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GMM estimation of the covariance structure of longitudinal data on earnings

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Abstract. In this article, we discuss generalized method of moments estimation of the covariance structure of longitudinal data on earnings, and we introduce and illustrate a Stata program that facilitates the implementation of the generalized method of moments approach in this context. The program, `gmmcovearn`, estimates a variety of models that encompass those most commonly used by labor economists. These include models where the permanent component of earnings follows a random growth or random walk process and where the transitory component can follow either an AR(1) or an ARMA(1,1) process. In addition, time-factor loadings and cohort-factor loadings may be incorporated in the transitory and permanent components.

Keywords: st0001, gmmcovearn, permanent inequality, transitory inequality, generalized method of moments, GMM, covariance structure of earnings

1 Introduction

In recent years, an increasing number of articles have examined the covariance structure of longitudinal data on earnings. Prior to work by Hause (1977) and Lillard and Weiss (1978, 1979), much of the literature on earnings differences focused on differences at a point in time, using cross-sectional data. However, with increasing availability of panel data, attention turned to the evolution of individual earnings over time—hence the term *covariance structure of earnings*.

In this literature, earnings at a point in time reflect both individual-specific time-invariant characteristics and also transitory but serially correlated error-term variation. Consequently, earnings differences at a point in time may reflect either permanent inequality or transitory inequality, and tackling these different sources of inequality requires different policy measures.

Econometric models of earnings covariance structures are widely used in both macroeconomics and labor economics. Macroeconomists use the estimates as inputs in dynamic stochastic general equilibrium models, where the persistence of shocks has implications for consumption and labor supply over the life cycle. Labor economists are particularly interested in the quantitative importance of the different variance components in explaining residual earnings inequality.

Covariance structure models are related to those used in factor analysis but have been developed in a distinct way by labor economists. Examples of recent research articles that fit earnings covariance structure models include Moffitt and Gottschalk (2002; 2008; 1995), Baker (1997), Baker and Solon (2003), Dickens (2000), Haider (2001), Ramos (2003), Cappellari (2004), Daly and Valletta (2008), Guvenen (2009), Gustavsson (2008), and Doris, O'Neill, and Sweetman (2010a,b).

Estimation of some covariance structure models is possible in Stata using `xtmixed`. However, `xtmixed` only allows fitting of basic models and requires strong parametric assumptions. For this reason, research in the area has favored a generalized method of moments (GMM) approach, which is more flexible. The GMM estimator was introduced into the econometrics literature by Hansen (1982); excellent surveys can be found in Hall (2005) and Cameron and Trivedi (2005).

Although Stata's `gmm` routine, in its moment-evaluator form, could be used to fit each of a number of covariance structure models on a case-by-case basis, this is relatively complicated to do—it involves writing an ado file for each specification—and there are benefits to having a specialist and integrated command that allows researchers to fit many specifications using a single common command syntax. In this article, we introduce `gmmcovearn`, a user-written program that meets this need.

The rest of the article is set out as follows. In section 2, we briefly review the general principles underlying GMM estimation, and in section 3, we consider this estimator in the context of the covariance structure of longitudinal data on earnings. Our `gmmcovearn` command is explained in section 4 and is illustrated in section 5 with panel data on earnings for the USA and for Germany. We conclude the article in section 6.

2 A review of GMM estimation

The GMM approach provides a computationally convenient method of performing inference without the need for distributional assumptions. The key to GMM estimation is a set of population moment conditions that are derived from the underlying statistical model. GMM is based on the analogy principle whereby population moment conditions are replaced by their sample analogues. This in turn provides a system of equations that form the basis for the derivation of the GMM estimator.

Suppose we have a $k \times 1$ vector m and a $p \times 1$ parameter vector θ such that for a given value θ_0 and data Y,

$$E\left\{m(Y;\theta_0)\right\} = 0$$

The GMM approach replaces the population expectation with the sample moments

$$\overline{m}\left(\theta\right) = \frac{1}{N}\sum_{i=1}^{N} m_i\left(y_i;\theta\right)$$

and chooses the value of θ that makes $\overline{m}(\theta)$ equal to or close to zero. Formally, the GMM estimator chooses the value of θ so as to minimize the criterion function

$$\overline{m}\left(\theta\right)' W_n \overline{m}\left(\theta\right)$$

where W_n is a positive-semidefinite weighting matrix that does not depend on θ.

For a just-identified model, any full-rank weighting matrix will lead to the same estimate. For over-identified models, different weighting matrices give rise to different estimators within the GMM class, all of which are consistent under regularity assumptions but differ in terms of efficiency.

The optimal GMM estimator weights by the inverse of the variance matrix of the sample moments. In practice, the optimal weighting matrix has to be estimated. Provided the estimated weighting matrix is consistent, this makes no difference asymptotically. However, estimation of the weighting matrix can lead to biases in finite samples. The problem arises because in small samples, there is a correlation between the moments and the weighting matrix.

Altonji and Segal (1996) and Clark (1996) consider the appropriate weighting matrix in relatively straightforward covariance models, and show that in finite samples the use of the identity matrix is preferable to the optimal weighting matrix. This approach has therefore become common practice when estimating the covariance structure of earnings and is adopted in `gmmcovearn` and in the remainder of this article. When the identity matrix is used, the GMM objective function resembles that of an equally weighted, nonlinear least-squares model.

The GMM estimator identifies model parameters if the probability limit of the GMM criterion function is uniquely minimized at the true parameter vector, θ_0. Stock, Wright, and Yogo (2002) provide a very useful summary of the identification issues that arise in GMM estimation, and Doris, O'Neill, and Sweetman (2010b) discuss these issues in the context of the covariance structure of earnings. Weak identification can lead to inconsistent estimates, nonnormality, and size distortions in hypothesis testing even in very large samples. However, if the model is well identified, then under suitable regularity conditions, it can be shown that the limiting distribution of the estimator $\widehat{\theta}_{\mathrm{GMM}}$ is as follows:

$$\sqrt{N}\left(\widehat{\theta}_{\mathrm{GMM}}-\theta_0\right) \xrightarrow{d} N\left\{0,\left(G_0'G_0\right)^{-1}\left(G_0'S_0G_0\right)\left(G_0'G_0\right)^{-1}\right\}$$

where $G_0 = \mathrm{plim}(1/N)\sum_{i=1}^{N}\left(\delta m_i/\delta\theta\left|_{\theta_0}\right.\right)$ and $S_0 = \mathrm{plim}(1/N)\sum_{i=1}^{N}\left(m_i m_i'\left|_{\theta_0}\right.\right)$.

In practice, G_0 and S_0 are estimated by evaluating the analytical expressions at the GMM estimate, $\widehat{\theta}_{\mathrm{GMM}}$. Hypothesis tests on individual parameters can then be carried out using a Wald test.

For more-general model specification tests, Newey's (1985) test of over-identifying restrictions, based on an adjusted residual sum of squares, can be used. However, as several authors have noted (Baker and Solon 2003; Ramos 2003; Gustavsson 2008), this test almost always rejects the specified model at conventional levels. In addition, Baker (1997) found that such specification tests have inflated size in small samples. For these reasons, this approach is not used as the basis for specification tests in this literature. However, the adjusted or unadjusted residual sum of squares is typically used as a measure of goodness of fit.

3 Estimating the covariance structure of earnings

In the literature on the covariance structure of earnings, the de-meaned logarithm of earnings of worker i at time t, y_{it}, is the sum of a permanent component (for example, due to individual-specific fixed characteristics such as the level of education) and a transitory component (reflecting temporary shocks that affect the individual directly or indirectly via the labor market). Thus in the very simplest specification,

$$y_{it} = \alpha_i + \nu_{it}$$

where α_i is the individual-specific permanent component with variance σ_α^2 and ν_{it} are serially uncorrelated transitory shocks with mean zero and variance σ_ν^2. It is assumed that $\mathrm{cov}(\alpha_i, \nu_{it}) = 0$. It follows that the total variance of (log) earnings is $\sigma_\alpha^2 + \sigma_\nu^2$ and the fraction of total inequality that is permanent is given by $\sigma_\alpha^2/\left(\sigma_\alpha^2+\sigma_\nu^2\right)$. Readers will recognize this as a simple random-effects model, estimable using `xtreg, re` or `xtmixed`. Economists have modified this specification in three main ways: 1) they have allowed the relative importance of the permanent and transitory components to change with calendar time and across birth cohorts; 2) they have allowed for persistence in transitory shocks to earnings, using a variety of autoregressive and moving-average specifications; and 3) they have allowed the permanent component of earnings to evolve over time.

When modeling the evolution of the permanent component, two main approaches have been adopted in the literature. In the first approach, individuals have common life-cycle profiles and are subject to shocks that permanently change the individual's place in the earnings distribution (Dickens 2000). This specification is interchangeably referred to as the *random walk* or *unit-root model* (Baker 1997; Ramos 2003). In the second approach, the *random growth model*, each worker has an individual-specific experience–earnings profile so that earnings growth rates vary across individuals in a systematic way (Haider 2001). In the random walk specification, current earnings are a sufficient statistic for future earnings, while in the random growth model, information in addition to current earnings (for example, initial earnings) may be informative about future earnings. For a detailed discussion and comparison of these two approaches, see Baker (1997) and Guvenen (2009). Additionally, a few articles have fit covariance models that combine the random walk model with the random growth model—see, for example, Ramos (2003) and Moffitt and Gottschalk (2008).

`gmmcovearn` allows researchers to fit models that incorporate the three key features of earnings dynamics discussed earlier: time and cohort effects in both the permanent and the transitory component; either an AR(1) or an ARMA(1,1) process for the transitory term; and a random growth or random walk model for the permanent component.

The general model we consider assumes that earnings for individual worker i, belonging to birth cohort c and with x years of experience at time t, y_{ict}, are given by

$$y_{ict} = q_c p_t \left(\alpha_i + \beta_i x_{it} + u_{it}\right) + s_c \lambda_t \nu_{it} \tag{1}$$

$$u_{it} = u_{i,(t-1)} + w_{it} \tag{2}$$

where $E(\alpha_i) = E(\beta_i) = E(w_{it}) = E(\nu_{it}) = 0$. α_i and β_i have variances σ^2_α and σ^2_β, respectively, and covariance $\sigma_{\alpha\beta}$. The first two terms inside the parentheses in (1) capture the random-growth component of earnings. Thus each individual may have a different permanent life-cycle growth rate of earnings, and this growth rate may be correlated with initial earnings. The final term inside the parentheses, u_{it}, follows a random walk process, with the variance of w_{it} given by σ^2_w and $E(u_{i,(t-1)} w_{it}) = 0$.

This process allows for random shocks that have permanent effects. The accumulated variance of the random walk process prior to entry to the labor market cannot be identified in this model and is incorporated into estimation of σ^2_α. This identification problem arises in any model with fixed effects and a unit-root process. p_t and λ_t are factor loadings that allow the permanent and transitory components, respectively, to change over time in a way that is common across individuals; q_c and s_c allow the permanent and transitory components, respectively, to differ by cohort. Thus the model allows for time, experience, and cohort effects. These parameters can be separately identified because of nonlinearities in the underlying model.

Serial correlation in the transitory shocks, ν_{it}, is modeled using either an AR(1) or an ARMA(1,1) process, with AR parameter ρ and MA parameter θ. Specifically,

$$\nu_{it} = \rho \nu_{i(t-1)} + \theta \epsilon_{i(t-1)} + \epsilon_{it} \tag{3}$$

where ϵ_{it} is a random variable with variance σ^2_ϵ. The recursive nature of the transitory process necessitates consideration of initial conditions. Panel datasets do not typically observe individuals from the start of their working lives, and the absence of this information has to be taken into consideration. Because working lives are not infinitely long, it is inappropriate to appeal to a long-run steady-state assumption to resolve the initial-conditions problem (which is the approach taken by `xtmixed` when fitting AR and MA specifications for the transitory component). Instead, we follow the approach suggested by MaCurdy (1982), which has been widely adopted by labor economists, and treat the variance at the start of our sample period, $\sigma^2_{\nu 1}$, as an additional parameter to be estimated.

The GMM estimator matches sample variances and covariances to their population counterparts. In the model specified by equations (1)–(3), the true variance–covariance matrix for cohort c has diagonal elements

$$\sigma^2_{c1} = \left\{q^2_c p^2_1 \left(\sigma^2_\alpha + \sigma^2_\beta X^2_{c1} + 2\sigma_{\alpha\beta} X_{c1} + \sigma^2_w X_{c1}\right)\right\} + \left(s^2_c \lambda^2_1 \sigma^2_{\nu 1}\right) \tag{4}$$

$$\begin{aligned}\sigma^2_{ct} = &\left\{q^2_c p^2_1 \left(\sigma^2_\alpha + \sigma^2_\beta X^2_{ct} + 2\sigma_{\alpha\beta} X_{c1} + \sigma^2_w X_{ct}\right)\right\} + \\ &\left\{s^2_c \lambda^2_t \left(\rho^{2t-2}\sigma^2_{\nu 1} + K \sum_{w=0}^{t-2} \rho^{2w}\right)\right\}, t > 1\end{aligned} \tag{5}$$

and off-diagonal elements

$$\begin{aligned}\mathrm{Cov}\left(y_{ct}, y_{c(t+s)}\right) = &q^2_c p_t p_{t+s} \left\{\sigma^2_\alpha + \sigma^2_\beta X_{ct} X_{c(t+s)} + \sigma_{\alpha\beta}\left(X_{ct} + X_{c(t+s)}\right) + \sigma^2_w X_{ct}\right\} + \\ &s^2_c \lambda_t \lambda_{t+s} \left(\rho^s \sigma^2_{\nu 1} + \rho^{s-1}\theta\sigma^2_\epsilon\right), t = 1 \text{ and } s > 0 \\ \mathrm{Cov}\left(y_{ct}, y_{c(t+s)}\right) = &q^2_c p_t p_{t+s} \left\{\sigma^2_\alpha + \sigma^2_\beta X_{ct} X_{c(t+s)} + \sigma_{\alpha\beta}\left(X_{ct} + X_{c(t+s)}\right) + \sigma^2_w X_{ct}\right\} + \\ &s^2_c \lambda_t \lambda_{t+s} \left(\rho^{2t+s-2}\sigma^2_{\nu 1} + \rho^2 K \sum_{w=0}^{t-1} \rho^{2w} + \rho^{s-1}\theta\sigma^2_\epsilon\right), t > 1 \text{ and } s > 0\end{aligned} \tag{6}$$

where $K = \sigma^2_\epsilon(1 + \theta^2 + 2\rho\theta)$, X_{ct} is the average experience of individuals in cohort c at time t, and X^2_{ct} is the average value of experience-squared for cohort c at time t.

The parameter vector to be estimated is given by
$\varphi = \left\{\sigma^2_\alpha, \rho, \sigma^2_{\nu 1}, \sigma^2_\epsilon, \lambda_2 \dots \lambda_T, p_2 \dots p_T, q_2 \dots q_C, s_2 \dots s_C, \sigma^2_\beta, \sigma_{\alpha\beta}, \sigma^2_w, \theta\right\}$. Identification requires a normalization of the factor loadings and in keeping with the literature, we set p_1, λ_1, q_1, and s_1 equal to one. We then use this parameter vector to recover the individual components of aggregate inequality. The permanent component at time t is given by the first term in curly braces in (4) or (5), as appropriate, while the second term in curly braces is the transitory component.

The model given in (1)–(3) above encompasses many, but not all, of the models that have been used in the empirical literature. Firstly, the program does not allow for heteroskedasticity in the specified variances, as used, for example, by Baker (1997) and Hoffmann (2009). However, because of identification issues, such specifications have been used only with extremely large administrative datasets, which are typically not publically available. Secondly, earlier work by MaCurdy (1982) and Abowd and Card (1989) found that an MA(2) specification best fits the covariance matrix of earnings differences. While `gmmcovearn` cannot fit this model directly, our random walk model in levels with an ARMA(1,1) transitory term implies an ARMA(1,2) model in differences (Moffitt and Gottschalk 1995). In practice, the estimated value of the AR parameter is such that the difference between this and the MaCurdy and Abowd–Card specifications is small.

4 The gmmcovearn command

`gmmcovearn` estimates the parameters of the covariance structure of earnings. The program requires that the panel dataset be in wide format and contain an earnings variable. This variable may be specified in logs or in levels and may refer to actual earnings or to residuals derived from a first-stage regression on a set of observed covariates. If a heterogeneous profiles model is specified (random growth or random walk), the data must also include a labor-market experience variable. For models with cohort effects, the data must contain a cohort indicator variable, and there must be data for every cohort in each year. The program works for both balanced and unbalanced data at the individual level.

The program begins by computing the earnings variances and covariances from the raw data. When the data are unbalanced, each sample moment is constructed using all the available observations for that moment. For each moment expression, average experience (and average-squared experience) is calculated using only the information on individuals who contribute to that moment. The program uses the sample moments; the population expressions given in (4), (5), and (6); and Stata's `nl` command to recover the parameter estimates. As mentioned earlier, this implies that the weighting matrix used is the identity matrix. The sample moments are provided as part of the program's output. The program adjusts the standard errors of the parameter estimates to take account of the number of observations used in the computation of each moment, following the approach suggested by Haider (2001); this yields valid standard errors whether the data are balanced or unbalanced.

4.1 Syntax for gmmcovearn

The syntax for `gmmcovearn` is as follows:

`gmmcovearn` *earningsvar* [*if*], `modeln(`#`)` `yearn(`#`)` [`expvar(`*exp*`)` `firstyr(`#`)`
`cohortn(`#`)` `cohortvar(`*cohvarname*`)` `firstcohort(`#`)` `cohvarname`
`stvalue(`*start_values*`)` `newdataname(`*momdataname*`)` `graph(`#`)`]

where *earningsvar* is the stub of the name of the earnings variable in the dataset. The numeric suffix attached to the stub identifies the year (or panel wave) of the earnings variable. The program assumes that these suffixes are consecutive integers running from `firstyr()` through `firstyr()` + `yearn()` − 1, where `firstyr()` is the year associated with the first year observed in the dataset and `yearn()` is the total number of years observed (see below).

4.2 Options

`modeln(`#`)` specifies the type of model to be fit. The default is `modeln(1)`. `modeln()` is required.

`modeln(1)`: AR(1), no growth heterogeneity ($\sigma_\beta^2 = \sigma_{\alpha\beta} = \sigma_w^2 = \theta = 0$)

`modeln(2)`: ARMA(1,1), no growth heterogeneity ($\sigma_\beta^2 = \sigma_{\alpha\beta} = \sigma_w^2 = 0$)

`modeln(3)`: AR(1), random growth ($\sigma_w^2 = \theta = 0$)

`modeln(4)`: ARMA(1,1), random growth ($\sigma_w^2 = 0$)

`modeln(5)`: AR(1), random walk ($\sigma_\beta^2 = \sigma_{\alpha\beta} = \theta = 0$)

`modeln(6)`: ARMA(1,1), random walk ($\sigma_\beta^2 = \sigma_{\alpha\beta} = 0$)

`modeln(7)`: AR(1), combined random growth and random walk ($\theta = 0$)

`modeln(8)`: ARMA(1,1), combined random growth and random walk

`yearn(`#`)` specifies the total number of years over which earnings are observed. `yearn()` is required.

`expvar(`*exp*`)` specifies *exp* as the stub of the name of the experience variable and must be used for specifications that allow for growth heterogeneity in the life-cycle earnings profile (`modeln(3)`–`modeln(8)`). In the dataset, these suffixes must follow the same convention as for *earningsvar*, described above.

`firstyr(`#`)` specifies the numeric suffix attached to the first year of earnings data observed in the sample. The default is `firstyr(1)`.

`cohortn(`#`)` specifies the number of cohorts used for the analysis. The default is `cohort(1)`.

`cohortvar(`*cohvarname*`)` identifies *cohvarname* as the variable that distinguishes the different cohorts. The default is `cohortvar(cohort)`.

`firstcohort(`#`)` specifies the value of `cohvarname` for the first cohort. The default is `firstcohort(1)`.

`cohvarname` is assumed to be coded in consecutive integers from `firstcohort()` to `firstcohort()` + `cohortn()` − 1. For example, in a model with four cohorts, the values of `cohvarname` could be 1 to 4 (in which case `firstcohort()` is 1) or 1994 to

1997 (in which case `firstcohort()` is 1994). Nonconsecutive suffixes, such as 1960, 1970, 1980, and 1990, would have to be recoded before being used.

`stvalue(`*start_values*`)` specifies the starting values for the estimation. For T years of data and C cohorts, the values are entered in the following order, separated by commas: `sigalpha`, `rho`, `sigv1`, `sige`, `l2`−`lT`, `p2`−`pT`, `q2`−`qC`, `s2`−`sC`, `sigbeta`, `covalphabeta`, `sigw`, `theta`, corresponding to parameters σ^2_α, $\rho, \sigma^2_{\nu 1}$, σ^2_ϵ, $\lambda_2 \dots \lambda_T$, $p_2 \dots p_T$, $q_2 \dots q_C$, $s_2 \dots s_C$, σ^2_β, $\sigma_{\alpha\beta}$, σ^2_w, and θ.

The default value for each of the `l`, `p`, `q`, and `s` parameters is 1; for `sigalpha` and `rho`, it is 0.5; for `sigv1` and `sige`, it is 0.1; for `sigbeta`, `covalphabeta`, and `sigw`, it is 0; and for `theta`, it is −0.5. The user should specify starting values only for the parameters actually estimated in the chosen model (see `modeln()` above).

`newdataname(`*momdataname*`)` allows the user to create the new dataset *momdataname* consisting of two or more variables, depending on the model used. The first variable, `moment`, contains the sample moments calculated by the program. The second variable, `nobsmoment`, contains the number of observations used in calculating the corresponding moment. If a random growth or a random walk model is specified (`modeln(3)`−`modeln(8)`), the dataset will also contain the average of experience, `aveexp`, and the average of squared experience, `aveexp2`.

For the `moment` variable, the first T observations are the T sample variances for the first cohort, the next T observations are the variances for the second cohort, and so on for each of the C cohorts. Starting at the $(T \times C + 1)$th element, the next $(T-1)$ elements are the $(T-1)$ first-order covariances for the first cohort, beginning with the earnings covariance between the first and second year. The next $(T-1)$ elements refer to the first-order covariances for the second cohort, and so on. This pattern is repeated for the higher-order covariances so that the final C observations are the $C(T-1)$th-order covariances.

`graph(`*#*`)` requests a graphical display of the predicted permanent and transitory components of inequality [calculated using (4) and (5)], along with predicted and actual aggregate inequality. Predicted aggregate inequality is simply the sum of the predicted permanent and transitory components. The default is `graph(0)`. The data underlying these graphs are available in the saved results (see below).

4.3 Saved results

`gmmcovearn` saves the following in `e()`:

Scalars

`e(numoment)`	number of moment conditions used in fitting the model

Macros

`e(cmd)`	`gmmcovearn`
`e(cmdline)`	command as typed
`e(properties)`	`b V`

Matrices

`e(b)`	coefficient vector: the parameters of the covariance structure model
`e(V)`	adjusted variance–covariance matrix of the parameters
`e(momentc)`	sample moments for the earnings variable for cohort c, $c =$ `firstcohort()`,...,`firstcohort()` + `cohortn()`−1 (these are presented in a variance–covariance matrix for each cohort)
`e(nobsc)`	observations used in calculating each of the sample moments for cohort c, $c =$ `firstcohort()`,...,`firstcohort()` + `cohortn()`−1 (these are presented in the same format as the corresponding variance–covariance matrices)
`e(permc)`	predicted permanent component of earnings variance in each of the `yearn()` years for cohort c, $c =$ `firstcohort()`,...,`firstcohort()` + `cohortn()`−1
`e(tempc)`	predicted transitory component of earnings variance in each of the `yearn()` years for cohort c, $c =$ `firstcohort()`,...,`firstcohort()` + `cohortn()`−1

5 Examples

In this section, we use `gmmcovearn` to analyze the covariance structure of earnings with panel datasets for the USA and Germany. The examples are for illustrative purposes only. The example for the USA uses publically available data and illustrates the use of our estimator when the data are provided in long format. The German example estimates a random growth model of earnings and also shows how cohort effects are accounted for in our estimation procedure. Both examples use unbalanced data.

5.1 National Longitudinal Survey (NLS) data

In this example, we use the NLS panel dataset used in Wooldridge (2002) and available for download within Stata. The dataset provides an unbalanced panel of data on earnings, schooling, and demographic information for 530 individuals from the NLS for the years 1981 to 1987.

```
. use http://www.stata.com/data/jwooldridge/eacsap/nls81_87.dta
```

To begin, we fit a random-effects model with an AR(1) transitory error term, `modeln(1)`, similar to that fit by Lillard and Weiss (1978). A simpler version of this model could be fit using `xtmixed`; however, `xtmixed` uses a steady-state assumption to handle initial conditions, which is unlikely to be appropriate, as discussed in section 3. In contrast, `gmmcovearn` follows the approach suggested by MaCurdy (1982) and treats the variance at the start of our sample period, $\sigma^2_{\nu 1}$, as an additional parameter to be estimated.

To fit this model with `gmmcovearn`, we first `reshape` the data into long format:

```
. keep id year lwage
. reshape wide lwage, i(id) j(year)
  (output omitted)
```

To illustrate the naming conventions used in our program, we next `describe` the variables in this dataset:

```
. describe
Contains data
  obs:           530
 vars:             8
 size:        15,900

                storage  display     value
variable name    type    format      label      variable label

id              int     %9.0g                  identifier
lwage81         float   %9.0g                  81 lwage
lwage82         float   %9.0g                  82 lwage
lwage83         float   %9.0g                  83 lwage
lwage84         float   %9.0g                  84 lwage
lwage85         float   %9.0g                  85 lwage
lwage86         float   %9.0g                  86 lwage
lwage87         float   %9.0g                  87 lwage

Sorted by:  id
```

This dataset contains seven years of earnings data, from 1981 to 1987. The program requires the user to input *earningsvar*, which is the stub of the name of the earnings variable. In this example, *earningsvar* is `lwage`. In these data, the numeric suffix attached to `lwage` refers to the year of observation. The lowest suffix is `firstyr()`, the numeric year attached to the first year of earnings data, which in this case is 81.

A summary of the variables used is provided below:

```
. summarize
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
          id |       530    991.3925    660.7559          4       2229
     lwage81 |       242    9.388543    .5187844    6.71772     11.503
     lwage82 |       261    9.457249    .4130265   7.855649   10.69415
     lwage83 |       312    9.498774    .4851994   6.579612   11.12857
     lwage84 |       349    9.518088    .4505686   7.406881   10.80471
-------------+--------------------------------------------------------
     lwage85 |       359    9.551186     .506268   6.031743   10.79433
     lwage86 |       361    9.643893    .5299772   7.141506   11.53664
     lwage87 |       379    9.704476    .5805771   7.841583   12.80161
```

This table clearly illustrates the unbalanced nature of the data. For example, there are 242 observations used to calculate the sample variance for 1981 but 379 observations used to calculate the final-year sample variance. To see the number of observations that will be used to calculate the covariance of earnings between 1981 and 1982, we can issue the `count` command, as follows:

```
. count if lwage81~=. & lwage82~=.
  193
```

As we will see later, all information on the number of observations used can be recovered from the program's saved results. The `gmmcovearn` command is issued as follows:

```
. gmmcovearn lwage, yearn(7) modeln(1) firstyr(81)
(obs = 28)
Iteration 0:  residual SS =  .1643298
Iteration 1:  residual SS =    .05161
Iteration 2:  residual SS =  .0017779
Iteration 3:  residual SS =  .0016158
Iteration 4:  residual SS =   .001615
Iteration 5:  residual SS =   .001615
Iteration 6:  residual SS =   .001615
Iteration 7:  residual SS =   .001615
Iteration 8:  residual SS =   .001615
Iteration 9:  residual SS =   .001615
      Source |       SS       df       MS
-------------+------------------------------         Number of obs =        28
       Model |  .796840451    16  .049802528         R-squared     =    0.9980
    Residual |  .001614961    12   .00013458         Adj R-squared =    0.9953
-------------+------------------------------         Root MSE      =  .0116009
       Total |  .798455413    28  .028516265         Res. dev.     = -193.8376

------------------------------------------------------------------------------
      moment |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   /sigalpha |   .0683058   .0088071     7.76   0.000     .0491168    .0874948
        /rho |   .3130349   .0608686     5.14   0.000     .1804137    .4456561
      /sigv1 |    .201089   .0144893    13.88   0.000     .1695194    .2326586
       /sige |   .0588356   .0375667     1.57   0.143    -.0230152    .1406864
        /l2  |   1.209775   .3380594     3.58   0.004     .4732069    1.946343
        /l3  |   1.497133    .496198     3.02   0.011     .4160105    2.578256
        /l4  |   1.142064   .3835471     2.98   0.012     .3063868    1.977742
        /l5  |   1.317238   .4160183     3.17   0.008     .4108118    2.223664
        /l6  |   1.438042    .464473     3.10   0.009     .4260424    2.450042
        /l7  |   1.706241   .5657103     3.02   0.011     .4736643    2.938818
        /p2  |   .9159306   .0803285    11.40   0.000     .7409098    1.090951
        /p3  |   1.112308   .0998032    11.15   0.000      .894856    1.329761
        /p4  |   1.307378   .1214745    10.76   0.000     1.042708    1.572048
        /p5  |   1.449588   .1411007    10.27   0.000     1.142156     1.75702
        /p6  |   1.466273   .1388268    10.56   0.000     1.163796    1.768751
        /p7  |   1.470464   .1267276    11.60   0.000     1.194348     1.74658
------------------------------------------------------------------------------
 Estimated parameters with corrected standard errors below
```

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
sigalpha	.0683058	.024722	2.76	0.006	.0198516	.1167599
rho	.3130349	.077011	4.06	0.000	.1620961	.4639736
sigv1	.201089	.0402344	5.00	0.000	.122231	.279947
sige	.0588356	.0456725	1.29	0.198	-.0306809	.1483521
l2	1.209775	.4187484	2.89	0.004	.3890433	2.030507
l3	1.497133	.6406559	2.34	0.019	.2414706	2.752796
l4	1.142064	.4497494	2.54	0.011	.2605716	2.023557
l5	1.317238	.5380952	2.45	0.014	.2625906	2.371885
l6	1.438042	.5672408	2.54	0.011	.3262706	2.549814
l7	1.706241	.6850775	2.49	0.013	.3635139	3.048968
p2	.9159306	.1452523	6.31	0.000	.6312413	1.20062
p3	1.112308	.2170894	5.12	0.000	.6868211	1.537796
p4	1.307378	.237724	5.50	0.000	.8414475	1.773308
p5	1.449588	.2657075	5.46	0.000	.9288111	1.970366
p6	1.466273	.270121	5.43	0.000	.9368458	1.995701
p7	1.470464	.3122724	4.71	0.000	.8584214	2.082507

The output header includes the standard information on goodness of fit produced by the `nl` command. This includes the residual sum of squares (unadjusted) which, as noted earlier, is sometimes used as a measure of fit in the empirical literature.

The parameters `sigalpha`, `rho`, `sigv1`, `sige`, `l2`–`l7`, and `p2`–`p7` correspond to σ^2_α, ρ, $\sigma^2_{\nu 1}$, σ^2_ϵ, $\lambda_2 - \lambda_7$, and $p_2 - p_7$. The result for `rho` indicates moderate persistence in the transitory shock; the factor loadings `l2`–`l7` and `p2`–`p7` indicate rising transitory and permanent variances over time.

The standard errors reported in the top panel of the output are incorrect: they fail to take into account the number of individuals used when calculating the sample moments. Correctly adjusted standard errors are reported in the bottom panel. The corresponding adjusted variance–covariance matrix is saved in `e(V)`.

Hypothesis tests can be carried out using `test` after running the `gmmcovearn` command. For example, a test that the permanent factor loadings, p_t, are constant over time can be carried out using a Wald test, as follows:

```
. test _b[p2]=_b[p3]=_b[p4]=_b[p5]=_b[p6]=_b[p7]=1
 ( 1)  p2 - p3 = 0
 ( 2)  p2 - p4 = 0
 ( 3)  p2 - p5 = 0
 ( 4)  p2 - p6 = 0
 ( 5)  p2 - p7 = 0
 ( 6)  p2 = 1
           chi2(  6) =   12.14
         Prob > chi2 =    0.0588
```

In this example, we reject constant permanent factor loadings at the 6% significance level.

To illustrate the saved results from `gmmcovearn`, we use `ereturn list`.

```
. ereturn list
scalars:
          e(numoment) =  28
macros:
               e(cmd) : "gmmcovearn"
           e(cmdline) : "lwage, yearn(7) modeln(1) firstyr(81)"
        e(properties) : "b V"
matrices:
                 e(b) :  1 x 16
                 e(V) :  16 x 16
             e(temp1) :  7 x 1
             e(perm1) :  7 x 1
             e(nobs1) :  7 x 7
           e(moment1) :  7 x 7
```

As noted above, `gmmcovearn` begins by computing the earnings covariances from the raw data. Because we have only one cohort in this dataset, there is only one sample covariance matrix saved in `e(moment1)`.

```
. matrix list e(moment1)
symmetric e(moment1)[7,7]
           81          82          83          84          85          86          87
81  .26913726
82  .14437909  .17059092
83  .08859929  .11214142  .23541845
84  .12305372  .09880327  .13358462  .20301206
85  .09517703  .08657077  .13002885  .15184436  .25630733
86  .10260867  .09372365  .11757618  .13149989   .1893622  .28087588
87   .0913199  .09370207  .11586105  .13049657  .16674991  .19970039  .33706979
```

The number of observations reported in the header of the output table for `gmmcovearn` refers to the number of moment conditions used in the analysis and not the number of individuals used in the estimation. The number of individuals contributing to each of the sample moments is saved in `e(nobs1)`.

```
. matrix list e(nobs1)
symmetric e(nobs1)[7,7]
     81   82   83   84   85   86   87
81  242
82  193  261
83  206  229  312
84  209  232  282  349
85  213  227  274  300  359
86  206  219  265  290  315  361
87  212  230  270  302  314  315  379
```

These sample sizes are used in the calculation of the corrected variance–covariance matrix for our parameter estimates, which are saved in `e(V)`.

The components of the variance decomposition are saved in `e(perm1)`, which contains the permanent component of inequality, and `e(temp1)`, which is the transitory component. To recover the permanent components, we simply type

```
. matrix list e(perm1)

e(perm1)[7,1]
                    perm
cohort1year1  .06830577
cohort1year2  .05730368
cohort1year3  .08450995
cohort1year4  .11675074
cohort1year5  .14353134
cohort1year6  .14685449
cohort1year7  .14769514
```

The results reported above indicate a steady increase in permanent inequality over this time period.

5.2 German earnings data

To illustrate the use of `gmmcovearn` for a more complicated model that includes cohort effects, we use a data extract from the eight waves of the European Community Household Panel for Germany. The years covered by the survey are 1994–2001. As in the previous example, the data are unbalanced. The earnings variable for each year is the residual from a first-stage regression of earnings on potential experience and potential experience squared. The earnings variables are `yi1994`–`yi2001` and the individual experience variables are `potexp1994`–`potexp2001`. In the dataset, the cohort indicator variable is `birthcoh`, which takes the values 1–4 corresponding to the four cohorts in the data. The number of individuals in each cohort is given below.

```
. tab birthcoh

   birthcoh |      Freq.     Percent        Cum.
------------+-----------------------------------
          1 |      1,650       24.41       24.41
          2 |      1,109       16.41       40.82
          3 |      1,547       22.89       63.71
          4 |      2,453       36.29      100.00
------------+-----------------------------------
      Total |      6,759      100.00
```

The following shows the estimates of a random growth model with cohorts and an AR(1) specification for the transitory error term.

```
. gmmcovearn yi, yearn(8) modeln(3) cohortn(4) expvar(potexp) firstyr(1994)
> cohortvar(birthcoh)
(obs = 144)

Iteration 0:  residual SS =  .4483834
Iteration 1:  residual SS =  .0186083
Iteration 2:  residual SS =  .0086681
Iteration 3:  residual SS =  .0070441
Iteration 4:  residual SS =  .0070062
Iteration 5:  residual SS =  .0070061
Iteration 6:  residual SS =  .0070061
Iteration 7:  residual SS =  .0070061
Iteration 8:  residual SS =  .0070061
```

Source	SS	df	MS
Model	2.16612446	26	.083312479
Residual	.007006143	118	.000059374
Total	2.17313061	144	.015091185

Number of obs	=	144
R-squared	=	0.9968
Adj R-squared	=	0.9961
Root MSE	=	.0077055
Res. dev.	=	-1021.378

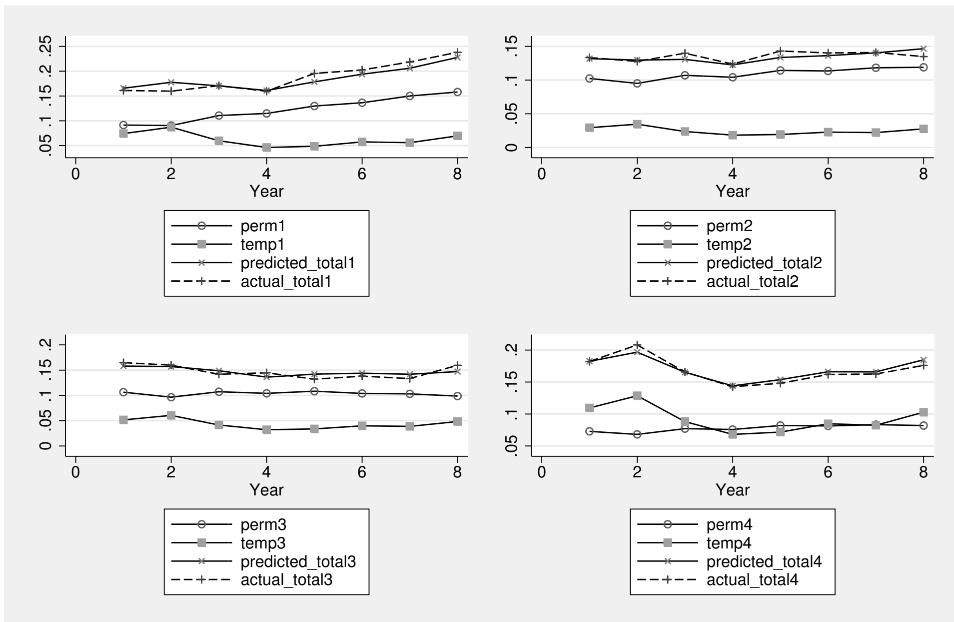
moment	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
/sigalpha	.4386704	.0605217	7.25	0.000	.318821	.5585198
/rho	.3417541	.0361788	9.45	0.000	.2701102	.4133979
/sigv1	.0742456	.0062163	11.94	0.000	.0619355	.0865556
/sige	.0301694	.0112735	2.68	0.009	.0078448	.0524939
/l2	1.498371	.2427683	6.17	0.000	1.017624	1.979118
/l3	1.312379	.2569202	5.11	0.000	.8036072	1.821151
(output omitted)						
/sigbeta	.0003872	.0000489	7.92	0.000	.0002904	.0004841
/covalphab~a	-.012158	.001729	-7.03	0.000	-.0155819	-.0087341

Parameters with corrected standard errors below

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
sigalpha	.4386704	.1706614	2.57	0.010	.1041801	.7731606
rho	.3417541	.0361377	9.46	0.000	.2709254	.4125827
sigv1	.0742456	.0118533	6.26	0.000	.0510135	.0974776
sige	.0301694	.0104324	2.89	0.004	.0097223	.0506164
l2	1.498371	.2208264	6.79	0.000	1.065559	1.931183
l3	1.312379	.2312379	5.68	0.000	.8591613	1.765597
l4	1.162531	.1944341	5.98	0.000	.7814474	1.543615
l5	1.193413	.1942618	6.14	0.000	.8126667	1.574159
l6	1.297056	.2167269	5.98	0.000	.8722795	1.721833
l7	1.279366	.2119609	6.04	0.000	.8639299	1.694801
l8	1.428739	.2584156	5.53	0.000	.9222535	1.935224
p2	.9842998	.0313717	31.38	0.000	.9228123	1.045787
p3	1.07534	.0433522	24.80	0.000	.9903711	1.160309
p4	1.08418	.050535	21.45	0.000	.9851328	1.183226
p5	1.148706	.0694794	16.53	0.000	1.012529	1.284883
p6	1.168077	.0789378	14.80	0.000	1.013362	1.322793
p7	1.205651	.0887034	13.59	0.000	1.031796	1.379507
p8	1.219382	.0922591	13.22	0.000	1.038558	1.400207
q2	.989772	.111584	8.87	0.000	.7710713	1.208473
q3	.7322496	.1311763	5.58	0.000	.4751487	.9893505
q4	.4866869	.1032265	4.71	0.000	.2843667	.689007
s2	.6293472	.0628975	10.01	0.000	.5060703	.7526241
s3	.8336288	.0684399	12.18	0.000	.6994891	.9677684
s4	1.214287	.0917179	13.24	0.000	1.034523	1.394051
sigbeta	.0003872	.0001758	2.20	0.028	.0000426	.0007319
covalphabeta	-.012158	.005614	-2.17	0.030	-.0231613	-.0011547

The parameters `sigalpha` to `covalphabeta` correspond to σ^2_α, ρ, $\sigma^2_{\nu 1}$, σ^2_ϵ, $\lambda_2 \dots \lambda_8$, $p_2 \dots p_8$, $q_2 \dots q_4$, $s_2 \dots s_4$, σ^2_β, and $\sigma_{\alpha\beta}$. The cohort loadings show that the youngest cohort (`birthcoh` $= 4$) has the lowest permanent variance and the highest transitory variance. The variance of the random growth parameter, `sigbeta`, is significant, which supports the heterogeneous growth-profiles model. The covariance between the random effect and the random growth parameter, `covalphabeta`, is negative and significant, indicating that those with lower initial earnings have higher earnings growth rates.

Adding `graph(1)` as an option in the above command returns a graph of the predicted transitory and permanent components of inequality [calculated using (4) and (5)] as well as the predicted and actual aggregate inequality, as shown below.



The data for the predicted permanent and transitory components used in these graphs are saved in `e(perm1)`–`e(perm4)` and `e(temp1)`–`e(temp4)`.

Because researchers in this area sometimes report convergence problems, it is good practice for users to experiment with a range of starting values by using the `stvalue()` option to check the robustness of the reported parameter estimates. In the following example, we use a starting value of ρ of 0.9, which is far away from our previous parameter estimate. In this example, the model quickly converges to point estimates that are practically identical to those reported earlier.

```
gmmcovearn yi, yearn(8) modeln(3) cohortn(4) firstyr(1994) cohortvar(birthcoh)
> expvar(potexp)
> stvalue(.5,.9,.1,.1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,.01,-.01)
(obs = 144)

Iteration 0:  residual SS =  5.183927
Iteration 1:  residual SS =  .6704438
Iteration 2:  residual SS =  .3053995
Iteration 3:  residual SS =  .0147202
Iteration 4:  residual SS =  .0079693
Iteration 5:  residual SS =  .0070087
Iteration 6:  residual SS =  .0070062
```

```
Iteration 7:  residual SS =  .0070061
Iteration 8:  residual SS =  .0070061
Iteration 9:  residual SS =  .0070061
Iteration 10:  residual SS =  .0070061

      Source |       SS       df       MS
-------------+------------------------------         Number of obs =       144
       Model |  2.16612446    26  .083312479         R-squared     =    0.9968
    Residual |  .007006143   118  .000059374         Adj R-squared =    0.9961
-------------+------------------------------         Root MSE      =  .0077055
       Total |  2.17313061   144  .015091185         Res. dev.     = -1021.378

------------------------------------------------------------------------------
      moment |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
   /sigalpha |   .4386703   .0605215     7.25   0.000     .3188213    .5585194
        /rho |   .3417535   .0361787     9.45   0.000     .2701098    .4133973
      /sigv1 |   .0742456   .0062163    11.94   0.000     .0619356    .0865557
       /sige |   .0301693   .0112735     2.68   0.009     .0078446     .052494
         /l2 |   1.498373   .2427693     6.17   0.000     1.017624    1.979122
         /l3 |   1.312381   .2569215     5.11   0.000     .8036066    1.821156
  (output omitted)
    /sigbeta |   .0003872   .0000489     7.92   0.000     .0002904    .0004841
/covalphab~a |   -.012158    .001729    -7.03   0.000    -.0155819   -.0087341
------------------------------------------------------------------------------

Parameters with corrected standard errors below
------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    sigalpha |   .4386703   .1706613     2.57   0.010     .1041804    .7731602
         rho |   .3417535   .0361377     9.46   0.000      .270925     .412582
       sigv1 |   .0742456   .0118533     6.26   0.000     .0510136    .0974777
        sige |   .0301693   .0104324     2.89   0.004     .0097221    .0506165
          l2 |   1.498373    .220828     6.79   0.000     1.065558    1.931188
          l3 |   1.312381   .2312394     5.68   0.000     .8591603    1.765602
          l4 |   1.162533   .1944351     5.98   0.000     .7814469    1.543618
          l5 |   1.193415   .1942636     6.14   0.000     .8126652    1.574165
          l6 |   1.297058   .2167281     5.98   0.000     .8722788    1.721837
          l7 |   1.279368   .2119622     6.04   0.000     .8639294    1.694806
          l8 |   1.428741   .2584167     5.53   0.000     .9222536    1.935228
          p2 |   .9842999   .0313717    31.38   0.000     .9228124    1.045787
          p3 |    1.07534   .0433522    24.80   0.000     .9903712    1.160309
          p4 |    1.08418    .050535    21.45   0.000     .9851329    1.183226
          p5 |   1.148706   .0694793    16.53   0.000     1.012529    1.284883
          p6 |   1.168077   .0789377    14.80   0.000     1.013362    1.322793
          p7 |   1.205651   .0887032    13.59   0.000     1.031796    1.379506
          p8 |   1.219382    .092259    13.22   0.000     1.038558    1.400207
          q2 |   .9897719    .111584     8.87   0.000     .7710713    1.208473
          q3 |   .7322497   .1311767     5.58   0.000     .4751481    .9893513
          q4 |   .4866869   .1032268     4.71   0.000     .2843661    .6890078
          s2 |   .6293474   .0628975    10.01   0.000     .5060705    .7526243
          s3 |   .8336278   .0684398    12.18   0.000     .6994883    .9677673
          s4 |   1.214286   .0917179    13.24   0.000     1.034522     1.39405
     sigbeta |   .0003872   .0001758     2.20   0.028     .0000426    .0007319
covalphabeta |   -.012158    .005614    -2.17   0.030    -.0231613   -.0011547
------------------------------------------------------------------------------
```

6 Conclusion

Models of the earnings covariance structure are widely used in both labor economics and macroeconomics. The most common approach to fitting these models entails the use of GMM. However, there is no routine that allows for easy and fast estimation of these models. `gmmcovearn` is a user-written Stata program that is designed to meet this need.

The program first computes earnings variances and covariances from the raw data. These sample moments, combined with appropriate population expressions are then used with Stata's `nl` command to recover the parameter estimates. The program uses these parameter estimates to decompose aggregate inequality into its permanent and transitory components and provides a graphical display of this decomposition.

The program has a number of attractive features. First, it is not written for one specific data structure; it allows for balanced and unbalanced data and has flexibility with respect to the number of time periods and the number of cohorts. In addition, a wide range of models can be fit, covering the majority of those used in the empirical literature. Moreover, it calculates standard errors that are correct for the given data structure. The program also allows the user to easily experiment with alternative starting values, which may be important in practice. These features combine to facilitate easy and fast estimation of earnings covariance models.

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