NUI MAYNOOTH
Ollscoil na hÉireann Má Nuad

FACTOR SUBSTITUTION IN THE IRISH MANUFACTURING SECTOR

submitted by

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to

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Summary

In this paper I use a flexible specification of a cost function to estimate the elasticities of demand for skilled labour, unskilled labour and capital in the Irish manufacturing sector for the years 1979 to 2005. An understanding of the magnitude of these elasticites can play an important role in developing approrpariate policies for promoting growth and employment in the economy. The research updates previous work and examines whether recent changes in Ireland relating to Foreign Direct Investment, increased trade and increased inwards migration have altered the input demand elasticities. The main findings of the study are that in most industries in the Irish manufacturing sector, own demand elasticities are less than unity, indicating that an exogenous shock that increases wages and the cost of using physical capital will not be expected to have a large influence on the demand for labour and capital. Of the demands for the three inputs, the demand for capital was the least elastic, while the demand for unskilled labour tended to be the most elastic. In most cases, I find that all inputs are substitutes while there is evidence of relative capital skilled labour complementarity. Based on the results, I also conclude that the changes in the economic performance of the Irish economy beginning of the 1990's do not, for the most part, seemed to have changed the elasticity of demand for input. I discuss some potential reasons for this finding and suggest possible avenues for future research.

1. Introduction

In this paper I estimate the elasticities of demand for three types of inputs used in the production process: skilled labour, unskilled labour and capital in the Irish manufacturing sector for the years 1979 to 2005. I do this first by estimating a system of three input demand equations that are derived from an optimisation problem of minimising the total cost of production incurred by a typical firm subject to a fixed level of output. The parameter estimates from this exercise are then used to calculate own demand elasticities, elasticities of substitution and cross price elasticities for the three inputs. An analysis of inter-temporal changes in own demand elasticities over the period will also be carried out.

The Irish economy experienced significant changes during the period from 1980 to 2005. After going through difficult times in the 1980s, Ireland experienced unprecedented economic growth rates from the beginning of the 1990s. There were three noticeable features associated with this development. Firstly, Ireland experienced a large increase in Foreign Direct Investment (FDI). The second development was an increase in trade. Finally, there was an influx of immigrants, mainly from Eastern Europe, who came in response to this economic growth as well as in response to changes in legislation, which relaxed restrictions placed on immigrants from other parts of Europe looking for jobs in the Irish market. The purpose of this study is to update earlier estimates of elasticities of demand for labour for Ireland and to examine the extent to which the changes noted above affected these elasticities.

It is important to know about wage elasticities since the magnitudes of elasticities indicate by how much a shock to the input prices would change the demand for those inputs and the substitute inputs. When elasticities are high, employers substitute labour for a cheaper input, which implies redundancies for that type of labour. Through social welfare services, it implies that the burden on government resources would have increased. High magnitudes of labour demand elasticities can also reduce union bargaining power. The reason why unions cannot bargain for and win large wage gains for their members if the demand elasticity is high is that the decline in employment associated with an increase in wages is too large. Another policy implication regards the impact of the minimum wage.

In a competitive labour market, the impact of the minimum wage on employment depends on the elasticity of labour demand. When labour demand elasticity is high, firms respond to the higher wages imposed by the minimum wage with larger reductions in employment. Establishing the extent to which demand elasticities vary across sectors can help us identify which sectors are most likely to be adversely affected by the minimum wage legislation. Finally, when unemployment grows there are often calls for government intervention programmes such as wage subsidies. One can show that the distribution of the effects of a wage subsidy between wages and employment will depend on the labour demand elasticity. Consequently, it may be important for governments to know the magnitude of these elasticities in order to predict the likely impact of their labour market policy.

The results I get show own demand elasticities that are less than unity. All factor demands are inelastic, while capital is the least elastic. Inputs are in most cases substitutes while there is evidence of relative capital-skilled labour complementarity. One other result is that elasticities do not seem to have changed over the period despite the developments mentioned earlier. A number of reasons can account for this observation, one being the assumption that output is fixed. It means that another important determinant, the scale effect has not been taken into account in this analysis.

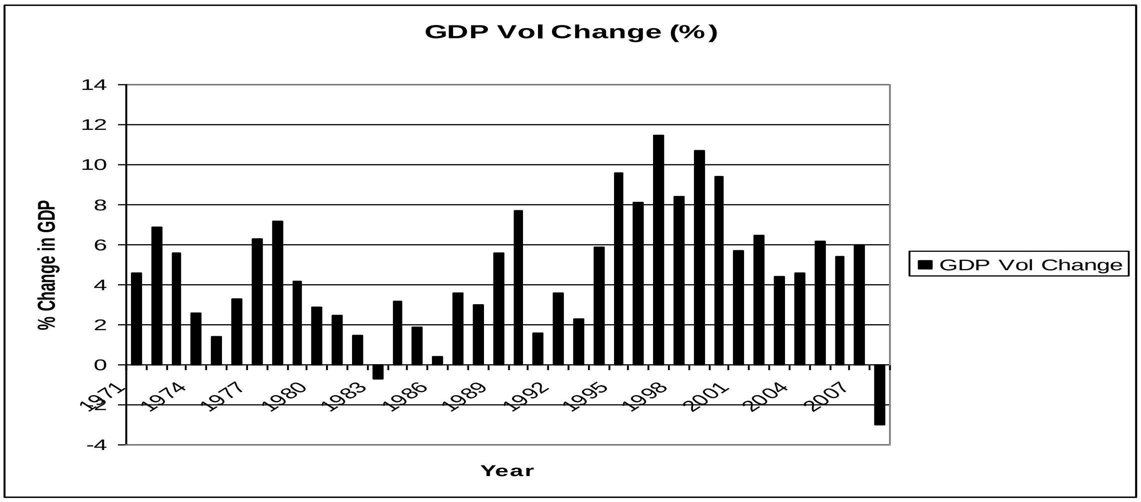
The rest of the document is divided as follows. The next section reviews changes that took place in the Irish economy that can be expected to affect elasticities. Section 3 reviews the existing literature on input demand elasticities. The theoretical framework is discussed in section 4. Methodology is presented in section 5. The description of data used in this study is in section 6. Section 7 presents the empirical results and the discussion of the results. Policy implications are discussed in section 8 and the conclusion is in the last section.

2. Review of Irish Economy

To understant the potential effect of changes in the Irish economy on the elasticity of factor demand it is helpful to know the key determinants of factor demand elasticities. According to the Hicks-Marshall Laws of Derived Demand, the elasticity of demand for an input depends on the following: First is the share of the total cost of production that can be attributed to that particular input. In terms of labour, this measures how much is paid to labour as a reward for engaging labour in the production process as a proportion of total costs. The bigger this cost share is, the bigger will be the absolute value of the demand elasticity. This effect is apparent is unconditional demand elasticities and results from the large increases in costs that follow a factor price increase when the input's share in total cost is high. When output is held fixed (i.e. conditional factor demand elasticities) then the effect may be reversed. When an input's share in total cost is large and output is fixed, producers may have no option but to continue to use the existing inputs if it is to maintain output. The second determinant is the size of the substitution effect i.e. how easy it is for labour to be substituted in production by a cheaper input (substitute input). The larger this substitution effect is, the larger the elasticity of demand for labour. The third determinant is the size of the price elasticity of demand for domestically produced goods. The larger this price elasticity of demand for products is, the larger is the elasticity of demand for the labour used to produce such goods. This elasticity of demand for an input can also be affected by the elasticity of supply of another input such as capital. If the supply of this input is highly elastic, demand for labour will also be highly elastic. Suppose wages increase. Firms would want to substitute towards capital. If it is possible for the firm to increase its capital stock without the price of capital increasing substantially, i.e. if the supply curve of capital is elastic, then the demand curve for labour will be more elastic.

In this section, I review the major changes in the Irish economy over the period of our study and discuss their likely impacts on labour demand elasticity, in light of the above discussion. The macroeconomic problems of the economy of Ireland back in early 1980s have been well documented. Low rates of economic growth characterised the economy. The economy grew at an average rate of 2.2% between 1979 and 1987. The economy then experienced unprecedented growth rates. Between 1994 and 2000, it grew at an average rate of 9.09%. During this period, industrial production also increased from EUR35, 566m in 1995 to EUR 95, 931m in the year 2000. Figure 2.1 below shows the growth rates between 1971 and 2008.

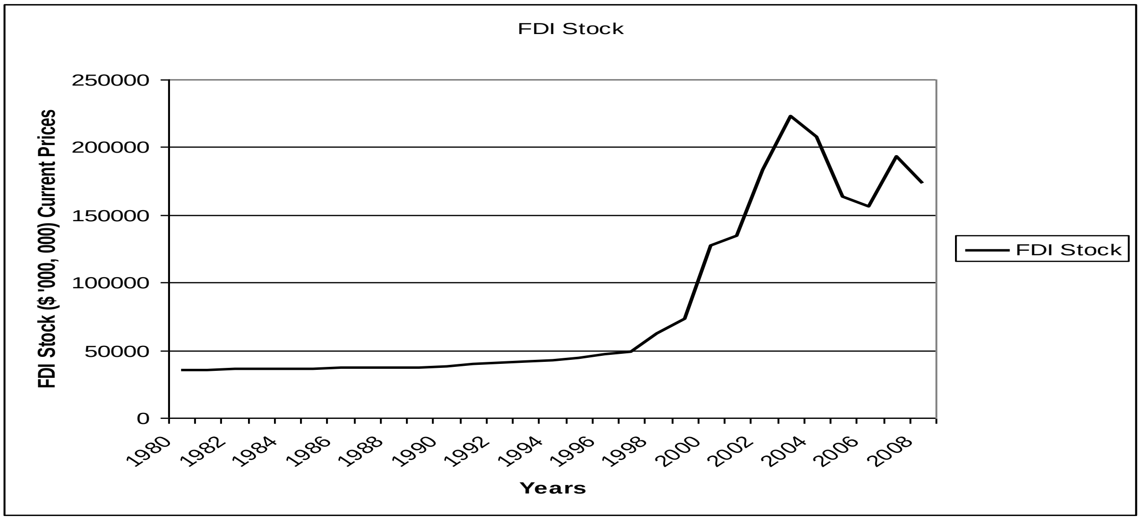
Figure 2.1



Foreign Direct Investment (FDI)

Underlying this high economic growth was an increase in the inflow of FDI into Ireland. The reduction in corporate tax to 12.5% for most companies in the late 1980s in order to attract Multinational Enterprises (MNEs) to set up operations in Ireland led to an improvement in Ireland's competitiveness by reducing the costs of setting up businesses. This led to an increase in the number of incoming firms. The effect of this increased FDI inflow was to increase the value of investment in the economy. Figure 2.2 below shows how this affected the stock of FDI in Ireland. There is a massive increase of the stock of FDI in the late 1990s. From the year 2000 though, the graph shows a fall in the stock of FDI, and the reason for this is that Ireland had to repay the loans to foreign parent companies (Duff, 2007).

Figure 2.2 Direct Investment in Ireland (FDI inward).



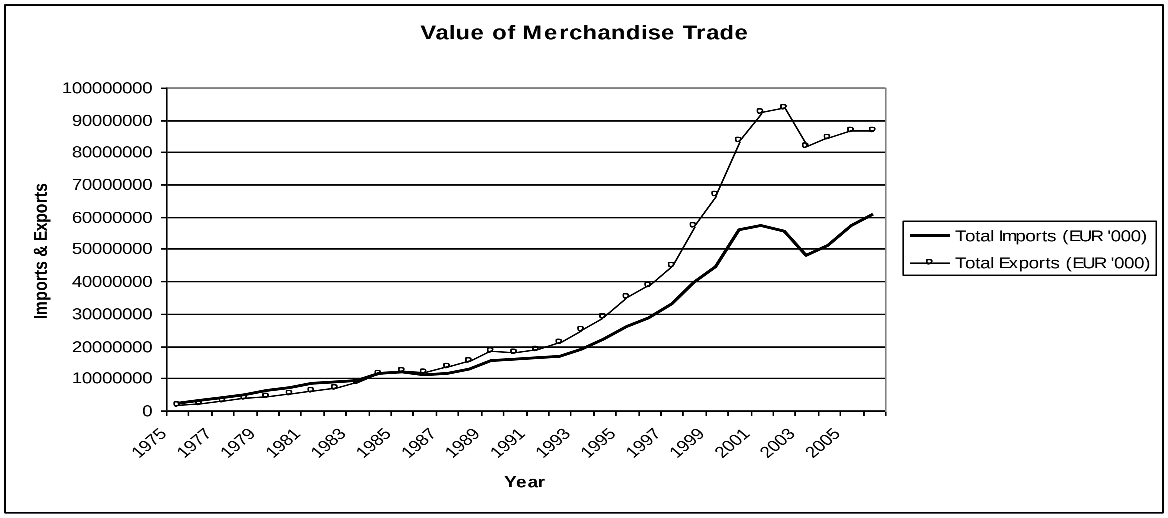
Constructed with FDI data (Ireland) from UNCTAD. FDI stock is defined as the value of the share of their capital and reserves (including retained profits) attributable to the parent enterprise, plus the net indebtedness of affiliates to the parent enterprises.

Since the nature of MNEs is such that they can more easily relocate operations if production costs rise, their labour demand elasticities may be higher than those associated with national enterprises. An example is that of Dell closing down its factory in Limerick leading to 1,900 people losing their jobs in January 2009, and relocating to Poland. To the extent that the skill composition of MNEs differs from local firms, this may also lead to differences in labour demand elasticities between the two firms.

Trade

Not only did this increase in FDI lead to more employment but it also led to an increase in output, evidence of which is given by the increases in growth rates discussed earlier. The result of this increase in output was an increase in the activity in the external sectors as shown in figure 2.3. Exports and imports increased by a large margin. From 1979 to 1990, total volume of exports increased from €4,415,816 to €18,203,873. From 1990 to 2002, these exports increased from €18,203,873 to €93,675,200.

Figure 2.3



Constructed from the data on the Volume of Exports from the CSO website

Trade also has its effects on labour demand elasticities. Trade affects labour demand elasticities in two ways: via the scale effect and via the substitution effect. The scale effect results if trade opens up opportunities for domestic consumers to acquire cheaper foreign produced substitute goods, thereby increasing the elasticity of output demand and consequently labour demand elasticities. A second effect, the substitution effect, results when it becomes easier to substitute foreign inputs for domestic inputs in the event that domestic wages increase, again leading to a more elastic demand for labour.

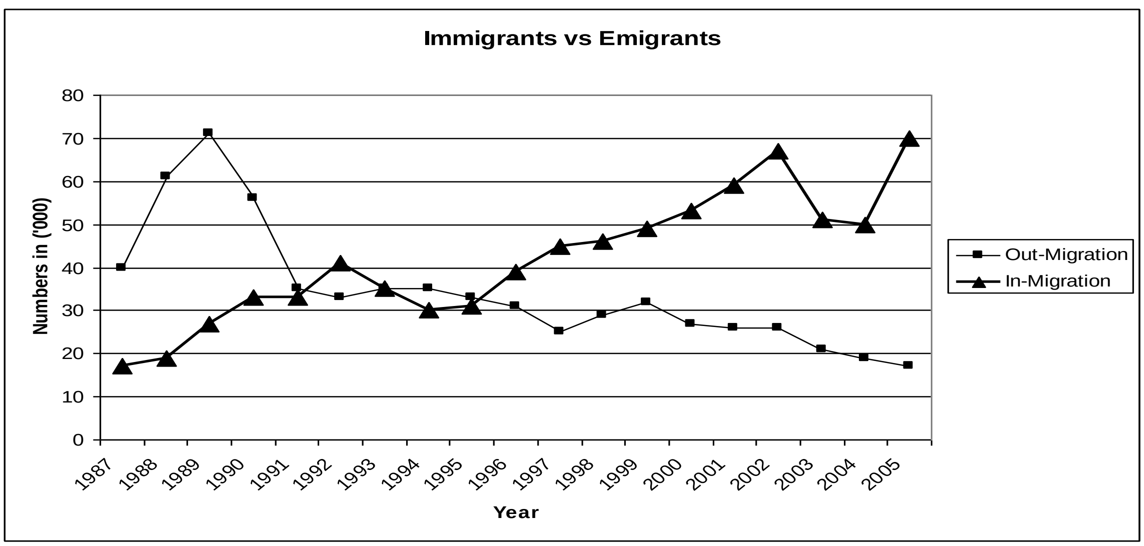
Migration

The process of migration in Ireland can be divided into two periods. First were periods when there were large outflows of labour that outweighed the number of incoming immigrants. This was followed by periods when incoming migration far outweighed outward migration movements. FitzGerald and Kearney (1999) report that between 1992 and 1997 on average, 0.9% of the population emigrated while on average, 1% of the population was made up of incoming immigrants. During the time when the economy was not doing well, people with higher skills left Ireland to seek greener pastures elsewhere, especially in the UK and the US[1]. It is estimated by the authors that one third of the

[1] FitzGerald and Kearney (1999) show that of those who emigrated, 45% went to the UK; 18% to the US and 18% to other EU countries.

emigrants had a third level education in the late 1980s. Between 1987 and 1990, the number of people emigrating from Ireland rose from 40,200 to 56,300. When the economy started picking up after that, there was a remarkable fall in the numbers migrating out (in fact, the number of returning emigrants increased) and then the trend levelled off from around 1992 onwards. Regarding immigrants, in 1987 the number of immigrants was 17, 200 but by 2005, this number had increased to 84, 600. Most of these immigrants were from Eastern Europe. The approval of laws among EU countries that would facilitate free movement of people took place in 1993 (europa.eu). Later, the accession of Eastern European countries to the EU resulted in free movement of their citizens to countries such as Ireland, Sweden and the UK, which had opened up their labour markets to them (www.euractiv.com).

Figure 2.4



Source: Constructed with data from CSO's releases and publications; population and migration estimates

To see how migration can affect the elasticity of demand, recall that one of Marshall's laws of derived demand says that the elasticity of input demand is higher if the supply of another input is also highly elastic. Let us suppose that the supply of capital is inelastic, because we have only one machine in the market that can be used in production. Then suppose that the wages increase, therefore the average and the marginal cost of production increase. The first thing that the firm will want to do is to substitute away from labour towards a cheaper input, which in this case is capital. But then because of the fact that the supply of machines is highly inelastic, this has an effect of pushing up the price of capital, which means that it also becomes more costly to engage more capital, thereby reducing incentives to switch. Changes which facilitate the switch to capital will clearly increase the elasticity of demand for labour. Looking at immigration in this context, we note that the change in legislation that allowed workers from Eastern Europe to work in Ireland led to an influx of immigrants into Ireland. These immigrants came in and settled in unskilled jobs which at the time; the Irish workers did not want to take. By increasing the available stock of unskilled labour and perhaps creating a pool of unskilled labour that was more responsive to wage changes, this influx of migrants may have increased the elasticity of unskilled labour supply. As discussed above, such a change would potentially increase the elasticity of demand for skilled labour and capital. Firms faced with a rise in price of these inputs now find it easier to susbstitute to unskilled labour without driving up costs.

3. Literature Review of Labour Demand Elasticities

A number of studies have calculated labour demand elasticities in an effort to analyse the impact of policy reforms or any major events such as a sudden change in the population composition due to say labour movements. It is important to investigate how certain changes may have affected the way the economy has been operating. Research in this area falls mainly into two categories; studies that are just interested in establishing how sensitive the demand for each input is to changes within the economy such as when there is trade reform, and those studies that just determine how different inputs, such as production workers and non-productions workers interact in the economy during production.

The approach that is used in this study has been used in a number of previous studies. Many estimate the demand for inputs in a similar manner as this study using a range of functional forms. Some of the functions impose some restrictions on the eventual elasticities while a more flexible form, called the translog function is used for this study. A comprehensive review of studies that calculate elasticities of substitution between capital, skilled labour and unskilled labour, where both skilled and unskilled labour are defined in accordance with their occupations can be found in Hamermesh (1993). The only thing about Hamermesh's work is that all the studies are about the US or UK. Before I look at studies from other countries, especially Europe, it is important to mention some of the conclusions he makes about elasticities of demand for labour. Although there are a number of diverging results regarding the elasticity of demand for labour, Hamermesh concludes that the demand elasticity for homogenous labour with respect to labour's own price is within the interval – 0.15 to – 0.75, with – 0.3 being the best estimate.

Koschel (2000) estimates own demand elasticities for labour, capital, material electricity and fossil fuels in the producing sector and the services sector in Germany. His data which are taken from national accounts statistics and the input-output table for Germany is a pooled time series and cross section that covers the period 1978 – 1990. Elasticity of demand for labour in the services sector was – 0.76. Own price elasticities were – 0.155

and – 0.133 for Non-energy intensive manufacturing sectors and energy intensive manufacturing sectors respectively.

Griffin (1996) uses the US firm level data for 1550 relatively large firms obtained from the annual reports from Equal Employment Opportunity Commission (EEOC) for the year 1980 to estimate labour demand elasticities for a number of groups of workers as well as capital. Using a felxible specificaiton for the cost function, he finds that the wage elasticity of demand for white male workers is – 1.637 as against the black males whose elasticity was – 0.641. Elasticities for white females and black females were – 6.262 and – 1.791 respectively. He finds it suprising that the demand for black males and black females seem less elastic than those of white males and white females respectively. He believes that the high elasticities for the demand for whites as against the demand for blacks are caused by existence of constraints imposed on hiring, such as the affirmative action. Elasticity for demand for capital was – 0.275.

Min, Hong-Ghi (2007) estimated the elasticities of demand for labour for countries in Africa and countries in Latin America. He regresses the log of labour demand on the log of wages. He finds that for Africa's manufacturing sector, the wage elasticity of demand for labour is – 0.2.

Konings and Lehmann (2002) estimated the elasticities of demand for labour for the period 1996 – 1997 for the Russian Federation. Their goal was to investigate how important the changes in wages have been in shaping the employment adjustment process in Russia as a result of a change to the market economy in the early 1990s. They first do this by estimating a relationship between the log of employment and wages and this gives an elasticity of – 0.181 for the whole economy. They then estimate the same relationship by sector, and the elasticities of labour demand for the manufacturing, construction and trade sectors are – 0.156, - 0.228 and – 0.188 respectively.

Slaughter (2001) uses the US manufacturing time series data to study the effects of trade on the labour demand elasticities. In terms of the method, he uses the constant-output elasticity of demand for labour. He was investigating how elasticities have been evolving over time between 1961 and 1991. He does this by estimating the own demand elasticities for production and non-production workers for the whole economy and then he estimates own demand elasticities at the disaggregated level where he has eight industries within the manufacturing sector. He reports that as a result of non-wage factor prices being presented only in indexes in the data, and not on levels, comparison across industries would not be possible. To get around this, he uses data in time differences and not levels. Own demand elasticity for production workers is – 0.7 while that of non-production workers is – 0.63. On the effect of trade on elasticities, he finds mixed results since the elasticities seem to fluctuate a lot during that time period. Specifically regarding non-production labour, the demand for it did not become more elastic over time.

Using a different methodology but still looking at the effects of globalisation on demand elasticities for labour, Bruno, Falzoni and Helg (2003) goal is to investigate the effects of globalisation on labour demand elasticities industry-year panel data for a number of European countries, Japan and the US for the period 1970 – 1996. Using openness as a measure of trade, they use a Least Squares Dummy Variables (LSDV) estimator to estimate their functions. Elasticities are – 0.26 for France; - 1.19 for Germany; - 0.77 for Italy and – 1.04 for the UK. Just like Slaughter for the US, the authors conclude that in general they do not find any significant impact of trade on labour demand elasticities.

Like Bruno, Falzoni and Helg (2003) above, Sebastien (2000) had used a degree of openness as a measure of trade in the study where he investigates the effects of international trade on labour demand elasticities for skilled and unskilled workers in France. He finds that price elasticity of unskilled labour that is associated with the scale effect increases if an assumption is made that the magnitude of the substitution effect is increasing. The elasticities are - 0.047 for 1977; – 0.071 for 1985 and – 0.051 for 1993.

A closely related issue of inward FDI and its effects on labour demand elasticities has been studied by Checchi, Navaretti and Turrini (2003) for a number of European countries. Their argument is that labour demand in the multinational enterprises (MNEs) is likely to be less elastic as they are characterised by higher intensity of skilled labour, which happens to be less reposnsive to changes in wages. They go on to show that on the other hand, if the skill composition of labour demand is kept equal, labour demand elasticities in MNEs will be higher than in national enterprises since MNEs have an option relocating production activities if wages were to change. Using panel data that covers the period 1993 – 2000 for eleven European countries, they find that the long run wage elasticities in MNEs in the Netherlands, the UK and Belgium were – 0.55; - 0.47; and – 0.56 respectively while for the national enterprises, the elasticities were – 2.55; - 3.55; and - 5.77 respectively. This large difference between elasticities of the MNEs against elasticities of national firms is attributed to the difference in skill intensity of the workforce. Finland registered a wage elasticity of – 0.53 for the MNEs, which was slightly higher than elasticities associated with national enterprises. Gorg *et al*. (2009) use plant level panel data from the Irish Economy Expenditure (IEE) Survey to estimate labour demand elasticities by foreign owned MNEs and the domestically owned enterprises in order to investigate the links between nationality of ownership and labour demand elasticities. Their goal is to find out whether labour demand becomes less elastic when a plant has backward linkages with the local economy. They find that foreign owned MNEs register an elasticity of demand for labour that is in absolute terms higher than that of national enterprises by – 0.055. They conclude that the extent of local linkages leads to less elastic labour demand for foreign owned enterprises.

Just like Checchi, Navaretti and Turrini (2003), Li and Girma (2006) estimate the wage elasticities of labour demand by foreign MNEs, exporting firms and non-exporting firms. Their goal is to investigate the differences in the speed of adjustment of labour demand among the three firms from the UK manufacturing sector. They do this by estimating a log linear conditional labour demand function where the log of the desired level of employment is regressed on wages and other control variables. They estimate wage elasticities of demand for labour of – 0.574; - 1.106; and – 1.239 for foreign MNEs, exporting firms and non-exporting firms respectively.

With respect to Irish data, two studies that are closest to this study are that of Boyle and Sloane (1982) and Kearney (1997). The difference between them is on the fact that Boyle and Sloane have done a long run study of the demand for factor inputs while Kearney was more interested in the short run dynamics of the demand for factors in high growth sectors; medium growth sectors and the declining sectors. Boyle and Sloane studied the demand for production labour; non-production labour and physical capital within the manufacturing sector using the translog cost function. They found that own demand elasticities were smaller than unity, leading them to conclude that a general wage increase might not bring about substantial changes in the demand for the two types of labour. The elasticity of substitution between production workers and capital were larger than those between skilled labour and capital in less than 1 in 4 of the industries studied, providing evidence of capital skill complementarity. They found the elasticity of substitution between unskilled labour and capital to be high but generally less than unity.

The study by Kearney (1997) used a dynamic framework for the demand for skilled labour, unskilled labour and clerical workers in the Irish manufacturing sector during the 1980s. She separates industries into three categories: high growth sectors; medium growth sectors and the declining sectors. Using translog cost function, she starts off with the long run study, in which she finds that within the medium group of sectors, skilled labour, unskilled labour and capital are all substitutes in production. She also found capital and skilled labour to be complements within the high growth sectors.

FitzGerald and Kearney (2000) estimated the demand for skilled labour and unskilled and calculated the elasticities of substitution between skilled labour and unskilled labour over time. The study itself was investigating the reasons behind the failure by the Irish economy to converge to the performance of other European countries. The authors estimate a translog cost function where the share of high skilled labour is explained by the log of wages, output and a measure of technical progress. This Irish data covers the period 1967 – 2000. They found that labour intensive output had been declining and this has been coupled with the increase in the employment of more skilled workers while the employment of unskilled workers had declined over time. The elasticities of substitution are calculated using the share of high skilled labour and the coefficient of the log of wages over time. Their calculation of the elasticities of substitution between skilled labour and unskilled labour between 1967 and 2000 reveal that skilled labour has become less and less substitutable with unskilled labour. They find that the elasticity of substitution between skilled and unskilled labour falls so remarkably over the years that by late 1990s, both inputs are basically not substitutable.

4. Theoretical Background

In this section, I consider the firm's production decision in more detail. The demand for factors of production in the long run can be analysed through scale effects and substitution effects. Assuming that the output of the firm remains unchanged, substitution effect refers to the situation where the firm now switches to the employment of more of a cheaper input in the wake of an input price change. The problem of maximising output given the cost of engaging inputs in the production process can be represented cost minimising given the optimal level of output. In cost minimisation, based on the cost of using each input, the firm looks for the optimal combination of inputs that attains a fixed level of output. This dual nature of production and costs allows us to look at the firm's problem through the cost function. The problem of the firm is therefore to minimise the cost of production subject to that fixed level of output.

Let $Y = f(X_1, X_2, ..., X_M)$ be the production function for the firm. The firm seeks to minimise costs according to:

$$\min\ C = \sum_{i=1}^{M} X_i P_i \ \text{ subject to } Y = f(X_1, X_2, ..., X_M) \quad (i = 1,2,...,M)$$

where X_i are factors of production; P_i are factor prices corresponding to factor i.

The solution to the above problem is a set of cost minimising conditional demand for inputs given by:

$$x_i^* = x_i(P_1, P_2, ..., P_M, Y)$$

These conditional factor demands can be obtained by Shephard's Lemma, which shows that:

$$x_i^* = \frac{\partial C(P_1, P_2, ...P_M, Y)}{\partial P_i}$$

Where the cost function $C(P_1, P_2, ...P_M, Y)$, is defined as the minimum cost of producing output Y, given the observed prices. This is obtained by substituting the conditional factor demand functions into the cost expression. This cost function must be a concave function and summarises the firm's production possibilities.

The elasticity of substitution between factor i and factor j, is a summary measure of the substitution possibilities given the firm's technology. With two inputs it is defined as the percentage change in the factor ratio when the factor price ratio changes by 1%, and is given by:

$$\sigma_{ij} = \frac{P_j / P_i}{x_i / x_j} \frac{\partial(x_i / x_j)}{\partial(P_j / P_i)}$$

It is possible to write the elasticity of substitution in terms of the cost function as follows:

$$\sigma_{ij} = \frac{CC_{ij}}{C_i C_j} \qquad (4.1)$$

where C_i is the partial derivative of the cost function with respect to the price of factor i, C_j is the partial derivative of the cost function with respect to the price of factor j, and

$$C_{ij} = \frac{\partial^2 C}{\partial P_i \partial P_j}.$$

To prove how σ_{ij} can be expressed as in *(4.1)*, first note that through Shephard's Lemma, the conditional demand for both factor i and factor j can be written as:

$$x_i = C_i(P_i, P_j, Y) \quad \text{and} \quad x_j = C_j(P_i, P_j, Y) \qquad (4.2)$$

Since also the cost function is homogenous of degree one, then the cost function can be written as:

$$C(P_i, P_j, Y) = P_j C\left[\frac{P_i}{P_j}, 1, Y\right] \qquad (4.3)$$

Differentiating *(4.3)* with respect to P_i, I get:

$$C_i(P_i, P_j, Y) = \frac{d}{dP_i}\left(P_j C\left[\frac{P_i}{P_j}, 1, Y \right] \right)$$

$$= \frac{dP_j}{dP_i} C\left[\frac{P_i}{P_j}, 1, Y \right] + P_j \frac{d}{dP_j}\left(C\left[\frac{P_i}{P_j}, 1, Y \right] \right)$$

$$= 0 + P_j \left(\frac{1}{P_j} \right) C_i \left[\frac{P_i}{P_j}, 1, Y \right]$$

$$C_i(P_i, P_j, Y) = C_i \left[\frac{P_i}{P_j}, 1, Y \right] \qquad \textit{(4.4)}$$

Differentiating *(4.3)* with respect to P_j I get:

$$C_j(P_i, P_j, Y) = \frac{d}{dP_j}\left(P_j C\left[\frac{P_i}{P_j}, 1, Y \right] \right)$$

$$= \frac{dP_j}{dP_j} C\left[\frac{P_i}{P_j}, 1, Y \right] + P_j \frac{d}{dP_j}\left(C\left[\frac{P_i}{P_j}, 1, Y \right] \right)$$

By chain rule and then quotient rule:

$$= C\left[\frac{P_i}{P_j}, 1, Y \right] + P_j (P_i)(-1) \frac{C_i \left[\frac{P_i}{P_j}, 1, Y \right]}{P_j^2}$$

$$= C\left[\frac{P_i}{P_j}, 1, Y \right] - \frac{P_i}{P_j}\left(C_i \left[\frac{P_i}{P_j}, 1, Y \right] \right) \qquad \textit{(4.5)}$$

Using *(4.2)*, *(4.4)* and *(4.5)*, input i relative to input j can be written as:

$$\frac{x_j}{x_i} = \frac{C_j(P_i, P_j, Y)}{C_i(P_i, P_j, Y} = \frac{C\left[\frac{P_i}{P_j}, 1, Y \right] - \left(\frac{P_i}{P_j} \right) C_i \left[\frac{P_i}{P_j}, 1, Y \right]}{C_i \left[\frac{P_i}{P_j}, 1, Y \right]}$$

which is
$$\frac{x_j}{x_i}=\frac{C_j\left(P_i,P_j,Y\right)}{C_i(P_i,P_j,Y}=\frac{C\left[\frac{P_i}{P_j},1,Y\right]}{C_i\left[\frac{P_i}{P_j},1,Y\right]}-\frac{P_i}{P_j} \qquad (4.6)$$

Differentiating *(4.6)* with respect to $\frac{P_i}{P_j}$ I get:

$$\frac{\partial\left(x_i/x_j\right)}{\partial\left(P_i/P_j\right)}=\frac{C_i\left(\frac{P_i}{P_j},1,Y\right)\frac{d}{d\left(\frac{P_i}{P_j}\right)}C\left(\frac{P_i}{P_j},1,Y\right)-C\left(\frac{P_i}{P_j},1,Y\right)\frac{d}{d\left(\frac{P_i}{P_j}\right)}C_i\left(\frac{P_i}{P_j},1,Y\right)}{C_i^2\left(\frac{P_i}{P_j},1,Y\right)}-1$$

$$=\frac{C_i^2\left(\frac{P_i}{P_j},1,Y\right)-C\left(\frac{P_i}{P_j},1,Y\right)C_{ii}\left(\frac{P_i}{P_j},1,Y\right)}{C_i^2\left(\frac{P_i}{P_j},1,Y\right)}-1$$

$$=\frac{C_i^2\left(\frac{P_i}{P_j},1,Y\right)}{C_i^2\left(\frac{P_i}{P_j},1,Y\right)}-\frac{C\left(\frac{P_i}{P_j},1,Y\right)C_{ii}\left(\frac{P_i}{P_j},1,Y\right)}{C_i^2\left(\frac{P_i}{P_j},1,Y\right)}-1$$

$$=1-\frac{C\left(\frac{P_i}{P_j},1,Y\right)C_{ii}\left(\frac{P_i}{P_j},1,Y\right)}{C_i^2\left(\frac{P_i}{P_j},1,Y\right)}-1$$

$$=\frac{C\left(\frac{P_i}{P_j},1,Y\right)C_{ii}\left(\frac{P_i}{P_j},1,Y\right)}{C_i^2\left(\frac{P_i}{P_j},1,Y\right)} \qquad (4.7)$$

Differentiating *(4.3)* with respect to P_j I get:

$$C_{ij}(P_i, P_j, Y) = \frac{d}{dP_j}\left(C_i\left[\frac{P_i}{P_j}, 1, Y\right]\right)$$

By chain rule and then quotient rule: $$= \frac{(-1)(P_i)}{P_j^2}\left(C_{ii}\left[\frac{P_i}{P_j}, 1, Y\right]\right)$$

$$C_{ij}(P_i, P_j, Y) = -\frac{P_i}{P_j^2}\left(C_{ii}\left[\frac{P_i}{P_j}, 1, Y\right]\right) \qquad \textit{(4.8)}$$

Multiplying *(4.7)* by $\dfrac{\left(P_i / P_j \right)}{\left(x_i / x_j \right)}$ to get σ:

$$\sigma = \frac{\left(P_i / P_j \right)}{\left(x_i / x_j \right)} \frac{\partial \left(\dfrac{x_i}{x_j} \right)}{\partial \left(\dfrac{P_i}{P_j} \right)} = -\frac{\left(P_i / P_j \right)\left[C\left(\dfrac{P_i}{P_j}, 1, Y\right) C_{ii}\left(\dfrac{P}{P_j}, 1, Y\right)\right]}{\left(x_i / x_j \right) C_i^2\left(\dfrac{P_i}{P_j}, 1, Y\right)} \qquad \textit{(4.9)}$$

Since $x_i = C_i(P_i, P_j, Y)$; $x_j = C_j(P_i, P_j, Y)$ and $C_i(P_i, P_j, Y) = C_i\left[\dfrac{P_i}{P_j}, 1, Y\right]$, the denominator in *(4.9)* is given by:

$$\left(x_j / x_i \right) C_i^2\left(\frac{P_i}{P_j}, 1, Y\right) = \left[\frac{C_j(P_i, P_j, Y)}{C_i\left(\dfrac{P_i}{P_j}, 1, Y\right)}\right] C_i^2\left(\frac{P_i}{P_j}, 1, Y\right) = C_j(P_i, P_j, Y) C_i(P_i, P_j, Y) \qquad \textit{(4.10)}$$

Also, using *(4.8)*, x_i , x_j and $C_i(P_i, P_j, Y)$ above, the numerator in *(4.9)* is given by:

$$-\left(P_i \big/ P_j\right)\left[C\left(\frac{P_i}{P_j},1,Y\right)C_{ii}\left(\frac{P_i}{P_j},1,Y\right)\right]=\left(-\frac{P_i}{P_j}\right)\left(-\frac{P_j^2}{P_i}\right)C\left(\frac{P_i}{P_j},1,Y\right)C_{ij}(P_i,P_j,Y)$$

$$=C\left(\frac{P_i}{P_j},1,Y\right)P_jC_{ij}(P_i,P_j,Y) \qquad (4.11)$$

but $C\left(\frac{P_i}{P_j},1,Y\right)=\frac{1}{P_j}C(P_i,P_j,Y)$. It therefore implies that *(4.11)* is given by:

$$=C\left(\frac{P_i}{P_j},1,Y\right)P_jC_{ij}(P_i,P_j,Y)=\frac{1}{P_j}C(P_i,P_j,Y)P_jC_{ij}(P_i,P_j,Y)$$

$$=C(P_i,P_j,Y)C_{ij}(P_i,P_j,Y) \qquad (4.12)$$

Using *(4.10)* and *(4.12)*: $\sigma=\frac{C(P_i,P_j,Y)C_{ij}(P_i,P_j,Y)}{C(P_i,P_j,Y)C(P_i,P_j,Y)}=\frac{CC_{ij}}{C_iC_j}$

I also derive an expression for the own price elasticity and the cross-price elasticities of demand. To start with, note that under perfect competition, the demand function must be downward sloping. To see this, differentiate the conditional factor demands with respect to their corresponding factor prices gives:

$$\frac{\partial x_i}{\partial P_i}=C_{ii}\leq 0 \quad \text{as cost functions are concave.}$$

To examine the sensitivity of demand to prices in more detail I consider the own price elasticity defined as

$$\frac{\partial x_i}{\partial P_i}\frac{P_i}{x_i}=\eta_i^i$$

and the cross-price elasticity defined as

$$\frac{\partial x_i}{\partial P_j}\frac{P_j}{x_i} = \eta_j^i \quad \textit{(4.13)}$$

With some additional work I can derive expressions for each of these elasticities.

Define the share of factor i in the total labour cost is given by $s = \frac{P_i x_i}{C}$ and $\frac{\partial x_i}{\partial P_j} = C_{ij}$ then by rearrenging, the cross price elasticity *(4.13)* can be written as

$$\eta_j^i = \frac{P_j}{x_i} C_{ij} \qquad \textit{(4.14)}$$

If I solve for C_{ij} in *(4.1)* above, then I get $C_{ij} = \frac{C_i C_j}{C}\sigma$. Substituting this C_{ij} into *(4.14)* then $\eta_j^i = \frac{P_j}{C_i}\frac{C_i C_j}{C}\sigma = \frac{P_j C_j}{C}\sigma$ but $\frac{P_j C_j}{C} = (1-s)$ where $(1-s)$ is the share of factor j leading to $\eta_j^i = (1-s)\sigma$.

The conditional factor demand $x_i(P_j, P_i)$ is homogenous of degree zero in the input prices P_i and P_j, then its total differentiation leads to $0 = P_j \frac{dx_i}{dP_j} + P_i \frac{dx_i}{dP_i}$. Therefore $-\frac{P_j}{P_i}\frac{dx_i}{dP_j} = \frac{dx_i}{dP_i}$. If I then multiply both sides by $\frac{P_i}{x_i}$ then I end up with

$$\frac{dx_i}{dP_i}\frac{P_i}{x_i} = -\frac{P_j}{P_i}\frac{P_i}{x_i}\frac{dx_i}{dP_j}$$

which means that $$\eta_i^i = -\eta_j^i$$

This gives me the following expression:

$$\eta_i^i = -\eta_j^i = -(1-s)\sigma.$$

What this expression says is that if the elasticity of substitution (σ) is high, then the elasticity of demand for factor i with respect to the price of factor j is large in absolute terms. The high value of σ implies that for the producer to still produce the same amount of output, if the price ratio $\frac{P_i}{P_j}$ changes, then the producer can decrease by a large amount the use of the factor whose price has increased and increase by a large amount the use of the factor whose price has decreased or was unchanged.

From this, we can begin to understand the factors that influence this elasticity as summarised by the Hicks-Marshall Law of Derived demand, namely a factor's share in total cost and the ease of substitution between factors in the production process. In this calculation, it has been assumed that the elasticity of supply of other inputs was perfectly elastic. However, if we relax this assumption we can also find that own price elasticity of input becomes less elastic as the supply of other inputs become more inelastic. The derivation of these elasticities also ignore scale effects. Scale effects measure the change in factor demand resulting from changes in a firm's output level. The scale effect of an increase in wages results from the increased cost of producing goods that follow wage increases. If costs increase, this in turn leads to firms to try and increase the price of such products. The demand for the products then falls and this in turn increases the elasticity of demand for labour. When scale effects are considered, we find that the elasticity of the unconditional labour demand function also depends on the price elasticity of the product that has been produced using this labour is high.

5. Methodology

Method of Estimation

When looking at the firm's production decision, we can start by specifying the firm's production function. However, since profit maximisation implies cost minimisation, we can also consider production decisions by specifying and estimating a firm's cost function. Duality implies that the firm's cost function summarises all the economically relevant information about a firm's technology, (Varian (1992)). Binswanger (1974) cites a number of associated with cost function approach. These are:

a) the cost function has factor prices as regressors, which is appropriate because we assume perfect competition here. In this case, firms do not have control over factor prices, but with the production function, factors quantities are the regressors even though firms are at liberty to change factor quantities whenever circumstances change.
b) the cost function is homogeneous of degree one in factor prices, and this property is independent of the homogeneity properties of the underlying production function. Thus we do not have to impose restrictive assumptions on our technology to arrive at estimating equations.
c) in production function estimation, high collinearity between inputs may cause estimation problems.
d) Estimation of elasticites using the production function requires inversion of the Hessian matrix of the entire system. Errors in any one paramter may therefore bias estimates of every elasticity.

Having chosen to adopt the cost function approach, the next step is to specify a functional form for the cost function from which coefficient estimates that are needed for calculation of elasticities can be obtained. The transcendental logarithmic (translog) function falls within a class of flexible functional forms since it allows substitution among inputs to be unrestricted. In particular, one can view the translog cost function as a second approximation to any arbitrary cost function.

To see this, assume that the cost function of this manufacturing sector can be approximated by a twice differentiable cost function. A second order taylor series expansion around the point $\ln P_i = 0$ and $\ln Y = 0$ is carried out on a logarithm of the cost function.

If the minimum cost function is given by $\ln C(P_1, P_2, \ldots P_M, Y)$, then 2^{nd} order Taylor series expansion yields:

$$\ln C = \ln \alpha_0 + \sum_{i=1}^{M} \left[\frac{\partial \ln C}{\partial \ln P_i} \right] \ln P_i + \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} \left[\frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j} \right] \ln P_i \ln P_j \qquad (5.1)$$

Letting $\frac{\partial \ln C}{\partial \ln P_i} = \alpha_i$ and $\frac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j} = \gamma_{ij}$ and subsituting these partial derivatives into *Eq. 5.1*, assuming that $M = 3$, then we end up with the following equation:

$$\begin{aligned} &\ln C = \ln \alpha_0 + \alpha_1 \ln P_1 + .\alpha_2 \ln P_2 + \alpha_3 \ln P_3 + \frac{1}{2}\gamma_{11}(\ln P_1)^2 + \gamma_{12} \ln P_1 \ln P_2 + \\ &\gamma_{13} \ln P_1 \ln P_3 + \gamma_{21} \ln P_2 \ln P_1 + \frac{1}{2}\gamma_{22}(\ln P_2)^2 + \gamma_{23} \ln P_2 \ln P_3 + \gamma_{31} \ln P_3 \ln P_1 + \quad (5.2) \\ &\gamma_{32} \ln P_3 \ln P_2 + \frac{1}{2}\gamma_{33}(\ln P_3)^2 \end{aligned}$$

Equation (5.2) above is called the translog cost function.

From *(5.1)* above, a few manipulations on the second order cross partial derivatives above leads to the following formulas for calculating elasticities of substitution, own demand elasticities and the cross price elasticities:

$$\sigma_{ij} = \frac{\gamma_{ij}}{S_i S_j} + 1 \qquad (5.3)$$

$$\sigma_{ii} = \frac{(\gamma_{ii} + S_i^{\ 2} - S_i)}{S_i} \qquad (5.4)$$

$$\eta_{ii} = \frac{\gamma_{ii}}{S_i} + S_i - 1 \qquad (5.5)$$

$$\eta_{ij} = \frac{\gamma_{ij}}{S_i} + S_j \qquad (5.6)$$

The above formulas are derived in the following way:

ELASTICITY OF SUBSTITUTION (σ_{ij}):

From the symmetry condition in (5.1), $\gamma_{ij} = \dfrac{\partial^2 \ln C}{\partial \ln P_i \partial \ln P_j} = \dfrac{\partial}{\partial P_j}\left[\dfrac{\partial \ln C}{\partial \ln P_i}\right]\dfrac{P_j}{1}$

$$\gamma_{ij} = \frac{\partial}{\partial P_j}\left[\frac{\partial C}{\partial P_i}\frac{P_i}{C}\right]\frac{P_j}{1} = \frac{P_j}{1}\left[\frac{P_i}{C}\frac{\partial}{\partial P_j}\left(\frac{\partial C}{\partial P_i}\right) + \frac{\partial C}{\partial P_i}\frac{\partial}{\partial P_j}\left(\frac{P_i}{C}\right)\right] \text{ by product rule.}$$

Apply quotient rule:

$$\gamma_{ij} = \frac{P_j}{1}\left[\frac{P_i}{C}\frac{\partial^2 C}{\partial P_i \partial P_j} + \frac{\partial C}{\partial P_i}\left(\frac{C\dfrac{\partial P_i}{\partial P_j} - P_i\dfrac{\partial C}{\partial P_j}}{C^2}\right)\right] \quad \text{but} \quad \frac{\partial P_i}{\partial P_j} = 0$$

$$\gamma_{ij} = \frac{P_j}{1}\left[\frac{P_i}{C}\frac{\partial^2 C}{\partial P_i \partial P_j} - \frac{\partial C}{\partial P_i}\frac{\partial C}{\partial P_j}\left(\frac{P_i}{C^2}\right)\right] = \frac{P_i P_j}{C}\frac{\partial^2 C}{\partial P_i \partial P_j} - \frac{P_i P_j}{C^2}\frac{\partial C}{\partial P_i}\frac{\partial C}{\partial P_j}$$

$$\text{but } \frac{\partial C}{\partial P_i} = X_i \text{ and } \frac{\partial C}{\partial P_j} = X_j$$

$$\gamma_{ij} = \frac{P_i P_j}{C}\frac{\partial^2 C}{\partial P_i \partial P_j} - \frac{P_i P_j}{C^2} X_i X_j$$

Rearrange: $$\gamma_{ij} + \frac{P_i P_j}{C^2} X_i X_j = \frac{P_i P_j}{C}\frac{\partial^2 C}{\partial P_i \partial P_j}$$

$$\frac{\partial^2 C}{\partial P_i \partial P_j} = \frac{C}{P_i P_j}\left[\gamma_{ij} + \frac{P_i P_j}{C^2} X_i X_j\right] = \frac{C}{P_i P_j}\left[\gamma_{ij} + \frac{P_i X_i}{C}\frac{P_j X_j}{C}\right]$$

where $\frac{P_i X_i}{C} = S_i$ and $\frac{P_j X_j}{C} = S_j$

$$\frac{\partial^2 C}{\partial P_i \partial P_j} = \frac{C}{P_i P_j}\left[\gamma_{ij} + S_i S_j\right]$$

Since $\sigma_{kr} = \frac{\sum P_i X_i}{X_k X_r}\frac{\partial^2 C}{\partial P_r \partial P_k}$ substitute for $\frac{\partial^2 C}{\partial P_r \partial P_k}$

$$\sigma_{ij} = \frac{\sum P_i X_i}{P_i X_i P_j X_j} C\left(\gamma_{ij} + S_i S_j\right) = \frac{C}{P_i X_i}\frac{C}{P_j X_j}\left(\gamma_{ij} + S_i S_j\right) \text{ where } \frac{C}{P_i X_i} = \frac{1}{S_i}$$

$$\sigma_{ij} = \frac{1}{S_i}\frac{1}{S_j}\left(\gamma_{ij} + S_i S_j\right)$$

$$\sigma_{ij} = \frac{\gamma_{ij} + S_i S_j}{S_i S_j} = \frac{\gamma_{ij}}{S_i S_j} + 1$$

ELASTICITY OF SUBSTITUTION (σ_{ii}):

From the symmetry condition in Eq. 51, $\gamma_{ii} = \frac{\partial^2 \ln C}{\partial \ln P_i^2} = \frac{\partial}{\partial P_i}\left[\frac{\partial \ln C}{\partial \ln P_i}\right]\frac{P_i}{1}$

$$\gamma_{ii} = \frac{\partial}{\partial P_i}\left[\frac{\partial C}{\partial P_i}\frac{P_i}{C}\right]\frac{P_i}{1} = \frac{P_i}{1}\left[\frac{P_i}{C}\frac{\partial}{\partial P_i}\left(\frac{\partial C}{\partial P_i}\right) + \frac{\partial C}{\partial P_i}\frac{\partial}{\partial P_i}\left(\frac{P_i}{C}\right)\right] \text{ by product rule.}$$

Apply quotient rule:

$$\gamma_{ii} = \frac{P_i}{1}\left[\frac{P_i}{C}\frac{\partial^2 C}{\partial P_i^2} + \frac{\partial C}{\partial P_i}\left(\frac{C\frac{\partial P_i}{\partial P_i} - P_i\frac{\partial C}{\partial P_i}}{C^2}\right)\right] \quad \text{but} \quad \frac{\partial P_i}{\partial P_i} = 1$$

$$\gamma_{ii} = \frac{P_i}{1}\left[\frac{P_i}{C}\frac{\partial^2 C}{\partial P_i^2} + \frac{\partial C}{\partial P_i}\left(\frac{C - P_i\frac{\partial C}{\partial P_i}}{C^2}\right)\right] = \frac{P_i P_i}{C}\frac{\partial^2 C}{\partial P_i^2} + \frac{P_i}{C}\frac{\partial C}{\partial P_i} - \frac{P_i P_i}{C^2}\frac{\partial C}{\partial P_i}\frac{\partial C}{\partial P_i}$$

but $$\frac{\partial C}{\partial P_i} = X_i$$

$$\gamma_{ii} = \frac{P_i P_i}{C}\frac{\partial^2 C}{\partial P_i^2} + \frac{P_i X_i}{C} - \frac{P_i P_i}{CC} X_i X_i = \frac{P_i P_i}{C}\frac{\partial^2 C}{\partial P_i^2} + \frac{P_i X_i}{C} - \frac{P_i X_i}{C}\frac{P_i X_i}{C} \quad \text{where} \quad \frac{P_i X_i}{C} = S_i$$

Rearrange: $\gamma_{ii} + S_i^2 - S_i = \frac{P_i P_j}{C}\frac{\partial^2 C}{\partial P_i^2}$

$$\frac{\partial^2 C}{\partial P_i^2} = \frac{C}{P_i P_i}\left[\gamma_{ii} + S_i^2 - S_i\right]$$

Since $\sigma_{kr} = \frac{\sum P_i X_i}{X_k X_r}\frac{\partial^2 C}{\partial P_r \partial P_k}$ substitute for $\frac{\partial^2 C}{\partial P_r \partial P_k}$

$$\sigma_{ii} = \frac{\sum P_i X_i}{P_i X_i P_i X_i} C\left(\gamma_{ii} + S_i^2 - S_i\right) = \frac{C}{P_i X_i}\left(\gamma_{ij} + S_i^2 - S_i\right) \quad \text{where} \quad \frac{C}{P_i X_i} = \frac{1}{S_i}$$

$$\sigma_{ii} = \frac{1}{S_i}\left(\gamma_{ii} + S_i^2 - S_i\right)$$

$$\sigma_{ii} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i}$$

CROSS PRICE ELASTICITY (η_{ij}):

$$S_i = \frac{P_i X_i}{C} = \alpha_i + \sum_{i=1}^{M} \gamma_{ij} \ln P_j$$

$$\eta_{ij} = \frac{\partial \ln X_i}{\partial \ln P_j} = \frac{P_j}{X_i}\frac{\partial X_i}{\partial P_j} \quad \text{where} \quad \frac{CS_i}{P_i} = X_i$$

$$\eta_{ij} = \frac{P_j}{X_i}\frac{\partial}{\partial P_j}\left(\frac{CS_i}{P_i}\right)$$

By product rule: $$\eta_{ij} = \frac{P_j}{X_i}\left(\frac{C}{P_i}\frac{\partial S_i}{\partial P_j} + \frac{1}{P_i} S_i \frac{\partial C}{\partial P_j}\right)$$

From $$S_i = \frac{P_i X_i}{C} = \alpha_i + \sum_{i=1}^{M} \gamma_{ij} \ln P_j\ , \quad \frac{\partial S_i}{\partial P_j} = \gamma_{ij}\frac{1}{P_j}\ ; \ \frac{\partial C}{\partial P_j} = X_j \ \text{ and } \ \frac{P_i X_i}{C} = S_i$$

$$\eta_{ij} = \frac{P_j}{X_i}\frac{C}{P_i}\frac{\gamma_{ij}}{P_j} + \frac{P_j}{X_i}\frac{S_i}{P_i} X_j = \gamma_{ij}\left[\frac{C}{P_i X_i}\right] + \frac{P_j X_j}{P_i X_i} S_i = \gamma_{ij}\frac{C}{P_i X_i} + \frac{P_j X_j}{P_i X_i}\left[\frac{P_i X_i}{C}\right]$$

but $$\frac{P_i X_i}{C} = S_i \ \text{ and } \ \frac{C}{P_i X_i} = \frac{1}{S_i}$$

so, $$\eta_{ij} = \frac{\gamma_{ij}}{S_i} + S_j$$

Alternatively:

$$\frac{\partial \ln C}{\partial \ln P_i} = \frac{P_i}{C}\frac{\partial C}{\partial P_i} = X_i \frac{P_i}{C} = S_i \quad \text{which implies that } \frac{CS_i}{P_i} = X_i$$

$$\eta_{ij} = \frac{\partial \ln X_i}{\partial \ln P_j} = \frac{\partial \ln\left(\frac{S_i C}{P_i}\right)}{\partial \ln P_j} = \frac{\partial \ln S_i}{\partial \ln P_j} + \frac{\partial \ln C}{\partial \ln P_j} - \frac{\partial \ln P_i}{\partial \ln P_j}$$

but $$\frac{\partial \ln P_i}{\partial \ln P_j} = 0 \quad \text{and} \quad \frac{\partial \ln C}{\partial \ln P_j} = S_j$$

$$\frac{\partial \ln S_i}{\partial \ln P_j} = \frac{\partial S_i}{\partial P_j}\frac{P_j}{S_i} \quad \text{where, from the formula } S_i = \frac{P_i X_i}{C} = \alpha_i + \sum_{i=1}^{M} \gamma_{ij} \ln P_j\,, \qquad \frac{\partial S_i}{\partial P_j} = \frac{\gamma_{ij}}{P_j}$$

$$\frac{\partial \ln S_i}{\partial \ln P_j} = \frac{\gamma_{ij}}{P_j}\frac{P_j}{S_i}$$

$$\eta_{ij} = \frac{\gamma_{ij}}{P_j}\frac{P_j}{S_i} + S_j$$

$$\eta_{ij} = \frac{\gamma_{ij}}{S_i} + S_j$$

OWN PRICE ELASTICITY (η_{ii}):

$$\eta_{ii} = \frac{\partial \ln X_i}{\partial \ln P_i} = \frac{\partial \ln\left(\frac{S_i C}{P_i}\right)}{\partial \ln P_i}$$

$$\eta_{ii} = \frac{\partial \ln S_i}{\partial \ln P_i} + \frac{\partial \ln C}{\partial \ln P_i} - \frac{\partial \ln P_i}{\partial \ln P_i} \quad \text{where} \quad \frac{\partial \ln P_i}{\partial \ln P_i} = 1 \text{ and } \frac{\partial \ln C}{\partial \ln P_i} = \frac{\partial C}{\partial P_i}\frac{P_i}{C} = S_i$$

From $$S_i = \alpha_i + \sum_{i=1}^{M} \gamma_{ij} \ln P_j \qquad \frac{\partial \ln S_i}{\partial \ln P_i} = \frac{\gamma_{ii}}{P_i}\frac{P_i}{S_i} = \frac{\gamma_{ii}}{S_i}$$

$$\eta_{ii} = \frac{\gamma_{ii}}{S_i} + S_i - 1$$

To get to the formulae used in my estimation, (5.2) above is differentiated with respect to the logarithm of factor prices, then cost minimising input shares S_i are:

$$S_i = \frac{\partial \ln C}{\partial \ln P_i} = \frac{\partial C}{\partial P_i}\frac{P_i}{C} = \frac{X_i P_i}{C} = \alpha_i + \sum_{i=1}^{M} \gamma_{ij} \ln P_j \qquad (5.7)$$

where α_i's and γ_{ij}'s are the parameters to be estimated. Once we have estimated these parameters we can then use them to calculate the required elasticities.

Estimation via Seemingly Unrelated Regression Equations (SURE)

Based on *(5.7)* above, I estimate a system of three seemingly unrelated equations, each equation corresponding to each one of the three inputs, skilled labour, unskilled labour and capital. This yields the following:

$$\begin{aligned}
S_1 &= \frac{X_1 P_1}{C} = \alpha_1 + \gamma_{11} \ln P_1 + \gamma_{12} \ln P_2 + \gamma_{13} \ln P_3 + u_1 \\
S_2 &= \frac{X_2 P_2}{C} = \alpha_2 + \gamma_{21} \ln P_1 + \gamma_{22} \ln P_2 + \gamma_{23} \ln P_3 + u_2 \qquad (5.8) \\
S_3 &= \frac{X_3 P_3}{C} = \alpha_3 + \gamma_{31} \ln P_1 + \gamma_{32} \ln P_2 + \gamma_{33} \ln P_3 + u_3
\end{aligned}$$

The seemingly unrelated regression equations (SURE) are a set of equations that look unrelated to one another but in fact are all connected through the correlation in their disturbances. Exogenous shocks to the demand for skilled labour do not only affect the demand for skilled labour, but also affect demand for unskilled labour and capital as well. A method of estimation that takes advantage of this relationship between equations, and estimates the equations as a system rather than individually, is therefore needed. In this way, estimating these equations as a system combines information from all the equations. The appropriate method here is Generalised Least Squares (GLS). This method is an extension to OLS but allows for disturbances to be correlated across equations. The advantage of using GLS is that it improves efficiency, which means that it leads to lower standard errors of the coefficient estimates. One should note that GLS and OLS yield identical results when, in the absence of cross equation restrictions, all the equations have identical regressors on the right hand side. The results are also identical if when estimating by GLS, we get a diagonal covariance matrix with the only non-zero elements being on the main diagonal, and zeros everywhere else.

The assumptions underlying this method are that:

1. $E[u_i]=0$ where $i=1,....M$ (M=3 in my case). The assumption implies that within each equation i, the disturbances associated with each observation average out to zero.

2. $E[u_{it}u_{js}^{'}]=\sigma_{ij}$ if $t=s$ and $E[u_{it}u_{js}^{'}]=0$ if $t\neq s$; t and s denote time. It implies that the t^{th} observation in the i^{th} equation can be correlated with the t^{th} observation in the j^{th} equation; otherwise errors are uncorrelated across observations. This assumption implies $E[u_i u_j^{'}]=\sigma_{ij}I_T$ about the errors. T is the number of observations. This is the covariance between errors of the i^{th} equation and errors in the j^{th} equation.

Let $\sum = [\sigma_{ij}]$ be an $M\times M$ covariance matrix of the disturbances for the t^{th} observation. GLS method of estimation assumes that $\sum$ is known, which is not usually the case. To get around this problem, the estimate of $\sum$ is usually used. In this case, the method of estimation is Feasible Generalised Least Squares (FGLS).

When estimating this system we must be aware of some of the restrictions imposed by theory. The factor shares have to add up to unity and this adding up condition implies that

$$\sum_i \alpha_i = 1$$

Also, I have mentioned earlier that the cost function is homogeneous of degree one in factor prices. This linear homogeneity in factor prices implies that:

$$\sum_i \gamma_{ij} = 0 \qquad \text{and} \qquad \sum_j \gamma_{ij} = 0$$

Theory also requires the equality of the cross price effects to hold, and this is the symmetry constraint. It implies that the effect of an increase in the price of factor j on the demand for factor i should be equal to the effect of an increase in the price of factor i on the demand for factor j. This restriction requires that $\gamma_{ij} = \gamma_{ji}$.

To impose linear homogeneity note that $\gamma_{11} + \gamma_{12} + \gamma_{13} = 0$ implies that $-(\gamma_{11} + \gamma_{12}) = \gamma_{13}$.

Imposing this on the first equation of (5.8) yields:

$S_1 = \dfrac{X_1 P_1}{C} = \alpha_1 + \gamma_{11} \ln P_1 + \gamma_{12} \ln P_2 - (\gamma_{11} + \gamma_{12}) \ln P_3 + u_1$ and collecting like terms gives:

$$S_1 = \frac{X_1 P_1}{C} = \alpha_1 + \gamma_{11} \ln P_1 - \gamma_{11} \ln P_3 + \gamma_{12} \ln P_2 - \gamma_{12} \ln P_3 + u_1$$

$S_1 = \dfrac{X_1 P_1}{C} = \alpha_1 + \gamma_{11}(\ln P_1 - \ln P_3) + \gamma_{12}(\ln P_2 - \ln P_3) + u_1$ but $\ln X - \ln Y = \ln\left[\dfrac{X}{Y}\right]$

so, $$S_1 = \frac{X_1 P_1}{C} = \alpha_1 + \gamma_{11} \ln\left[\frac{P_1}{P_3}\right] + \gamma_{12} \ln\left[\frac{P_2}{P_3}\right] + u_1$$

The same procedure applies to the rest of the equations, giving a new system of equations

$$\begin{aligned} S_1 &= \frac{X_1 P_1}{C} = \alpha_1 + \gamma_{11} \ln\left[\frac{P_1}{P_3}\right] + \gamma_{12} \ln\left[\frac{P_2}{P_3}\right] + u_1 \\ S_2 &= \frac{X_2 P_2}{C} = \alpha_2 + \gamma_{21} \ln\left[\frac{P_1}{P_3}\right] + \gamma_{22} \ln\left[\frac{P_2}{P_3}\right] + u_2 \\ S_3 &= \frac{X_3 P_3}{C} = \alpha_3 + \gamma_{31} \ln\left[\frac{P_1}{P_3}\right] + \gamma_{32} \ln\left[\frac{P_2}{P_3}\right] + u_3 \end{aligned} \tag{5.9}$$

This gives us a new system of equations which has linear homogeneity imposed. However since cost shares must sum to 1, the resulting error covariance matrix is singular. The singularity problem is solved by dropping the M^{th} equation (S_3 in my case) since the estimates for its parameters can be derived from the homogeneity restriction above.

To proceed with the calculation of elasticities, I estimate the system of factor demand equations (5.9) above. These parameter estimates, together with the shares of each input in the total cost of production will be used to calculate the partial elasticity of substitution between factor i and factors j (σ_{ij}); the own elasticity of substitution for factor i (σ_{ii}); own demand elasticity for factor i (η_{ii}); and cross elasticity between factor i and factor j (η_{ij}) as derived above.

6. Data

The data I utilise, are taken from the Census of Industrial Production (CIP) and were obtained from the Central Statistics Office (CSO) of Ireland. This is an annual survey that covers the whole nation and the CSO has been collecting this data from all enterprises from 1953. The response rate is usually very high and stands at around 74% and most of the non-respondents typically employ less than twenty workers. Estimates are imputed for those non-respondents. I make use of data covering the period from 1979 to 2005. A study similar to this one had been done by Boyle and Sloane (1982) and covered the period 1953 – 1973. A study by Kearney (1997) focused on the period 1979 – 1990. I chose the period 1979 – 2005 because of the changes that took place in the economy after 1990. The CIP is made up of two components, the Census of Industrial Enterprises and the Census of Industrial Local Units. The former is described as consisting of all enterprises which are mainly or only engaged in industrial production, and employ three or more persons. The Census of Local Industrial units on the other hand can also be involved in industrial production but are identified by their geographical location. This means that while most enterprises that fall within the local industrial units category can be found in industrial enterprises category, some only fall within the local units only. Those that fall within local units but not in industrial enterprises may not mainly be involved in industrial production and are part of non-industrial sectors such as transport and construction.

In this study I use the data from Census of Industrial Local Units. This local units dataset contains data on the number of local units (enterprises) in each year and each industry; additions to capital assets; gross output; net output; number of workers engaged, and salaries and wages paid out to the workers. These data categorises workers and their wages based on occupations according to the following groups:

a) Proprietors and family workers
b) Managerial and technical staff
c) Clerical staff

d) Industrial workers[2]
e) Apprentices
f) Outside piece-workers

I have omitted data for proprietors and family workers, and outside piece-workers because data on some of the wages are not provided, especially in the case of proprietors and family workers. I am then left with four categories: Managerial and technical staff; clerical staff; industrial workers and apprentices. I chose to form the skilled labour category by grouping together managerial and technical staff and clerical staff. In that way, the unskilled labour category is made up of industrial workers and apprentices. The distinction between skilled and unskilled workers based on occupation rather than educational attainments can be problematic because as Hamermesh (1993, pg 65) points out, the fact that there is an overlap in production and non-production earnings (i.e. some of the highest earners amongst the unskilled workers earn more than some of the lowest earners among skilled), makes the distinction less clear, and he suggests the distinction to be made on the basis of age or experience. This problem has been taken into consideration but there was no data that allowed an alternative grouping based on Hamermesh's suggestion. In one exercise, I separated clerical workers from the skilled labour group and created three skill levels, unskilled labour, skilled labour and high skill labour. The results from this exercise are not much different from when I have only two skill levels.

As mentioned earlier, the data I use is disaggregated according to industries within the manufacturing sector. Even though it was possible to obtain data disaggregated further, the choice of industry level of aggregation was dictated by the fact that CSO, in an attempt to maintain confidentiality and to protect firm information, merged some of classifications with the fewest elements with some of the bigger groups.

One potential problem with the CIP is that in 1991, the CSO had to adopt the NACE Rev. 1 classification in accordance with the EU legislation. This change altered the

[2] Industrial workers are made up of manual supervisory staff and manual workers. It was not possible to separate industrial workers into the two before the 1991 reclassification of NACE groups.

classifications from what they had been before 1991, making it difficult to make a before-1991 and after-1991 match for some of the classified groups. For example, after 1991, NACE 30-33 classification was made up of manufacture of electrical and optical equipment. Before 1991 under NACE 70 classifications, category 34 was called 'electrical engineering', which was the only one closest to NACE 30-33. Upon constructing a time series graph of this category for the period 1979 to 1990, there were big jumps in 1991 indicating that the after-1991 classification included some items that may have been left out in the before-1991 classification. NACE groups that exhibited this type of a jump, I have excluded from the dataset I use. As a result of this problem, I had to use data on twelve industries within the manufacturing sector. The official names and codes given to the industries are: Mining and Quarrying (10-14); Manufacture of Food Products, Beverages and Tobacco (15-16); Manufacture of Textile and Textile products (17-18); Manufacture of Leather and Leather products (19); Manufacture of Wood and Wood products (20); Manufacture of Pulp, Paper and Paper products, Publishing and Printing (21-22); Manufacture of Chemicals, Chemical products and Man-Made Fibres (24); Manufacture of Rubber and Plastic products (25); Manufacture of other Non-Metallic Mineral products (26); Manufacture of Basic Metals and Fabricated Metal products (27-28); Manufacture of Transport Equipment (34-35); and Electricity, Gas and Water Supply (40-41).

Variable Construction:

Skilled and unskilled factor prices

For every year and for every industry, CSO provided me with the total annual wage bill in thousands of euros corresponding to all the afore-mentioned occupational categories. These wages and salaries include overtime and shift allowances. Since the total wage bill for each category is obtained by multiplying the wages paid out to workers and the number of workers employed, the factor prices are then obtained by just dividing the total wage bill by the number of workers engaged. This is done for every industry separately and for every year separately. To get the total factor prices for skilled workers, I add the wage bills of Managerial and Technical staff together with the wage bill of clerical staff. The next step is to divide this total skilled wage bill by the number of Managerial and

Technical staff and Clerical staff added together. That is the price if skilled labour $P_{Skilled}$ is given by:

$$\frac{(P_{Mg} * M) + (P_{Cl} * C)}{(M + C)} = P_{Skilled}$$

where P_{Mg} is the price of Managerial and Technical staff and P_{Cl} is the price of Clerical staff. M and C are the numbers of Managerial and Techinical staff, and Clerical staff employed respectively. $(P_{Mg} * M)$ and $(P_{Cl} * C)$ are the wage bills of Managerial and Technical staff and Clerical staff respectively.

The same procedure is followed when calculating the price of unskilled labour.

Price of capital: The following formula (*Eq 6.1*) for the user cost of capital at time *t* was adopted from Romer (2001). The advantage of calculating the cost of capital in this manner is that the formula takes into account all those aspects of the economy that have a bearing on the price of capital, such as the rate of corporation tax, depreciation rate and the interest rates:

$$C_K(t) = \left[r(t) + \delta - \frac{\dot{P_K}(t)}{P_K(t)} \right] (1 - f\tau) P_K(t) \qquad (6.1)$$

$r(t)$ is the prime lending rate; δ is the depreciation rate, which has been assumed to be fixed at 0.026; $P_K(t)$ is the price of investment goods; $\dot{P_K}(t)$ is the rate at which the prices of investment goods change; τ is the rate of corporation tax and f is the capital allowances that have been written off against tax.

I obtained the series on prime lending rate and the rate of corporation tax from the ESRI databank while I got data on capital allowances from the Revenue Commissioner's office[3]. Before 1992, the prime lending rate was the average of AAA overdraft rates. In 1979, it was around 14% but declined with time and by 2005, it was below 3%. Regarding the rate of corporation/ company tax, two series were obtained from ESRI databank. One was on tax charged on companies that do not export and that charged on those that export their produce. I used an average of both that was charged on manufacturing companies, which for almost the entire period, was 10%. The 0.026 depreciation rate was assumed and used by Geary and McDonnell (1979) and I decided to use the same value. The CSO also provided me with data on Industrial Producer Price Index, which I used as the price of investment goods. The CSO also had the wholesale price index for capital goods, which I could have used but the series was only from 1995. The industrial producer price index has steadily been increasing over the period.

Total cost of capital: As discussed in the theoretical section that the process of maximising output given a fixed level of production costs is identical to the process of minimising production costs given a fixed level of output to be produced. This duality theory is important because it makes it possible to calculate the total cost of using capital. This total cost of using capital is calculated by subtracting both the total wage bill for skilled labour and unskilled labour from output.

Cost shares of skilled and unskilled labour: The cost shares of each of the inputs is then derived by dividing the cost of the input by the total cost of production given above. Summary statistics for each of the key variables used in this analysis are given in table 6.1 below. These data are as expected, with the share of capital relatively high in the chemical sector and the share of unskilled labour high in the textile and leather products.

[3] I thank Jackie Mahon for the data.

Table 6.1
SUMMARY STATISTICS

Variable	Obs	Mean	Std. Dev.	Min	Max
NACE (10 – 14) - Mining and Quarrying					
$P_{Capital}$	27	896.1701	340.9199	359.6014	1470.788
$P_{Skilled}$	27	29.13283	10.36381	9.935216	47.17196
$P_{Unskilled}$	27	22.05194	9.470543	6.850239	38.31799
$S_{Skilled}$	27	.1021523	.0163929	.0681556	.1329974
$S_{Unskilled}$	27	.2926278	.062261	.2185775	.4574354
$S_{Capital}$	27	.6052198	.0706824	.4095672	.7107829
NACE (15 – 16) - Manufacture of Food Products, Beverages and Tobacco					
$P_{Capital}$	27	979.9084	312.4828	458.6573	1722.573
$P_{Skilled}$	27	25.36827	11.02556	7.613026	47.92715
$P_{Unskilled}$	27	16.27656	5.849956	5.798568	27.31404
$S_{Skilled}$	27	.0744936	.0140721	.0572852	.1019171
$S_{Unskilled}$	27	.1680667	.0677769	.0836118	.2876069
$S_{Capital}$	27	.7574396	.0817642	.610476	.859103
NACE (17 - 18) - Manufacture of Textiles and Textile Products					
$P_{Capital}$	27	879.5856	334.1954	335.197	1447.502
$P_{Skilled}$	27	20.19817	8.435912	6.405549	36.34612
$P_{Unskilled}$	27	12.02815	4.318875	4.239784	20.65371
$S_{Skilled}$	27	.1267438	.0227038	.0950961	.1800999
$S_{Unskilled}$	27	.3993947	.054226	.274321	.473751
$S_{Capital}$	27	.4738615	.0441451	.4017891	.5615268
NACE (19) - Manufacture of Leather and Leather Products					
$P_{Capital}$	27	805.7094	328.8365	284.9777	1289.832
$P_{Skilled}$	27	17.91101	7.951139	5.099448	32.41964
$P_{Unskilled}$	27	11.64196	3.801365	4.707227	19.99405
$S_{Skilled}$	27	.1369443	.064694	.0559665	.3090463
$S_{Unskilled}$	27	.4181364	.0898194	.2565493	.6772588
$S_{Capital}$	27	.4449193	.0972074	.2150538	.6091819
NACE (20) - Manufacture of Wood and Wood Products					
$P_{Capital}$	27	961.2818	314.945	427.8389	1692.175
$P_{Skilled}$	27	18.61035	8.969588	5.614958	37.74223
$P_{Unskilled}$	27	12.8488	5.982677	4.081361	24.9502
$S_{Skilled}$	27	.1129119	.0074634	.0986188	.1251098
$S_{Unskilled}$	27	.3416827	.0471682	.2759286	.4225375
$S_{Capital}$	27	.5454054	.0478735	.4523527	.6155567
NACE (21 - 22) - Manufacture of Pulp, Paper and Paper Products; Printing					
$P_{Capital}$	27	981.042	344.3079	413.4521	1806.518
$P_{Skilled}$	27	24.56389	10.67244	7.714071	46.29298
$P_{Unskilled}$	27	18.49581	6.785194	6.239779	31.81774
$S_{Skilled}$	27	.118246	.0641105	.0360248	.2004027
$S_{Unskilled}$	27	.1884152	.1393724	.0236225	.3698276
$S_{Capital}$	27	.6933388	.2029986	.4421909	.9401404

NACE (24) - Manufacture of Chemicals, Chemical Products and Man-Made Fibres					
$P_{Capital}$	27	819.2305	356.6884	272.7797	1465.115
$P_{Skilled}$	27	27.8441	11.59919	8.12834	51.1444
$P_{Unskilled}$	27	21.53688	8.640622	6.831573	40.63616
$S_{Skilled}$	27	.0500847	.0223552	.0171025	.0833979
$S_{Unskilled}$	27	.0710799	.0404203	.0187162	.1388821
$S_{Capital}$	27	.8788353	.0624074	.7827609	.9641368
NACE (25) - Manufacture of Rubber and Plastic Products					
$P_{Capital}$	27	967.5665	351.6893	382.0524	1684.758
$P_{Skilled}$	27	22.82753	9.280404	7.83165	42.83027
$P_{Unskilled}$	27	15.08654	5.322183	5.638242	25.20048
$S_{Skilled}$	27	.1181062	.0155633	.0990799	.1516991
$S_{Unskilled}$	27	.3087443	.024269	.2752533	.3727061
$S_{Capital}$	27	.5731495	.0278611	.4927547	.6104366
NACE (26) - Manufacture of Other Non-Metallic Mineral Products					
$P_{Capital}$	27	872.6722	359.4342	316.5621	1532.616
$P_{Skilled}$	27	25.31021	8.978737	8.453246	42.7007
$P_{Unskilled}$	27	18.27134	6.457499	6.250845	30.76139
$S_{Skilled}$	27	.1090895	.0141224	.0813321	.1442322
$S_{Unskilled}$	27	.2875918	.0330031	.2307356	.3757229
$S_{Capital}$	27	.6033187	.0378287	.4800449	.6591735
NACE (27 - 28) - Manufacture of Basic Metals and Fabricated Metal Products					
$P_{Capital}$	27	901.1062	329.6309	364.6166	1494.353
$P_{Skilled}$	27	21.62569	9.810555	6.60179	40.72514
$P_{Unskilled}$	27	15.29107	6.201056	5.11552	27.62407
$S_{Skilled}$	27	.1314288	.0143069	.1084026	.1637303
$S_{Unskilled}$	27	.3495947	.0346898	.2948416	.4146413
$S_{Capital}$	27	.5189765	.0363393	.4501069	.5793149
NACE (34 - 35) - Manufacture of Transport Equipment					
$P_{Capital}$	27	966.1165	313.2602	434.3075	1686.587
$P_{Skilled}$	27	24.40281	9.4723	8.526078	42.20551
$P_{Unskilled}$	27	19.2146	7.411963	6.922682	34.38324
$S_{Skilled}$	27	.139177	.0181692	.1017251	.1702129
$S_{Unskilled}$	27	.5109422	.0834786	.3815622	.6809512
$S_{Capital}$	27	.3498808	.0920766	.1535239	.4653762
NACE (40 - 41) – Electricity, Gas and Water Supply					
$P_{Capital}$	27	877.1113	345.4224	325.8035	1459.492
$P_{Skilled}$	27	30.02163	14.1675	8.945281	59.55918
$P_{Unskilled}$	27	24.28537	13.29378	7.456497	51.61488
$S_{Skilled}$	27	.1324009	.020615	.094174	.1685588
$S_{Unskilled}$	27	.1736282	.0389408	.1045266	.3119902
$S_{Capital}$	27	.6939709	.0495322	.5381638	.795929

7. Empirical Results and Discussion

In this section we present estimates of own demand elasticities; cross price elasticities and elasticities of substitution for our three inputs. A discussion of these estimates will also be made in this section, followed by the analyses of changes over time.

Table 7.1: Elasticities of Substitution; Own Demand Elasticities; and Cross Price Elasticities

	Mining & Quarrying	Food; Beverages & Tobacco	Textiles & Textile Products	Leather & Leather Products	Wood & Wood Products	Pulp; paper; Publishing	Chemicals & Man-made Fibres	Rubber & Plastic Products	Non-Metallic Minerals	Meta & Fabricated Metals	Transport Equipment	Electricity; Gas & Water
Own Demand Elasticities												
η_{ss}	-0.471	-0.750	-0.903	-1.277	-0.963	-0.791	-0.734	-0.637	-0.469	-0.626	-0.468	-0.702
η_{uu}	-0.477	-0.996	-0.712	-0.819	-0.843	-1.170	-0.945	-0.702	-0.624	-0.609	-0.511	-0.979
η_{kk}	-0.271	-0.323	-0.560	-0.506	-0.550	-0.564	-0.168	-0.452	-0.387	-0.448	-0.831	-0.342
Cross Price Elasticities												
η_{su}	0.116	-0.142	0.528	1.067	0.429	-0.326	-0.438	0.138	-0.0136	0.230	0.128	0.095
η_{sk}	0.355	0.892	0.374	0.210	0.534	1.117	1.172	0.499	0.483	0.386	0.340	0.607
η_{us}	0.0404	-0.063	0.168	0.350	0.142	-0.205	-0.309	0.053	-0.005	0.090	0.035	0.073
η_{uk}	0.437	1.058	0.545	0.470	0.701	1.375	1.254	0.649	0.630	0.519	0.476	0.906
η_{ks}	0.060	0.088	0.100	0.065	0.111	0.190	0.067	0.103	0.087	0.098	0.135	0.116
η_{ku}	0.211	0.235	0.459	0.442	0.439	0.374	0.101	0.350	0.300	0.350	0.696	0.227
Elasticities of Substitution												
σ_{su}	0.395	-0.845	1.322	2.553	1.255	-1.730	-6.156	0.446	-0.047	0.686	0.251	0.549
σ_{sk}	0.587	1.178	0.790	0.471	0.979	1.610	1.334	0.871	0.801	0.744	0.971	0.874
σ_{uk}	0.722	1.397	1.150	1.056	1.286	1.983	1.427	1.132	1.044	1.000	1.362	1.305
Own Elasticities of Substitution												
σ_{ss}	-4.608	-10.071	-7.122	-9.324	-8.527	-6.685	-14.659	-5.393	-4.304	-4.764	-3.361	-5.304
σ_{uu}	-1.630	-5.923	-1.784	-1.960	-2.467	-6.210	-13.301	-2.272	-2.171	-1.743	-1.001	-5.636
σ_{kk}	-0.448	-0.426	-1.180	-1.137	-1.008	-0.813	-0.191	-0.789	-0.642	-0.862	-2.375	-0.493

Discussion

Table 7.1 above presents the calculated elasticities. Own demand elasticities for all inputs all have expected signs confirming a downward sloping demand curve. It is clear from the table that in all the industries, changes in the wages of workers and changes in the cost of using capital will not trigger very large responses by firms in the demand for these inputs. This is because all the elasticity estimates are below unity with the exception of own demand elasticity for skilled labour in leather industry (- 1.277) and wage elasticity of demand for unskilled labour in the paper and publishing industries (- 1.17). This result is similar to what was obtained by Boyle and Sloane (1982) as they also found that demand for inputs was mostly inelastic.

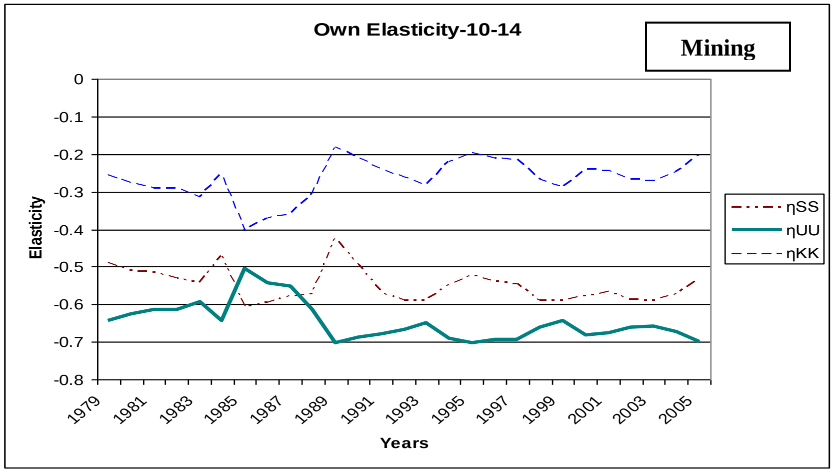
Out of forty industries that they included for their study, Boyle and Sloane had found demand for skilled labour to be inelastic in thirty nine industries, while demand for unskilled labour was found to be inelastic in all the industries, while the demand for capital was found to be inelastic in all the industries. Regarding the demand for skilled labour their result is comparable to my own since I found the demand for skilled labour to be elastic only in Leather and Leather products industry. Furthermore, my results and theirs also agree in terms of the demand for unskilled labour. I found that the demand for unskilled labour was elastic only in Pulp, Paper and Paper products industry and it was unit elastic in Food and Beverages industry. The results of own demand elasticities from table 7.1 reflect that in eight out of twelve industries, demand for unskilled labour is more elastic than the demand for skilled labour. This result was found by Boyle and Sloane (1982) and also noted in Hamermesh (1993).

The demand elasticities for skilled labour in the textiles industry and wood products industry are close to unity. The industries of food and beverages as well as chemical products have the elasticities of demand for unskilled labour that are closest to unity. The demand for capital in the chemicals industry and the mining and quarrying industry are the lowest of all own elasticities of demand.

Regarding the elasticities of substitution, in most of the industries inputs appear to be substitutes of each other. If an estimate of the elasticity of substitution is less than zero, then the two inputs are said to be *p*-complements. This means that if the price of one input increases while the price of the other remains constant, that will decrease the demand for both inputs. On the other hand, if the elasticity of substitution between two inputs is positive, the two inputs are said to be *p*-substitutes. This means that when the price of one of the inputs increases while holding the price of the other input, this will have an effect of increasing the demand for the input whose price did not change. Skilled labour and unskilled labour are *p*-complements in the three industries of food, beverages and tobacco; paper and publishing; and non-metallic minerals industries. In all other industries, all pairs of inputs seem to be *p*-substitutes.

Another important result involves capital-skill complementarity (CSC). Almost all research work in this area of input elasticities, which involve capital, skilled labour and unskilled labour results in skilled labour being more easily substitutable with capital than with unskilled labour. In all industries in table 1, we had found this to be the case i.e. ($\eta_{SK} < \eta_{UK}$). This result is similar to the ones found by Boyle and Sloane (1982) and Kearney (1997). Boyle and Sloane also found that σ_{UK} are generally high but less than unity. While I also find most of σ_{UK} to be less than unity, their magnitudes are not high. Kearney had found that within the medium growth sector, skilled labour, unskilled labour and capital are all substitutes, while there was evidence of capital-skilled complementarity in the high growth sector.

Another objective had been to show what happened to own demand elasticities throughout this period. The following graphs therefore depict the evolution of those elasticities over time:


Own Elasticity-10-14
Mining
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
Elasticity
1979
1981
1983
1985
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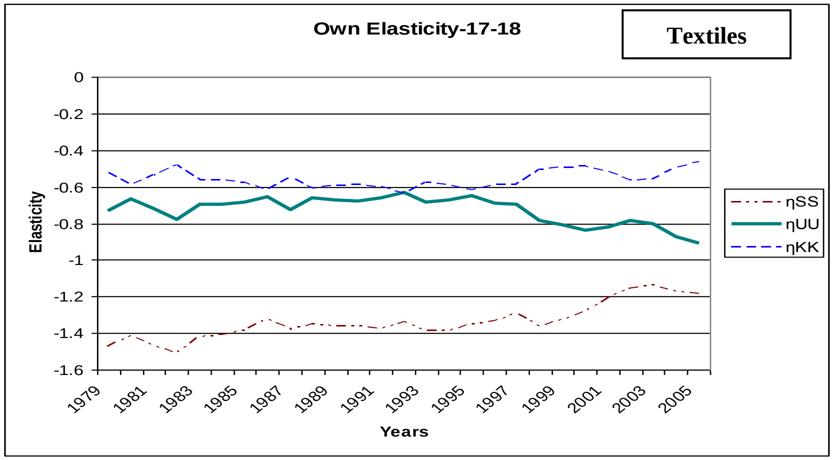
Own Elasticity-17-18
Textiles
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-0.2
-0.4
-0.6
-0.8
-1
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Elasticity
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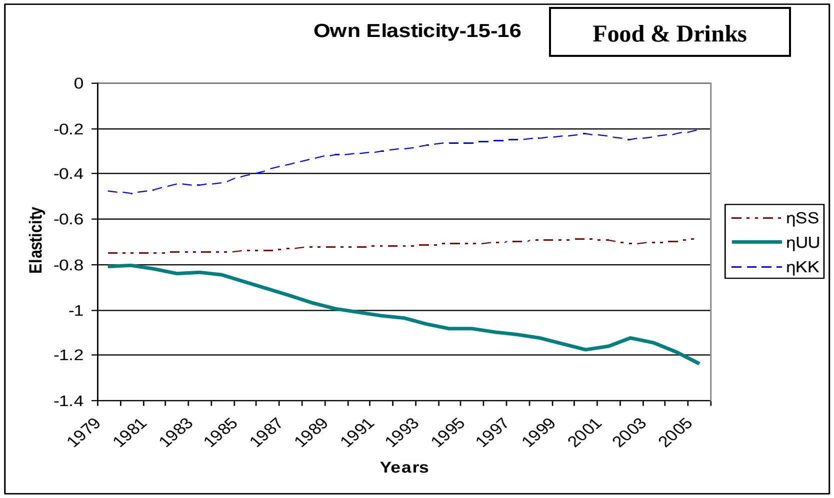
Own Elasticity-15-16
Food & Drinks
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-0.4
-0.6
-0.8
-1
-1.2
-1.4
Elasticity
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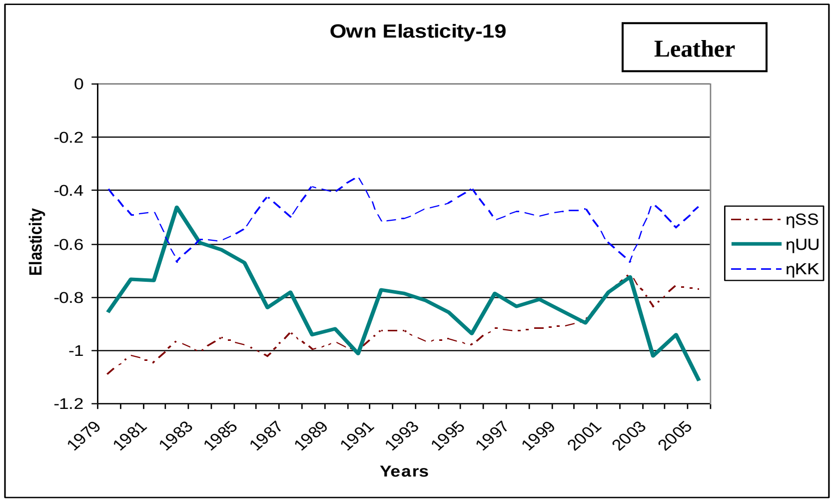
Own Elasticity-19
Leather
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-0.2
-0.4
-0.6
-0.8
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Elasticity
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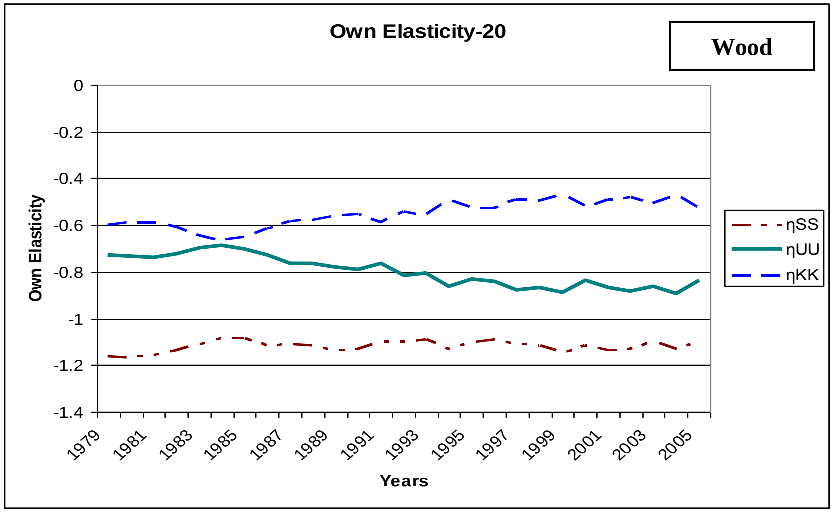
Own Elasticity-20
Wood
Own Elasticity
Years
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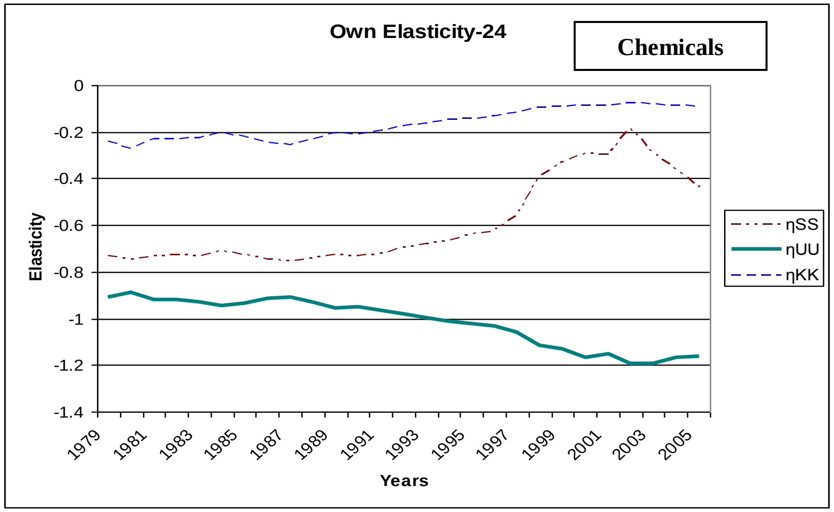
Own Elasticity-24
Chemicals
Elasticity
Years
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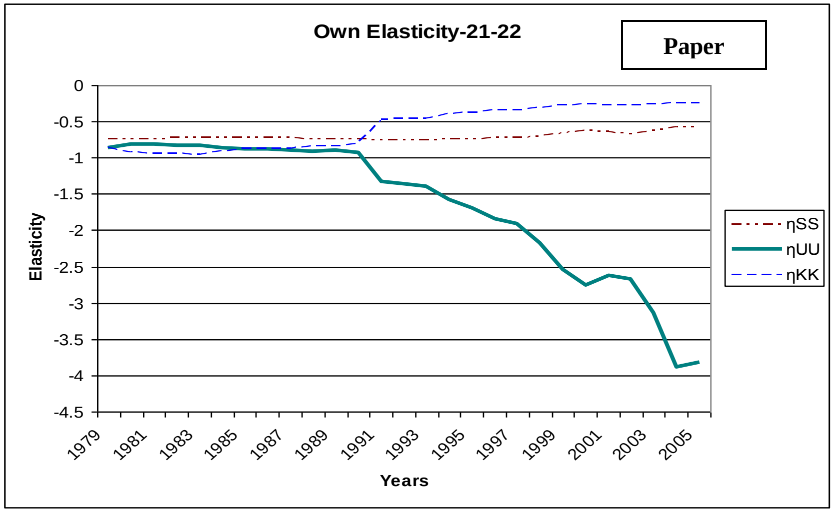
Own Elasticity-21-22
Paper
Elasticity
Years
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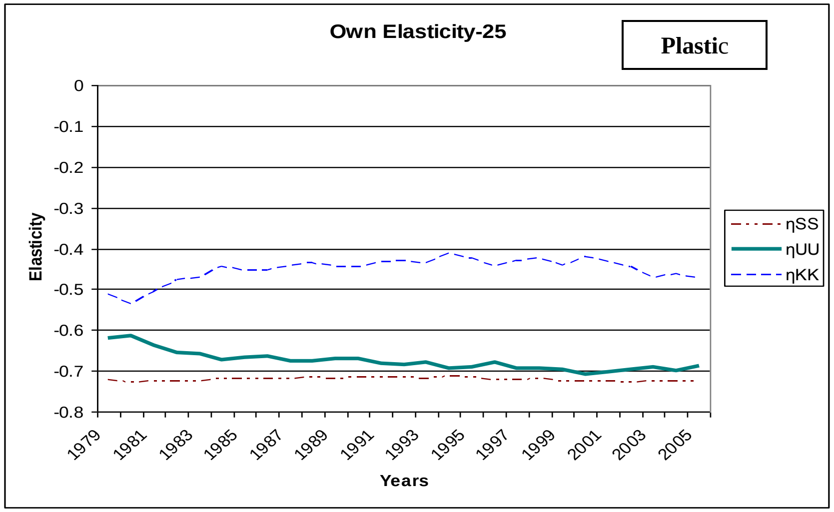
ηKK


Own Elasticity-25
Plastic
Elasticity
Years
ηSS
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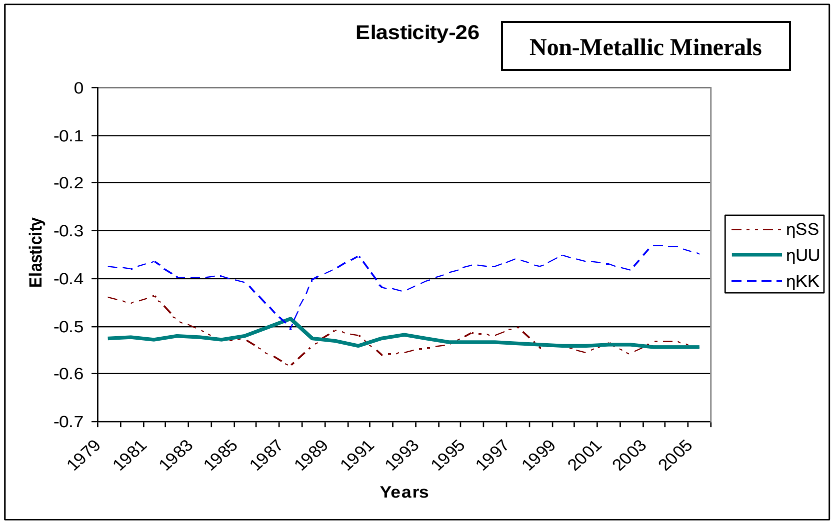
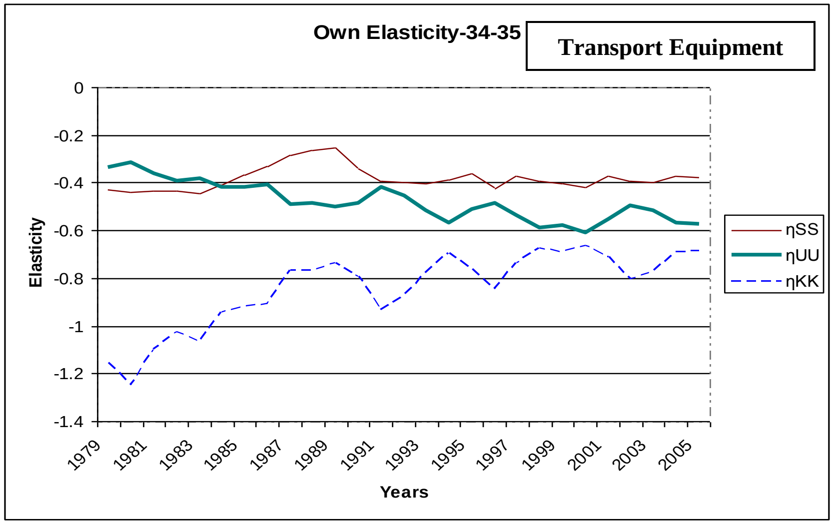
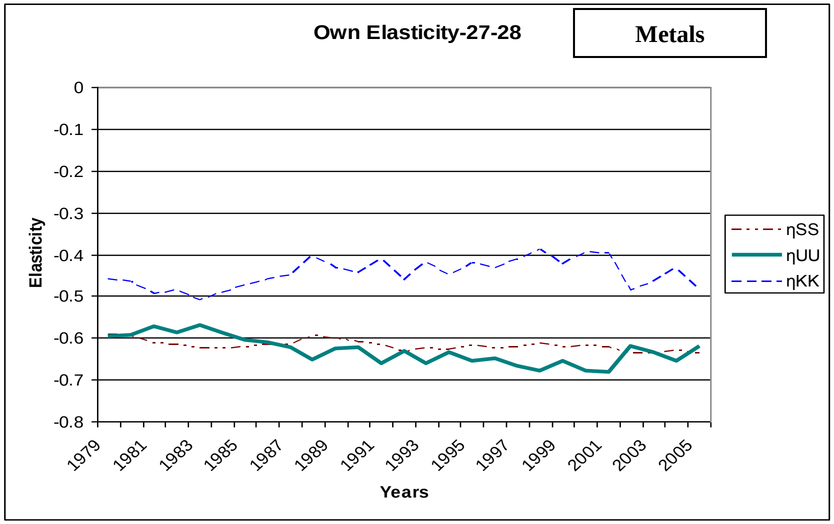


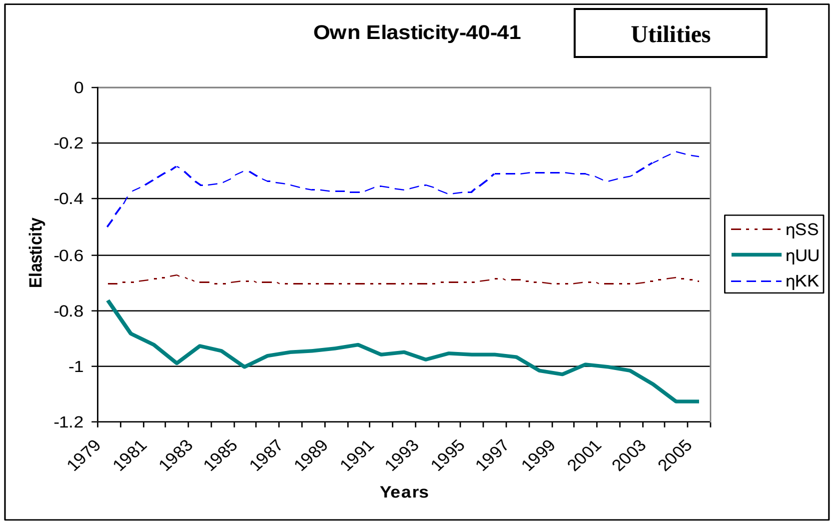
Elasticity-26
Non-Metallic Minerals
Elasticity
Years
ηSS
ηUU
ηKK



Own Elasticity-34-35
Transport Equipment
Elasticity
Years
ηSS
ηUU
ηKK



Own Elasticity-27-28
Metals
Elasticity
Years
ηSS
ηUU
ηKK



Own Elasticity-40-41
Utilities
Elasticity
Years
ηSS
ηUU
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Examination of the graphs does not reveal any major changes in the evolution of own demand elasticities over this period. The demand elasticity for unskilled labour in the Manufacture of Paper industry (NACE 21-22) was just about inelastic up until the early 1990's but since then it has registered a remarkable increase that in the early 2000, it made the demand become much more elastic over time, and infact it is -1.17 on average for the whole period (table 7.1). From the graph of own demand elasticities in the paper industry above, one notes a large increase in the elasticity of demand for unskilled labour and a decline in the elasticity of demand for capital. These large changes seem to reflect the changes in the shares of both capital and skilled labour in the total cost. During that time, the chemical and paper industries experienced a surge in the use of modern technology and possibly a decline in the use of unskilled labour.[4] In Manufacture of Food and Drinks and Manufacture of Chemicals the demand for unskilled labour stays inelastic up until the beginning of 1990 where it becomes highly elastic. The reason for this may have been the fact that capital is in general more substitutable with unskilled labour than it is with skilled labour. Since these are modern sectors there was an increased adoption of more advanced technology, therefore this capital may have substituted unskilled labour in production. For all the other industries, demand elasticities do not change by much and all of them stay inelastic. Except for Manufacture of Leather products, demand for skilled labour for all other industries remained inelastic for the whole period. Demand for capital was inelastic for all industries and for the whole of that period. The demand for capital in the Transport Equipment industry starts off elastic but becomes inelastic after 1984.

So, even though such important changes took place in the Irish economy, apart from a small number of notable exceptions the figures do not show much change in labour demand elasticities over time. The following may be possible explanations of why we do not observe large changes in the elasticities over time:

1. One possible reason why I am not observing large changes in the elasticity trends over time may be the fact that the calculation of elasticities in this paper follows from the

[4] This possibile explanation was suggested to me by researchers at the CSO when I enquired about this sudden fall.

an earlier assumption that when the typical firm minimises production costs, it does so holding output fixed. As it has been discussed, the magnitude of the labour demand elasticity depends on the substitution effect and the scale effect. The result of of making the afore-mentioned assumption about output in the calculation of elasticities is that it eliminates the part played by the scale effect in the magnitude of the labour demand elasticity.

2. In the same manner that the scale effect has not been taken into account in this study, the study does not also consider the effect of the supply elasticity of immigrants on the observed own elasticity of demand for labour. There are four Marshall's Rules of Derived Demand but in order that it is more convenient and not too complicated to undertake the study of this nature, some of the factors that determine the magnitude of labour demand elasticities are assumed to be constant and therefore do not play any part. For example, One of the laws of derived demand for labour is that the wage elaticity of the demand for labour will be high if the supply elasticity of another input is highly elastic. For the puporses of this study, the supply elasticities of other inputs is assumed away. This may also be another reason why I do not observe large changes in the evolution of these labour demand elasticities. Consequently, the impact of any change that works through the scale effect will not be captured in my estimates.

3. A third reason may be that the observed changes are based on the Census of Industrial Production data and not the whole economy. It is true that as a result of the observed changes in the economy, industrial output increased, and so was the production in the whole economy. I have shown that the increase in the FDI inflow was mainly made up of high tech corporations from the US. This therefore means that more changes in employment and output will have been observed in the pharmaceutical companies, in companies that produce high tech products such as computers, as well as in the financial sectors, which this study does not take into account. Although there was an increase in output and employment, I believe more reasonable changes in the labour demand elasticities would have taken place in those sectors that experienced a surge in FDI inflows.

8. Policy Implications

The following policy implications are based on the results obtained. From table 7.1, it is clear that a general wage increase is not expected to bring about big changes in the demand for both unskilled and skilled workers. The reason for this being that in most industries, the demand for labour seems to be inelastic. Any policy decision that increases the general wage level, such as an increase in the minimum wage, may not have that much effect on the demand for labour, because of inelastic magnitudes. Results show that demand for unskilled labour is elastic in the Food; Beverages and Tobacco industry as well as the Pulp; Paper; Priniting and Publishing industry. This elasticity is very close to unity though.

My results have implications for specific policies. A policy measure that subsidises employment costs of unskilled workers can therefore be affected by the findings of a study like this one. In Ireland, the Employment Subsidy Scheme had been introduced, which ran up until 2010. This scheme offered employment subsidies of up to EUR 200 per week per full time worker, where full time is defined as 35 hours per week for positions that would otherwise be made redundant as a result of the most recent economic downturn. To qualify, at the time of application, companies must have been in manufacturing or internationally traded sevices that employ more than 10 workers and intended to retain all workers up to 2010. A policy of this nature can be expected to lower the employment costs thereby lowering the share of production costs that can be attributed to employment of labour. But given the results, the change in employment as a result of this policy can be expected to be small.

As another example I consider changes to the income tax structure and changes to other important schemes such as the Pay Related Social Insurance (PRSI) will affect the costs of production by firms. PRSI is paid as a contribution into the national social insurance fund by employers, employees and self employment workers above 16 years of age and are in any employment contract in Ireland. A mandatory increase in the employers' contribution into this scheme will increase the share of costs associated with engaging labour into production. On the other hand reductions in contributions such as those proposed in the

Employer job (PRSI) Initiative Scheme, which exempts employers from the liability to pay their share of their workers' contributions to the PRSI for the next two years of their employment, may be expected to lower costs for the firm.

Based on the Hicks-Marshall law of derived demand for labour, and if there are close substitutes whose supply elasticity is high, then this would change the demand for labour. But, as it has been mentioned in section 7, based on the findings of this study, the effects of these changes will be small.

A policy measure that increases capital allowances and grants will lead to a decline in the demand for both skilled and unskilled labour while production will tend to be more capital intensive. This is because the cost of using capital will have gone down. Skilled labour and unskilled labour are substitutes in production except in Food industry; Paper industry; Chemicals industry and non-metallic minerals industry. While a change in the cost of using physical capital will lead to changes in both skilled and unskilled labour in all industries, unskilled labour is more substitutable for capital than skilled labour. In other words, skilled labour is more complementary to capital than unskilled labour. This result supports what most studies of complementarity between skilled labour and capital have found. Capital complements skilled labour, and this is considered to be one of the reasons why a divergence of skilled labour and unskilled labour wages in most developed countries was observed from the late 1980s. This means that an increase in the adoption of technology may lead to an increased wage premium for skilled labour, at the expense of unskilled labour.

One interested party in all these would be the labour unions. Unions are interested in securing the highest wages for their members as they can, but they are also conscious of the fact that high wages can also reduce employment for their members and their potential members. If the elasticity of labour is high, unions know that they only stand to get very small gains in the wages for their members from wage bargaining process, otherwise the number of their members employed might decline. Unions would therefore always want to take such actions as would lead to lower wage elasticity of demand for labour. They would win larger wage gains from the bargaining process if the demand for labour is inelastic.

9. Conclusion

In this study, I estimate the demand for three factors of production: skilled workers; unskilled workers and physical capital in the Irish manufacturing sector. This is achieved by estimating a set of three demand equations, each equation corresponding to the demand for each one of the mentioned factors of production. Then the coefficient estimates from estimating this system of equations are used to calculate the own demand elasticity for those factors; the elasticity of substitution between the three factors; and the cross price elasticities. I also analyse the evolution of own demand elasticities over the period under study so as to determine if the changes that took place in the Irish economy during that time may have led to changes in elasticities or not.

A study of this nature is important because of the afore mentioned implication associated with its findings. Other than shedding light on the nature of interaction of factors in a production process, the changes in the demand for these inputs have an influence in the eventual policy decisions.

The main findings of this study are that in most industries in the Irish manufacturing sector, own demand elasticities are less than unity, indicating that an exogenous shock that increases wages and the cost of using physical capital will not be expected to have a large influence on the demand for labour and capital. Of the demands for the three inputs, the demand for capital was the least elastic. Another result is that in most cases, all inputs are substitutes while there is also evidence of relative capital skilled labour complementarity.

Based on the results, I also conclude that the changes in the economic performance of the Irish economy beginning of the 1990's does not for the most part seem to have changed the elasticity of demand for inputs all that much. On average, results seem to be in line with what was found to the studies that looked at elasticities within the Irish manufacturing sector. I believe the reason why my results may be similar to the results from previous studies even though the economic environments are different is that more FDI may have led to much more increases in employment in the services sector and the financial sector. With the boom, also came increased activity in the construction sector.

Another large section of the engaged workforce may have joined that sector. However neither of these sectors fall within the scope of my study but may be worth analysis in a separate study.

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