New AdS non Abelian black holes with superconducting horizons

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Abstract

We present arguments for the existence of higher dimensional asymptotically AdS non Abelian black holes with a Ricci flat event horizon and analyze their basic properties. Unlike higher dimensional black holes with a curved horizon, of the usual Einstein-Yang-Mills system, these solutions have finite mass-energy. Below some non-zero critical temperature, they are thermodynamically preferred over the Abelian configurations.

1 Introduction

In recent years it became clear that various well-known, and rather intuitive, features of self-gravitating solutions with Maxwell fields in $d = 3 + 1$ spacetime dimensions are not shared by their counterparts with non Abelian gauge fields. For example, in contrast to the Abelian situation, self-gravitating Yang-Mills (YM) fields can form particle-like configurations [1]. The Einstein-Yang-Mills (EYM) equations also admit black hole solutions that are not uniquely characterised by their mass, angular momentum and YM charges [2]. Therefore the uniqueness theorem for electrovacuum black hole spacetimes ceases to exist for EYM systems. As a result, the literature on gravitating solutions with non Abelian fields has steadily grown up in the last two decades, including solutions with a cosmological constant Λ (see e.g. [3, 4] and references therein). The asymptotically anti–de Sitter (AAdS) solutions are of particular interest, since gauged supergravity theories playing an important role in AdS/CFT, generically contain non Abelian matter fields in the bulk, although to date mainly Abelian truncations are considered in the literature. Notably, non Abelian AAdS solutions exhibit new features which are absent for $\Lambda = 0$. For example, stable solutions with a global magnetic charge are known to exist even in the absence of a Higgs field [5], [6].

In the context of the AdS/CFT correspondence [7], Klebanov and Witten have proposed a mechanism of spontaneously breaking gauge symmetry [8]. This mechanism has recently been exploited by Gubser *et. al.* [9]–[12] to explain important phenomena in condensed matter physics, in particular superconductivity and critical phenomena. This mechanism results, with no recourse to supersymmetry, in a symmetry breaking boundary theory of a bulk gravitational theory with negative cosmological constant, the temperature of the black hole being nonzero.

From our point of view, the most interesting development in this domain is the recent discovery in [9] that some AAdS non Abelian black hole solutions with a Ricci flat event horizon may posses superconducting horizons which are thermodynamically preferred below some non-zero critical temperature. Such solutions exhibit hair of the 'electric' part of the gauge field on the AdS boundary, manifesting the gauge symmetry breaking mechanism; at the same time the condensate of the 'magnetic' part floats above the horizon of the black hole. This mechanism was further exploited in subsequent works [10, 11].

The only case discussed so far pertain to four dimensional AAdS spacetimes with Ricci flat horizon, and relatively little is known about such higher dimensional solutions with non Abelian matter fields. Naturally, it is always of interest to see how the dimensionality of the spacetime affects the physical consequences of a given theory. In particular, it would be interesting to see how general is the mechanism discovered in [9]. Besides, it is known from the work of [13], [14], [15], [16], that static spherically symmetric solutions of the usual gravitating YM system in spacetime dimensions $d > 4$ do not have finite energy as a result of their scaling propertes. Finite energy solutions exist only when the usual YM system is augmented with higher derivative corrections in the non Abelian action [16], [17]. Therefore the examination of higher dimensional gravitating non Abelian solutions with a different topology (in this case with Ricci flat) of the event horizon is a pertinent task.

Our objective in the present work is to extend this type of symmetry breaking to non Abelian EYM solutions used by Gubser in [9], to arbitray $d = D + 1$ dimensional AAdS spacetimes. For this we exploit, qualitatively, previous results on d dimensional finite energy AAdS solutions for EYM systems given in [16]. In fact, the actual model considered here differs from those of [16] in that the latter is described by the purely 'magnetic' components of the YM field[1] whereas here our model will include also the 'electric' components which play an essential role as is the case of [9]. The salient feature of the YM models in [16] is the presence of higher order terms in the YM curvature, whose role is to supply the necessary (Derick) scaling properties of the Lagrangian to enable the existence of finite energy solutions. There is however one major departure between the models in [16] and those exploited here. While the metric Ansatz employed in [16] describes a spacetime with an $S^{d-2} \times R$ boundary at infinity, the one here describes in contrast a flat Minkowkian boundary. One consequence of this is that the appropriate gauge group here is $SO(D-1)$, *i.e.* $SO(d-2)$, differing from the choice of $SO(D)$, *i.e.* $SO(d-1)$, in [16]. This feature is a reminder of the fact that the electric component A_0 of the YM connection takes the role of a Higgs field. The other consequence of a flat Minkowkian boundary is that inclusion of higher order YM curvature terms are no longer necessary for the solutions to describe finite energy configurations, as was the case when the black hole horizon had a nonzero Ricci tensor. This is a result of the much wider range of scaling properties satisfied when the metric has a Ricci flat event horizon instead of the more restrictive scaling properties of the system when the horizon is spherical. Inclusion of higher order YM curvature terms, while not necessary for achieving finite energy, is still possible here, resulting only in quantitative effects. We have eschewed this option here since it is not qualitatively important. In addition, although we have restricted our attention here to spacetime dimensions $5 \leq d \leq 8$ for simplicity, it is obvious that this limitation is unimportant.

The metric Ansatz we use is a direct extension of that in [5, 9, 20], to dimensions with a larger number of spacelike coordinates. To implement our procedure it is necessary to devise an Ansatz for the YM connection, generalising that used in previous work on YM fields in AdS spacetime. Here, we have found two distinct Ansätze which we have verified to be consistent. These generalise the distinct YM connection Ansätze of [9] and of [10], respectively.

2 General formalism

2.1 The field equations and the abelian solution

Instead of specializing to a particular supergravity model, we shall consider the pure EYM theory with negative cosmological constant in $d \geq 4$ spacetime dimensions

$$S = \int d^d x \sqrt{-g}\, \left(\frac{1}{16\pi G}(R - 2\Lambda) - \frac{1}{4}\, F^a_{\mu\nu} F^{a\mu\nu} \right), \tag{1}$$

where the cosmological constant is $\Lambda = -(d-2)(d-1)/2\ell^2$. Although it seems that the model (1) is non-supersymmetric in itself (at least[2] for $d > 4$), it usually enters the gauged supergravities as the basic building block. Therefore one can expect the basic features of its solutions to be generic.

Variation of the action (1) with respect to the metric $g^{\mu\nu}$ and the gauge field A_μ leads to the EYM equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad D_\mu F^{\mu\nu} = 0, \tag{2}$$

where $T_{\mu\nu}$ is the YM stress-energy tensor $T_{\mu\nu} = F^a_{\mu\alpha} F^a_{\nu\beta} g^{\alpha\beta} - \frac{1}{4} g_{\mu\nu} F^a_{\alpha\beta} F^{a\alpha\beta}$, and $D_\mu = \partial_\mu + ig\,[A_\mu, \cdot]$ (with g the gauge coupling constant).

[1] Inclusion of the 'electric' components of the YM field can readily be made, as *e.g.* in the case of Euclidean signature in [19].

[2] The case $d = 4$, with $\Lambda/(16\pi G) = -3g^2$ corresponds to a consistent truncation of $\mathcal{N} = 4$ gauged supergravity and may be uplifted to $d = 11$ supergravity [21], [22],[3].

We shall consider black hole solutions of the above equation with locally flat horizons, which asymptotically approach a locally AdS spacetime with a boundary at infinity R^{d-1}. The simplest such configuration with a nonzero gauge field is represented by the Reissner-Nordström-AdS (RNAdS) black hole with

$$ds_d^2 = \frac{dr^2}{\frac{r^2}{\ell^2} - \frac{2M_0}{r^{d-3}} + \frac{8\pi G(d-3)}{g^2(d-2)}\frac{q^2}{r^{2(d-3)}}} + r^2 d\gamma^2 - (\frac{r^2}{\ell^2} - \frac{2M_0}{r^{d-3}} + \frac{8\pi G(d-3)}{g^2(d-2)}\frac{q^2}{r^{2(d-3)}})dt^2, \quad A = (c - \frac{1}{g}\frac{q}{r^{d-3}})\mathcal{T}dt, \quad (3)$$

where M_0, c, q are constants, $\mathcal{T}$ is an element of the gauge group while $d\gamma^2$ is the line element of the $d-2$ euclidean space. The parameters M_0 and q are proportional to the mass and electric charge of the solution. The black hole horison is located at $r = r_h$, with $r_h^2/\ell^2 - 2M_0/r_h^{d-3} + (8\pi G(d-3)/g^2(d-2))q^2/r_h^{2(d-3)} = 0$. When taking $q = 0$, the Schwarzschild-AdS (SAdS) black hole with a planar horizon is recovered.

2.2 The Ansatz

We are interested in non Abelian configurations whose magnetic gauge potential vanishes asymptotically, such that the abelian configuration (3) is approached in that limit. Moreover, we shall suppose that our configurations present a dependence only on a suitable radial coordinate r which is orthogonal to the boundary of the spacetime.

The choice of the gauge group compatible with these assumptions (and the corresponding YM Ansatz) is quite flexible. For the $d = 4$ case, two different[3] non Abelian Ansätze have been proposed in the literature, both of them for a gauge group SU(2). The first Ansatz used in [9] corresponds to a "circular polarisation" of the magnetic YM connection and leads to an isotropic energy momentum tensor for the components on a surface of constant (r, t), with t the time coordinate. This is not the case for the second YM Ansatz proposed in [10], where a particular direction in the R^2 subspace is choosen, leading to a more complicated metric Ansatz.

A straightforward generalisation of the isotropic Ansatz in [9] is found for a gauge group $SO(D)$ (with $D < d$), the Ansatz for the YM connection being stated by

$$A(r) \quad = \quad u(r) n_\alpha^i \Gamma_{ij} m_\alpha^j dt + w(r) dx^i \Gamma_{iD}, \quad (4)$$

in which $\Gamma_{AB} = (\Gamma_{ij}, \Gamma_{iD})$ are the gamma matrices in D dimensions and the indices α, β run over the range $\alpha, \beta = 1, 2, ..(D-1)/2$. In the above relation, x^i are the coordinates parametrizing a surface of constant r, t. Also, the sets (n_α^i, m_α^j) appearing in (4) form a complete and orthonormal basis of constant valued vectors of unit length,

$$\sum_{i=1}^{D-1} n_\alpha^i m_\beta^i = 0, \quad \sum_{i=1}^{D-1} n_\alpha^i n_\beta^i = \delta_{\alpha\beta} \quad , \quad \sum_{i=1}^{D-1} m_\alpha^i m_\beta^i = \delta_{\alpha\beta}, \quad \sum_{\alpha=1}^{\frac{D-1}{2}} n_\alpha^i n_\alpha^j + \sum_{\alpha=1}^{\frac{D-1}{2}} m_\alpha^i m_\beta^j = \delta^{ij}.$$

The last completeness condition is very important and means that $D-1$ is even. Therefore, for $d = D+1$, the YM Ansatz (4) is defined[4] only for an even number of space dimensions d. Of course, it can also be used for an arbitrary spacetime dimension by adding n codimensions y^k, with the non Abelian potential identically zero in the subspace labeled by the extra-coordinates. Obviously, this Ansatz leads to an isotropic energy momentum tensor in both x^i and y^k-directions, with $T_{x^i}^{x^i} \neq T_{y^k}^{y^k}$.

However, it is possible to define a different non Abelian Ansatz (Ansatz II in what follows), valid for all even and odd dimensions d. In this case we employ only one (constant valued) set of orthonormal unit vectors n^i, in terms of which

$$A(r) = u(r) n^i \Gamma_{i,d-1} dt + w(r) n^j \Gamma_{ji} dx^i \ . \quad (5)$$

[3]This contrasts with the case of solutions with a $R \times S^{d-2}$ boundary at infinity, where the choice of Ansatz is unique, see *e.g.* [6].

[4]The construction of a consistent isotropic Ansatz for $d = 2k+1$ necessitates the enlargement of the gauge group, resulting in a YM connection with more than one magnetic potential. For example for the most interesting $d = 5$ case, the minimal gauge group is SO(5), while the Ansatz would contain six non Abelian potentials.

While the components of the connection (5) take their values fully in the algebra of $SO(d-1)$, only $d-2$ components of the YM connection are effective since $n^i A_i = 0$. Thus we have one additional radius, but the fields depend only on the radial coordinate r, so one of the magnetic components is zero. This results also on an anisotropic energy mementum tensor of the YM field.

For an arbitrary spacetime dimension, a metric form compatible with the above two YM ansätze is given by

$$ds_d^2 = A(r)dr^2 + F_1(r)\sum_{i=1}^{d-n-2} dx^i dx^i + F_2(r)\sum_{k=1}^{n} dy^i dy^i - B(r)dt^2, \tag{6}$$

where we have found convenient to take

$$A(r) = \frac{1}{N(r)}, \quad B(r) = N(r)\sigma^2(r), \quad F_1(r) = r^2 f^2(r), \quad F_2(r) = r^2(f(r))^{2(n+2-d)/n} \tag{7}$$

with

$$N(r) = -\frac{2m(r)}{r^{d-3}} + \frac{r^2}{\ell^2}, \tag{8}$$

the function $m(r)$ being related to the local mass-energy density up to some $d-$dependent factor. For the case of the first Ansatz (4), the YM field is defined on a subspace labeled by the $(r,t;x^i)$-coordinates, where $i = 1,\ldots,d-n-2$ (with $n \geq 0$). The second YM ansatz corresponds to an arbitrary d, with $n = 1$ (Here we assume without any loss of generality $n^i = \delta^i_{d-2}$ and write $x^{d-2} = y^1$.)

2.3 The equations of motion and asymptotic solutions

Within these Ansätze, the EYM field equations reduce to a set of five ordinary differential equations which can be expressed in a unified form as[5] (where one takes $c = 1/2$ for the Ansatz I and $c = 1/(d-3)$ for Ansatz II)

$$\begin{aligned}
&w'' = \left(\frac{2f'}{f} - \frac{d-4}{r} - \frac{\sigma'}{\sigma} - \frac{N'}{N}\right)w' + \left(\frac{u^2}{N\sigma^2} - \frac{(d-n-3)w^2}{r^2f^2}\right)\frac{w}{N} = 0,\\
&u'' + \left(\frac{d-2}{r} - \frac{\sigma'}{\sigma}\right)u' - \frac{1}{c}\frac{uw^2}{r^2Nf^2} = 0,\\
&m' = \frac{d-n-2}{2n}r^{d-2}N\frac{f'^2}{f^2} + \alpha^2 r^{d-4}\left(\frac{Nw'^2}{f^2} + c\frac{r^2u'^2}{\sigma^2} + \frac{(d-n-3)w^4}{2r^2f^4} + \frac{u^2w^2}{N\sigma^2f^2}\right),\\
&\sigma' = \frac{2\sigma}{f^2 r}\left(\frac{d-n-2}{2n}r^2f'^2 + \alpha^2(w'^2 + \frac{w^2u^2}{N^2\sigma^2})\right),\\
&f'' = \frac{2n\alpha^2}{(d-n-2)r^2f}\left(\frac{u^2w^2}{N^2\sigma^2} - \frac{(d-n-3)w^4}{r^2f^2N} - w'^2\right) - \left(\frac{d-2}{r} - \frac{f'}{f} + \frac{N'}{N} + \frac{\sigma'}{\sigma}\right)f',
\end{aligned} \tag{9}$$

where $\alpha^2 = 8\pi G/(g^2(d-2))$. For $d = 4$, $n = 0$, the above equations reduce to those derived in [9] albeit for a different metric Ansatz[6]. The Abelian Reissner-Nordström solution (3) is found for $m(r) = M_0 - c(d-3)q^2\alpha^2/2r^{d-3}$, $f(r) = \sigma(r) = 1$, $w(r) = 0$, $u(r) = u_0 - q/r^{d-3}$.

Unfortunately, no exact non Abelian solutions of this system are yet known. However, one can analyse their properties by using a combination of analytical and numerical methods, which are sufficient for most

[5]Here and in what follows, the relations for the case with no codimensions are found by formally setting $f \equiv 1$, followed by the limit $n \to 0$.

[6]$d = 4$ AAdS non Abelian black holes with a Ricci flat horizon were discussed previously in [20].

purposes. The solutions have the following expansion[7] at $r = r_h > 0$ near the event horizon, located at r_h:

$$\begin{aligned}
&m(r) = \frac{r_h^{d-1}}{2\ell^2} + m_1(r-r_h) + O(r-r_h)^2,\ \sigma(r) = \sigma_h + \sigma_1(r-r_h) + O(r-r_h)^2,\\
&f(r) = f_h + f_1(r-r_h) + O(r-r_h)^2,\\
&w(r) = w_h + w_1(r-r_h) + O(r-r_h)^2,\ \ u(r) = u_1(r-r_h) + u_2(r-r_h)^2 + O(r-r_h)^3.
\end{aligned} \tag{10}$$

Here, following [9] we interpret w_h as a 'magnetic' condensate. All coefficients in the above relation can be expressed in terms of the real constants σ_h, w_h, f_h, u_1. One finds *e.g.*

$$\begin{aligned}
&m_1 = \frac{r_h^{d-6}\alpha^2}{2\sigma_h^2 f_h^2}(2cf_h^4 u_1 r_h^4 + (d-n-3)\sigma_h^2 w_h^4), \qquad f_1 = -\frac{2\alpha^2(d-n-3)nr_h^{d-5}\ell^2 w_h^2}{(d-n-2)f_h^2((d-1)r_h^d - 2m_1 r_h^2\ell^2)},\\
&\sigma_1 = \frac{1}{f_h^2}\left(\frac{2\sigma_h w_1^2\alpha^2}{r_h} + \frac{(d-n-2)}{n}f_1^2 r_h\sigma_h + \frac{2\alpha^2 r_h^{2d-3}\ell^4 u_1^2 w_h^2}{((d-1)r_h^d - 2m_1 r_h^2\ell^2)^2\sigma_h}\right),\\
&w_1 = \frac{(d-n-3)\ell^2 w_h^3}{f_h^2 r_h^3(d-1-\frac{2m_1\ell^2}{r_h^{d-2}})}, \qquad u_2 = \frac{u_1}{2}\left(-\frac{d-2}{2} + \frac{\sigma_1}{\sigma_h} + \frac{\ell^2 w_h^2}{cf_h^2 r_h^3(d-1-2m_1 r_h^{2-d}\ell^2)}\right).
\end{aligned} \tag{11}$$

Note also that the physical condition $N'(r_h) > 0$ implies the following condition on the boundary data $2cf_h^2 u_1^2/\sigma_h^2 + (d-n-3)w_h^4/(r_h^4 f_h^2) < (d-1)/(\alpha^2\ell^2)$.

We are interested in solutions of the EYM equations approaching at infinity the Abelian RNAdS solution (3). This implies the following asymptotic expansion as $r \to \infty$

$$\begin{aligned}
&m(r) = M_0 - \frac{(d-3)\alpha^2}{\ell^2}(J^2 + cq^2\ell^2)\frac{1}{r^{d-3}} + O(1/r^{d-1}),\ \sigma(r) = 1 - \frac{(d-3)^2}{(d-2)}\frac{J^2}{r^{2(d-2)}} + O(1/r^{2(d+2)}),\\
&f(r) = 1 - \frac{\bar{f}}{r^{d-1}} + O(1/r^{2(d-2)}),\ \ w(r) = \frac{J}{r^{d-3}} + O(1/r^{d-1}),\ \ u(r) = u_0 - \frac{q}{r^{d-3}} + O(1/r^{2(d-2)}),
\end{aligned} \tag{12}$$

with $M_0, J, q, \bar{f}$ real constants. The holographic interpretations of u_0, q, and J are as follows: u_0 is the chemical potential, q is the electric charge, and J is that component of the current J_i on the boundary connected with the spontaneously broken part of the bulk gauge symmetry [9]. In other words, we have a D-dimensional conformal field theory described on the boundary of AdS_{D+1} space equipped with $SO(D-1)$ conserved currents, which satisfy their own current algebra[8]. The normalisable boundary value of the 'magnetic' field $w(r) = J/r^{d-3} + O(1/r^{d-1})$ corresponds to the vacuum expectation value of the boundary currents proportional to J arising after symmetry breaking, and, the existence of the horizon 'magnetic' condensate w_h.

The case $d = 4,\ n = 0$ considered in [9] is special from the point of view of the asymptotic expansion, since the finite energy requitements are compatible with a nonvanishing value of the magnetic potential at infinity. The condition $w(\infty) = 0$ is imposed there by requiring w to make a finite contribution to the norm of the non Abelian potential [9]. However, one can easily see from the field equation (9) that for $d > 4$, $w(\infty) \neq 0$ results in a divergent value of the mass function $m(r)$, which gives further justification to the choice (12).

The constant u_0 in the asymptotic expansion (12) corresponds to the electrostatic potential $\Phi = u_0/g$, while q fixes the electric charge density $Q_e = (d-3)q/g$. Other quantities of interest are the mass-energy density M, Hawking temperature T and entropy density S,

$$M = \frac{(d-2)M_0}{8\pi G},\ \ T = \frac{N'(r_h)\sigma_h}{4\pi} = \frac{\sigma_h}{4\pi}\left(\frac{(d-1)r_h}{\ell^2} - \frac{2m_1}{r_h^{d-3}}\right),\ \ S = \frac{1}{4G}r_h^{d-2}. \tag{13}$$

The constant J which enters the asymptotics of the magnetic non Abelian potential $w(r)$ corresponds to an order parameter describing the deviation from the Abelian solution.

[7] The case of Ansatz II for $d = 4$ is special, as the near horizon expansions of $w(r)$ and $f(r)$ are different in that case: $w(r) = w_h + w_2(r-r_h)^2 + O(r-r_h)^3$, $f(r) = f_h + f_2(r-r_h)^2 + O(r-r_h)^3$. However, the horizon data is still determined by σ_h, w_h, f_h, u_1.

[8] The dual CFT global current J_i is defined through $\mathrm{Tr}A_iJ_i$, where A_i here is the asymptotic YM connection.

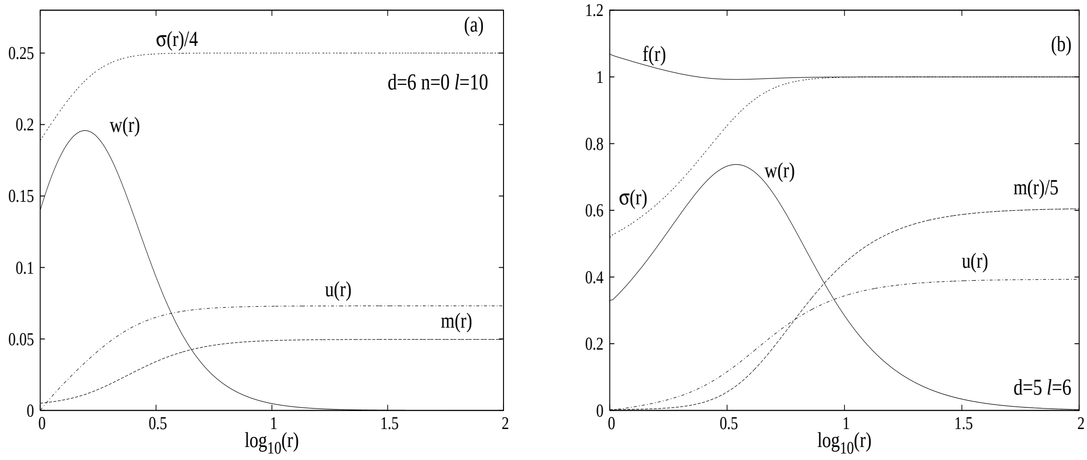

Figure 1: The profiles of typical solutions are shown as a function of radial coordinate for Ansatz I (a) and Ansatz II (b).

3 Numerical solutions

3.1 Scaling properties and general features

We start by noticing that the equations (9) are not affected by the transformation:

$$r \to \lambda r, \quad m \to \lambda^{d-3} m, \quad \ell \to \lambda \ell, \quad u \to \lambda u, \quad \alpha \to \alpha/\lambda \tag{14}$$

while w, σ and f remain unchanged. Thus, in this way one can always take an arbitrary positive value for α. The usual choice is $\alpha = 1$, which fixes[9] the EYM length scale $L = \sqrt{8\pi G/(g^2(d-2)}$, while the mass scale is fixed by $\mathcal{M} = (8\pi G/(g^2(d-2))^{(d-3)/2}/G$. All other quantities get multiplied with suitable factors of L. However, in what follows, to avoid cluttering our expressions with a complicated dependence of (G, g, d), we take a *unit* value for α and ignore the extra-factors of g and G in the expressions of various global quantities.

For solutions with a spherical event horizon, the event horizon radius and the value of the magnetic potential on the horizon are independent parameters (see e.g. [6]). This is not the case for the solutions here, in which case one can always set $r_h = 1$ without any loss of generality. This is a consequence of the following scaling symmetry of the system (9):

$$r \to \lambda r, \quad w \to \lambda w, \quad u \to \lambda u, \quad m \to \lambda^{d-1} m, \tag{15}$$

while σ, f and the cosmological constant remain unchanged[10].

The system (9) presents in addition two more scaling symmetries associated with the functions σ and f (*e.g.* $\sigma \to \lambda\sigma$, $u \to \lambda u$, $t \to \lambda t$ etc.). In the numerical procedure these symmetries are used to set $\sigma(\infty) = f(\infty) = 1$ and thus to fix the horizon values of the functions σ and f. Together with the other symmetries mentioned above, this leaves us with three numerically relevant parameters: w_h, u_1 and the AdS length scale ℓ. Since equations (9) are invariant under the transformation $w \to -w$, only values of $w_h > 0$ are considered.

The equations (9) with boundary conditions implied in turns by (10), (12) have been solved numerically, using a standard shooting method. As expected, the properties of the solutions obtained for the two distinct YM Ansätze (4), (5) are rather similar and thus we have preferred to present them together. For the first case, families of solutions have been constructed in a systematic way for $d = 4, 6, 8$ with $n = 0$, and $d = 5, 7$

[9]These are the units usually used in the literature on EYM solutions [23]. Note also that (14) is a generic property of the EYM system, shared by solutions with a different event horizon topologies.

[10] The global quantities scale as follows: $M \to \lambda^{d-1}M$, $T \to \lambda T$, $S \to \lambda^{d-2}S$, $Q_e \to \lambda^{d-2}Q_e$, $\Phi \to \lambda\Phi$, $J \to \lambda^{d-2}J$.

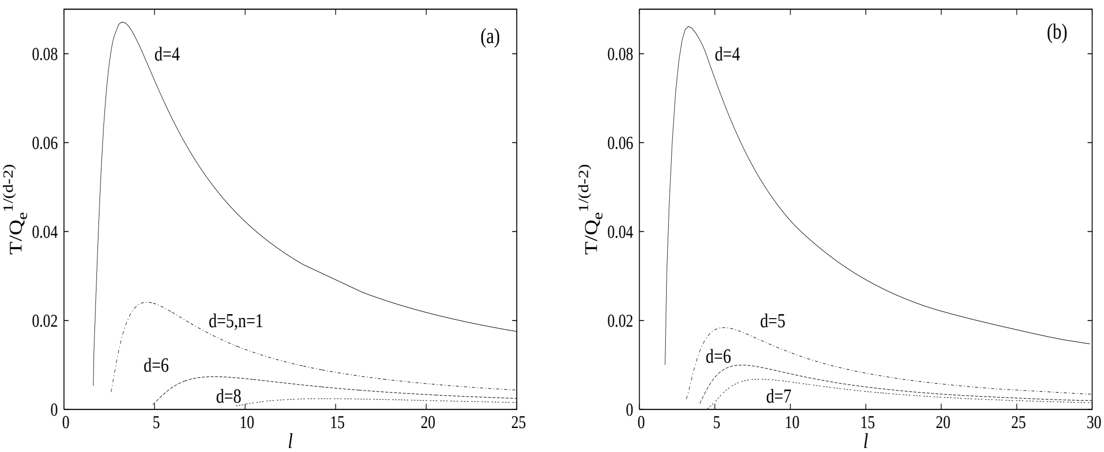

Figure 2: The phase diagram of the non Abelian solutions is plotted for several dimensions, for Ansatz I (a) and Ansatz II (b). The value of n is zero, except for the $d = 5$ curve in Figure 2a.

with $n = 1$. Several configurations with $d = 6$, $n = 2$ have been constructed as well. When choosing instead the YM Ansatz (5), we have constructed solutions in $d = 4, 5$ and 6 dimensions. For every considered value of ℓ, we could find regular black hole solutions for only one interval $0 \leq w_h < w_h^c$. The value of w_h^c increases as ℓ decreases, $w_h = 0$ corresponding to the RNAdS solution (3) .

In all these cases, we noticed a number of common features. The behaviour of solutions for generic initial data is such that $w \to w_0 \neq 0$ at large r (in which case the total mass-energy diverges), or else there is a singularity at finite r. Given (w_h, ℓ), solutions with the right asymptotic behaviour (12) exist only for a discrete set of values of u_1. As in the well known case of the Bartnik-McKinnon solutions [1], the solutions here are also indexed by the node number of the magnetic potential $w(r)$. It turns out that the configurations with nodes represent excited states whose energy is always greater than the energy of the corresponding nodeless configurations, and are therefore ignored in what follows.

For all solutions the functions $m(r)$, $\sigma(r)$ and $u(r)$ always increase monotonically with growing r. However, $f(r)$ and $w(r)$ feature a more complicated behaviour. Tyical solutions are presented in Figure 1 for both Ansätze. For sufficiently small ω_h, all field variables remain close to their values for the Abelian configuration with the same r_h. Significant differences occur for large enough values of ω_h and the effect of the non Abelian field on the geometry becomes more and more pronounced.

3.2 Thermal properties and superconducting horizons

For all cases we considered, finite energy solutions were found only for values of the Hawking temperature less than a critical temperature T_c. As in the $d = 4$ case in [9] this T_c is, within numerical error, the temperature at which the RNAdS solution admits a static linearised perturbation, with nonvanishing but infinitesimally small[11] w. Also, our numerical results indicate that T_c goes to zero for some critical value of the AdS length scale ℓ, but the corresponding solutions do not appear to have a singular behaviour there.

These features are shown in Figure 2 for several values of d. For a given dimension the part of the parameter space above the curve corresponds to the unbroken phase, where only Abelian solutions exist.

In Figure 3, we plot several quantities which are invariant under the transformation (15) as a function of the ratio T/T_c. ΔF there is the difference in the free energy density, $M - TS$, between a non Abelian solution and the the RNAdS solution with the same T and Q_e. As usual, $\Delta F < 0$ means that the non Abelian solution is thermodynamically favoured.

In all cases there is a second order phase transition with simple critical exponents, from RNAdS solutions to solutions with normalisable non Abelian condensates. We have verified that for $T_m < T < T_c$, (with T_m

[11] Our numerical code usually provided good quality solutions for $T \gtrsim T_c/2$.

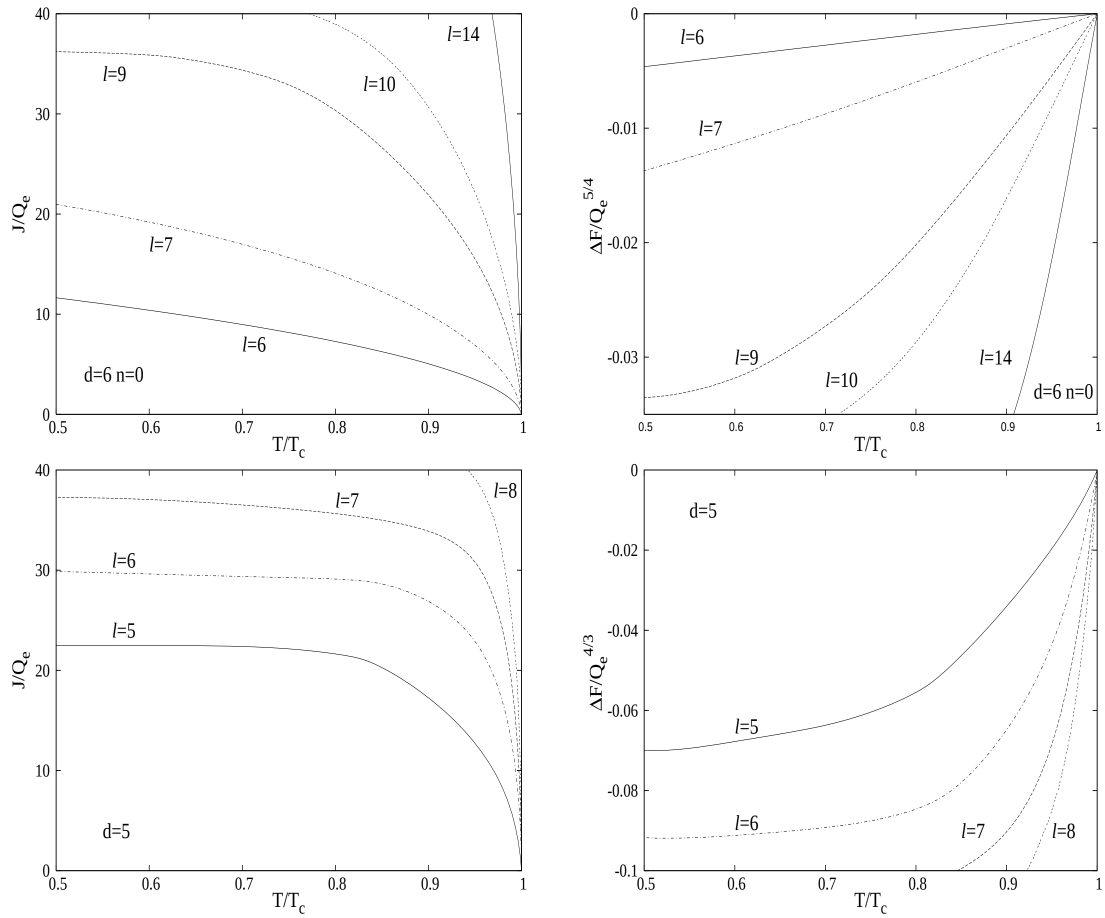

Figure 3: J/Q_e and $\Delta F/Q_e^{(d-1)/(d-2)}$ are plotted as a function of T/T_c for $d = 5$ (Ansatz I) and $d = 6$ (Ansatz II) non Abelian solutions.

always around $0.7\,T_c$), the solutions satisfy the universal relation $J/Q_e = j_{1/2}\sqrt{1 - T/T_c}$, where $j_{1/2}$ depends on the model.

4 Further remarks

In this work we have presented arguments that the $d = 4$ picture discovered in [9] is generic for the higher dimensional case as well. Considering several values of $d \geq 5$, we have found evidence for the existence of a second order phase transition with simple critical exponents, from the (electrically charged) RNAdS solutions with a flat event horizon to non Abelian configurations with a nontrivial magnetic field.

One should also note that the existence of these finite energy non Abelian solutions with $d > 4$ represents a surprise in itself, since it contradicts the expectation based on the no-go theorems in [13], [14], [15],[16]. The negative results in the latter were proven for configurations with a spherical topology of the event horizon, in which case the electric potential necesarily vanishes when $d \geq 5$. Purely magnetic EYM solutions with finite mass were found by considering corrections to the YM Lagrangian consisting in higher order terms of the Yang–Mills hierarchy of the form $L_p = \mathrm{T}r\ F(2p)^2$ where $F(2p)$ is the $2p$-form p-fold totally antisymmetrised product of the $SO(d)$ YM curvature 2-form $F(2)$ (see e.g. [17] for asymptotically flat configurations and [16], [18] for solutions with a cosmological constant). Such systems occur in the low energy effective action of

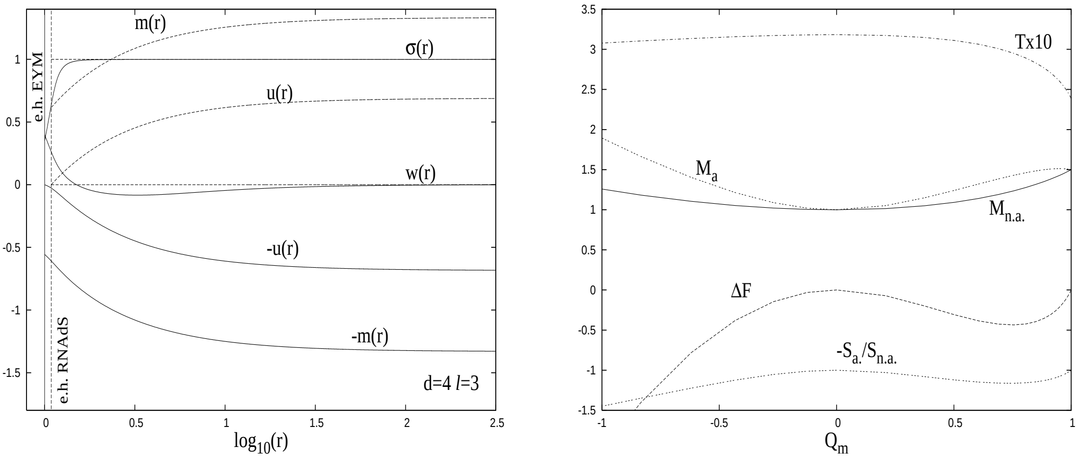

Figure 4: *Left*: The profiles of a $d = 4$ dyonic non Abelian EYM solution with spherical horizon (continuous line) is plotted together with a RNAdS configuration with the same Hawking temperature and electric charge. *Right*: A number of quantities are presented as function of the magnetic charge for non Abelian monopoles and RNAdS solutions with the same temperature for $d = 4$ solutions with $\ell = 1$. We have ploted the Hawking temperature (multiplied with a factor of ten, for better visualisation), the ratio between the entropies of the Abelian and the SU(2) solutions, the black hole masses and the difference between the free energies.

string theory and are, in some sense, the non Abelian counterparts of the Lovelock gravitational hierarchy [12]. By contrast, we have verified that to construct finite energy EYM solutions with Ricci flat horizons, inclusion of higher derivative terms in the YM curvature to the action (1) is not necessary. For this reason we have restricted our attention to the usual gravitating YM system, as inclusion of higher order YM terms results only in (unimportant) quantitative changes.

An important question is, whether it is only for solutions planar event horizon that the non Abelian solution is themodynamically favoured over the Abelian one? To answer this question, consider simply the case of the $d = 4$ dyonic SU(2) black holes with spherical event horizon topology originally discussed in Ref. [6]. These solutions are found within the Ansatz

$$
\begin{aligned}
ds^2 &= \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) - \sigma^2(r)N(r)dt^2, \quad \text{with} \quad N(r) = 1 - \frac{2m(r)}{r} + \frac{r^2}{\ell^2}, \\
A &= \frac{1}{2}\Big\{u(r)\tau_3 dt + w(r)\tau_1 d\theta + (\cot\theta\tau_3 + w(r)\tau_2)\sin\theta d\varphi\Big\},
\end{aligned}
\tag{16}
$$

where τ_a are the Pauli matrices. Without any loss of generality, by using the symmetry (14), one can set $4\pi G = g = 1$. The problem reduces in this case to a system of four coupled ordinary differential equations. The properties of these solutions including the boundary conditions and the asymptotic expansion can be found[13] *e.g.* in [6]. The generic behaviour of the solutions is such that they have a nonvanishing magnetic charge $Q_m = 1 - w^2(\infty)$. Non Abelian solutions with $Q_m = 1$ are found for special values of $u'(r_h)$ and have $g_{rr}g_{tt} \neq -1$. The dyonic Abelian RNAdS solution with unit magnetic charge corresponds to $w(r) = 0$, $u(r) = u_0 + q/r$, $\sigma(r) = 1$, $m(r) = M_0 - (1 + q^2)/2r$. Considering again the case of a canonical ensemble, we have found numerical evidence for the existence of non Abelian solutions which are thermodynamically favoured over the Abelian ones. An example of such a situation is presented in Figure 4 for an AdS length scale $\ell = 3$. There, the Hawking temperature and the electric charge are $T \simeq 0.0053$ and

[12] However, inclusion of members of this gravitational hierarchy turns out to be of no practical utility because subject to the symmetries imposed such terms with the requisite scaling properties vanish.

[13] The function $p(r)$ in [6] corresponds to $1/\sigma(r)$ in the Ansatz (16).

$Q_e \simeq 0.751$ for both solutions, while the masses are slightly different: M(RNAdS)$\simeq 1.334$ and $M \simeq 1.33$ for the non Abelian counterpart. This implies $\Delta F < 0$ and thus the existence of a phase transition[14].

One should also remark that, for $d = 4$, the presence of a non Abelian electric field is not crucial for the existence of a phase transition between Abelian and non Abelian solutions. Setting $u(r) = 0$ in the Ansatz (16), our numerical results indicate the existence of purely magnetic non Abelian configuration which are thermodynamically favoured over the abelian solutions with the same magnetic charge[15] and temperature. In Figure 4b we plot a number of relevant quatities for a family of $d = 4$ EYM monopole black holes with $r_h = 1$ and $\ell = 1$ and the corresponding RNAdS solutions. The gauge potential $w(r)$ is nodeless for all solutions there. The solution with $Q_m = 0$ corresponds to the Schwarzschild-AdS (SAdS) black hole with a spherical horizon. One can see that for $\Lambda = -3$ all non Abelian solutions with $r_h = 1$ have $\Delta F < 0$. However, the generic picture is more complicated, with a nontrivial dependence on ℓ, r_h.

A study of these aspects is beyond the purposes of this work and will be presented elsewhere.

We close by remarking that the asymptotic AdS structure of the spacetime is crucial for the existence of such solutions. As proven in [25] for $d = 4$, the asymptotically flat EYM solutions have no magnetic charge while their electric part vanishes identically. Moreover, by using the data in [2], one can easily verify the difference between the free energy of a SU(2) hairy black hole and the Schwarzschild solution with the same temperature is always positive.

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References

[1] R. Bartnik and J. McKinnon, Phys. Rev. Lett. **61** (1988) 141.

[2] M. S. Volkov and D. V. Galtsov, Sov. J. Nucl. Phys. **51** (1990) 747 [Yad. Fiz. **51** (1990) 1171];
P. Bizon, Phys. Rev. Lett. **64** (1990) 2844;
H. P. Kuenzle and A. K. M. Masood- ul- Alam, J. Math. Phys. **31** (1990) 928.

[3] R. B. Mann, E. Radu and D. H. Tchrakian, Phys. Rev. D **74** (2006) 064015 [arXiv:hep-th/0606004].

[4] E. Winstanley, arXiv:0801.0527 [gr-qc].

[5] E. Winstanley, Class. Quant. Grav. **16** (1999) 1963 [arXiv:gr-qc/9812064].

[6] J. Bjoraker and Y. Hosotani, Phys. Rev. D **62** (2000) 043513 [arXiv:hep-th/0002098].

[7] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231 [Int. J. Theor. Phys. **38** (1999) 1113] [arXiv:hep-th/9711200];
S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428** (1998) 105 [arXiv:hep-th/9802109];
E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253 [arXiv:hep-th/9802150].

[8] I. R. Klebanov and E. Witten, Nucl. Phys. B **556** (1999) 89 [arXiv:hep-th/9905104].

[9] S. S. Gubser, Phys. Rev. Lett. **101** (2008) 191601 [arXiv:0803.3483 [hep-th]].

[10] S. S. Gubser and S. S. Pufu, JHEP **0811** (2008) 033 [arXiv:0805.2960 [hep-th]].

[11] M. M. Roberts and S. A. Hartnoll, JHEP **0808** (2008) 035 [arXiv:0805.3898 [hep-th]].

[14] Note also that, for these parameters there exists only one Abelian configuration (this is a property of all symmetry breaking solutions we have found so far). However, as discussed in [24], the general picture is much more complicated, with the possible existence of several branches of Abelian configurations. Moreover, similar to the case of black holes with a flat horizon, thermodynamically favoured non Abelian solutions seem to exist only for a limited region of the parameter space.

[15] Due to the existence of electric-magnetic duality in $d = 4$ Einstein-Maxwell theory, one can consider electrically charged RNAdS solutions as well.

[12] S. S. Gubser and F. D. Rocha, "The gravity dual to a quantum critical point with spontaneous symmetry breaking," arXiv:0807.1737 [hep-th];
S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, Phys. Rev. Lett. **101** (2008) 031601 [arXiv:0803.3295 [hep-th]];
S. S. Gubser, "Breaking an Abelian gauge symmetry near a black hole horizon," arXiv:0801.2977 [hep-th];
T. Albash and C. V. Johnson, JHEP **0809** (2008) 121 [arXiv:0804.3466 [hep-th]];
W. Y. Wen, "Inhomogeneous magnetic field in AdS/CFT superconductor," arXiv:0805.1550 [hep-th];
E. Nakano and W. Y. Wen, Phys. Rev. D **78** (2008) 046004 [arXiv:0804.3180 [hep-th]].

[13] N. Okuyama and K. i. Maeda, Phys. Rev. D **67** (2003) 104012 [arXiv:gr-qc/0212022].

[14] M. S. Volkov, Phys. Lett. B **524** (2002) 369 [arXiv:hep-th/0103038].

[15] Y. Brihaye, B. Hartmann and E. Radu, Phys. Rev. D **72** (2005) 104008 [arXiv:hep-th/0508028].

[16] E. Radu and D. H. Tchrakian, Phys. Rev. D **73** (2006) 024006 [arXiv:gr-qc/0508033].

[17] Y. Brihaye, A. Chakrabarti and D. H. Tchrakian, Class. Quant. Grav. **20** (2003) 2765 [arXiv:hep-th/0202141];
Y. Brihaye, A. Chakrabarti, B. Hartmann and D. H. Tchrakian, Phys. Lett. B **561** (2003) 161 [arXiv:hep-th/0212288].

[18] Y. Brihaye, E. Radu and D. H. Tchrakian, Phys. Rev. D **75** (2007) 024022 [arXiv:gr-qc/0610087].

[19] E. Radu, D. H. Tchrakian and Y. Yang, Phys. Rev. D **77** (2008) 044017 [arXiv:0707.1270 [hep-th]].

[20] E. Radu, Phys. Rev. D **67** (2003) 084030 [arXiv:hep-th/0211120];
J. J. Van der Bij and E. Radu, Phys. Lett. B **536** (2002) 107 [arXiv:gr-qc/0107065].

[21] C. N. Pope, Class. Quant. Grav. **2** (1985) L77.

[22] M. Cvetic, H. Lu and C. N. Pope, Nucl. Phys. B **574** (2000) 761 [arXiv:hep-th/9910252].

[23] M. S. Volkov and D. V. Gal'tsov, Phys. Rept. **319** (1999) 1 [arXiv:hep-th/9810070].

[24] A. Chamblin, R. Emparan, C. V. Johnson and R. C. Myers, Phys. Rev. D **60** (1999) 104026 [arXiv:hep-th/9904197];
A. Chamblin, R. Emparan, C. V. Johnson and R. C. Myers, Phys. Rev. D **60** (1999) 064018 [arXiv:hep-th/9902170].

[25] P. Bizon, O.T. Popp, Class. Quant. Grav. **9** (1992) 193;
A.A. Ershov, D.V. Galtsov, Phys. Lett. A **150** (1990) 159;
D. V. Galtsov and A. A. Ershov, Phys. Lett. A **138** (1989) 160.