How Risk Averse are Fund Managers?
Evidence from Irish Mutual Funds

Abstract

This paper investigates the degree of risk aversion exhibited by Irish fund managers. Assuming a mean-variance optimising manager, we employ the dynamic conditional correlation specification (Engle, 2002) of the multivariate GARCH model to estimate the coefficient of relative risk aversion. We find that fund managers whose remit is to "aggressively" manage their portfolios have coefficients lying between 1.69 and 2.42, while the risk aversion parameter of "balanced" managed funds range from 3.24 to 3.69. Finally we discuss the implications of these numbers on the likelihood of these managers partaking in risky investments.

Keywords: Risk aversion; Fund managers; Dynamic conditional correlations.
JEL Classification: G11, G15, C32, G20.

I. Introduction

Risk aversion is a central tenet in financial economics. However, the debate as to the magnitude of the coefficient of relative risk aversion (CRRA) is one that has long been to the forefront of the field and the economics of uncertainty in general. In simulating many of the popular models in finance, the coefficient of risk aversion is a free parameter that requires calibration. In their famed paper on the 'equity premium puzzle', Mehra and Prescott (1985) argue that values greater than 10 are implausibly large. Both Mankiw and Zeldes (1991) and Lucas (1994) state that even 10 is an extreme case, with Lucas arguing that any 'solution' to the equity premium puzzle that relies on a CRRA greater than 2.5 is unlikely to be broadly accepted.

Since the early 1970s, research on the CRRA has spawned a voluminous literature. In his seminal work, Arrow (1971) argued that due to the bounding conditions of the utility function, the coefficient should be close to unity. Ever since, there have been numerous studies, spanning different fields of economics providing estimates of this parameter and values needed to match the data in simulated models.

Friend and Blume (1975) use information on asset holdings, income and other demographics for a large cross-section of households and conclude that the CRRA is greater than unity and "is more likely to be in excess of two". Generally, estimates from finance applications tend to be large. An exception is Hansen and Singleton (1982) who report estimates between 0.35 and 1. However, Mehra and Prescott (1985) require the CRRA to be in excess of 10 (and may be as high as 50) to reconcile the large premium paid by equity with theoretical models. Szpiro (1986) using data from insurance markets finds support for constant relative risk aversion with a coefficient between 1.2 and 1.8. However, Blake (1996) finds estimates vary with wealth level, with the poorest and richest groups exhibiting CRRA of 47.60 and 7.88 respectively. Clare et al. (1998) investigate the appropriateness of the CAPM for the UK market and fail to reject a CRRA of 2, an often-hypothesised value in calibrated models. More recently, Aït-Sahalia and Lo (2000) provide estimates of CRRA using option-pricing models and find estimates ranging from 1 to 60, with a weighted average of 12.7.

In testing the CAPM, Engel and Rodrigues (1989), Giovannini and Jorion (1990) and Thomas and Wickens (1993) generate estimates of the CRRA. However these are generally highly implausible, often negative for a static covariance matrix and not statistically significantly different from zero for time-varying specifications of the conditional covariance matrix.

Our paper sheds new light on the issue by focusing exclusively on estimating the CRRA. We use a simple mean-variance framework and show that by fully covering the range of assets in a typical portfolio and employing time-varying covariance matrices as risk measures, even such a simple model can provide estimates of CRRA that are consistent with theoretical values. Previously, estimation of time-varying covariance matrices for a broad range of assets proved difficult but here we adopt the highly flexible dynamic conditional correlation (DCC) specification of the multivariate GARCH model due to Engle (2002). This allows us to capture changes in the investment opportunity set and assess the reaction of portfolio managers.

Our approach is closest in spirit to Thomas and Wickens (1993), Engle and Rodrigues (1989) and Giovannini and Jorion (1989), but differs in a number of important aspects that are likely to influence the parameter of interest in our analysis. Firstly, our paper is the only one to focus exclusively on estimating the coefficient of risk aversion. The others concentrate on tests of the CAPM with the CRRA being a by-product rather than the focus of the test. Secondly, we use the actual weights employed by portfolio managers as opposed to the CAPM weights. Therefore we are not imposing any restrictions on the portfolio allocations. Given that observed asset weights differ substantially from those implied by the CAPM, our analysis represents actual financial market behaviour and hence should provide a better of estimate of risk aversion amongst fund managers. The CRRA from the other studies indicates the degree of risk aversion required for the CAPM to hold rather than that displayed by market participants. Thirdly, employing the highly flexible DCC version of the multivariate GARCH model allows us to increase the asset coverage in the analysis. Other studies constrain their asset coverage to include only the largest markets. While this is a legitimate approach, the portfolio effects of the smaller and often less correlated markets are inevitably omitted. In our model, the attractiveness of such markets is captured through the (time-varying) covariance terms. The decision of the fund manager as to whether or not to invest in such assets can be quite revealing as to their attitudes to risk.

We focus on two classes of funds; aggressively managed and balanced managed funds. Both undertake significant international diversification and are therefore most consistent with theoretical models. Irish funds are worthy of attention for a number of reasons. Firstly, the domestic equity market is small, accounting for less than 1% of world market capitalisation, making international investment a necessary vehicle for portfolio choice. Secondly, Ireland's tradition and culture mean that agents may be more familiar with foreign markets and less prone to overstating the risk of foreign assets. Assuming that fund managers are mean-variance optimisers, we estimate their implied CRRA. Our results show that aggressively managed funds exhibit lower risk aversion with CRRA estimates ranging from 1.69 to 2.42. Balanced managed funds typically hold more riskless assets and consequently, CRRA estimates vary between 3.21 and 3.78.

The remainder of the paper is structured as follows; section 2 outlines the mean-variance framework on which our estimations are based. Section 3 discusses the econometric model and the data employed. Section 4 presents our results and discusses their implications while Section 5 contains our concluding remarks.

II. Mean-variance framework

We assume that fund managers adopt a simple mean-variance framework[1] (as in Engel and Rodrigues, 1989; Giovannini and Jorion, 1990; Thomas and Wickens, 1993) to allocate funds among various asset classes. This is consistent with myopic investment and a single period model such as the CAPM. Even in a multi-period setting, Shleifer and Vishney (1997) argue that fund managers can be motivated to take a myopic view in their investing strategies if less sophisticated investors use short-term returns to evaluate their performance or competence. Hence we argue that our assumed framework is justified. We have a representative manager who seeks to maximise end-of-period real wealth, given information available at the beginning of the period.

$$MaxU[E_t(W_{t+1}),V_t(W_{t+1})], \quad U_1 > 0, U_2 < 0 \tag{1}$$

where E_t is the conditional expectation of end-of-period wealth, W_{t+1}, and V_t is the conditional variance. We can write

$$E_t(W_{t+1}) = W_t + W_t x_t^{'} E_t r_{t+1} + W_t(1 - x_t^{'} i) r_f \tag{2}$$

and its variance as

$$V_t(W_{t+1}) = W_t^2 x_t^{'} V_t(r_{t+1}) x_t . \tag{3}$$

x_t, r_{t+1} and i are n-vectors of portfolio asset weights, asset returns and ones respectively. The risk free rate is denoted by r_f. $V_t(r_{t+1})$ refers to the conditional variance-covariance matrix of asset returns. The excess return on the portfolio between t and $t+1$ is given by;

$$r_{p,t+1} - r_f = x_t^{'}(r_{t+1} - r_f). \tag{4}$$

Substituting Eqs. (2) and (3) into (1) and maximising with respect to x_t gives the first order conditions:

$$\frac{dU}{dx_t} = U_1 W_t (E_t r_{t+1} - r_f) + U_2 W_t^2 V_t(r_{t+1}) x_t = 0 \tag{5}$$

Defining the coefficient of relative risk aversion, $\rho_t = -2\frac{U_2}{U_1} W_t$, and re-arranging the above expression, we get the following condition;

$$E_t(r_{t+1} - r_f) = \rho_t V_t(r_{t+1}) x_t \tag{6}$$

Assuming that agents are rational, we get the equation that we want to estimate:

$$r_{t+1} - r_f = \rho_t V_t(r_{t+1}) x_t + \varepsilon_{t+1}. \tag{7}$$

This equation gives us a relationship between asset returns, the risk associated with each asset, the correlation structure between each pair of assets, the coefficient of relative risk aversion and the portfolio weight attributed to each asset.

III. Econometric Model and data

The model

A key feature of Eq. (7) is that we require an estimate of the conditional variance of asset returns. There is now ample evidence that this matrix is time varying (Bollerslev et al., 1988; Clare et al., 1998 among others). The development of the family of (G)ARCH models (Engle, 1982; Bollerslev, 1986) has made it possible to allow the covariance matrix to be continuously changing. They also capture other features of asset returns such as thick tails and volatility clustering. As our focus is on portfolio diversification, it's necessary to adopt a multivariate GARCH specification. A well-documented problem of estimating these models lies in the vast number of potential parameters to be estimated simultaneously.[2] A recent advance due to Engle (2002) combines the parsimony of earlier specifications with a model sufficiently flexible to incorporate time-varying conditional correlations. For an *n*-vector of asset returns, the model requires the estimation of *n* variances but it is assumed that the time variation of the covariance elements stems from a common source and can be captured by just two parameters. Thus the *n(n-1)/2* covariance terms can be modelled for the price of two additional parameters. This is the technique adopted here.

We estimate a multivariate GARCH-*in-mean* model. It is specified as follows:

$$\begin{aligned} r_{t+1} &= \rho V_t(r_{t+1})x_t + \varepsilon_{t+1} \\ \varepsilon_{t+1} &\sim N(0, H_{t+1}) \\ H_{t+1} &= D_{t+1}\Gamma_{t+1}D_{t+1}. \end{aligned} \tag{8}$$

D is a diagonal matrix of conditional standard deviations, which is generated by

$$D_t = V'V + A(\varepsilon_{i,t-1}\varepsilon_{i,t-1})A' + B(D_{t-1})B'. \tag{9}$$

Γ is a time-varying correlation matrix with typical element given by

$$\gamma_{ij,t} = \frac{h_{ij,t}}{\sqrt{h_{ii,t}h_{jj,t}}}, \text{ where}$$
$$h_{ij,t} = \bar{\gamma}_{ij}(1-\alpha-\beta) + \alpha(\varepsilon_{i,t-1}\varepsilon_{j,t-1}) + \beta(h_{ij,t-1}). \tag{10}$$

where $\bar{\gamma}_{ij}$ is the unconditional expectation of the correlation between *i* and *j*.

The data

Our goal is to estimate the CRRA from Eq. (8). We use data on asset holdings of two classes of Irish mutual funds; aggressively managed and balanced managed funds. The asset holdings for both funds are monthly averages of all the investment firms operating in this market. Average behaviour is taken to be more indicative of market behaviour. Approximately 20 and 50 funds operate in the aggressively and balanced managed categories respectively[3]. This data is obtained from Moneymate and we also rely on their fund classifications. Moneymate categorise aggressively managed funds as those with a mix of equities, fixed interest, property, cash and a minimum 65% real asset exposure. Balanced managed funds also contain a mix of the above asset types but only require a 40% real asset exposure. All funds are monitored on a monthly basis.

Our sample extends from January 1993 to December 2002. Figures 1 and 2 plot the asset holdings of aggressively and balanced managed funds respectively. As expected, balanced funds have relatively larger holdings in the risk-free asset. Consistent with the phenomenon of "home bias" in portfolio composition, Irish funds disproportionately hold domestic assets. The degree of international diversification is less than suggested by financial theory. However, the allocation to Irish equity has fallen over time, with an offsetting growth in other Euro zone equities.

Asset holdings are not given by individual assets but by geographical breakdown. Therefore we assume that the foreign asset holdings have a beta of unity with respect to their regional index. Returns on these assets are computed using Datastream constructed indices for each region. We work with rates of return in excess of the risk-free rate to prevent volatility in this variable from overstating portfolio risk. The risk free rate is proxied by the 1-month money market rate. Nominal returns are converted to real returns using monthly inflation calculated from the CPI for all items.

IV. Results

Discussion of results

The model outlined above was estimated using the Quasi-maximum likelihood approach of Bollerslev and Wooldridge (1992). Table 1 summarises our results. We begin with an analysis of the aggressively managed funds. Using the asset weights as in Figure 1, our estimate of the CRRA is 1.69. Furthermore, it is quite precisely estimated with a standard error of 0.005. Therefore managers of aggressively managed funds exhibit a degree of risk aversion that is consistent with theoretical models. A similar analysis for the balanced managed funds shows these managers are more risk averse. However, the estimate of 3.21, is still at the lower end of theoretically acceptable parameters. Table 2 reports estimates of the coefficients in the second-order moments. All are statistically significant at conventional levels.

However, the reported asset holdings omit a section of the investment opportunity set. In particular, the emerging markets of Latin America do not feature in the geographical breakdown. We re-estimate Eq. (7) including an index of emerging markets with a zero weighting. As expected, we find that the CRRA is higher for both categories of fund. In the case of aggressively managed funds the estimate grows to 2.42, while for balanced managed funds, it increases to 3.79.

Aggressively managed funds, which undertake more international diversification and hence would appear to be most consistent with theoretical models, have coefficients between 1.69 and 2.42. These estimates are in the range suggested by Mehra and Prescott (1985) and also within the more restricted range of Mankiw and Zeldes (1991) and Lucas (1994). The estimated coefficients for the balanced managed funds are higher and outside of the latter range but are still statistically significantly less than 4.

Implications of our results

We begin by analysing the implications for the utility specification. It is common in finance applications to adopt a power utility function as it displays many properties that are consistent with investment behaviour[4]. All of our estimated CRRAs are positive and statistically different from zero and are thus consistent with strictly concave, upward sloping utility functions. This function also nests another of the great workhorses of finance theory, the log utility function. Log utility requires that the CRRA equals one but this hypothesis is rejected in all cases. Hence we find no support for the adoption of log utility.

Next, we examine the implications of our results for portfolio selection. In particular, we focus on the willingness of fund managers to undertake risky investments. We calculate the required probability of winning an actuarially fair gamble to induce a risk-averse individual to participate in a lottery. If an agent is risk neutral, this probability will simply be 0.5, but a risk averse agent will require a premium. The required probability premium is represented by the second term on the right hand side of Eq (11): [5]

$$\pi(W,\Theta) = \frac{1}{2} + \frac{1}{4}\Theta\rho \,. \tag{11}$$

π is the probability of winning the gamble and Θ is the proportion of wealth at risk. The other variables are defined as before. It is clear that with odds of 0.5, a risk-neutral agent (ρ=0) will participate in the lottery. However, for positive values of the CRRA, the probability must be greater than one half to induce the agent to gamble. Focussing on aggressively managed funds, we find that for the lower estimate of 1.69 managers would be willing to gamble any proportion of the portfolio provided the odds of winning are sufficiently stacked in their favour. This is presented in Figure 3. We can see that to induce a fund manager to gamble 50% of their portfolio value, the odds of winning would have to be 0.712, while odds of over 0.92 are required before the manager would gamble the entire portfolio. Figure 4 conveys a similar story for balanced managed funds. With a CRRA of 3.21, the fund manager would require a 90% probability of winning before gambling half of the fund. Complete certainty is required to induce the manager to gamble 60% of the fund.

When the emerging market index is included, the CRRA of the aggressively managed fund implies that there is now a maximum proportion of the fund that a manager is willing to gamble. Without absolute certainty of winning (π=1), the manager will never gamble amounts in excess of 80% of the fund value. In contrast to the previous case, the agent requires 80% chance of winning before gambling 50% of the portfolio. With a CRRA of 3.78 for the balanced managed fund, a 97.3% probability of winning is required before the manager would gamble half of the fund, while only complete certainty would induce the manager to gamble 52% of the fund. Figures 5 and 6 present these scenarios.

V. Conclusions

We focus exclusively on the estimation of the CRRA. Even using a simple mean-variance framework, we obtain estimates of the CRRA that are consistent with theoretically acceptable values. The innovations in our approach are that we cover the entire range of assets in a typical portfolio; secondly we use actual portfolio holdings as opposed to those implied by the CAPM; and thirdly we capture the continuously changing nature of financial markets through the DCC multivariate GARCH model (Engle, 2002). This technique allows us to model the time-varying conditional covariance matrix required by our framework.

We use data on two categories of funds; aggressively and balanced managed. Aggressively managed funds are more internationally diversified and hence have lower levels of risk aversion. The CRRA exhibited by these fund managers lies between 1.69 and 2.42. Compared to previous studies in the finance literature, our estimates are small and nearer to the magnitudes suggested by theory and often used in model calibration. However, we can reject the hypothesis that CRRA is 1 and hence find no evidence to support the use of log utility. For the balanced managed funds, CRRA is in the range 3.21 – 3.78. These are still relatively low and lie close to the generally accepted range of values.

We investigate the implications of our estimates for the behaviour of a representative fund manager. We compute the probability of success required by such an agent to participate in an actuarially fair gamble. In many cases, complete certainty is required to induce managers into large bets on the value of their funds.

Endnotes

[1] This framework is compatible with any utility function as long as returns are multivariate normally distributed.

[2] Bollerslev et al. (1994) provide an excellent review of this topic along with a number of parsimonious parameterisations used in the literature.

[3] Further details on the funds are available from the author.

[4] For example, Cass and Stiglitz (1970) show that fund managers offering an identical portfolio to clients with different initial wealth is only consistent with utility functions that exhibit constant relative (or absolute) risk aversion.

[5] For a full derivation of this equation, see Danthine and Donaldson (2002), pp 44-46.

References

Aït-Sahalia, Y. and Lo, A. (2000) Nonparametric risk management and implied risk aversion, *Journal of Econometrics*, 94, 9-51.

Arrow, K. (1971) *Essays in the Theory of Risk Bearing*, Markham, New York.

Blake, D. (1996) Efficiency, risk aversion and portfolio insurance: an analysis of financial asset portfolios held by investors in the United Kingdom, *Economic Journal*, 106, 1175-92.

Bollerslev, T., Engle, R.F. and Nelson, D.B. (1994) ARCH Models, in Handbook of Econometrics (Eds) Engle, R.F. and McFadden, D.L., North Holland Press, Amsterdam, pp. 2959-3038.

Bollerslev, T., Engle, R.F. and Wooldridge, J.M. (1988) A capital asset pricing model with time varying covariances, *Journal of Political Economy*, 96, 116-30.

Bollerslev, T. and Wooldridge, J.M. (1992) Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances, *Econometric Reviews*, 11, 143-72.

Cass, D. and Stiglitz, J.E. (1970) The structure of investor preference and asset returns and separability in portfolio allocation: a contribution to the pure theory of mutual funds, *Journal of Economic Theory*, 2, 122-60.

Clare, A.D., O'Brien, R., Thomas, S.H. and Wickens, M.R. (1998) Macroeconomic shocks and the domestic CAPM: evidence from the UK stockmarket, *International Journal of Finance and Economics*, 3, 111-26.

Danthine, J.P. and Donaldson, J.B. (2002) *Intermediate Financial Theory*, Prentice Hall, New Jersey.

Engel, C. and Rodrigues, A.P. (1989) Tests of international CAPM with time-varying covariances, *Journal of Applied Econometrics*, 4, 119-38.

Engle, R. (2002) Dynamic conditional correlation – a simple class of multivariate GARCH models, *Journal of Business and Economic Statistics*, 20, 339-50.

Friend, I. and Blume, M.E. (1975) The demand for risky assets, *American Economic Review*, 65, 900-22.

Giovannini, A. and Jorion, P. (1989) The time variation of risk and return in the foreign exchange and stock markets, *Journal of Finance*, 44, 307-25.

Hansen, L.P. and Singleton, K.J. (1982) Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica*, 50, 1269-86.

Lucas, D. (1994) Asset pricing with undiversifiable risk and short sales constraints: deepening the equity premium puzzle, *Journal of Monetary Economics*, 34, 325-42.

Mankiw, N.G. and Zeldes, S.P. (1991) The consumption of stockholders and non-stockholders, *Journal of Financial Economics*, 29, 97-112.

Mehra, R. and Prescott, E. (1985) The equity premium: a puzzle, *Journal of Monetary Economics*, 15, 145-61.

Shleifer, A. and Vishney, R. (1997) The Limits of Arbitrage, *Journal of Finance*, 52(1), 35-55.

Szpiro, G.S. (1986) Measuring risk aversion: an alternative approach, *Review of Economics and Statistics*, 68, 156-59.

Thomas, S. and Wickens, M.R. (1993) An international CAPM for bonds and equities, *Journal of International Money and Finance*, 12, 390-412.

Table 1. Summary of results

	Estimated CRRA
Aggressively Managed Funds	1.69 (0.00)
Aggressively Managed (inc. Emerging markets)	2.42 (0.00)
Balanced Managed Funds	3.21 (0.00)
Balanced Managed (inc. Emerging markets)	3.78 (0.00)

*Numbers in parentheses are p-values.

Table 2: Estimated parameters of the time-varying covariance matrix.

	Aggressively	**Aggressively***	**Balanced**	**Balanced***
V11	0.003 (0.00)	0.002 (0.00)	0.002 (0.00)	0.003 (0.00)
V22	0.002 (0.00)	0.002 (0.00)	0.002 (0.00)	0.002 (0.00)
V33	0.003 (0.00)	0.003 (0.00)	0.003 (0.00)	0.003 (0.00)
V44	0.001 (0.00)	0.001 (0.00)	0.001 (0.00)	0.001 (0.00)
V55	0.003 (0.00)	0.004 (0.00)	0.005 (0.00)	0.005 (0.00)
V66	0.0005 (0.00)	0.0003 (0.00)	0.0002 (0.00)	0.0003 (0.00)
V77	-	0.003 (0.00)	-	0.003 (0.00)
A11	0.067 (0.00)	-0.069 (0.00)	0.026 (0.00)	-0.049 (0.00)
A22	-0.004 (0.53)	0.009 (0.00)	0.054 (0.00)	0.095 (0.00)
A33	-0.024 (0.00)	-0.023 (0.00)	0.059 (0.00)	0.037 (0.00)
A44	0.376 (0.00)	0.303 (0.00)	0.303 (0.00)	0.258 (0.00)
A55	0.260 (0.00)	0.016 (0.00)	0.128 (0.00)	-0.003 (0.00)
A66	-0.279 (0.00)	0.046 (0.00)	0.080 (0.00)	0.095 (0.00)
A77	-	0.039 (0.00)	-	-0.014 (0.00)
B11	0.079 (0.00)	0.248 (0.00)	-0.039 (0.00)	0.114 (0.00)
B22	0.039 (0.00)	0.091 (0.00)	0.271 (0.00)	0.077 (0.00)
B33	-0.053 (0.00)	0.001 (0.02)	-0.071 (0.00)	-0.015 (0.00)
B44	0.222 (0.00)	0.298 (0.00)	0.087 (0.00)	0.314 (0.00)
B55	0.006 (0.24)	0.007 (0.00)	0.039 (0.00)	-0.060 (0.00)
B66	-0.055 (0.00)	0.017 (0.00)	0.087 (0.00)	0.062 (0.00)
B77	-	0.005 (0.00)	-	-0.010 (0.00)
α	0.022 (0.018)	0.067 (0.00)	0.039 (0.00)	0.071 (0.00)
β	0.064 (0.00)	0.484 (0.00)	0.799 (0.00)	0.528 (0.00)

Numbers in parentheses are p-values. Starred columns refer to portfolios including the emerging market index.

Figure 1: Geographical breakdown of Aggressively Managed Funds

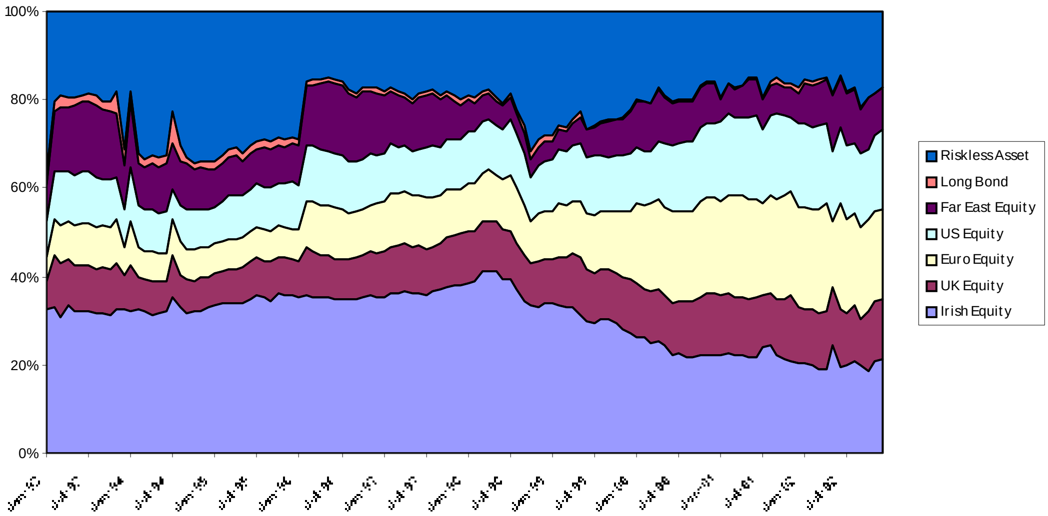

Figure 2: Geographical breakdown of Balanced Managed Funds

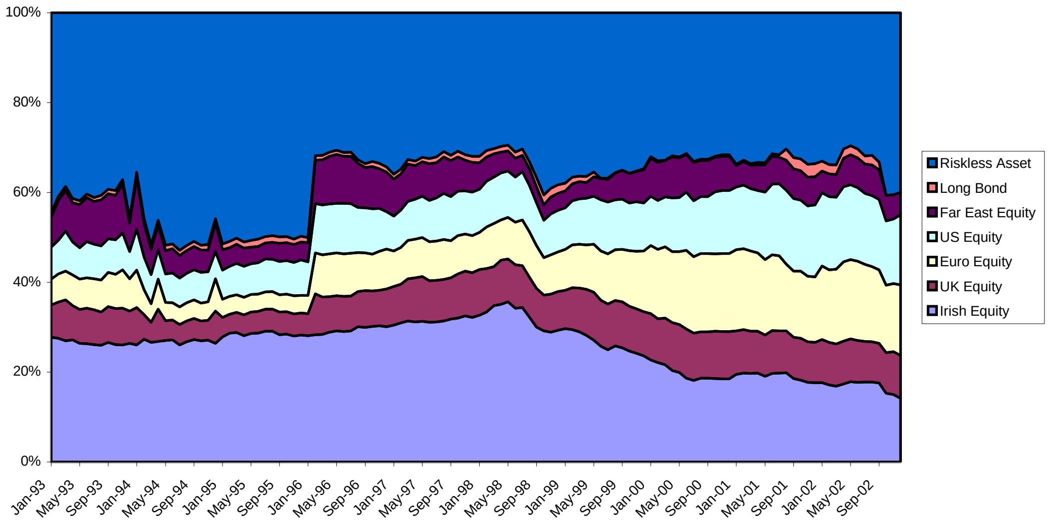

Figure 3: Odds required by Aggressive Funds Manager to participate in actuarially fair gamble

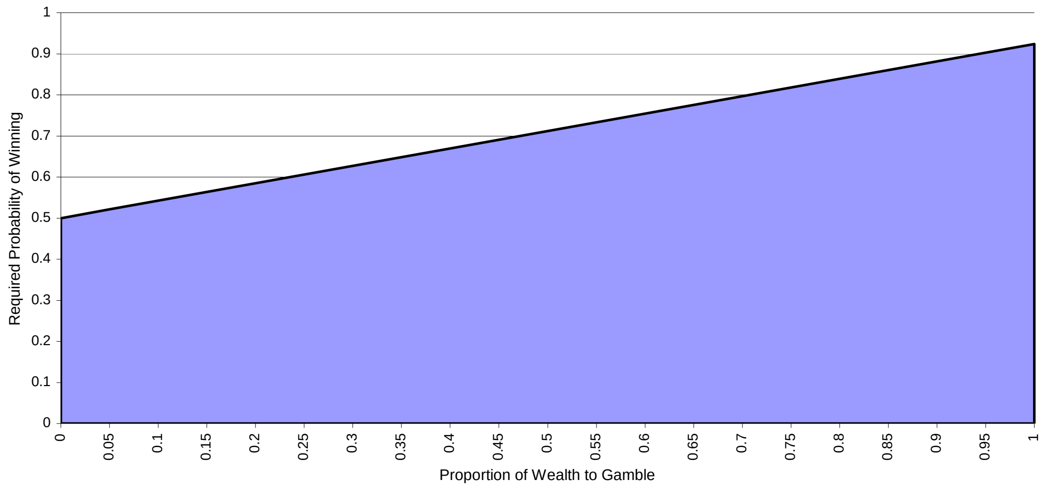

Figure 4: Odds required by Balanced Funds Manager to participate in actuarially fair gamble

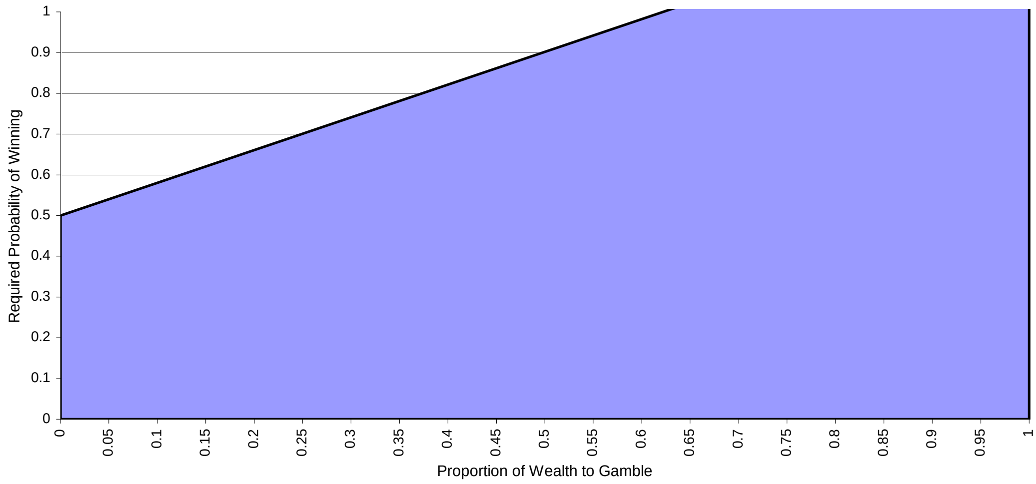

Figure 5: Odds required by Aggressive Funds (Inc. Emerging Markets) Manager to participate in actuarially fair gamble

Figure 6: Odds required by Balanced Funds (Inc. Emerging Markets) Manager to participate in actuarially fair gamble