Twin suppression in digital holography by means of speckle reduction.

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Abstract -- **A method for numerically removing the twin image in on-axis digital holography, based on multiple digital holograms, is discussed. The digital holograms under examination are captured experimentally using an in-line modified Mach-Zehnder interferometric setup and subsequently reconstructed numerically. The technique is suitable for both transmission and reflection geometry. Each individual hologram is recorded with a statistically independent diffuse illumination field. This is achieved by shifting a glass diffuser in the *x-y* plane of the object path. By recording the holograms in this manner the twin image, from a numerical reconstruction, appears as speckle. By reducing this speckle pattern the twin image can be effectively removed in the reconstruction plane. A theoretical model is developed and experimental results are presented that validate this model.**

Index Terms—**Digital Holography, On- Axis, Speckle, Twin reduction**

I. Introduction

In recent years there has been a great deal of interest in the field of Digital Holography (DH)[1-5] and 3D display[6, 7] and capture technology. This is apparent by the number of papers published in the literature. Holography[8, 9] is a method for capturing the complex field of an object and thus limited three-dimensional structures can be obtained. Recent technological improvements, such as CCD cameras, high-powered desktop computers and spatial light modulators, have made DH a viable alternative to traditional holography. DH boasts advantages such as digital storage, processing and compression of holograms, transmission over existing digital infrastructure. In this paper we will examine a method of twin removal in digital holography based on a speckle[10, 11] reduction technique.

II. Theory

In the following section we present a simple theoretical model to describe the behaviour of our optical system. Our aim here is not to conduct a fully rigorous examination of the complex interaction of multiple speckle fields and various apertures in the system but rather to present a plausible description of the complex behaviour of the system. A more complete analysis would take us far beyond the scope of this manuscript. A colluminated plane wave is generated using a spatial filter and lens as depicted in Figure. 1. This colluminated plane wave is then incident on a diffuser. We assume that the diffuser is optically rough and imparts a random phase to the plane wave front that emerges from the diffuser. This random phase field now propagates to the object plane where it illuminates our transmissive target. We describe the random field that illuminates our object as

$$U_R(X) = a_R(X)\exp\left[j\phi_R(X)\right], \tag{1}$$

where $a_R(X)$ and $\phi_R(X)$ are random amplitude and phase values respectively. We note that the random phase field at the output of the diffuser gives rise to both random amplitude and phase values (a_R and ϕ_R) at our object plane, due to diffraction introduced by the finite extent of the diffuser. For simplicity however we assume that the diffuser is sufficiently large and the distance d_1 sufficiently small (see Figure 1) such that the resulting speckle field in the object plane may be assumed to be delta correlated. We describe the effect of our transmissive object as

$$U_T(X) = a_T(X)\exp\left[j\phi_T(X)\right], \tag{2}$$

and write the field immediately after our object as

$$U(X) = U_T(X)U_R(X). \tag{3}$$

This combined field, $U(X)$ is now allowed to propagate to the CCD plane where it interfers with an ideal unit amplitude plane wave, $R(x) = \exp[j2\pi(z-z_c)/\lambda]$ and the resulting interference pattern is recorded. We write the continious intensity distribution incident upon the camera face as

$$H(x) = \left|u_z(x) + R(x)\right|^2, \tag{4 a}$$

$$H(x) = I_z + I_R + u_z(x)R^*(x) + u_z^*(x)R(x), \tag{4 b}$$

where I_z, and I_R are the DC terms corresponding to the object and reference intensities respectively and '*' denotes complex conjugate operation. The latter two terms in Eq. (4 b) correspond to the real and twin image terms respectively. The field u_z is related to our object field $U(X)$ by a Fresnel transform

$$u_z(x) = \Im_z\left\{U(X)\right\}(x),$$

$$u_z(x) = \frac{1}{\sqrt{j\lambda z}}\int U(X)\exp\left[\frac{j\pi}{\lambda z}(x-X)^2\right]dX, \tag{5}$$

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where $\Im_z\{\ \}$ is the Fresnel transform operator. We now make some more simplifying approximations. In practical DH systems the continuous intensity field, Eq. (4 b) is recorded by a camera of finite physical extent using finite size pixels located at fixed distances from each other. Each of these factors, act to significantly limit the imaging performance of DH systems and we refer the reader to [12] for more detail. However for our purposes we do not need to consider these aspects of the imaging process to get across the essence of our idea and so for simplicity we assume that the continuous field $H(x)$, Eq. (4 b) is available to us. We note that the DC terms can be removed either numerically[13] or by recording the reference and object intensities separately and subtracting them from the captured hologram. Setting $z = z_c$ in Eq. (4 b), removing the dc terms and performing an inverse Fresnel transform yields the following result

$$A(x) = U(X) + \Im_{-z}\{u_z^*(x)\}(X), \tag{6 a}$$

$$A(x) = U(X) + \tilde{U}(X) \tag{6 b}$$

where $\tilde{U}(X)$ is the twin image term.

To remove the twin image term requires that we capture multiple digital holograms using a series of statistically independent speckle fields to illuminate our object. We assume that each of these statistically independent fields has the same average intensity M. Each of the resulting digital holograms are then reconstructed and averaged on an intensity basis in the object plane. We will now examine what happens to the real and twin terms as we average them on an intensity basis in the object plane. Let us first consider the real image term. Using a random speckle field, denoted 'n', to illuminate our object the resulting reconstructed intensity pattern of the real image term is given by

$$I_n(X) = U(X)U^*(X),$$
$$I_n(X) = |a_T a_{Rn}|\exp[j(\phi_T + \phi_{Rn})]|a_T a_{Rn}|\exp[-j(\phi_T + \phi_{Rn})],$$
$$I_n(X) = (|a_T||a_{Rn}|)^2. \tag{6}$$

We now average N of these intensity distributions formed by N statistically different speckle fields

$$I^{AR}(X) = \frac{I_1 + I_2 + \ldots.. + I_N}{N},$$
$$I^{AR}(X) = |a_T|^2\left(\frac{|a_{R1}|^2 + |a_{R2}|^2 + \ldots + |a_{RN}|^2}{N}\right),$$
$$I^{AR}(X) = |a_T|^2(M) \tag{7}$$

where I^{AR} represents the result of averaging together N real image intensity reconstructions. We turn our attention to the term in round brackets in Eq. (7) and note that the sum of N statistically different intensity patterns as N goes to infinity reduces to the average intensity value for an individual speckle distribution. Therefore we may replace the term in the round brackets in Eq. (7), by the average intensity value for a given speckle field, M. It is important to note that the intensity distribution for our object field, $|a_T|^2$, is contained in Eq. (7).

We now consider the twin image term. Examining the derivation of Theorem 3 in Ref. [14] we find that $u_z^*(x) = \Im_{-z}\{U^*(X)\}(x)$. Using this result we re-write the twin image term $\tilde{U}(X)$ as

$$\tilde{U}(X) = \Im_{-2z}\{U^*(X)\}(X). \tag{8}$$

The corresponding intensity is given by

$$\tilde{I}_n(X) = U_n(X)U_n^*(X)$$
$$\tilde{I}_n(X) = \left(\int a_{Rn}a_T \exp[j(\phi_T + \phi_{Rn})]\exp\left[\frac{j\pi}{2\lambda z}(X - X_1)^2\right]dX_1\right)$$
$$\times\left(\int (a_{Rn}a_T)^* \exp[-j(\phi_T + \phi_{Rn})]\exp\left[\frac{-j\pi}{2\lambda z}(X - X_2)^2\right]dX_2\right) \tag{9}$$

This result means that each intensity distribution due to the twin image term generates is a statistically independent speckle pattern. Like in the previous case the $a_T\exp(j\phi_T)$ term remains constant however now each component is multiplied by a random phase and then Fresnel transformed. Thus averaging over N intensity patterns gives the following result

$$\tilde{I}^{TR}(X) = \frac{\tilde{I}_1 + \tilde{I}_2 + \ldots.. + \tilde{I}_N}{N}$$
$$\tilde{I}^{TR}(X) = M. \tag{10}$$

This latter equation suggests that the twin image becomes gradually reduced as more intensity distributions are averaged together. Finally it is important to consider that cross-terms (interference between the real and the twin image) that arise when we calculate the intensity of Eq. (6 b). This interfernce term can be re-expressed as

$$CT = |U||\tilde{U}|\cos(\theta_R) \tag{11}$$

where θ_R can be shown to be a random variable. Thus we see that the interference term described in Eq. (11) will average to zero and can be neglected. Although the analysis here is presented for transmissive objects (see Ref. [15]), it also seems to apply to reflective objects too as a new series of experimental results indicate. We would like to acknowledge that the theoretical description provided is relatively simplistic however it does capture some essence of the underlying physical behavior of the system, as we shall now demonstrate with a series of experimental results

III. Results

The experimental set-up is shown in Figure 1. A 678nm laser is used. The wave-plate in this set-up is used in conjunction with a polarising beam splitter to allow the laser power between the two paths to be adjusted. A piezo mirror, electronically controlled, is employed to impart a phase-shift into the object path of the set-up.

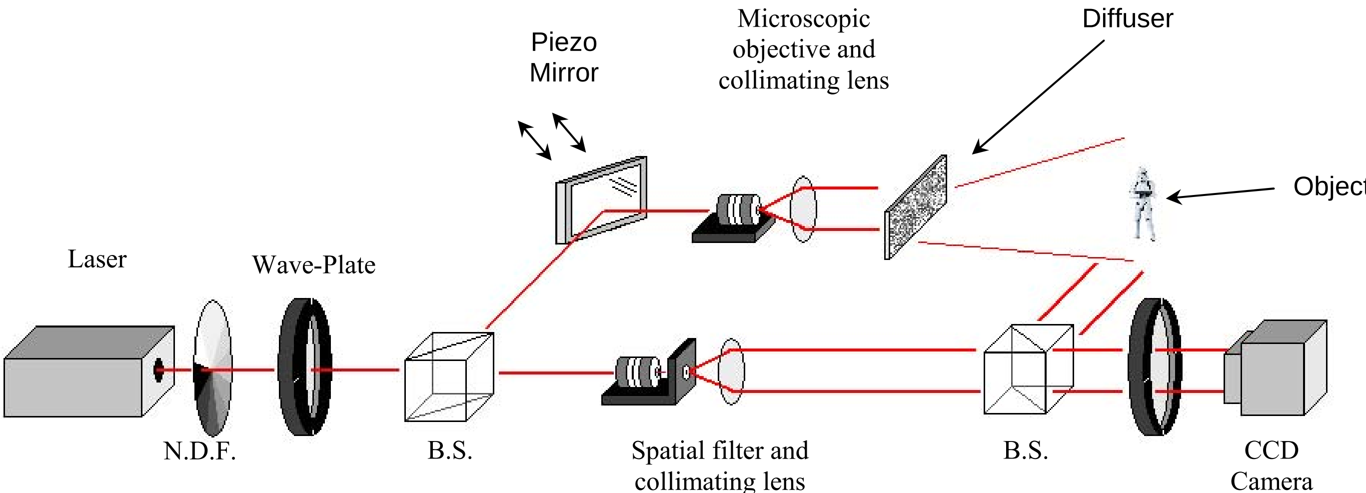
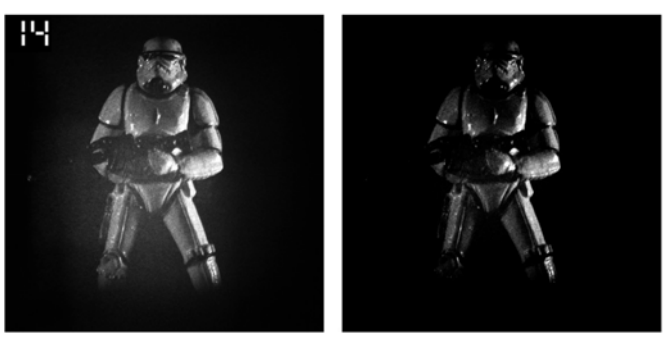


Figure 1. Experimental set-up used in the recording of reflective digital holograms. Where N.D.F. is a neutral density filter and B.S. is a polarising beam splitter.

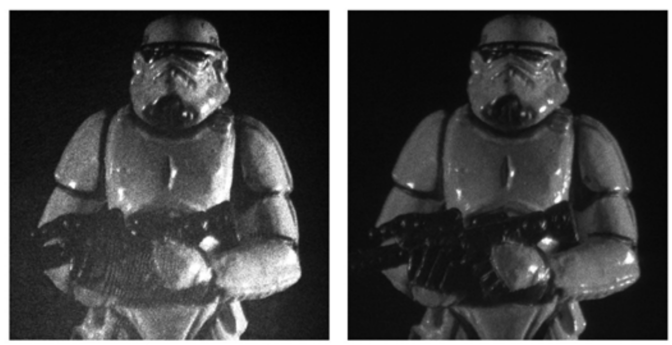
This allows a Phase Shift Interferometry (PSI)[16, 17] digital hologram to be captured. The *x-y* position of the diffuser is moved in between each capture of a digital hologram to provide a different and statistically independent speckle pattern on each hologram.

All the holograms presented in Figure 2 are reflection holograms and were recorded using the experimental set-up shown in Figure 1. The reconstruction distance for these holograms is 285mm. They have been numerically reconstructed using the direct method (Fast Fourier transform based technique) to implement the discrete Fresnel transform[14 & 18], (also see Eq 5).

(a) (b)



(c) (d)



(e) (f)

Figure 2. Numerical reconstructions of a digital hologram showing (a) the DC term, twin and object, (b) single hologram and twin, (c) 14 holograms added together, (d) post-processing of (c), (e) zoomed in portion of (c) for comparison with 14 PSI holograms added together in (f).

Figure 2(a) shows a reconstruction that contains a strong DC component (or zero order term), the twin term and the original object. The DC component arises due to the intensity terms that appear as a product of the holographic process. In Figure 2(b) the DC component had been removed by a numerical high-pass filter and it can be clearly seen that the resultant reconstruction contains the original object and the twin term, which has been reconstructed as a speckle pattern due to the introduction of the diffuser (see Figure 1). Figure 2(c) shows the results when 14 separate holograms have been added together on an intensity basis. It can be seen that the twin term has been significantly reduced when compared with Figure 2(b). The process of addition has produced a background DC term, which has been removed in Figure 2(d). This DC term has been removed by subtracting the mean value of the background from the entire reconstruction. Figure 2(e) and (f) show a comparison between the speckle reduction method, (e), and a PSI method, (f).

IV. CONCLUSION

The presence of a twin image in on-axis digital holography is a fundamental property of a holographic imaging system. The removal or reduction of this twin image is of principal importance in digital holography as it is present in the reconstructed image as a source of noise. In this paper we have examined a method of twin removal based on a speckle reduction technique. We have shown that this method can be applied to reflective objects in digital holography.

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