Chiral fermions and torsion in the early Universe

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Torsion arising from fermionic matter in the Einstein-Cartan formulation of general relativity is considered in the context of Robertson-Walker geometries and the early Universe. An ambiguity in the way torsion arising from hot fermionic matter in chiral models should be implemented is highlighted and discussed. In one interpretation, chemical potentials in chiral models can contribute to the Friedmann equation and give a negative contribution to the energy density.

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It is often the case that quantum matter acts as a source for a classical field in situations where quantum aspects of the field itself can be ignored. This approximation has proven extremely useful for Einstein's equations where

$$G_{ab} = 8\pi G < T_{ab} >, \tag{1}$$

works well when the matter source is degenerate fermionic matter, where $<>$ is a quantum expectation value, and for thermal radiation, where $<>$ is a thermal average of photons. There are difficulties with this approach however, not least that the singularities inherent in fully fledged quantum field theory for the sources render (1) ambiguous and some criterion for cutting off the integrals must be introduced. For example it is well known that a naïve calculation of the vacuum energy density of the standard model of particle physics leads to far too high a value of the cosmological constant to be compatible with observations [1]. Nevertheless (1) seems to work well in the early Universe when the dynamics is dominated by radiation, as long as temperatures are well below the Planck temperature. In the radiation dominated Universe Einstein's equations boil down to the Friedmann-Robertson-Walker (FRW) equation, ignoring spatial curvature this is

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} < T_{00} >= N_{eff}\frac{4\pi^3}{45}\frac{T^4}{m_{Pl}^2} \tag{2}$$

where $a(t)$ is the cosmological scale factor and N_{eff} is the effective number of degrees of freedom in the relativistic gas,

$$N_{eff} := N_B + \frac{7}{8}(N_+ + N_-) \tag{3}$$

with N_B the number of bosonic degrees of freedom (2 for photons), and N_+ and N_- the number of positive chirality and negative chirality fermionic degrees of freedom respectively (for a standard model neutrino $N_+ = 2$, $N_- = 0$; for a Dirac fermion $N_+ = N_- = 2$), [2]. The Planck mass, $m_{Pl}^2 = G^{-1}$ (we use units with $\hbar = c = 1$), appears in (2) not because we are considering a theory of quantum gravity but because of the quantum nature of the source for classical gravity.

In the Einstein-Cartan formulation of general relativity fermionic matter is expected to induce torsion (recent bounds on the magnitude of torsion have been derived from tests of violation of Lorentz invariance [3] and from cosmic microwave polarization [4]). When the connection is varied in the Einstein-Cartan action the torsion two-forms $\tau^a = \frac{1}{2}\tau^a{}_{bc}e^b \wedge e^c$ are determined by a spinor field Ψ via the algebraic equation

$$\tau^a = 2\pi G\epsilon_{abcd}(\overline{\Psi}\gamma^5\gamma^d\Psi)e^b \wedge e^c, \tag{4}$$

$a, b, c, \dots$ are orthonormal indices (for a review of torsion in Einstein-Cartan formulation in general see [5] and [6]).

In the spirit of (1) the equation of motion (4) would be interpreted as

$$\tau_{a,bc} = 4\pi G\epsilon_{abcd} < \overline{\Psi}\gamma^5\gamma^d\Psi >= -4\pi G\epsilon_{abcd} < j_5^d >, \tag{5}$$

where $j_5^a = \overline{\Psi}\gamma^a\gamma^5\Psi$ is the axial current. As is well known fermions generate torsion in the anti-symmetric class of tensors, according to the classification of [7].

We shall examine the effect of torsion arising from relativistic fermions in the early Universe, assuming isotropy and spatial homogeneity of both the geometry and the matter. It will be assumed that the metric of Robertson-Walker type and that the energy-momentum is of the form

$$T_{ab} = \begin{pmatrix} \rho & 0 \\ 0 & P\delta_{ij} \end{pmatrix} \tag{6}$$

where the density ρ and pressure P are homogeneous and depend only on time and $i, j = 1, 2, 3$ are space-like indices.

The Riemann tensor involves the square of the connection and the net effect of including the torsion (4) into the gravitational connection is that Einstein's equations are modified to

$$3\left(\frac{\dot{a}^2}{a^2} - \frac{\tau^2}{4}\right) = 8\pi G\rho \quad (7)$$

$$-\frac{2\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{\tau^2}{4} = 8\pi G P, \quad (8)$$

where $a(t)$ is the Robertson-Walker scale factor and

$$\tau^2 = -\tau^a\tau_a = -16\pi^2 G^2(\overline{\Psi}\gamma^5\gamma^a\Psi)(\overline{\Psi}\gamma^5\gamma_a\Psi) \quad (9)$$

(the metric signature is $(-,+,+,+)$, the Clifford algebra convention is $\{\gamma^a,\gamma^b\} = -2\eta^{ab}$, with γ^0 hermitian, and spatial curavature is taken to be zero). Equations (7) and (8) are not independent and are related by the (first) Bianchi identity which is analysed below. Eliminating τ gives the usual relation between the acceleration and the density and pressure,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P). \quad (10)$$

Since fermions constitute quantum matter, it seems natural to interpret (4) in the early Universe as meaning a thermal average (5). However there is an ambiguity as to whether (9) should be interpreted using

$$< (\overline{\Psi}\gamma^5\gamma^a\Psi)(\overline{\Psi}\gamma^5\gamma_a\Psi) > = < j_5^a j_{5,a} > \quad (11)$$

or

$$< \overline{\Psi}\gamma^5\gamma^a\Psi > < \overline{\Psi}\gamma^5\gamma_a\Psi > = < j_5^a > < j_{5,a} > \quad (12)$$

These are different in general. The former can be Fierz re-arranged to give

$$< (\overline{\Psi}\gamma^5\gamma^a\Psi)(\overline{\Psi}\gamma^5\gamma_a\Psi) > = 4 < (\Psi_+^\dagger\Psi_-)(\Psi_-^\dagger\Psi_+) > \quad (13)$$

where Ψ_+ and Ψ_- are the positive and negative chirality components of Ψ. In a first quantised theory this is positive definite for Dirac spinors and vanishes for Weyl spinors [5], hence $\tau^2 < 0$ in (9), making the torsion space-like (in the classification of [7] this is denoted As). The cosmological consequences of this formulation in inflationary models are explored in [8]. The same philosophy, applied to spin densities rather than the pseudo-vector $\overline{\Psi}\gamma^5\gamma^a\Psi$, is followed in [9] and [10].

We reach a radically different conclusion if we use (12), which follows from taking the thermal average of (4) before calculating the Riemann tensor. Applying the usual Robertson-Walker assumptions of spatial homogeneity and isotropy to the connection, and hence the torsion, we would conclude that, in the cosmic frame,

$$< j_5^i > = 0, \quad (14)$$

while

$$< j_5^0 > = (n_+ - \overline{n}_+) - (n_- - \overline{n}_-) \quad (15)$$

where n_+, $\overline{n}_+$, n_- and $\overline{n}_-$ are thermal averages of the number densities of positive chirality particles, positive chirality anti-particles, negative chirality particles and negative chirality anti-particles respectively. In contrast the vector current $j^a = \overline{\Psi}\gamma^a\Psi)$ has

$$< j^0 > = (n_+ - \overline{n}_+) + (n_- - \overline{n}_-) = n - \overline{n}, \quad (16)$$

where $n = n_+ + n_-$ is the number density of all particles and $\overline{n} = \overline{n}_+ + \overline{n}_-$ the number density of anti-particles. The integral of j^0 over a co-moving 3-dimensional volume V gives the total number of fermions in that volume, counting anti-particles as negative,

$$\int_V j^0 a^3(t) d^3x = \mathcal{N} - \overline{\mathcal{N}}, \quad (17)$$

where $\mathcal{N} = \int_V n a^3(t) d^3x$, etc. The corresponding number for the axial current,

$$\int_V j_5^0 a^3(t) d^3x = \mathcal{N}_+ - \mathcal{N}_- - (\overline{\mathcal{N}}_+ - \overline{\mathcal{N}}-) := \mathcal{N}_A \quad (18)$$

we shall call the axial particle number in the volume V.

In the abscence of chemical potentials, for a thermal distribution with the temperature much greater than any particle masses,

$$n_\pm = \overline{n}_\pm = \frac{3}{4}\frac{\zeta(3)}{\pi^2}T^3 \quad (19)$$

so $\mathcal{N} = \overline{\mathcal{N}}$ and the torsion vanishes. A non-zero chemical potential is necessary for any asymmetry between particle and anti-particle numbers. We see here that a chemical potential can also generate torsion, since then

$$n_\pm - \overline{n}_\pm = \frac{1}{6}\mu_\pm T^2 + o\left(\frac{\mu_\pm}{T}\right)^2 T^3 \quad (20)$$

for $\mu_\pm << T$, where μ_+ and μ_- are chemical potentials for positive and negative chirality particles, see *e.g.* [2]. In a chiral theory, such as the Standard Model, μ_+ and μ_- can be different in general and

$$< j_5^0 > = \frac{1}{6}(\mu_+ - \mu_-)T^2, \quad (21)$$

where we have ignored terms $o\left(\frac{\mu_\pm}{T}\right)^2$. In a Robertson-Walker Universe undergoing adiabatic expansion, $a \propto 1/T$, the thermal average of the axial current $\nabla_a < j_5^a > = 0$ is conserved, and hence $\mathcal{N}_A$ is constant, if and only if $(\mu_+ - \mu_-) \propto T$ is linear in T. The total fermion number, $\mathcal{N} - \overline{\mathcal{N}}$, is constant if and only if $(\mu_+ + \mu_-) \propto T$ (particle masses are being ignored here).

To summarise, in general equations (5) and (12) give

$$\tau^2 = \frac{4\pi^2 G^2}{9}(\mu_+ - \mu_-)^2 T^4 \quad (22)$$

which is positive in any chiral model for matter with $\mu_+ \neq \mu_-$. For a model with N_f different types of fermion each fermions species can have different chemical potentials $\mu_\pm^{(k)}$, where we label the species with an integer k, and then $\mu_\pm$ is always understood below to mean $\mu_\pm = \sum_{k=1}^{N_f} \mu_\pm^{(k)}$. Since positive and negative chirality particles in a chiral model can have different weak charges one expects that $\mu_+^{(k)} \neq \mu_-^{(k)}$ in general, and the torsion can be non-zero, at least for the era before the electro-weak phase transition, [11].

Define

$$\tau = \frac{2\pi G}{3}(\mu_+ - \mu_-)T^2, \qquad (23)$$

in terms of which the non-vanishing components of the torsion are

$$\tau_{i,jk} = \epsilon_{ijk}\tau, \qquad (24)$$

(in the classification of [7] this is time-like, At). Rotational invariance of the thermal average is not incompatible with the conclusion of [12], where classical solutions of the Weyl equation were analyzed in spherical symmetric space-times with torsion — thermal averages do not necessarily have the same symmetries as solutions of the equations of motion. The general form of the torsion compatible with Robertson-Walker symmetries was given in [13]. The fact that chiral fermions can have interesting consequences when torsion is taken into consideration was noted in the context of anomalies for lepton currents in the Standard Model of particle physics in [14].

Both (11) and (12) have interesting, though very different, cosmological consequences. The form (12), being the square of a vector, has a dual description as the square of a 3-form and as such is in the class of models described in [15]. Indeed a term of this form is present in the Landau-Ginsparg models discussed in [15], though the stabilising quartic term is absent and there is no kinetic term here. A kinetic term would require time derivatives of the torsion and so would go beyond Einstein-Cartan theory — such terms would be expected to appear in an effective action description of gravity involving higher derivatives and powers of the Riemann tensor.

So which should one use (11) or (12)? Weinberg [16] takes the point of view that there is nothing special about torsion: it is just another tensor and one can always move it to the right hand side of Einstein's equations and consider it to be part of the matter rather than part of the geometry. We see here that, in the context of (1), there is an ambiguity. If the torsion terms are absorbed into the energy momentum tensor before expectation values are taken then it would seem that (11) is appropriate. In the Einstein-Cartan formulation however the torsion is determined by the equation of motion (4), in which the square of the torsion does not appear. If the gravitational field itself is not quantised, it is hard to see any interpretation of the equation of motion (4) other than (5). When the Riemann tensor is calculated it is then (12) that arises and not (11). Much of the literature has focused on (11), in a cosmological context for example (11) was used in [8]. In this paper the consequences of (5) and (12) will be explored and developed.

We shall see that, in the context of the early Universe, the torsion can give a negative contribution to the energy density. The mechanism here is different to torsion induced avoidance of the initial singularity due to spin fluids considered previously, [17–21], in which the spin density necessarily breaks either rotational or translation invariance.

When there is torsion the Bianchi identity does not require that G_{ab} be co-variantly constant, in general one has

$$\nabla_b G^{ba} = -\tau^c{}_{bc}G^{ba} + \frac{1}{2}\widetilde{R}^{adbc}\tau_{d,bc}, \qquad (25)$$

where $\widetilde{R}^{adbc} := \frac{1}{4}\epsilon^{ada'd'}R_{a'd'b'c'}\epsilon^{bcb'c'}$. In the case of Friedmann-Robertson-Walker (FRW) Universes under study here only the second term on the right hand side contributes giving

$$\nabla_b G^{b0} = -\frac{3}{2}\frac{\tau}{a}\frac{d}{dt}(\tau a), \qquad \nabla_b G^{bi} = 0. \qquad (26)$$

One strategy is to demand $\nabla_b G^{ba} = 0$ and use this to determine the torsion, implying that $\tau \propto 1/a$ [22], but this is too restrictive for our purposes. Instead we take thermodynamic averages as above and use (23) for the form of the torsion.

Assuming adiabatic expansion $T \propto 1/a$, [2], a can be eliminated from Einstein's equations in favour of T to give

$$\frac{\dot{T}^2}{T^2} = \frac{8\pi G}{3}\rho + \frac{\pi^2 G^2}{9}(\mu_+ - \mu_-)^2 T^4, \qquad (27)$$

$$\frac{\ddot{T}}{T} = \frac{4\pi G}{3}(5\rho + 3P) + \frac{2\pi^2 G^2}{9}(\mu_+ - \mu_-)^2 T^4. \qquad (28)$$

These two equations are not independent, the Bianchi identity (26) gives an equation relating ρ, P, T and $\mu_\pm$. Expressing time derivatives as temperature derivatives, $\frac{d}{dt} = \dot{T}\frac{d}{dT}$, differentiating (27) and using (28) to eliminate $\ddot{T}$ gives

$$T^3 \frac{d}{dT}\left\{(\mu_+ - \mu_-)^2 T^2\right\} = \frac{24}{\pi G}\left(3h - T\frac{d\rho}{dT}\right), \qquad (29)$$

where $h = \rho + P$ is the enthalpy density. For standard equations of state (relativistic gas, dust, cosmological constant) the right hand side of (29) vanishes, so the torsion described here necessarily requires a modification of the equation of state. Let $\rho = \rho_0 + \Delta\rho$, $P = P_0 + \Delta P$ and $h = h_0 + \Delta h$, where ρ_0 etc., satisfy a standard equation of state. Then

$$T^3 \frac{d}{dT}\left\{(\mu_+ - \mu_-)^2 T^2\right\} = \frac{24}{\pi G}\left(3\Delta h - T\frac{d\Delta\rho}{dT}\right). \qquad (30)$$

For a relativistic gas with the full energy-momentum tensor traceless $\Delta P = \frac{\Delta\rho}{3}$ and (30) reduces to

$$\frac{d}{dT}\left\{(\mu_+ - \mu_-)^2 T^2\right\} = -\frac{24}{\pi G}T^2\frac{d}{dT}\left(\frac{\Delta\rho}{T^4}\right). \quad (31)$$

Two special cases are:

- If axial particle number is conserved, $(\mu_+ - \mu_-) = bT$ with b a constant, so $\Delta\rho$ must have a T^6 component,

$$\rho = \frac{\pi^2}{30}N_{eff}T^4 - \frac{\pi G b^2}{12}T^6, \quad (32)$$

  where the T^4 term is assumed to have the usual form for massless particles with N_{eff} degrees of freedom. In particular the torsion necessarily gives a negative contribution to the energy density in this case, though in realistic models this is a small effect. We must take $b << 1$ to be consistent with our assumption that $(\mu_+ - \mu_-) << T$. This is compatible with the observed value of the current ratio of Baryon to photon densities

$$\frac{n - \bar{n}}{n_\gamma} = \frac{\pi^2}{12\zeta(3)}\left(\frac{\mu_+ + \mu_-}{T}\right) \approx 10^{-9}, \quad (33)$$

  with $n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3$. Unless μ_+ and μ_- have opposite sign and the smallness of this number is due to a delicate cancellation between two large numbers, this requires b to be of the order of, or less than, 10^{-9} at the present time. Then the ratio of the two terms in (32) is of order $\frac{b^2}{N_{eff}}\left(\frac{T}{m_{Pl}}\right)^2$ which is very small for $T << m_{Pl}$ (obviously the analysis here is only valid for $T << m_{Pl}$ where quantum gravity effects are assumed to be very small).
  This behaviour $\tau \sim 1/a^3$ for torsion arising from spin in the early Universe has been studied before, [20, 21].

- For any period during which $\mu_+ - \mu_-$ is constant, independent of temperature, equation (30) gives a logarithmic correction to the the energy density,

$$\rho \approx \frac{\pi^2}{30}N_{eff}T^4 - \frac{\pi G}{12}(\mu_+ - \mu_-)^2 T^4 \ln\left(\frac{T}{T_0}\right), \quad (34)$$

  where T_0 is an arbitrary constant that can be absorbed into the definition of N_{eff}. Axial particle number is not conserved in this case and one expects Baryon/Lepton number violating processes unless $\mu_+ = -\mu_-$.

Of course the generation of Baryon or Lepton number requires P and CP violation as well as a period out of thermal equilibrium, according to the Sakharov conditions, [25]. The details would depend on the particular chiral model generating the chemical potentials and taking the Universe through a period during which it is out of thermal equilibrium.

At first sight if might seem disconcerting that energy-momentum does not appear to be conserved in this formalism — because of (26) and the Einstein equations T_{ab} cannot be co-variantly constant unless $a\tau$ is constant. However an "improved" energy-momentum tensor, which is conserved, can be defined. We make the co-variant decomposition of the Einstein tensor

$$G_{ab} = \overset{0}{G}_{ab} + \Delta G_{ab} \quad (35)$$

where $\overset{0}{G}_{ab}$ is the Einstein tensor constructed from the torsion-free connection. We similarly decompose the connection one-forms as

$$\omega^a{}_b = \overset{0}{\omega}{}^a{}_b + \Delta\omega^a{}_b \quad (36)$$

with $\overset{0}{\omega}{}^a{}_b$ the torsion-free connection. Expanding $\Delta\omega^a{}_b = \Delta\omega^a{}_{b,c}e^c$ the components $\Delta\omega^a{}_{b,c}$, being the difference of two connections, constitute a tensor field so (36) is again a co-variant decomposition. $\overset{0}{G}_{ab}$ is the zero torsion Einstein tensor for which the first Bianchi identity implies

$$\overset{0}{\nabla}_b \overset{0}{G}{}^{ba} = 0 \quad (37)$$

where $\overset{0}{\nabla}_b$ is the co-variant derivative using $\overset{0}{\omega}{}^a{}_b$. From this follows

$$\nabla_b G^{ba} = \overset{0}{\nabla}_b\left(\Delta G^{ba}\right) + \Delta\omega^b{}_{c,b}G^{ca} + \Delta\omega^a{}_{c,b}G^{bc}. \quad (38)$$

We also have, by definition,

$$\nabla_b T^{ba} = \overset{0}{\nabla}_b T^{ba} + \Delta\omega^b{}_{c,b}T^{ca} + \Delta\omega^a{}_{c,b}T^{bc} \quad (39)$$

for T_{ab}. Einstein equations, $G^{ab} = 8\pi G T^{ab}$, now imply

$$\overset{0}{\nabla}_b\left(\Delta G^{ba}\right) = 8\pi G\,\overset{0}{\nabla}_b T^{ba}. \quad (40)$$

An "improved" energy-momentum tensor can be defined

$$\mathcal{T}^{ab} := T^{ab} - \frac{1}{8\pi G}\Delta G^{ab} \quad (41)$$

which is conserved using the torsion free connection,

$$\overset{0}{\nabla}_b \mathcal{T}^{ba} = 0. \quad (42)$$

For example if $\tau = \left(\frac{2\pi G b}{3}\right)T^3$ in a radiation dominated Universe, the improved energy-momentum tensor for FRW space-time with torsion is

$$\mathcal{T}_{ab} = \begin{pmatrix} \rho_0 & 0 \\ 0 & \frac{\rho_0}{3}\delta_{ij} \end{pmatrix} - \frac{b^2\pi G}{24}T^6\delta_{ab} \quad (43)$$

with $\rho_0 = \frac{\pi^2}{30} N_{eff} T^4$. In fact both terms in (43) are separately conserved with the torsion-free connection.

Finally we observe that the geometrical significance of non-zero τ follows from the anti-symmetrised action of two co-variant derivatives on an arbitrary vector field with components U^a,

$$[\nabla_a, \nabla_b] U^c = -\tau^d{}_{ab} \nabla_d U^c + R^c{}_{dab} U^d. \qquad (44)$$

In addition to the algebraic (rotation) term involving the Riemmann tensor there is a derivative term involving the torsion — a deficit displacement implying that parallelograms generated by parallel transport do not close. The deficit displacement in Robertson-Walker space-time described here is compatible with 3-dimensional rotational symmetry — a vector field with Robertson-Walker symmetries must have $U^i = 0$ in which case

$$[\nabla_i, \nabla_j] U^0 \;=\; -\tau\, \epsilon_{ij}{}^k \nabla_k U^0 \qquad (45)$$

and this vanishes if U^0 independent of position. For any field compatible with the Robertson-Walker symmetries space-like parallelograms close with the torsion studied here. However they need not close for fields that do not share the symmetries of the background metric.

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