Dependent Children and Aged Parents:

Funding Education and Social Security in an Aging Economy

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In the last few decades in the United States birth rates have declined and longevity has risen while productivity growth has slowed. Given such changes, the increasing burden of funding programs for the elderly is likely to shift resources away from the young and toward the elderly. This paper uses an overlapping generations framework to examine the effects of tax policies on an aging economy. We find that if the quality of the education system is sufficiently high then raising the education tax rate and subsequently lowering the social security tax rate enhances growth and welfare.

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1. Introduction

The demographic profile of the United States is undergoing profound change. Birth rates are falling and longevity is increasing. As a result the elderly are projected to comprise an increasing proportion of the population. This has led to an increasing focus on issues relevant to the welfare of the elderly, such as Social Security. Meeting the demands of an aging population may, at the same time, reduce resources allocated to the young, such as education expenditures.

In this paper we extend Pecchenino and Utendorf (1999) to examine the interconnections between funding for programs benefiting the elderly, represented by Social Security, and funding for programs benefiting the young, represented by public education. We utilize a Diamond (1965) style overlapping generations model to analyze possible intergenerational linkages. In our model growth, along the transition or balanced growth paths, is endogenously fueled by individuals' investments in both physical capital, to fund their retirements, and human capital, to fund their children's education, and by the government's investment in human capital via public education expenditures. Individuals face uncertainty over their longevity. All old agents receive social security benefits, which are funded in a pay-as-you-go manner. We examine how policies aimed at a specific target group, e.g. the elderly or the young, affect current and future welfare of the economy as a whole.

Our model is similar in construct to Kaganovich and Zilcha (1999), which also examines the effects of the public funding of social security and education on economic growth. They, as we, find that shifting tax revenues from social security benefits to education can be welfare improving. We, however, take the constraints of the social security system (that benefits are determined as a replacement rate on wages, so benefits determine taxes) explicitly into account in our analysis. In addition, we assume that the government, effectively, faces two budget constraints: a social security constraint and an education constraint, rather than the unified constraint with the explicit tradeoff (more for social security implies less for education) assumed by Kaganovich and Zilcha. Further, our model incorporates uncertainty over the length of life as well as population growth, allowing us to analyze the demographic transition, an important issue completely ignored in their analysis.[1]

The connection between social security and education is also central to a model sketched in Mulligan and Sala-i-Martin (1999). In this model social security is an explicit return when old on investments made in the human capital of the young and is a formalization of the observation found in Pogue and Sgontz (1977) and Becker and Murphy (1988) that social security is a dividend paid to the old for investing in the human capital of the current workers when they were young. Our model differs in that we do not make this political linkage since in practice Social Security is a Federal program while education is by and large a local program.

Our model differs from many models of social security (see, for example, Diamond (1977), Imrohoroglu, Imrohoroglu, and Joines (1995) among others) in that these models ignore child welfare when assessing the effects of various social security programs. Other work, while neither ignoring children nor education, makes expenditures on children exogenous, as in Wildasin (1991), or claims that human capital formation is independent of direct expenditures on children so that parent's decisions concerning their children's education are not important to the analysis, as in Sala-i-Martin (1996). In still other work education decisions are central to the analysis, as in Glomm and Ravikumar (1992), while abstracting from any spillovers of these decisions on the utility of the aged. In our model the interactions between social security programs and expenditures on children's education play a central role through the effects of education on individual and aggregate productivity.

We model education as being utility enhancing for parents and productivity enhancing for the individual and society. Given our assumption of representative individuals we exclude the possibility of education as a signal, as in Spence (1973). We instead concentrate on the effects of the quality of education on individual and social welfare where middle-aged individuals value education for their children's sake and these children are rewarded by a higher market return for their labor. That the quality of education a child gets matters, both in terms of the market return to schooling and parental utility, is supported by empirical studies such as Hanushek (1986), Card and Krueger (1990), Ehrenberg et al. (1989), and Altonji and Dunn (1995) and by the casual empiricism that parents often relocate to "good" school districts.

Because education is productivity enhancing, it provides the impetus for growth in our model. This connection between education and growth has its roots in the work of Schultz (1961) and the more recent theoretical work of Lucas (1988), Rebelo (1991) and others who, building on Schultz's insight, suggest that human capital formation as evidenced by educational attainment is an important ingredient in explaining economic growth. This theoretical position is supported empirically by Barro and Lee's (1993) analysis. Hanushek and Kim (1996) extend this analysis, providing evidence that not only does the amount of schooling matter but so does the quality of that schooling. Since education may be growth inducing the failure to adequately provide for and educate children may lead to slower growth or economic decline. Our model allows us to explore this possibility.

Analysis of our model yields the following results. In the steady-state representations, increases in the social security tax rate leads to reductions in physical and human capital accumulation, output and the rate of balanced growth while the opposite may be true for increases in the education tax rate. Increases in life expectancy increase capital accumulation, output and the rate of balanced growth if the positive longevity effect overwhelms the negative tax and bequest effects. These results suggest that there is the potential for Pareto-improving changes in tax policy. Using a balanced-growth version of the model we show that in an aging economy if the efficiency of public expenditures on education is sufficiently high increasing public expenditures on education is both growth and welfare enhancing. Thus, shifting public resources toward the young may benefit all generations.

2. The Model

Preliminaries

The model developed below is an extension of Pecchenino and Utendorf (1999),[2] and is similar to Kaganovich and Zilcha (1999). There is an infinitely-lived economy composed of finitely-lived individuals, firms, and a government. A new generation is born at the beginning of each period and lives for at most three periods divided into youth, middle age, and old age. At each period t, N(t) identical agents of generation t enter workforce. The workforce grows at the rate $[N(t)/N(t-1)]-1=n(t)$.

Consumers

At date *t*, agents in the first period of their lives, the young, do not consume, nor do they produce. They are, however, endowed with one unit of time that they combine inelastically with resources provided by their parents, *e(t)*, and the government, $e^g(t)$, to develop their human capital, $h_{t+1}(t+1)$. Agents in the second period of their lives, the middle-aged, supply their effective labor, the product of their one unit of time and their human capital developed in youth, inelastically to firms, for which they receive wage income, $w(t)h_t(t)$. They divide their after social security tax, $\tau(t)$, and school tax, $\omega(t)$, wage income plus any bequests, *B(t)*, they may receive, tax free, from their parents, between funding their children's human capital development, *e(t)*, their current consumption, $c_t(t)$, and saving, *s(t)*, for consumption when old, $c_t(t+1)$. Agents in the final period of their lives, the old, supply their savings, *s(t-1)*, inelastically to firms and consume their social security benefits, *T(t)*, and the return to their savings, $(1+\rho(t))s(t-1)$. With probability *p(t-1)* an agent who worked during period *t-1* will live throughout old age, and with probability *(1-p(t-1))* the agent will die at the onset of old age. If an agent dies at the onset of old age, his saving is bequeathed to the members of generation *t*, $B(t)=\{[1+\rho(t)]/[1+n(t)]\}s(t-1)$.

Let the preferences of a representative middle-aged worker at time *t* be represented by

$$U_t=\ln c_t(t)+p(t)\ln c_t(t+1)+\delta[1+n(t+1)]\ln h_{t+1}(t+1) \tag{1}$$

Parents get utility from consumption and from educating their children, the value of this education is summarized by the child's human capital. This utility is derived from an altruistic link between parent and child, that is their "love of" or "duty to" their children rather than any personal return they may reap from their investment or any other strategic motive (see Cremer, et al., 1992). This *inter vivos* bequest motive encompasses the lifetime bequest motive.[3] Since agents do not know when they will die, additional unintentional bequests may be forthcoming.[4,5]

Parental and government human capital investments are both are essential for human capital formation. If a parent invests *e(t)* in his child, and the government invests $e^g(t)$, then the child's human capital will be

$$h_{t+1}(t+1) = e_t(t)^{\theta_1(t)}\, e^g(t)^{\theta_2(t)} \tag{2}$$

where the parameters $\theta_1(t)$ and $\theta_2(t)$ measure the elasticity of parental and government expenditures on human capital, respectively. This method of modeling educational attainment follows Hanushek's (1992) achievement function. Family and school expenditures, *e(t)* and $e^g(t)$, respectively, as well as the efficiency of those expenditures, $\theta_1(t)$ and $\theta_2(t)$, which determine the quality of the component of education received from one's parent and the government, respectively, matter for human capital development. The utility a parent receives from his dependent children's human capital is $\delta[1+n(t+1)]\ln h_{t+1}(t+1)$, where δ is the discount factor.

The representative agent takes as given his human capital, wages, return on saving, the social security and education tax rates, social security benefits, bequests, and government expenditures on education. The agent then chooses saving and education expenditures to maximize lifetime utility as given by equation (1) subject to (2) and the following budget constraints

$$c_t(t) = w(t)h_t(t)[1-\tau(t)-\omega(t)] - s(t) - [1+n(t+1)]e(t) + [1-p(t-1)]B(t) \tag{3}$$

$$c_t(t+1) = [1+\rho(t+1)]s(t) + T(t+1) \tag{4}$$

where constraint (3) encompasses the assumption that bequests are allocated equally across all members of a generation so that the bequest dependent wealth distribution is uniform, as in Hubbard and Judd (1987). This assumption allows us to conduct a representative agent analysis, and restricts uncertainty to the timing of death alone.

The first-order conditions for this maximization problem, with respect to $s(t)$ and $e(t)$, respectively, are

$$-\frac{1}{c_t(t)}+\frac{p(t)[1+\rho(t+1)]}{c_t(t+1)}=0 \tag{5}$$

$$-\frac{1}{c_t(t)}+\frac{\delta\theta_1}{e(t)}=0\,. \tag{6}$$

Utility is maximized by equating the marginal rate of substitution between consumption and saving with the marginal rate of substitution between consumption and parental expenditures on their children's human capital development.

Firms

The firms are perfectly competitive profit maximizers that produce output using the production function $Y(t) = A(t)K(t)^{\alpha}H(t)^{1-\alpha}$, $\alpha \in (0, 1)$. $K(t)$ is the capital stock at t, which depreciates fully in the production process. $H(t)$ is the effective labor input at t, $H(t) = N(t)h_t(t)$, where $N(t)$ is labor hours. $A(t) > 0$ is a productivity scalar. The production function can be written in intensive form

$$y(t)=A(t)h_t(t)^{1-\alpha}k(t)^{\alpha} \tag{7}$$

where $y(t)$ is output per worker and $k(t)$ is the capital labor ratio.

Since firms are competitive they take the wage, $w(t)$, and rental rate, $R(t)$, as given. The firms then hire labor and capital up to the point where their marginal products equal their factor prices.

$$(1-\alpha)A(t)h_t(t)^{-\alpha}k(t)^{\alpha}=w(t) \tag{8}$$

$$\alpha A(t)\, h_t(t)^{1-\alpha}\, k(t)^{\alpha-1} = R(t) \tag{9}$$

The Government

The government administers the social security program and funds education. It levies proportional income taxes, $\tau(t)$ and $\omega(t)$, on the middle-aged to finance social and education expenditures, respectively. Social security benefits are specified as a replacement rate on current workers' wages. So,

$$T(t) = \xi(t-1)w(t)h_t(t)$$

where $T(t)$ are the transfers to the old at date t and $\xi(t-1)$ is the replacement rate on wages for agents in middle age at $t-1$. There is no debt in the model so expenditures on social security and education must be fully funded through tax receipts at each period t. Since total social security benefits to the old must be balanced by social security tax revenues

$$\frac{p(t-1)}{1+n(t)}T(t) = \frac{p(t-1)\xi(t-1)}{1+n(t)}w(t)h_t(t) = \tau(t)w(t)h_t(t)\,. \tag{10}$$

Solving equation (10) for $\tau(t)$ yields

$$\tau(t) = \frac{p(t-1)\xi(t-1)}{1+n(t)}\,. \tag{11}$$

That is, the government sets $\xi(t-1)$ to achieve the desired level of social security benefits, $T(t)$, for the old at t and the social security tax rate endogenously adjusts. Since social security benefits "replace" not one's own wages but one's children's wages, the old benefit from the human capital investments they made in their children during their working lives. Total spending on education must equal total education tax revenues

$$e^g(t) = \frac{\omega(t)}{1+n(t+1)}w(t)h_t(t)\,. \tag{12}$$

The Goods Market

The goods market clears when the demand for goods for either consumption or savings equals the supply of goods.

$$c_t(t)+\frac{p(t-1)}{1+n(t)}c_{t-1}(t)+s(t)+[1+n(t)]e(t)+[1+n(t)]e^g(t)=w(t)h_t(t)+R(t)k(t) \tag{13}$$

Substituting equations (3), (4), (8), (9), (11) and (12) into (13), and making use of the fact that by arbitrage the return on capital must equal the return on saving

$$R(t)=1+\rho(t) \tag{14}$$

yields

$$s(t\text{-}1)=[1+n(t)]k(t). \tag{15}$$

Savings at time *t-1* determine the capital stock at time *t*.

3. Equilibrium

Definition: A competitive equilibrium for this economy is a sequence of prices and taxes $\{\mathrm{w(t)}, \rho(\mathrm{t}), \mathrm{R(t)}, \tau(\mathrm{t}), \omega(\mathrm{t})\}_{\mathrm{t}=0}^{\infty}$, a sequence of allocations $\{c_t(t), c_t(t+1)\}_{t=0}^{\infty}$ and a sequence of human and physical capital stocks, $\{h_t(t), k(t)\}_{t=0}^{\infty}$, $k(0)$, $h_0(0)>0$ given, such that given population growth, agents and the government's expectations over longevity, and given these prices, allocations, and capital stocks, agents' utility is maximized, firms' profits are maximized, the government budget constraints are satisfied, and markets clear.

Substituting equations (2)-(4), (8), (9), (11), (12), (14) and (15) into the first order conditions given by equations (5) and (6) results in the following set of difference equations in *k(t+1), e(t)* and predetermined variables.

$$\frac{p(t)}{\left(1+n(t+1)+\frac{(1-\alpha)\xi(t)}{\alpha}\right)k(t+1)}-$$

$$\frac{1}{A(t)\left[\left(1-\frac{p(t-1)\xi(t-1)}{1+n(t)}-\omega(t)\right)(1-\alpha)+[1-p(t-1)]\alpha\right]e(t-1)^{\theta_1(t-1)(1-\alpha)}e^{g}(t-1)^{\theta_2(t-1)(1-\alpha)}k(t)^{\alpha}-[1+n(t)][e(t)+k(t+1)]}=0 \quad (16)$$

and

$$\frac{\delta\theta_1}{e(t)}-$$

$$\frac{1}{A(t)\left[\left(1-\frac{p(t-1)\xi(t-1)}{1+n(t)}-\omega(t)\right)(1-\alpha)+[1-p(t-1)]\alpha\right]e(t-1)^{\theta_1(t-1)(1-\alpha)}e^{g}(t-1)^{\theta_2(t-1)(1-\alpha)}k(t)^{\alpha}-[1+n(t)][e(t)-k(t+1)]}=0 \quad (17)$$

The equilibrium is fully characterized by these difference equations.

Steady-State and Balanced-Growth Equilibria

A steady-state equilibrium exists if the production function exhibits diminishing returns to all factors of production: $\theta_1+\theta_2<1$. A balanced-growth equilibrium exists if the production function exhibits constant returns to all factors of production: $\theta_1+\theta_2=1$. Assume that all time dependent parameters are time invariant, so $x(t)=x$ for all parameters x. Steady-state and balanced-growth solutions as well as the proofs of the following comparative static and dynamic results are found in the appendix.[6]

<u>Proposition 1</u>: *Economies with higher social security replacement rates, ξ, have lower (higher) steady state physical and human capital stocks and output, or rates of balanced growth if*

$$\frac{\theta_1(1-\alpha)}{1+n+\left(\frac{1-\alpha}{\alpha}\right)\xi}<(\theta_1+\theta_2\alpha)\left[\frac{1+\delta\theta_1(1+n)}{\left[1+n+\left(\frac{1-\alpha}{\alpha}\right)\xi\right][1+\delta\theta_1(1+n)]+p(1+n)}+\frac{\frac{\alpha p}{(1+n)}}{\left(1-\frac{p\xi}{1+n}-\omega\right)(1-\alpha)+(1+p)\alpha}\right],$$

$$\left(\frac{\theta_1(1-\alpha)}{1+n+\left(\frac{1-\alpha}{\alpha}\right)\xi} > (1-\theta_2(1-\alpha)) \left[\frac{1+\delta\theta_1(1+n)}{\left[1+n+\left(\frac{1-\alpha}{\alpha}\right)\xi\right]\left[1+\delta\theta_1(1+n)\right]+p(1+n)} + \frac{\frac{\alpha p}{(1+n)}}{\left(1-\frac{p\xi}{1+n}-\omega\right)(1-\alpha)+(1+p)\alpha} \right] \right).$$

The increase in the social security replacement rate, ξ, raises the social security tax rate thereby reducing the after-tax income of the middle-aged while increasing the social security benefits received by the elderly. Both effects lead to a reduction in saving in terms of human capital and physical capital (the right hand side of the inequality). The negative income effects are offset by a positive substitution effect on parental expenditures on education since, while they are proportional to saving, the factor of proportionality is increasing in the social security replacement rate (the left hand side of the inequality). If the income effects dominate the substitution effect, the first inequality in the proposition is satisfied and the negative effects of higher social security replacement rates on both human and physical capital reduce steady-state output or the rate of balanced growth. If the substitution effect is strong enough, then higher social security replacement rates can lead to increases in both human and physical capital, thus steady state output or the rate of balanced growth.

The inequalities in Proposition 1 can be solved to yield critical values of ξ in terms of the parameters of the model. This involves solving a quadratic equation. The restrictions implied do not lend themselves to intuitive interpretations. However, searching the parameter space we find, for the baseline parameter values set out in Section 4, below, and letting $\alpha \leq .01$ then for $\xi \leq .04$ or > 3.5, that is outside the feasible parameter space, increasing ξ leads to increases in both human and physical capital, thus steady state output or the rate of balanced growth. Whether such parameter values are likely to be encountered in practice is an empirical question.

<u>Proposition 2</u>: *Economies with higher tax rates to finance education, ω, have higher (lower) steady-state*

human and physical capital accumulation and output, or rates of balanced growth if

$$\theta_2\left[\left(1-\frac{p\xi}{1+n}\right)(1-\alpha)+(1-p)\alpha\right]=\hat{\omega}>\omega,\quad \left(\frac{\theta_2}{\theta_1+\theta_2}\left[\left(1-\frac{p\xi}{1+n}\right)(1-\alpha)+(1-p)\alpha\right]=\tilde{\omega}<\omega\right).$$

The education tax, ω, represents the marginal cost of public education while the marginal benefit to the taxpayer is the marginal increase in income in middle-age, $[1-p\xi/(1+n)](1-\alpha)+(1-p)\alpha$, discounted by the marginal efficiency of the government's educational input, θ_2. If the discounted marginal benefit exceeds the marginal cost, agents receive a positive steady-state income effect from an increase in the education tax rate, leading to increases in saving and investment in one's children's human capital. However, if the marginal cost exceeds the discounted marginal benefit (where the discount factor is now the marginal efficiency of the government's educational input relative to the sum of the marginal efficiency of government and parental educational inputs), that is $\omega>\tilde{\omega}$, both saving and human capital investment fall, and with them output or the rate of balanced growth. This suggests that simply throwing money at education may indeed have detrimental effects. It also indicates, as Hanushek and Kim (1996) suggest, the economic benefits from education are higher the higher is the quality of the education, here measured by θ_2.

<u>Proposition 3</u>: *Economies with higher longevity, p, have higher (lower) physical and human capital stocks and output or rates of balanced growth if the longevity effect dominates (is dominated by) the bequest and social security tax effects:*

$$\frac{\alpha}{p}>[1-\theta_2(1-\alpha)]\left[\frac{1+n}{\left[1+n+\left(\frac{1-\alpha}{\alpha}\right)\xi\right][1+\delta\theta_1(1+n)]+p(1+n)}+\frac{\frac{\xi(1-\alpha)}{(1+n)}+\alpha}{\left(1-\frac{p\xi}{1+n}-\omega\right)(1-\alpha)+(1-p)\alpha}\right]$$

$$\left(\frac{\alpha}{p}<[\theta_1+\theta_2\alpha]\left[\frac{1+n}{\left[1+n+\left(\frac{1-\alpha}{\alpha}\right)\xi\right][1+\delta\theta_1(1+n)]+p(1+n)}+\frac{\frac{\xi(1-\alpha)}{(1+n)}+\alpha}{\left(1-\frac{p\xi}{1+n}-\omega\right)(1-\alpha)+(1-p)\alpha}\right]\right).$$

Increasing the expected life span has both negative and positive effects on saving. The second bracketed right hand side term of the inequalities is the sum of the bequest and tax effects. When agents live longer, then, all else equal (including the age of retirement), they consume a higher proportion of their savings and leave less to their children. This implies a negative bequest effect, which reduces expected income for middle-aged agents, leading to reductions in saving. Since social security taxes will increase as the population ages to keep the replacement rate constant, the middle-aged's income falls compounding the negative bequest effect with a negative tax effect. But, a longevity effect, the left hand side of the inequality, leads to increased saving since the probability of surviving until retirement is now higher.

For parents investing in their children, the bequest and tax effects reduce their incomes, and the higher expected lifetime reduces the factor of proportionality linking saving in terms of physical capital with saving in terms of human capital. Thus, parental education expenditures fall (the first bracketed term on the right hand side of the inequalities). Consequently, as the age distribution becomes more skewed to the right, the conflict between providing for one's children and for one's own retirement intensifies. Clearly, the resultant reduction in parental education spending (as a proportion of saving in terms of physical capital) does not signify a reduction in parents' concern for their children, but rather a rational response to the exigencies of a longer life. Further, whenever the longevity effect is dominant since saving rises, parent's education expenditures may rise, as will total expenditures on education. Thus human capital accumulation will rise. The positive effects of a longer life on both human and physical capital increase steady-state output or the rate of balanced growth.

Critical values of p can be found by solving a quadratic equation in p and other parameter values. Since interpretable conditions cannot be found, we search the parameter space for the critical values of p. For example, for the baseline parameter values set out in Section 4, below, for all parameters other than p, if p>.6 then an increase in p reduces human and physical capital accumulation and steady-state output or the rate of balanced growth. Here the effect of a longer life is inadequate to overcome negative tax, bequest, and education effects.

4. Demographics and Social Welfare

Propositions 1 and 2 together suggest that in this economy intergenerational transfers from the middle-aged to the old can reduce steady-state capital stocks, while intergenerational transfers from the middle-aged to the young can increase steady-state capital stocks. Thus, if under the current tax code the steady-state physical and human capital stocks are too low relative to the social optimum, then Pareto improving tax changes may be possible. These results may be confounded, however, by population aging. In this section we examine the implications of tax policy for an aging economy along a balanced growth path.

There are two ways the population can age in this model. Either p, the probability of living into old age, can increase, or n, the working-age population growth rate, can decrease. The former, all else equal, leads to a permanently higher aged dependency ratio, while the latter leads to a higher aged dependency ratio only in the transition to a new equilibrium working-age population growth rate.

The results of the previous section suggest that in an aging economy growth may be enhanced through changes in tax policy that raise the education tax rate. Whether or not welfare is enhanced by such tax changes depends on the social welfare function. We define a set of social welfare functions[7] such that welfare in period t is

$$W(t) = \ln c_t(t) + \nu \frac{p(t-1)}{1+n(t)} \ln c_{t-1}(t) + \delta(1+n(t+1)) \ln h_{t+1}(t) . \tag{18}$$

If ν>1, then society as a whole puts greater value on the living standards enjoyed by the elderly than on the living standards of the young or the middle-aged. This social valuation could be a result of the voting habits and political activity of the elderly. This notion that the old have greater influence than their population size would suggest is explored by Mulligan and Sala-i-Martin (1999). Or, it could be a reflection of a negative external effect on the welfare of the middle aged and young of low living standards of the elderly. Thus, while the young individually cannot affect this, society as a whole can.

[Insert Table 1]

To examine the effects of an aging economy on growth and welfare, we begin by calibrating the model to match the growth experience of the U.S. economy. The initial values for the parameters in the model are given in Table 1. Each period is a generation, which we set equal to 25 years. The share of physical capital, α, is 0.30. The weight given by parents, in their utility functions, to the human capital development of their children, δ, is 0.98. The education tax rate, ω, is 7.4 percent which is the 1990 ratio of public expenditures on all levels of education to GDP adjusted for labor's share in output (OECD, 1996). The replacement rate, ξ, is 43 percent which is the rate for those retiring in 2000 who earned the average wage while working.[8] The period population growth rate, n, is 50.8 percent – the increase in the number of workers contributing to the Social Security system between 1975 and 2000. The ratio of Social Security beneficiaries to contributors in 2000 is 25.3 percent. Multiplying this number by the gross working age population growth rate, $(1+n)$, gives the value for p.[9] For balanced growth $\theta_1+\theta_2=1$. Thus, choosing θ_2 determines the value for θ_1. We begin by assuming that $\theta_2=0.5$. Finally we assume that ν, the social welfare weight on the utility of the elderly, is 5 since this implies that the pay as you go social security system is welfare improving given the economy's current demographic structure.

[Insert Table 2]

Using these parameter values and setting the growth rate of the economy at 2.5 percent per year (85.4 percent per period), allows us to determine the value of the constant, A, in the production function. We then introduce aging into the population and re-simulate the model, keeping all other variables, including A, at their initial values. Aging implies a decline in $n(t)$ and an increase in $p(t-1)$. Specifically, population growth slows beginning with the cohort entering the workforce in period $j+1$, $n(j+1)<n(j)$. In addition life expectancy rises beginning with generation j, $p(j)>p(j-1)$. Both of these factors result in a reduction in the size of the working age population (generation [$j+1$]) relative to the size of the retired population (generation [j]). Aging continues for three periods, at which point $p(t)$ and $n(t)$ remain constant.

The decline in the population growth rate and rise in longevity correspond to the projections of the Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds (1999), and are shown in Table 2.

[Insert Table 3] [Insert Figure 1]

<u>Result 1</u>: *An aging population raises the growth rate of output per worker and increases welfare in the long-run relative to the baseline. Welfare in the short-run falls relative to the baseline. Furthermore, the share of the economy's resources devoted to social security rises relative to the share devoted to public education.*

Aging increases both physical and human capital, and hence the growth rate of output per worker rises relative to the baseline, as shown in the second column of Table 3. The initial (period *j*) effect of an increase in aging on welfare is negative (relative to the baseline), as shown in Figure 1, primarily as a result of the decrease in the population growth rate *n(j+1)*. Specifically, consumption of the middle-aged rises as the lower population growth rate reduces parental expenditures on children, although expenditures per child rise. The human capital of each child, $h_{j+1}(j)$, rises but the weight given to this generation in the welfare function falls, as *n(j+1)* decreases. The increased consumption of the middle-aged is not large enough to offset this negative effect and welfare in period *t=j* falls relative to the baseline. Over time, however, the increase in the growth rate of output per worker, resulting from increased physical and human capital accumulation per worker, raises consumption for all generations, and hence welfare.

The effects of aging on the share of output per worker devoted to social security and public education are indicated by the following two equations. First, combining equations (7) and (10) and making use of equation (8) gives social security expenditures as a fraction of output per worker

$$\frac{\frac{p(t-1)T(t)}{1+n(t)}}{y(t)} = \frac{p(t-1)}{1+n(t)}\xi(t-1)(1-\alpha)\,. \tag{19}$$

Similarly, combining equations (7) and (12) and making use of equation (8) gives public education expenditures as a percent of output per worker

$$\frac{e^{g}(t)}{y(t)} = \frac{\omega(t)(1-\alpha)}{1+n(t+1)}. \tag{20}$$

[Insert Figure 2]

Both the decrease in n and the increase in p result in a rise in the share of output per worker devoted to social security, holding ξ constant, while the decline in n raises the share of output per worker devoted to public education, holding ω constant.[10] Because the increase in p only affects Social Security expenditures, these grow more rapidly than expenditures on education and hence the relative share of output per worker devoted to social security increases, as shown in the top panel of Figure 2.

We next examined the effects of reversing this shift in resources, that is shifting public expenditures toward education. We consider two policies. The first raises ω from 7.4 percent to 10 percent. The second combines the increase in ω with a reduction in the social security tax rate. The latter is achieved by reducing ξ from 43 percent to 33 percent. We then examine the effects of these changes on growth and welfare relative to the economy absent any policy change.

[Insert Figure 3]

<u>Result 2</u>: *In an aging economy, a permanent rise in the education tax rate from* ω *to* $\overline{\omega}$, *such that* $\overline{\omega} < \hat{\omega}$, *raises the growth rate of output per worker and increases welfare. The share of output per worker devoted to public education rises.*

Increasing ω in period j reduces the income of the middle aged in that period (relative to income absent a change in ω) and hence lowers consumption of this group as well as parental expenditures on children. Period j human capital expenditures rise, however, as the increase in spending on public education more than offsets the decline in parental expenditures. As shown in the top panel of Figure 3, overall welfare rises as the positive effect of the increase in human capital expenditures is greater than negative effect of the decline in consumption of the middle-aged.

Increasing ω also provides a productivity boost to the economy that raises the growth rate of output per worker, as shown in the first column of Table 3. The increase in ω has no effect on the share of output per worker devoted to social security while raising the share devoted to public education, as can been seen from equations (19) and (20). Thus, overall government expenditures as a share of output per worker rise, as shown in the middle panel of Figure 2, as does the relative share of expenditures on public education.

<u>Result 3</u>*: In an aging economy, a permanent rise in the education tax rate in the education tax rate from* ω *to* $\overline{\omega}$, *such that* $\overline{\omega} < \hat{\omega}$, *followed by a permanent reduction in the social security replacement rate raises the growth rate of output per worker and increases welfare relative to no policy change. Public education expenditures rise relative to social security expenditures, but total government expenditures as a share of output per worker falls.*

As noted in Result 2, increasing ω raises welfare in all periods. In contrast, while a decrease in ξ (holding ω fixed) will raise the long run growth rate of the economy and hence welfare, it lowers welfare in the short-run. Decreasing the replacement rate for generation j shifts their expenditures away from current consumption and spending on their children toward saving for retirement. These two factors both lower consumption of the middle-aged and the young, reducing welfare in period j. The subsequent increase in the capital stock in period $j+1$ in conjunction with the reduction in the social security tax rate, boosts consumption of the middle-aged and the young. Consumption of the old, however, falls -- increased saving by generation j is not enough to compensate for the reduction in social security benefits. Because of the high weight attached to the elderly in the social welfare function, the decrease in their consumption predominates and welfare in period $j+1$ falls. By the following period, output per worker rises sufficiently to boost consumption for all three generations and welfare rises from then on. Combining a period j

increase in ω with a decrease in the replacement rate for generation j (the old in period $j+1$) results in an increase in welfare in all periods. While the lower ξ reduces consumption of the middle-aged and their own expenditures on their children, the increase in ω causes a sharp rise in the overall level of human capital expenditures ensuring a rise in welfare in period j. In period $j+1$, consumption of the old falls, but now the increased human capital stock combined with the increase in the physical capital stock, both dampens this decline while boosting the consumption of both the middle-aged and young sufficiently to overcome the negative effect on welfare. Hence welfare in all periods increases.

As equation (19) demonstrates, a decline in ξ lowers social security expenditures relative to output per worker, while as noted in Result 2, the increase in ω raises education expenditures relative to output per worker. The overall share of government expenditures in output per worker falls and the share of education expenditures in the overall expenditures of the government rises relative to the no policy change. This can be seen by comparing the top and bottom panels of Figure 2. In order for this policy to be Pareto improving the increase in ω must precede the decline in the social security tax rate. As a result, the ratio of government expenditures to output per worker rises in period j relative to this ratio in the no policy change scenario, before falling thereafter.

The strength of the altruistic bond between parents and their children and the productivity of government expenditures on education are of utmost importance in determining whether a change in tax policy is welfare improving. If either δ or θ_2 is small then shifting government expenditures toward education is not Pareto improving since welfare in at least one period falls.[11] A low δ implies that parents, and hence society, place little weight on the human capital development of the young. Setting δ=.05 eliminates most of the inter-generational altruism in the model. As shown in the middle panel of Figure 3, when δ=0.05 raising ω alone or in combination with a decrease in ξ lowers welfare initially. As noted in Result 2 increasing ω in period j reduces the consumption of the middle-aged while raising human capital expenditures on the young. If the weight on the latter in the welfare function is low then the negative effect dominates and welfare falls. The effect is worsened by combining a rise in ω with a decrease in ξ since both reduce the consumption of the middle-aged while the latter decreases human capital expenditures on the young. After the initial period, welfare rises as the increase in physical and human capital boosts output per worker (see the third column of Table 3) and hence consumption of all generations.

A low θ_2 makes raising welfare through shifting government expenditures toward education even more problematic since it reduces the efficiency of these expenditures. As shown in the bottom panel of Figure 3, when θ_2=0.1 an increase in ω lowers welfare for all generations. The efficiency of the additional expenditures on education is too low to provide a sufficient productivity boost to offset the drag on output resulting from the tax increase, and hence the growth rate of output per worker falls, as shown in the last column of Table 3. If the increase in ω is combined with a decrease in ξ then the long-run growth rate of output per worker and hence welfare rises. That is, the long-run effects on the economy of increased saving, as a result of the lower ξ, eventually offset the negative effects of the rise in ω. Initially, however, the decline in ξ exacerbates the negative effects on consumption and welfare falls.

5. Conclusion

As a population ages there is increasing pressure to shift resources away from programs directly benefiting the young (education) and toward those directly benefiting the elderly (social security). The results of this paper indicate that such policies may be shortsighted. Because of the productivity enhancing effects of education and the saving reducing effects of social security, growth and welfare can both be increased by increasing the resources dedicated to educating the young.

The key determinant of whether dedicating more resources to the young is growth enhancing is the quality of the education system, measured by the effectiveness of government expenditures on education. When the efficiency of public education expenditures is low the productivity gains from an increase in spending are slight and do not offset the reduction in social security benefits from the expenditure switching policy for the initial generations.

The key determinants of whether dedicating more resources to the young is welfare enhancing in all periods are the quality of the education system and the degree of inter-generational altruism. Because the efficiency of government expenditures on education is linked with the growth rate of the economy it affects consumption and hence welfare. In an economy that places little or no importance on the young, policies that shift public resources toward the young will result in initial declines in welfare as the first effects of such policies are to raise the consumption of the young while reducing that of the middle-aged. Moreover, while in this paper the education tax rate is exogenous, this tax rate is likely linked to the weight that society places on the education of its youth. Thus, in economies with low levels of altruism, aging is more likely to lead to decreases rather than increases in taxes to fund education. Such a policy will raise consumption of the current generation of workers and thus current welfare to the detriment of future generations.

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Table 1 **Baseline Parameter Values** **for the Simulations**		
Description	Parameter	Value
Share of physical capital in output	α	0.30
Weight given to children's human capital in utility	δ	0.98
Education tax rate	ω	0.074
Social security replacement rate	ξ	0.43
Working age population growth rate	n	0.508
Probability of reaching old age	p	0.382
Elasticity of government education expenditures	θ_2	0.5
Weight given to elderly in social welfare	ν	5

Table 2
Parameter Values for an Aging Economy

Generation	n(t)	p(t)
j-1	0.508	0.382
j	0.508	0.424
j+1	0.122	0.447
j+2	0.049	0.482
j+3	0.025	0.482
j+4	0.025	0.482

Table 3 **Long-run Growth Rate of Output Per Worker**			
Scenario	$\theta_2 = 0.5$, $\delta = 0.98$	$\theta_2 = 0.5$, $\delta = 0.05$	$\theta_2 = 0.1$, $\delta = 0.98$
Baseline	2.50%	2.50%	2.50%
Aging Population	3.57	3.24	3.32
Aging Population, $\uparrow\omega$	3.93	3.60	3.30
Aging Population, $\uparrow\omega$, $\downarrow\xi$	4.17	3.82	3.59

Appendix

Steady-State and Balanced Growth Solutions for physical and human capital and output.

From (16) and (17)

$$e(t) = \varphi(t)k(t+1) \tag{A1}$$

where $\varphi(t) = \dfrac{\delta\theta_1(t)}{p(t)}\left[1 + n(t+1) + \dfrac{(1-\alpha)\xi(t)}{\alpha}\right]$.

Combining (A1) and (16) yields

$$\begin{aligned} k(t+1) &= \frac{\delta\theta_1(t)}{\varphi(t)+\delta\theta_1(t)[1+n(t+1)][1+\varphi(t)]}\times \\ &\left[\left(1-\frac{p(t-1)\xi(t-1)}{1+n(t)}-\omega(t)\right)(1-\alpha)+[1-p(t-1)]\alpha\right]A(t)\varphi(t-1)^{\theta_1(t-1)(1-\alpha)}e^g(t-1)^{\theta_2(t-1)(1-\alpha)}k(t)^{\alpha+\theta_1(t-1)(1-\alpha)} \end{aligned} \tag{A2}$$

From (2), (12), and (A2)

$$e^g(t) = \frac{(1-\alpha)\omega(t)k(t+1)}{\dfrac{\delta\theta_1(t)[1+n(t+1)]}{\varphi(t)+\delta\theta_1(t)[1+n(t+1)][1+\varphi(t)]}\left[\left(1-\dfrac{p(t-1)\xi(t-1)}{1+n(t)}-\omega(t)\right)(1-\alpha)+[1-p(t-1)]\alpha\right]} \tag{A3}$$

Lag (A3) and substitute it into (A2) to yield

$$k(t+1) = Z(t)k(t)^{\alpha+[\theta_1(t)+\theta_2(t)](1-\alpha)} \tag{A4}$$

where

$$Z(t) = \left(\frac{\delta\theta_1(t)\left[\left(1-\dfrac{p(t-1)\xi(t-1)}{1+n(t)}-\omega(t)\right)(1-\alpha)+[1-p(t-1)]\alpha\right]}{\varphi(t)+\delta\theta_1(t)[1+n(t+1)][1+\varphi(t)]}\right)A(t)\,\varphi(t-1)^{\theta_1(t-1)(1-\alpha)}\left(\frac{\omega(t-1)(1-\alpha)}{1+n(t)}\right)^{\theta_2(t-1)(1-\alpha)}\cdot$$

$$\left(\frac{\varphi(t-1)+\delta\theta_1(t-1)[1+n(t)][1+\varphi(t-1)]}{\delta\theta_1(t-1)\left[\left(1-\dfrac{p(t-2)\xi(t-2)}{1+n(t-1)}-\omega(t-1)\right)(1-\alpha)+[1-p(t-2)]\alpha\right]}\right)^{\theta_2(t-1)(1-\alpha)}.$$

When $Z > 1$ and $\theta_1 + \theta_2 = 1$, then $k(t+1) > k(t)\ \forall t$ and the economy converges to a balanced growth path;

when $\theta_1 + \theta_2 < 1$ the economy converges to a steady state.

Along a balanced growth path or in steady state

$$Z = \left(\frac{\delta\theta_1}{\varphi + \delta\theta_1(1+n)(1+\varphi)}\right)^{1-\theta_2(1-\alpha)} \left[\left(1 - \frac{p\xi}{1+n} - \omega\right)(1-\alpha) + (1-p)\alpha\right]^{1-\theta_2(1-\alpha)} \left(\frac{(1-\alpha)\omega}{1+n}\right)^{\theta_2(1-\alpha)} A\varphi^{\theta_1(1-\alpha)}.$$

In steady state

$$k^{ss} = Z^{\frac{1}{(1-\alpha)(1-\theta_1-\theta_2)}}.$$

$$h^{ss} = \varphi^{\theta_1}\left[\frac{(1-\alpha)\omega\{\varphi[1+\delta\theta_1(1+n)] + \delta\theta_1(1+n)\}}{\delta\theta_1(1+n)\left[\left(1-\frac{p\xi}{1+n} - \omega\right)(1-\alpha) + (1-p)\alpha\right]}\right]^{\theta_2} k^{ss(\theta_1+\theta_2)}.$$

$$y^{ss} = Ah^{1-\alpha}k^{\alpha}$$

The growth rate of output per worker is

$$\hat{y}(t+1) = \frac{y(t+1)}{y(t)} = \frac{A(t+1)h(t+1)^{1-\alpha}k(t+1)^{\alpha}}{A(t)h(t)^{1-\alpha}k(t)^{\alpha}}$$

$$= \frac{A(t+1)}{A(t)}\left(\frac{e(t)}{e(t-1)}\right)^{\theta_2(1-\alpha)}\left(\frac{e^g(t)}{e^g(t-1)}\right)^{\theta_2(1-\alpha)} Z(t)^{\alpha}$$

$$= \frac{A(t+1)}{A(t)}\left(\frac{\phi(t)}{\phi(t-1)} * \frac{k(t+1)}{k(t)}\right)^{\theta_2(1-\alpha)}\left(\frac{\chi(t)}{\chi(t-1)} * \frac{k(t+1)}{k(t)}\right)^{\theta_2(1-\alpha)} Z(t)^{\alpha}$$

$$= \frac{A(t+1)}{A(t)}\left(\frac{\phi(t)}{\phi(t-1)}\right)^{\theta_2(1-\alpha)}\left(\frac{\chi(t)}{\chi(t-1)}\right)^{\theta_2(1-\alpha)} Z(t)^{\alpha+(\theta_1+\theta_2)(1-\alpha)}$$

where

$$\chi(t) = \frac{(1-\alpha)\omega(t)}{\dfrac{\delta\theta_1(t)[1+n(t+1)]}{\varphi(t)+\delta\theta_1(t)[1+n(t+1)][1+\varphi(t)]}\left[\left(1-\dfrac{p(t-1)\xi(t-1)}{1+n(t)}-\omega(t)\right)(1-\alpha)+[1-p(t-1)]\alpha\right]}$$

Along a balanced growth path $\hat{y} = Z$.

Proof of Proposition 1: Let

$$M = \frac{[1+\delta\theta_1(1+n)]}{\left(1+n+\dfrac{(1-\alpha)\xi}{\alpha}\right)[1+\delta\theta_1(1+n)]+p(1+n)} + \frac{p\alpha}{(1+n)\left[\left(1-\dfrac{p\xi}{1+n}-\omega\right)(1-\alpha)+(1-p)\alpha\right]}$$

Then,

$$\frac{\partial Z}{\partial \xi} = Z\left[\left(\frac{\theta_1(1-\alpha)}{1+n+\dfrac{1-\alpha}{\alpha}\xi}\right) - [1-\theta_2(1-\alpha)]M\right]$$

$$\frac{\partial h}{\partial \xi} = h\left[\left(\frac{\theta_1(1-\alpha)}{1+n+\dfrac{1-\alpha}{\alpha}\xi}\right) - (\theta_1+\theta_2\alpha)M\right]$$

$$\frac{\partial y}{\partial \xi} = y\left[\left(\frac{\theta_1(1-\alpha)}{1+n+\dfrac{1-\alpha}{\alpha}\xi}\right) - [\theta_1(1-\alpha)+\alpha]M\right]$$

Since the sign of $\frac{\partial k}{\partial \xi}$ is the same as the sign of $\frac{\partial Z}{\partial \xi}$, then $\frac{\partial k}{\partial \xi}$, $\frac{\partial h}{\partial \xi}$, $\frac{\partial y}{\partial \xi}$ and $\frac{\partial Z}{\partial \xi}$ will all be negative if

$\left(\dfrac{\theta_1(1-\alpha)}{1+n+\dfrac{1-\alpha}{\alpha}\xi}\right) < M\{\min[1-\theta_2(1-\alpha), \theta_1+\theta_2\alpha, (1-\alpha)\theta_1+\alpha]\} = M(\theta_1+\theta_2\alpha)$ by $\theta_1+\theta_2 \leq 1$, and will all be positive if $\left(\dfrac{\theta_1(1-\alpha)}{1+n+\dfrac{1-\alpha}{\alpha}\xi}\right) < M\{\max[1-\theta_2(1-\alpha),\theta_1+\theta_2\alpha,(1-\alpha)\theta_1+\alpha]\} = M(1-\theta_2(1-\alpha))$ by

$\theta_1+\theta_2 \leq 1$.

Proof of Proposition 2: Let

$$M' = \frac{1}{\left(1-\frac{p\xi}{1+n}-\omega\right)(1-\alpha)+(1-p)\alpha}.$$

Then

$$\frac{\partial Z}{\partial \omega} = Z\left(\frac{\theta_2}{\omega}-[1-\theta_2(1-\alpha)]M'\right)$$

$$\frac{\partial h}{\partial \omega} = h\left(\frac{\theta_2}{\omega}-(\theta_1+\theta_2\alpha)M'\right)$$

$$\frac{\partial y}{\partial \omega} = Z\left(\frac{\theta_2}{\omega}-[\alpha+\theta_1(1-\alpha)]M'\right)$$

Since the sign of $\frac{\partial k}{\partial \omega}$ is the same as the sign of $\frac{\partial Z}{\partial \omega}$, then $\frac{\partial k}{\partial \omega}$, $\frac{\partial h}{\partial \omega}$, $\frac{\partial y}{\partial \omega}$ and $\frac{\partial Z}{\partial \omega}$ will all be positive if

$\frac{\theta_2}{\omega} > M'\{\max[1-\theta_2(1-\alpha),\theta_1+\theta_2\alpha,\alpha+\theta_1(1-\alpha)]\} = M'[1-\theta_2(1-\alpha)]$ by $\theta_1+\theta_2 \leq 1$, that is, if

$\theta_2\left[\left(1-\frac{p\xi}{1+n}\right)(1-\alpha)+(1-p)\alpha\right] > \omega$, and will all be negative if

$\frac{\theta_2}{\omega} < M'\{\min[1-\theta_2(1-\alpha),\theta_1+\theta_2\alpha,\alpha+\theta_1(1-\alpha)]\} = M'[\theta_1+\theta_2\alpha]$, that is if

$$\frac{\theta_2}{\theta_1+\theta_2}\left[\left(1-\frac{p\xi}{1+n}\right)(1-\alpha)+(1-p)\alpha\right] < \omega.$$

Proof of Proposition 3: Let

$$M'' = \frac{1+n}{\left[1+n+\left(\frac{1-\alpha}{\alpha}\right)\xi\right][1+\delta\theta_1(1+n)]+p(1+n)} + \frac{\frac{\xi(1-\alpha)}{(1+n)}+\alpha}{\left(1-\frac{p\xi}{1+n}-\omega\right)(1-\alpha)+(1+p)\alpha}.$$

Then

$$\frac{\partial Z}{\partial p} = Z\left(\frac{\alpha}{p} - (1-\theta_2(1-\alpha))M''\right)$$

$$\frac{\partial h}{\partial \omega} = h\left(\frac{\alpha}{p} - (\theta_1 + \theta_2\alpha)M''\right)$$

$$\frac{\partial y}{\partial \omega} = Z\left(\frac{\alpha}{p} - (\alpha + \theta_1(1-\alpha))M''\right)$$

Since the sign of $\frac{\partial k}{\partial p}$ is the same as the sign of $\frac{\partial Z}{\partial p}$, then $\frac{\partial k}{\partial p}$, $\frac{\partial h}{\partial p}$, $\frac{\partial y}{\partial p}$ and $\frac{\partial Z}{\partial p}$ will all be positive if

$\frac{\alpha}{p} > M''\{\max[1-\theta_2(1-\alpha), \theta_1+\theta_2\alpha, \alpha+(1-\alpha)\theta_1]\} = M''[1-\theta_2(1-\alpha)]$ by $\theta_1+\theta_2 \le 1$, and will all be

negative if $\frac{\alpha}{p} < M''\{\min[1-\theta_2(1-\alpha), \theta_1+\theta_2\alpha, \alpha+(1-\alpha)\theta_1]\} = M''[\theta_1+\theta_2\alpha]$ by $\theta_1+\theta_2 \le 1$.

ENDNOTES

[1] See Kaganovich and Zilcha (1999) for an extensive review of the literature on the interactions between education and Social Security.

[2] Pecchenino and Utendorf solve a similar model under general functional form assumptions. As the results are not affected by the functional form assumptions we will assume specific functional forms throughout.

[3] While other models endogenize both expenditures on children and bequests, e.g. Nishimura and Zhang (1992, 1995), they assume that income and interest rates are exogenous. In our model, where wages and interest rates are endogenous, introducing an end-of-life bequest motive makes the analysis intractable.

[4] The assumption of unintentional rather than altruistic bequests is consistent with empirical findings by numerous researchers as summarized by Hurd (1990). Laitner and Juster (1996) find support for intergenerational altruism but note that it is not the major explanation for saving.

[5] Assuming exogenous fertility and an altruistic bond between parents and their children runs counter to the empirical findings of Cigno and Rosati (1996). They find that parents are self-interested and choose their saving and fertility decisions without regard to their offspring. Given the similarities between our model and Pecchenino and Utendorf (1999), we believe that if we removed the altruistic bond between parents and children and funded education via intergenerational education loans, our results would continue to hold. Our model further differs from Barro (1974) and other dynastic models, such as Ehrlich and Lui (1991), in which parents internalize the lifetime utility of their children. In these models the effects of changes in taxes are negated via changes in bequests, and so are ill-suited to analyzing social security or publicly funded education. Our formulation is similar to the parent child utility linkage assumed in Boldrin (1993). We adopt it so that we can study the effects of changes in expected longevity on social security and education taxes in response to demographic changes.

[6] The following propositions provide sufficient conditions for all the key endogenous variables to rise or fall together with an increase in an exogenous variables. Weaker necessary conditions can be found that enable the comparative static/dynamic effects for the individual endogenous variables to be signed. These conditions are stated explicitly in the appendix. For the case of balanced growth the necessary and sufficient conditions coincide.

[7] A social security program may also be optimal in an economy with generation specific shocks. However, such a system would require the possibility of transfers from workers to retirees and vice versa. See Rangel and Zeckhauser (1999). Since existing social security programs do not allow for such transfers, we assume that social welfare considerations prevail.

[8] See the Committee on Ways and Means (1998), table 1-17, page 27.

[9] The data for n and p are taken from the Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds, 1999.

[10] The rise in education expenditures precedes the rise in Social Security expenditures.

[11] Although the analysis in the text is with respect to an increase in ω from .074 to 0.10 and a decrease in ξ from 0.43 to 0.33 this result generalizes to any increase in ω and decrease in ξ.