The Contribution of Growth and Interest Rate Differentials to the Persistence of Real Exchange Rates

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Abstract

This paper employs a new methodology for measuring the contribution of growth and interest rate differentials to the half-life of deviations from Purchasing Power Parity (PPP). Our method is based on directly comparing the impulse response function of a VAR model, where the real exchange rate is Granger caused by these variables with the impulse response function of a univatiate ARMA model for the real exchange rate. We show that the impulse response function of the VAR model is not, in general, the same with the impulse response function obtained from the equivalent ARMA representation, if the real exchange rate is Granger caused by other variables in the system. The difference between the two functions captures the effects of the Granger-causing variables on the half-life of deviations from PPP. Our empirical results for a set of four currencies suggest that real and nominal long term interest rate differentials and real GDP growth differentials account for 22% to 50% of the half-life of deviations from PPP.

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1 Introduction

Long-run Purchasing Power Parity (PPP) states that real exchange rates, defined as the relative price of a basket of goods expressed in a common currency, should be stationary, implying that changes in the real exchange rate should be arbitraged away in the long run. Yet, one characteristic of real exchange rates is that they are highly persistent processes. In other words, the speed at which a given shock to the real exchange rate dissipates is very slow. One measure of persistence is half-life, defined as the number of periods required for a given shock to reduce to half its initial value. A large number of empirical studies has found that real exchange rates are stationary, but highly persistent processes with half-lifes of deviations from PPP between three and five years.[1]

The empirical evidence of an extremely slow speed of convergence towards PPP cannot be easily reconciled with the stylized fact that short-term deviations from PPP are both large and volatile. Indeed, the short-term volatility of real exchange rates is of the same order of magnitude as the volatility of nominal exchange rates. Combined with this stylized fact, the finding of high persistence of the real exchange rate constitutes a puzzle as to the nature of the shocks driving real exchange rates.[2]

The majority of empirical studies compute half-lives of PPP deviations within a univariate framework, typically by estimating a first-order autoregressive, AR(1), model of the real exchange rate. In such a specification, the error term, which accounts for the variation of the real exchange rate, can be thought of as a 'composite shock' that incorporates various individual shocks, such as monetary shocks or shocks to tastes and technology. As a result, impulse response analysis (IRA) within the univariate framework cannot identify the effect of each individual shock, but simply tells us how fast the real exchange rate adjusts to a disturbance of unknown origins.

This paper aims to shed some light on the causes of persistence of real exchange rates. In particular, we are interested in quantifying the relative importance of a set of macroeconomic variables which are considered to be fundamental determinants of real exchange rates on the persistence

[1] See, e.g. Frankel (1986, 1990), Abuaf and Jorion (1990), Glen (1992), Froot and Rogoff (1995), Lothian and Taylor (1996) and Rogoff (1996), among others. Studies using panel data, find only slightly shorter half-lifes, see, e.g. Frankel and Rose (1996), Oh (1996), Wu (1996), Lothian (1997) and Papell (1997), among others. Recent work with panel data, however, casts doubt on the stationarity of real exchange rates, see e.g. O'Connel (1998) and Breuer et *al.* (2001, 2002).

[2] Rogoff (1996) termed this the "PPP puzzle".

of deviations from PPP. This set of variables includes output growth differentials and long-term interest rate differentials (both nominal and real) between the domestic and the foreign economy.

In order to measure the relative contribution of these variables to the persistence of deviations from PPP, we compare the half-life estimates obtained from a VAR model which includes these variables along with the real exchange rate with the half-life estimates obtained from univariate models of the real exchange rate. The difference between the two half-life estimates is a measure of the contribution of these variables to the persistence of the real exchange rate.

Our choise of macroeconomic determinants of real exchange rates has two motivations: First, sticky-price theories of exchange rates suggest that deviations from PPP are closely related to this set of macroeconomic variables.[3] Second, given the trend to globalization of both financial markets and economies, policymakers and practitioners are interested to know how much faster real exchange rates would revert towards PPP if business cycles and monetary policy were fully synchronized across major economies.

In order to motivate our method, let us first define the real exchange rate, y_{1t}, as the relative price of foreign goods in terms of domestic goods. In log form:

$$y_{1t} \equiv s_t - (p_t - p_t^*)$$

where s_t is the nominal exchange rate, measured in units of domestic currency per unit of foreign currency, and p_t (p_t^*) is the domestic (foreign) price index. Furthermore, let $Y_t = [y_{1t}, \mathbf{y}_{2t}]'$ be an $(n \times 1)-$vector of variables where $\mathbf{y}_{2t}$ is an $(n-1)$-vector of macroeconomic variables, which affect the dynamic adjustment of the real exchange rate towards the PPP level.

Let us further assume that Y_t follows a $n-$variate VAR(1) model.[4] It is well known that each variable in the VAR(1) model (including y_{1t}) has an equivalent univariate ARMA$(n, n-1)$ representation, where n and $n-1$ are the maximum orders of the autoregressive and moving average parts, respectively (see Lutkepohl, 1993). In view of this 'equivalence', there is no specification error involved in one's decision to employ the ARMA model for estimating the response of the real exchange rate to a unit shock in the error term, say e_t. The latter, however, is a combination of the

[3] See Dornbusch (1976, 1989), Frankel (1979) and Meese and Rogoff (1988).

[4] The VAR(1) model is assumed at this stage for expositional purposes only.

errors in the VAR model, which in turn implies that the origins of this shock cannot be identified. Assume for simplicity that there is no contemporaneous correlation among the elements of Y_t, and consider the first equation of the VAR model, that is the one for the real exchange rate. The error term in this equation, say ε_{1t}, describes the shocks in the real exchange rate not accounted for by $\mathbf{y}_{2t}$, that is it describes the effects of any other random factors that affect the exchange rate. The VAR-response, IR^V, of y_{1t} to a unit shock in ε_{1t} should now be faster than its equivalent ARMA-response, IR^A, to a unit shoch in e_t if the variables $\mathbf{y}_{2t}$ have actually a role to play. Indeed, the difference, $D = IR^A - IR^V$, describes the dynamic adjustment path of the real exchange rate which is solely due to the observed variables $\mathbf{y}_{2t}$. Obviously, the effects of other factors that influence the real exchange rate not taken into account in the VAR specification are captured by IR^A itself. The bigger D is, the more (less) important the role of $\mathbf{y}_{2t}$ (other factors) for the persistence of the real exchange rate will be.

To further clarify our point, assume that the half-life of PPP deviations, estimated within the ARMA model for the real exchange rate is 20 quarters. On the other hand, assume that the half-life estimate obtained from the VAR model, which includes y_{1t} and $\mathbf{y}_{2t}$ is only 12 quarters. This means that the contribution of $\mathbf{y}_{2t}$ to the half-life of y_{1t} is 20-12=8 quarters. The remaining 12 quarters is the number of periods required for y_{1t} to adjust (by half) to shocks in other factors. In such a scenario, $\mathbf{y}_{2t}$ accounts for 40% (=8/20) of the persistence of the real exchange rate.

The remainder of the paper is structured as follows. Section 2 focuses on the econometric methodology. In the context of a first-order bivariate VAR model, it compares the impulse response function (IRF) of the first variable of the VAR model with the IRF obtained from the univariate ARMA representation of this variable. It also derives conditions under which these two IRFs are identical. Section 3 motivates our choice of the macroeconomic variables in our empirical application. Section 4 reports the empirical results and section 5 concludes.

2 Impulse Response Analysis: Multivariate Models and their Equivalent Univariate Representations

This section highlights our main methodological point, namely that the impulse response analysis within a VAR model differs in general from that conducted within the equivalent univariate ARMA models. For illustrative purposes and in order to avoid unnecessary complications, we focus on the simplest possible case, namely that of a zero-mean bivariate VAR(1) model. The results extend to the case of a $k-$variate VAR(p) model in a straightforward way.

Let $Y_t = (y_{1t}, y_{2t})'$ follow a stable VAR(1) process:

$$Y_t = AY_{t-1} + U_t \tag{1}$$

where $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, $a_{ij} \in R$. The error vector $U_t = (u_{1t}, u_{2t})'$ is a white noise process, that is, $E(U_t) = 0$, $E(U_tU_t') = \Sigma_u = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$ and $E(U_tU_s') = 0$ for $t \neq s$. The covariance matrix Σ_u is assumed to be non-singular.

Following Lutkepohl (1993), each component series y_{it}, $i = 1, 2$ of Y_t has an equivalent univariate ARMA(p, q) representation where $p \leq 2$ and $q \leq 1$.[5] To be specific, the ARMA(2,1) representation of y_{1t} is as follows:

$$y_{1t} - (a_{11} + a_{22})y_{1t-1} + (a_{11}a_{22} - a_{21}a_{12})y_{1t-2} = e_{1t} + \gamma_1 e_{1t-1} \tag{2}$$

where $Var(e_{1t}) = \sigma_1^2$, $\gamma_1 = \frac{S \pm \sqrt{Q+R}}{F}$ and $\sigma_1^2 = \frac{G_1}{\gamma_1}$.[6]

Furthermore,

$S = (1 + a_{22}^2)\sigma_{11} - 2a_{12}a_{22}\sigma_{12} + a_{12}^2\sigma_{22}$,

$Q = (1 + a_{22}^4 - 2a_{22}^2)\sigma_{11}^2 + a_{12}^4\sigma_{22}^2 + (4a_{12}^2a_{22}^2 - 4a_{12}^2)\sigma_{12}^2 - 4(a_{12}a_{22}^3 - a_{22}a_{12})\sigma_{11}\sigma_{12}$,

$R = (2a_{12}^2 + 2a_{22}^2a_{12}^2)\sigma_{11}\sigma_{22} - 4a_{12}^3a_{22}\sigma_{12}\sigma_{22}$,

$F = 2(a_{12}\sigma_{12} - a_{22}\sigma_{11})$,

[5] For a proof, see Corollary 6.1.1. in Lutkepohl (1993), page 232.

[6] Note that we have to choose the invertible solution for γ_1, i.e. the value of γ_1 that satisfies $|\gamma_1| < 1$.

$G_1 = a_{12}\sigma_{12} - a_{22}\sigma_{11}$.

It is interesting to note that the MA error term, $w_{1t} \equiv e_{1t} + \gamma_1 e_{1t-1}$, is related to the original VAR errors as follows:

$$w_{1t} = u_{1t} - a_{22}u_{1t-1} + a_{12}u_{2t-1} \tag{3}$$

This relationship shows that the error in the univariate representation of y_{1t} can be thought of as an aggregation of the original errors in the VAR model. As a result, the variation of w_{1t} is due to the variation of either u_{1t} or u_{2t} or both. Furthermore the above relationships show that the variance, σ_1^2, of the error term, e_{1t}, is a complicated function of the VAR parameters. This means that the shock e_{1t} of y_{1t} in the context of the ARMA model is determined by the structure of the intertemporal interactions between y_{1t} and y_{2t} and the second moments of u_{1t} and u_{2t}. As a consequence, its 'origins' are far from clear.

Let us now examine the response of y_{1t} to a unit shock in its innovations, in the context of both the VAR(1) and the ARMA(2,1) models. Before we proceed any further, it is important to emphasize the role of $\sigma_{12} \neq 0$ on the interpretation of the errors in the VAR model. If $\sigma_{12} \neq 0$, then the error, u_{1t}, in the first equation of the VAR model, cannot be interpreted as the innovations driving y_{1t}. On the other hand, if $\sigma_{12} = 0$, then u_{1t} regains its status as 'the innovations' of y_{1t} in the VAR model and can be thought of as summarizing the factors that contribute to the variability of y_{1t}, other than y_{1t-1} and y_{2t-1}. We are interested in comparing the impulse response function, IRF_u, of y_{1t}, from the univariate model with the impulse response function, IRF_m, of y_{1t} from the multivariate model. Note that IRF_m refers to the response of y_{1t} to a unit shock in u_{1t}.[7] The cases $\sigma_{12} = 0$ and $\sigma_{12} \neq 0$ are analyzed in subsections 2.1 and 2.2 respectively.[8]

[7] In the case of the VAR model, a response in y_{1t} may be caused by an impulse in u_{2t}, even if $\sigma_{12} = 0$.

[8] The diagonality restrictions on the covariance matrix are tested in the empirical part of the paper for all the countries under consideration.

2.1 The Case of a Diagonal Covariance Matrix, $\sigma_{12} = 0$

Throughout this subsection we assume $\sigma_{12} = 0$. The impulse response functions under consideration, IRF_u and IRF_m, are defined as follows:

$$IRF_u(k) = \gamma_k + \sum_{j=1}^{k} a_j IRF_u(k-j)$$

where $k = 1, 2, 3, ...$, $IRF_u(0) = 1$, $\gamma_k = 0$ for $k > 1$, $a_1 = (a_{11} + a_{22})$, $a_2 = (a_{21}a_{12} - a_{11}a_{22})$ and $a_k = 0$ for $k > 2$. On the other hand, IRF_m is usually defined in the context of the infinite moving average representation of Y_t, that is $Y_t = \sum_{i=0}^{\infty} \Phi_i U_{t-i}$ where $\Phi_i = A^i$. Then, it is easy to show that

$$IRF_m(k) = \phi_{11,k}$$

where $\phi_{11,k}$ is the upper left element of Φ_k.

We are interested in comparing $IRF_u(k)$ with $IRF_m(k)$. We present our results in the form of the following propositions.

Proposition 1: $IRF_u(k)$ is in general not equivalent to $IRF_m(k)$ for some $k < \infty$.[9]

Proof: See Appendix.

Due to the presence of γ_1 in $IRF_u(k)$, it is analytically impossible to identify all the cases where $IRF_u(k) > IRF_m(k)$. If, however, we impose some additional parameter restrictions, then the following result can be established:

Proposition 2: If $a_{11} > 0$, $a_{22} > 0$ and $a_{12}a_{21} > 0$, $IRF_u(k) > IRF_m(k)$ for every $k \in N$.

Proof: See Appendix.

However, there is one case where $IRF_u(k) = IRF_m(k)$ for every k. Specifically, this case arises when y_{2t} does not Granger cause y_{1t}. Hence:

Lemma 1 *When $a_{12} = 0$, $IRF_u(k) = IRF_m(k)$ for every $k \geq 0$.*

Proof: See Appendix.

It is important to note that only when $a_{12} = 0$, the AR(1) model is the correct univariate specification for y_{1t}. In the opposite case, the AR(1) is a misspecified model, thus producing misleading

[9] Given the stability of (1), both IRF_u and IRF_m tend to zero as $k \longrightarrow \infty$.

results in every aspect of statistical inference. This has direct implications on the wide application of the AR(1) model as the univariate representation of the real exchange rate. In the presence of even a single Granger-causing variable for the real exchange rate, the AR(1) model is clearly inappropriate.

2.2 The Case of a Non-Diagonal Covariance Matrix, $\sigma_{12} \neq 0$

In this case, the error term, u_{1t}, in the first equation of the VAR(1) does not coincide with the innovations driving y_{1t}. Following standard practice, we restore the orthogonality of the errors by utilizing the Cholesky decomposition of Σ_u, that is $\Sigma_u = PP'$, where P is a lower triangular matrix. After some algebra, we obtain the following representation for Y_t:

$$\begin{aligned} y_{1t} &= a_{11}y_{1t-1} + a_{12}y_{2t-1} + v_{1t} \\ y_{2t} &= \frac{\sigma_{12}}{\sigma_{11}}y_{1t} + (a_{21} - \frac{\sigma_{12}}{\sigma_{11}}a_{11})y_{1t-1} + (a_{22} - \frac{\sigma_{12}}{\sigma_{11}}a_{12})y_{2t-1} + v_{2t} \end{aligned} \tag{4}$$

where $V_t = \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} - \frac{\sigma_{12}}{\sigma_{11}}u_{1t} \end{bmatrix}$ with covariance matrix $\Sigma_V = \begin{bmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} - \frac{\sigma_{12}^2}{\sigma_{11}} \end{bmatrix}$. This particular representation was obtained by assuming that y_{1t} is causally prior to y_{2t}. This means that the current values of y_{1t} do not react contemporaneously to changes in y_{2t}. The error term, v_{1t}, in the first equation of (4) is orthogonal to y_{1t-1} and y_{2t-1}, that is it can be thought of as summarizing all the other factors that contribute to the variability of y_{1t}, apart from y_{1t-1} and y_{2t-1}. Based on (4), we obtain the following infinite MA representation of Y_t:

$$Y_t = \sum_{i=0}^{\infty}\Theta_i W_{t-i}$$

where $\Theta_i = \Phi_i P$ and $W_t = (w_{1t}\ w_{2t})' = P^{-1}U_t$[10].

We now define the Impulse Response Function, $IRF_{mo,}$ of y_{1t} to be:

$$IRF_{mo}(k) = \frac{\theta_{11,k}}{\sqrt{\sigma_{11}}}$$

[10] By construction, the variance-covariance matrix of W_t is $\Sigma_W = I_2$.

where $\theta_{11,k}$ is the upper left element of Θ_k. By definition, $IRF_{mo}(k)$ is the response of y_{1t} to a unit shock in its innovations, v_{1t}, after k periods. Therefore, $IRF_{mo}(k)$ is directly comparable to $IRF_u(k)$. The following proposition holds:

Proposition 3 *In general, $IRF_{mo}(k) \neq IRF_u(k)$ for some finite k.*

Proof: See Appendix.

The following lemma provides the sufficient condition to obtain equivalence of $IRF_{mo}(k)$ and $IRF_u(k)$.[11]

Lemma 2 *When $a_{12} = 0$, $IRF_u(k) = IRF_{mo}(k)$ for every $k \geq 0$.*

Proof: See Appendix.

3 Choice of Economic Variables

Economic theory has identified two main sets of determinants of real exchange rates: (a) real variables which describe the evolution of tastes and technology and determine the long-run equilibrium real exchange rate,[12] and (b) monetary/aggregate demand variables which describe the deviations of real exchange rates from PPP.[13]

While real disturbances, such as changes in tastes and technology, are likely to explain long-term changes in the real exchange rate, medium- and short-term changes are more likely to reflect monetary or aggregate demand shocks. Such shocks can have substantial effects on the real economy in the presence of short-term nominal price rigidities. This is a central feature of the Dornbusch (1976) sticky-price monetary model. In this model, monetary disturbances lead to overshooting of the real exchange rate due to short-term price stickiness. During the adjustment to long-term equilibrium, deviations from PPP are related to output and interest rate differentials between the domestic and the foreign economy. Frankel (1979) derives an alternative representation of the real exchange rate in terms of real interest rate differentials.[14]

[11] Despite our best efforts, we have not yet succeeded in proving that $IRF_u(k) \geqq IRF_{mo}(k)$ for some sensible parameter configurations. Nevertheless, extentive simulation results seem to support such a conjecture.

[12] See, e.g. Balassa (1964) and Samuelson (1964). According to the so-called "Balassa-Samuelson hypothesis", the long-run equilibrium real exchange rate is determined by the share of nontradable goods in the consumer basket (i.e. by consumer preferences) and relative total factor productivity in the tradables and non-tradables sector.

[13] See, e.g. Dornbusch (1976, 1989) and Meese and Rogoff (1988).

[14] In an empirical paper, Baxter (1994) finds a strong correlation between real exchange rates and real interest rate differentials.

Guided by these theories, we choose GDP growth differentials and long-term interest rate differentials (both nominal and real) between the domestic and the foreign economy as the main driving forces of real exchange rates.

A disclaimer is in order. It is clear that these variables capture a combination of both real and monetary disturbances making it difficult to relate them to any particular theory of exchange rate determination. For example, GDP growth differentials are related to the relative business cycle position between the domestic and the foreign economy but also reflect productivity differentials. Consequently, they capture a mixture of both monetary/aggregate demand disturbances and real disturbances.

Since it is very difficult in practice to proxy monetary and real disturbances with two orthogonal sets of variables, our empirical work does not aim at identifying the contribution of monetary and real shocks on the persistence of real exchange rates and, hence, at resolving the so-called "PPP puzzle". However, conditional on choosing carefully the set of macroeconomic determinants of real exchange rates, our methodology opens the way to directly test different theories of exchange rate determination.

4 Empirical Results

4.1 Data

Our empirical analysis is based on post-1973, quarterly, real exchange rates for five major industrialized countries. Data for nominal exchange rates, consumer prices, long-term interest rates and real GDP are collected from International Financial Statistics (IFS CD-Rom, March 2002).[15] The business cycle position relative to the US is proxied by the 4-quarter real GDP growth differential between the home country and the US.

We consider four country pairs, with the US serving as the foreign country. The domestic country is represented by France, Germany, Italy and the UK, respectively. The bilateral real exchange rate is measured as the nominal exchange rate, defined in units of domestic currency per dollar, multiplied by the ratio between the US and the domestic consumer price index. Figure 1

[15] Nominal exchange rate: line ae.zf, long-term interest rate: line 61...zf, CPI: line 64...zf, real GDP: line 99BVRZF.

presents the relevant series.

Long-term interest rates are yields to maturity of 10-15 year government bonds. Ex ante returns on long-term bonds are difficult to compute since this requires a measure of expected inflation over the term of the bond. While these long-term inflation forecasts can be easily generated from time series models or filtering techniques, a drawback of these methods is that they produce time series for expected inflation that are very smooth, compared to realized inflation. An alternative method for computing real interest rates is to use past year realized inflation. In order to compute the real interest rate, we subtract consumer price inflation over the past four quarters from the nominal yield. Although this method for computing real interest rates is not entirely satisfactory, since inflation is not measured over the term of the bond, it avoids problems of overlapping observations, compared with the method of computing true ex post real interest rates.

4.2 Unit Root Tests

Inferences on the presence of unit roots in real exchange rates depends heavily on both the testing strategy and the sample employed. For example, Huizinga (1987) and Meese and Rogoff (1988) fail to reject the unit root null by means of standard unit root tests for the post-1973 period. The notorious low power of these tests may of course be the sole reason for not rejecting the null.[16] On the other hand, when longer-run time series are employed, blending fixed and floating rate data, the unit root hypothesis is rejected.[17] Similar evidence is obtained when the post-1973 data are expanded cross-sectionally, by means of panel data methods.[18] In the present case, the results from a variety of unit-root tests are, as usual, mixed.[19] When the null hypothesis of stationarity is tested, the KPSS test fails to reject the null for the real exchange rates as well as the other

[16] Some recent results by Taylor (2001) forcefully point towards the 'low-power' interpretation of not rejecting the unit root null. Specifically, sampling the data at low frequencies makes it impossible to identify an adjustment process occuring at high frequencies, thus producing the false impression of long or even infinite half-lives. In another recent paper, Imbs *et al.* (2005) show that estimates of persistence of real exchange rates suffer from a positive cross-sectional aggregation bias.

[17] See, for example, Abuaf and Jorion (1990), Frankel (1990), Lothian and Taylor (1996) and Cheung and Lai (1998, 2000).

[18] See, for example, Wei and Parsley (1995), Frankel and Rose (1996), Higgins and Zakrajsek (1999).

[19] The unit root null is tested by means of the following tests: the standard Dickey-Fuller test (Dickey and Fuller, 1979), the Dickey-Fuller test with GLS detrending (Elliott *et al.*, 1996), the Point Optimal test (Elliott *et al.*, 1996), the Phillips-Perron test (Phillips and Perron, 1988) and the Ng-Perron test (Ng and Perron, 2001). The stationary null hypothesis is tested by means of the KPSS test (Kwiatkowski *et al.*, 1992). Results are available upon request.

macroeconomic variables for all the countries under consideration. When the null hypothesis of a unit root is tested, the standard Dickey-Fuller (DF) or Phillips-Perron (PP) tests typically fail to reject the null. The GLS versions of the DF tests, however, being more powerful than the standard DF tests, reject the unit root null in many cases.

The general picture emerging from the empirical literature and our own tests suggests treating the real exchange rates and the macroeconomic variables as having a highly persistent but ultimately stationary univariate representation.

4.3 Univariate Models

The majority of studies employ the simplest univariate model, that is an AR(1) model, to estimate the half-life of deviations from PPP. Taylor (2001) refers to this as the 'basic model' in order to highlight the unanimity concerning the choices of models for the real exchange rate. In order, however, to relate our results to those of the existing literature, we begin our analysis by estimating an AR(1) model for each country under consideration. The estimation results along with the half-life estimates and their confidence intervals are reported in Table 1.

[INSERT TABLE 1]

It can be seen that the half-life estimates range from 9 to 14 quarters (2.25 years to 3.5 years). The shortest half-life corresponds to the UK pound, while the longest one to the German mark. The mean half-life for the four pairs of countries examined is 12.25 quarters which is very close to the estimate of Abuaf and Jorion (1990) for eight series of real exchange rates.

However, the AR(1) model is not an adequate representation of the real exchange rate, since serial correlation problems are encountered for all currencies examined. As a consequence, the half-life estimates are inconsistent since they are based on inconsistent estimates of the autoregressive coefficients. In order to specify the correct univariate model, we consider fourteen ARMA(p,q) models with p=1,...,4, q=1,2 and select p and q by means of the Schwartz Information Criterion (SIC). Table 2 reports the half-life estimates, calculated from the impulse response function of the selected model, along with their confidence intervals.

[INSERT TABLE 2]

The results suggest that half-lives are generally lower than in the AR(1) case, though not considerably: the mean half-life for the four pairs of countries is 10.75 quarters, compared to an estimate of 12.25 quarters from the AR(1) models.[20] Moreover, both asymptotic and Monte Carlo confidence intervals based on the selected univariate models are tighter than those based on the AR(1) models. While the average lower bounds remain approximately at 5 quarters, the average upper bounds reduce to 20 and 19 quarters from 32 and 25 quarters for the asymptotic and Monte Carlo confidence intervals, respectively.[21]

So far, we have estimated half-lifes of real exchange rate innovations based on univariate models, thus ignoring the interactions of real exchange rates with other macroeconomic variables. The results of Section 2 have shown that the impulse response analysis within univariate models is, in general, not equivalent to the impulse response analysis within multivariate models, even if the univariate models are correctly specified. Therefore, we proceed to estimate the half-life of PPP deviations within multivariate models.

4.4 Multivariate Models

The existing literature suggests that there is a number of macroeconomic variables that affect the dynamics of the real exchange rate. These variables include real or nominal interest rate differentials and GDP growth differentials between the home country and the US. We now attempt to assess the role of these macroeconomic variables in determining the degree of persistence of the real exchange rate by estimating VAR models in the real exchange rate and the set of the aforementioned macroeconomic variables. To select the appropriate multivariate model for each country, we proceed along the lines of the 'general-to-specific' methodology. Specifically, we start with a general VAR(1) model containing all the candidate variables and then we end up with a

[20] Similar results are obtained by Murray and Papell (2002) by comparing half-life estimates from Dickey-Fuller and augmented Dickey-Fuller regressions (see their Tables 5 and 6).

[21] Our results for the confidence intervals are consistent with those reported by Rossi (2005) which are constructed based on local to unity asymptotic theory that is robust to high persistence and small sample sizes.

parsimonious VAR(1) specification by excluding insignificant variables, i.e. variables that do not 'Granger cause' the real exchange rate. The estimated VAR models for each country are presented in Tables 3-6.

[INSERT TABLES 3-6]

It is interesting to note that the first-order models appear to be statistically adequate since no serial correlation is detected in any of the VAR(1) equations. Our estimates suggest that deviations from PPP are significantly related to some of the macroeconomic variables used in our analysis. More specifically, with the exception of Germany – where real GDP data are highly distorted due to the effect of unification in 1990 –, GDP growth differentials are in all countries significant determinants of real exchange rates. An increase in real GDP growth relative to the US is related to a real appreciation of the home currency both in the short-term and the long-term, in line with the theoretical predictions. Long-term interest rate differentials with the US are also an important determinant of real exchange rates. Our estimates suggest that in three out of four countries (France, Italy and UK), an increase in the real interest rate differential with the US is related to a real appreciation of the domestic currency. In Germany, we find that nominal long-term interest rate differentials are important in explaining deviations from PPP. As predicted by theory, an increase in the German nominal interest rate relative to the US is related to a real appreciation of the deutchmark.

As shown in Section 2, estimates of impulse response functions, and, hence, half-lifes of deviations from PPP, are, in general, different in the context of a VAR model, compared to estimates of univariate models. A condition for this to occur, is that (at least one of) the variables included in the VAR Granger cause(s) the real exchange rate. This condition can be tested using the standard t-test to assess the significance of the coefficients of macro-variables in the real exchange rate equation. The results reported in Tables 3-6 suggest that this condition is satisfied in all countries, providing evidence that estimates of half-life in multivariate models are different from those in univariate models. Our results provide evidence that the macroeconomic variables used in the VAR specification can partly account for the persistence of the real exchange rate.

Before proceeding to the calculation of the half-life of deviations from PPP in the VAR model, we test whether the contemporaneous correlation between innovations in the real exchange rate and other variables in each multivariate model is statistically different from zero. The importance of this condition was already discussed in Section 2. In the case of a zero correlation, we can compute half-life using the original VAR innovations, otherwise our calculations should be based on the orthogonal transformation of the VAR innovations. In order to test this assumption, we estimated both a restricted and an unrestricted model and computed the Likelihood Ratio (LR) statistic. The results, reported in Table 7, suggest that the orthogonality restriction, i.e. zero contemporaneous correlation between innovations in the real exchange rate and other variables included in the VAR model holds in all countries, but France.

[INSERT TABLE 7]

We now proceed to examine the dynamic characteristics of the system by examining the impulse response functions. We employ responses to a unit shock in the cases of Germany, Italy and the UK, where the orthogonality restriction between innovations in the real exchange rate and other variables is satisfied. In the case of France, we employ orthogonal impulse responses, since the orthogonality restriction was rejected. It is important to note that when orthogonal IRFs are considered, these are dependent on the ordering of the variables. To ensure comparability of multivariate IRFs with univariate IRFs, the real exchange rate is the first variable in the VAR. The IRFs for each of the countries are displayed in Figure 2. Estimated half-lifes along with their 95% asymptotic confidence intervals are presented in Table 8. In order to account for small sample effects, we also report Monte Carlo estimates of confidence intervals along with asymptotic ones.

[INSERT FIGURE 2 & TABLE 8]

Our results reported in Table 8 suggest that estimates of half-lives of deviations from PPP in the context of multivariate models are substantially lower than those of univariate models for all the countries considered. For example, the half-life for Germany reduces to 6 quarters from 12 quarters and for Italy to 7 quarters from 11 quarters. The mean half-life across the four country pairs is 7 quarters, compared with an average of 12.25 quarters from the AR(1) models and 10.75

quarters from the ARMA models. This suggests that real and nominal long term interest rate differentials and real GDP growth differentials account for a substantial fraction of the half-life of PPP deviations.

The difference between the ARMA estimate of half-life, HL_u (as reported in Table 2), and the VAR estimate of half-life, HL_m, is 3.75 quarters, in line with estimates of persistence of real exchange rates from calibrated international business cycle models with nominal price rigidities such as Chari *et al.* (2002). The remaining seven quarters of the half-life of deviations from PPP can be attributed to other (unspecified) sources of persistence. By comparing the half-life estimates of the multivariate models with the half-life estimates of their equivalent univariate representations, we can compute the fraction of half-life attributable to the set of macroeconomic variables included in the VAR model as $(HL_u - HL_m)/HL_u$. As reported in the last column of Table 8, the fraction of half-life due to real and nominal long term interest rate differentials and real GDP growth differential ranges from 22% in the UK to 50% in Germany, with an average across the four country-pairs of 34%.

The 95% confidence intervals of half-lifes are considerably tighter than in the univariate context, suggesting that our estimates of half-lifes are more precise. The lower bound of the asymptotic confidence intervals is estimated at four quarters for all country pairs, compared with 5-7 quarters in the univariate models. The upper bounds range from 13 to 30 quarters, compared to 16-23 in the univariate models. Interestingly, the Monte Carlo confidence intervals are tighter than those based on the asymptotic distribution of the impulse response function (lower bound: 3-4 quarters, upper bound: 12-22 quarters). It is important to note that our estimates break the consensus view at the lower end of its range without accounting for a series of potential econometric pitfalls, such as temporal aggregation bias,[22] nonlinear adjustment[23] or cross-sectional aggregation bias.[24] Correcting for these econometric issues would certainly reduce estimated half-lifes even further.

[22] For an extensive analysis of temporal aggregation bias in half-life estimates see Taylor (2001).
[23] See, for instance, Michael *et al.* (1997), Taylor and Peel (2000) and Taylor (2001).
[24] See, for instance, Imbs *et al.* (2005).

5 Conclusions

In this paper, we estimated the half-life of PPP deviations in the context of a Vector Autoregressive model, where the real exchange rate is allowed to interact with a set of macroeconomic variables, suggested by theories of exchange rate determination. By doing this, we were able to discern the relative effect of these variables on the speed of adjustment of the real exchange rate towards long-run PPP. We first showed that the impulse response function of a variable participating in the VAR model is not, in general, the same with the impulse response function obtained from the equivalent ARMA representation of this variable, if the latter is Granger caused by other variables in the system. The difference between the two impulse response functions captures the effect of the Granger-causing variables on the dynamic adjustment process of the variable of interest.

We investigate the implications of our analytical results for the speed of adjustment of four real exchange rates vis-a-vis the US dollar (French franc, German mark, Italian lira and UK pound) during the post-Bretton Woods period. Our empirical results suggest that real exchange rates are in fact Granger caused by these variables. As a result, the adjustment horizons of deviations from PPP decrease substantially. The average half-life estimate across the four pairs of real exchange rates is below two years, suggesting that real or nominal interest rate differentials and GDP growth differentials account for a significant fraction of deviations from PPP. Comparing the half-life estimates of the univariate models with the half-life estimates of the VAR model, we conclude that between 22% and 50% of the half-life of deviations from PPP is due to these variables.

Of course, although real or nominal interest rate differentials and GDP growth differentials explain a significant fraction of deviations from PPP, our results leave a good bit of variation in real exchange rates to unknown sources. These sources still account on average for a half-life of just below two years, hence, a puzzle remains as to whether real sources are volatile enough to explain the observed movements of real exchange rates. However, recent work on the PPP puzzle suggests that standard methods of estimation used in the literature largely overestimate the size of real exchange rates half-lifes because they fail to correct for a number of biases stemming from parameter heterogeneity, temporal aggregation and nonlinear adjustment.

Our method is not able to identify whether the persistence of real exchange rates is due to real or monetary shocks and, hence, does not address the so-called "PPP puzzle". However, it opens the way to assess the role of fundamental determinants of real exchange rates identified by different theories on the persistence of deviations from PPP. Further work is needed to address the issue of identification. Finally, our method is general enough to assess the importance of fundamental determinants on the observed persistence of a wide range of economic and financial variables, such as inflation, real wages, dividend-price ratios etc.

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Appendix

Proof of Proposition 1

It is easy to show that in the context of (1), $IRF_m(1) = a_{11}$. On the other hand, $IRF_u(1) = a_{11} + a_{22} + \gamma_1$. Similarly, $IRF_m(2) = a_{11}^2 + a_{12}a_{21}$, whereas $IRF_u(2) = (a_{11} + a_{22})(\gamma_1 + a_{11} + a_{22}) - a_{11}a_{22} + a_{21}a_{12}$. Similar results are obtained for $k > 2$. Therefore, in general, $IRF_u(k) \neq IRF_m(k)$.

Proof of Proposition 2

After some algebra we have that

$$\begin{aligned} IRF_m(k) - IRF_u(k) &= (2^{-1-k}((a_{11} + a_{22} - x)^k \\ &\quad -(a_{11} + a_{22} + x)^k)((-1 + a_{22}^2)\sigma_{11} - a_{12}^2\sigma_{22} + \\ &\quad +\sqrt{(\sigma_{11} + a_{22}^2\sigma_{11} + a_{12}^2\sigma_{22})^2 - 4a_{22}^2\sigma_{11}^2}))/(xa_{22}\sigma_{11}) \end{aligned}$$

or alternatively:

$$\begin{aligned} IRF_m(k) - IRF_u(k) &= (\frac{1}{2}(\lambda_2^k - \lambda_1^k)((-1 + a_{22}^2)\sigma_{11} - a_{12}^2\sigma_{22} + \\ &\quad +\sqrt{(\sigma_{11} + a_{22}^2\sigma_{11} + a_{12}^2\sigma_{22})^2 - 4a_{22}^2\sigma_{11}^2}))/(xa_{22}\sigma_{11}) \end{aligned}$$

where

$$x = \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}}$$

and λ_1 and λ_2 are the eigenvalues of A.[25] It is easy to show that $\lambda_1 > |\lambda_2|$ or $(\lambda_2^k - \lambda_1^k) < 0$ for every finite k. Then, what remains to be proved is that

$$((-1 + a_{22}^2)\sigma_{11} - a_{12}^2\sigma_{22} + \sqrt{(\sigma_{11} + a_{22}^2\sigma_{11} + a_{12}^2\sigma_{22})^2 - 4a_{22}^2\sigma_{11}^2}) > 0.$$

Indeed,

[25] $\lambda_1 = \frac{1}{2}(a_{11} + a_{22} + \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}})$ and $\lambda_2 = \frac{1}{2}(a_{11} + a_{22} - \sqrt{(a_{11} - a_{22})^2 + 4a_{12}a_{21}})$.

$$\sqrt{(\sigma_{11}+a_{22}^2\sigma_{11}+a_{12}^2\sigma_{22})^2-4a_{22}^2\sigma_{11}^2}=\sqrt{(\sigma_{11}-a_{22}^2\sigma_{11}+a_{12}^2\sigma_{22})^2+4a_{22}^2a_{12}^2\sigma_{11}\sigma_{22}}>$$

$$>\sqrt{(\sigma_{11}-a_{22}^2\sigma_{11}+a_{12}^2\sigma_{22})^2}=\sigma_{11}-a_{22}^2\sigma_{11}+a_{12}^2\sigma_{22}.$$

Thus,

$$((-1+a_{22}^2)\sigma_{11}-a_{12}^2\sigma_{22}+\sqrt{(\sigma_{11}+a_{22}^2\sigma_{11}+a_{12}^2\sigma_{22})^2-4a_{22}^2\sigma_{11}^2})>0$$

which in turn implies that $IRF_u(k)\geq IRF_m(k)$ for every $k\in N$.

Proof of Lemma 1

Before we prove this Lemma, we need to take an intermediate step, as described in the following remark:

Remark 1 *Let* $A=\begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22}\end{bmatrix}$ *where* $a_{ij}\in R$. *Then, for every integer* $d>0$, $A^d=\begin{bmatrix} a_{11}^d & 0 \\ q_1 & a_{22}^d\end{bmatrix}$ *where* q_1 *is a function of* a_{ij}.

Proof: We prove the remark by induction.

For $d=1$, $A^d=A=\begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22}\end{bmatrix}$, which is of the form: $\begin{bmatrix} a_{11}^d & 0 \\ q_1 & a_{22}^d\end{bmatrix}$ with $q_1=a_{21}$.

Assume that $A^d=\begin{bmatrix} a_{11}^d & 0 \\ q_1 & a_{22}^d\end{bmatrix}$ where q_1 is a function of a_{ij}. Then, we must show that $A^{d+1}=\begin{bmatrix} a_{11}^{d+1} & 0 \\ q_1' & a_{22}^{d+1}\end{bmatrix}$. Now,

$$A^{d+1}=A^dA=\begin{bmatrix} a_{11}^d & 0 \\ q_1 & a_{22}^d\end{bmatrix}\begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22}\end{bmatrix}=\begin{bmatrix} a_{11}^{d+1} & 0 \\ a_{11}q_1+a_{21}a_{22}^d & a_{22}^{d+1}\end{bmatrix}$$

which is of the form: $\begin{bmatrix} a_{11}^{d+1} & 0 \\ q_1' & a_{22}^{d+1} \end{bmatrix}$.

Now, we proceed with the proof of the lemma. We have defined $IRF_m(k)$ to be equal to the upper left element, $\phi_{11,k}$, of $\Phi_k = A^k$. By means of the previous remark, we have that Φ_k is of the form: $\begin{bmatrix} a_{11}^k & 0 \\ q_1 & a_{22}^k \end{bmatrix}$ where q_1 is a function of a_{ij}. Therefore, $IRF_m(k) = a_{11}^k$. Next, it is easy to show that when $a_{12} = 0$, i.e. y_{2t} does not Granger cause y_{1t}, the univariate representation of y_{1t} is the following AR(1) model: $y_{1t} = a_{11} * y_{1t-1} + e_{1t}$, which in turn implies that $IRF_u(k) = a_{11}^k$. Thus, $IRF_u(k) = IRF_m(k)$ for every k.

Proof of Proposition 3

It is straightforward to show that $IRF_{mo}(1) = a_{11} + a_{12}\frac{\sigma_{12}}{\sigma_{11}}$, which is in general different than $IRF_u(1) = a_{11} + a_{22} + \gamma_1$. Similarly,

$$IRF_{mo}(2) = a_{11}^2 + a_{11}a_{12}\frac{\sigma_{12}}{\sigma_{11}} + a_{12}a_{21} + a_{12}a_{22}\frac{\sigma_{12}}{\sigma_{11}}$$

whereas

$$IRF_u(2) = (a_{11} + a_{22})(\gamma_1 + a_{11} + a_{22}) - a_{11}a_{22} + a_{21}a_{12}$$

Similar results are obtained for $k > 2$. Therefore, in general, $IRF_u(k) \neq IRF_{mo}(k)$.

Proof of Lemma 2

We have already shown that when $a_{12} = 0$, $IRF_u(k) = a_{11}^k$, $k \geq 0$. In addition, Φ_k is of the form: $\begin{bmatrix} a_{11}^k & 0 \\ q_1 & a_{22}^k \end{bmatrix}$ (see lemma 1) where q_1 is a function of a_{ij}. Given that P is lower triangular, it is easy to show that $\Theta_k = \Phi_k P$ has the following form: $\Theta_k = \begin{bmatrix} a_{11}^k\sqrt{\sigma_{11}} & 0 \\ q_1 & q_2 \end{bmatrix}$, where q_1 and q_2 are functions of a_{ij} and σ_{ij}, $i, j = 1, 2$. Thus, $IRF_{mo}(k) = \frac{\theta_{11,k}}{\sqrt{\sigma_{11}}} = a_{11}^k = IRF_u(k)$.

Table 1. Estimated Half-lifes and 95% Confidence Intervals of AR(1) Models

	ar(1)	HL_u	95% Confidence Intervals				Adj. R^2
			Asymptotic		Monte Carlo		
			Lower	Upper	Lower	Upper	
France	0.947	13	6	35	5	27	0.898
Germany	0.948	14	6	37	5	30	0.900
Italy	0.945	13	5	34	5	26	0.893
UK	0.924	9	5	21	4	18	0.861
Average	0.941	12.25	5.5	31.75	4.75	25.25	

Notes: ar(1): estimate of autoregressive coefficient. HL_u: estimate of half-life. Data are quarterly from 1973:Q1 to 1998:Q4 for France, Germany and Italy and from 1973:Q1 to 2001:Q4 for the UK.

Table 2. Estimated Half-lifes and 95% Confidence Intervals of ARMA(p,q) Models

	Model	HL_u	95% Confidence Intervals				Adj. R^2
			Asymptotic		Monte Carlo		
			Lower	Upper	Lower	Upper	
France	AR(2)	11	6	21	5	18	0.911
Germany	AR(4)	12	7	21	6	19	0.918
Italy	ARMA(1,1)	11	5	23	5	20	0.903
UK	ARMA(4,4)	9	5	16	4	19	0.883
Average		10.75	5.75	20.25	5	19	

Notes: HL_u: estimate of half-life.

Table 3. VAR Estimates (France)

Variable	c	$y_1(-1)$	$y_2(-1)$	$y_3(-1)$	Adj. R^2
y_1	0.242	0.861	-0.859	-0.389	0.917
	(0.071)	(0.040)	(0.320)	(0.194)	
y_2	0.001	-0.0004	0.891	0.013	0.808
	(0.013)	(0.008)	(0.060)	(0.037)	
y_3	-0.006	0.002	0.032	0.838	0.728
	(0.020)	(0.012)	(0.092)	(0.056)	

Notes: y_1: real exchange rate, y_2: real long term interest rate differential, y_3: real GDP growth differential. Standard errors in parentheses below coefficient estimates.

Table 4. VAR Estimates (Germany)

Variable	c	$y_1(-1)$	$y_2(-1)$	Adj. R^2
y_1	0.048	0.891	-0.881	0.909
	(0.018)	(0.035)	(0.286)	
y_2	-0.004	0.005	0.974	0.928
	(0.002)	(0.004)	(0.031)	

Notes: y_1: real exchange rate, y_2: nominal long-term interest rate differential. Standard errors in parentheses below coefficient estimates.

Table 5. VAR Estimates (Italy)

Variable	c	$y_1(-1)$	$y_2(-1)$	$y_3(-1)$	Adj. R^2
y_1	0.793	0.892	-0.285	-0.565	0.904
	(0.316)	(0.042)	(0.218)	(0.193)	
y_2	0.241	-0.033	0.784	-0.143	0.854
	(0.075)	(0.010)	(0.052)	(0.046)	
y_3	-0.181	0.024	0.161	0.831	0.699
	(0.095)	(0.013)	(0.065)	(0.058)	

Notes: y_1: real exchange rate, y_2: real long term interest rate differential, y_3: real GDP growth differential. Standard errors in parentheses below coefficient estimates.

Table 6. VAR Estimates (UK)

Variable	c	$y_1(-1)$	$y_2(-1)$	$y_3(-1)$	Adj. R^2
y_1	-0.047	0.904	-0.445	-0.426	0.871
	(0.016)	(0.035)	(0.188)	(0.226)	
y_2	-0.047	-0.006	0.849	-0.183	0.736
	(0.005)	(0.009)	(0.051)	(0.061)	
y_3	0.008	0.024	0.183	0.659	0.543
	(0.004)	(0.010)	(0.055)	(0.066)	

Notes: y_1: real exchange rate, y_2: real long term interest rate differential, y_3: real GDP growth differential. Standard errors in parentheses below coefficient estimates.

Table 7. Orthogonality Restrictions

	Log Likelihood			
	Unrestricted	Restricted	LR-statistic	p-value
France	-109.617	-114.902	10.571	0.005
Germany	86.33	85.66	1.341	0.247
Italy	-154.555	-156.557	4.003	0.135
UK	-191.866	-194.109	4.487	0.106

Note: The Table tests the restriction that the covariance between innovations of the real exchange rate and innovations of the other variables included in the VAR is zero.

Table 8. Estimated Half-lifes and 95% Confidence Intervals of VAR(1) Models

	HL_m	95% Confidence Intervals				Ratio
		Asymptotic		Monte Carlo		
		Lower	Upper	Lower	Upper	
France	8	4	21	4	17	0.27
Germany	6	4	13	3	12	0.50
Italy	7	4	30	3	22	0.36
UK	7	4	22	4	17	0.22
Average	7	4	21.5	3.5	17	0.34

Notes: HL_m: half-life. *Ratio* is computed as $(HL_u - HL_m)/HL_u$, where HL_m is the half-life estimate of the VAR model and HL_u is the half-life estimate of the univariate ARMA(p,q) model, as reported in Table 2. Estimates for France are based on orthogonalized innovations (see text).

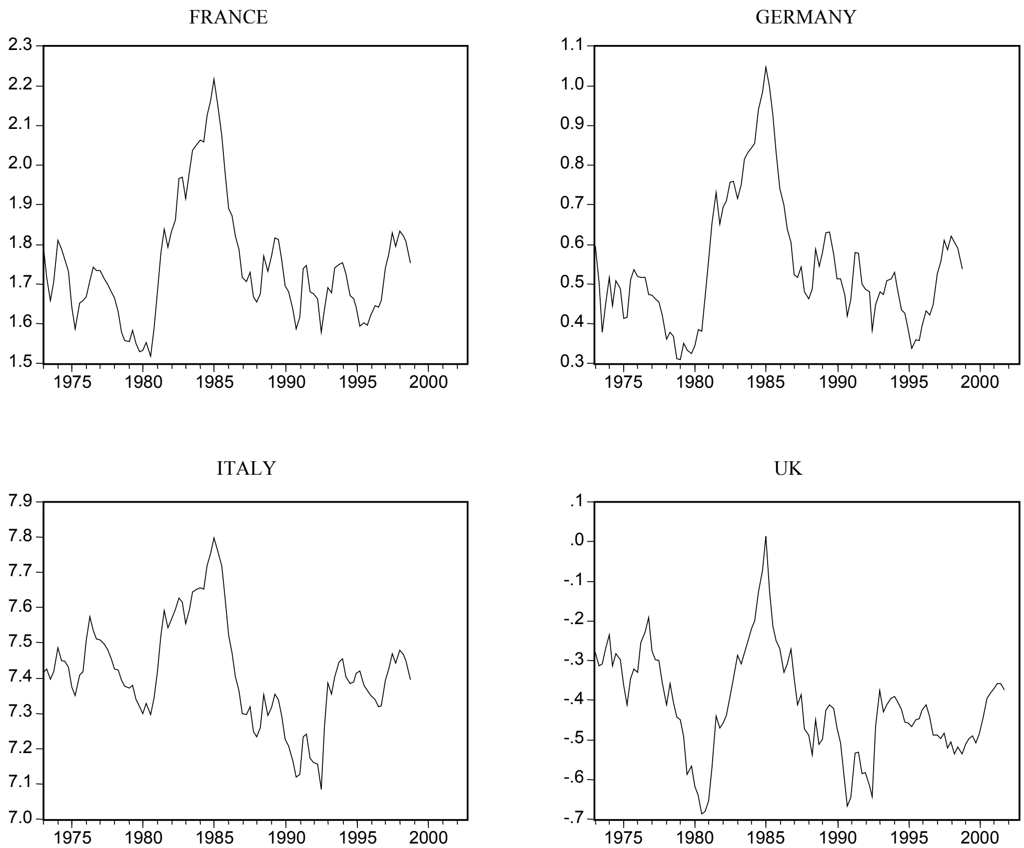

Figure 1: Real Exchange Rates

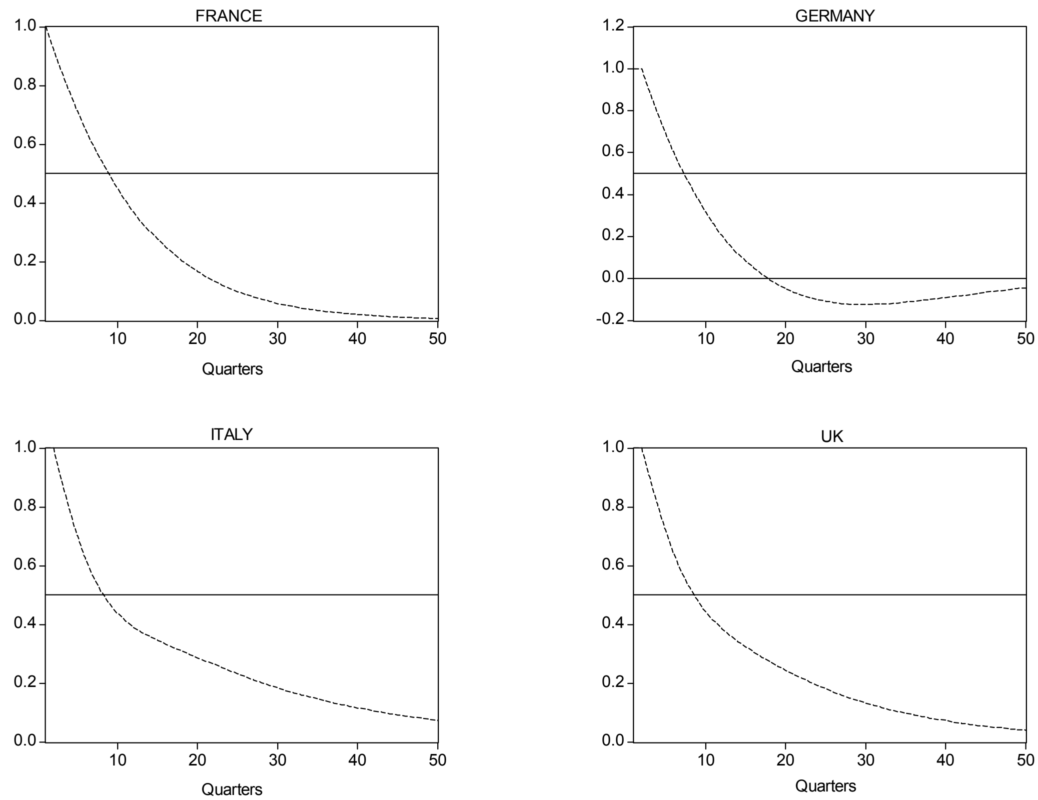

Figure 2: Impulse Responses to a Unit Shock