim(1,2);
num_samples = num_frames*stepsize;
reconstruct = zeros(1,num_samples);
ham_win = hanning(win_length);
start = 1;
stop = win_length;

for I = 1:1:num_frames
    reconstruct(start:stop) = real(ifft(spec(I,:)))  +
reconstruct(start:stop);
    start = start  + stepsize;
    stop = stop + stepsize;
    if stop > num_samples
        break
    end
end
```

Appendix B:

TI C6713 DSK Main C Program Implementation

echo.c echo with fixed delay and feedback

```
#include "DSK6713_AIC23.h"                          // codec support
Uint32 fs=DSK6713_AIC23_FREQ_8KHZ;                  // set sampling rate
#define DSK6713_AIC23_INPUT_MIC 0x0015
#define DSK6713_AIC23_INPUT_LINE 0x0011
Uint16 inputsource=DSK6713_AIC23_INPUT_MIC; // select input

#define GAIN 0.6                    // fraction (0 – 1) of output fed back
#define BUF_SIZE 2000               // this sets length of delay
short buffer[BUF_SIZE];             // storage for previous samples
short input,output,delayed;
int I;                              // index into buffer

interrupt void c_int11()        // interrupt service routine
{
   input = input_left_sample();    // read new input sample from ADC
   delayed = buffer[i];            // read delayed value from buffer
   output = input + delayed;       // output sum of input and delayed values
   output_left_sample(output);
   buffer[i] = input + delayed*GAIN; // store new input and a fraction
                                     // of the delayed value in buffer
   if(++I >= BUF_SIZE) i=0;        // test for end of buffer
   return;                         // return from ISR
}

void main()
{
   comm_intr();                    // init DSK, codec, McBSP
   for(i=0 ; i<BUF_SIZE ; i++)     // clear buffer
      buffer[i] = 0;
   while(1);                       //infinite loop
}
```

echo_control.c echo with variable delay and feedback

```
#include "DSK6713_AIC23.h"                          // codec support
Uint32 fs=DSK6713_AIC23_FREQ_8KHZ;                  // set sampling rate
#define DSK6713_AIC23_INPUT_MIC 0x0015
#define DSK6713_AIC23_INPUT_LINE 0x0011
Uint16 inputsource=DSK6713_AIC23_INPUT_MIC; // select input

#define MAX_BUF_SIZE 8000          // this sets maximum length of delay
float gain = 0.5;
short buflength = 1000;
short buffer[MAX_BUF_SIZE];        // storage for previous samples
short input,output,delayed;
int I = 0;                         // index into buffer

interrupt void c_int11()           // interrupt service routine
{
   input = input_left_sample();    // read new input sample from ADC
   delayed = buffer[i];            // read delayed value from buffer
   output = input + delayed;       // output sum of input and delayed values
   output_left_sample(output);
   buffer[i] = input + delayed*gain; // store new input and a fraction
                                     // of the delayed value in buffer
   if(++I >= MAX_BUF_SIZE)         // test for end of buffer
      I = MAX_BUF_SIZE – buflength;
   return;                         // return from ISR
}

void main()
{
   for(i=0 ; i<MAX_BUF_SIZE ; i++)    // clear buffer
      buffer[i] = 0;
   comm_intr();                       // init DSK, codec, McBSP
   while(1);                          //infinite loop
}
```

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