# Optimal Damping Profile for a Heaving Buoy Wave Energy Converter

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**Abstract:** This paper discusses optimal damping profiles for a heaving buoy Wave Energy Converter (WEC) with a single degree of freedom. The goal is to examine how the device can be controlled to harvest maximum energy from incident waves. Both latching and declutching strategies are allowed via a general parametrization of the damping force. Ultimately, the research attempts to determine the best control strategy to apply considering the relative resonant frequency of the device and the monochromatic wave frequency set.

*Keywords:* Wave energy; Heaving buoy; Latching; Declutching; Optimization.

## 1. INTRODUCTION

When the excitation force of the waves hitting a heaving buoy WEC is in phase with the velocity of the device, the energy captured is maximum (Falnes (2002)). Nevertheless, this phenomenon of resonance rarely occurs in real sea conditions. Consequently, to enhance the energy extracted, some control method have been investigated.
Among several strategies suggested since the past three decades, latching and declutching are categorized as phase control method. Latching was originally observed by Budal and Falnes (1975). Further work has been done by a variety of researchers (Falnes and Lillebekken (2003); Babarit *et al.* (2004); Korde (2002); Wright *et al.* (2003); Greenhow and White (1997)).
Declutching was introduced later on by Salter *et al.* (2002) and also by Wright *et al.* (2003), under the label of 'free-wheeling', while Babarit *et al.* (2009) has explored this control strategy in detail.
Control is not only made difficult by the randomness of the waves but also by the wave-device interaction as a process with memory. To overcome this challenge, control should be considered as a crucial issue to improve the efficiency of a point absorber.
To date, although several papers examine separately both latching and declutching, no one can reasonably state which strategy would be the most appropriate for a WEC and under which conditions. The following study aims to confront a broad range of damping profiles permitted by our parametrization including both latching and declutching. At the end, this paper aims to find out which damping profile is the most favorable in terms of capture of energy for each monochromatic excitation force frequency experienced.

Table 1. Hydrodynamic model parameters

| Parameter | Description |
|---|---|
| $A(\omega_w)$ | Amplitude of the excitation force |
| $B_{rad}(\omega_w)$ | Radiation damping |
| $B_{PTO}(t)$ | PTO damping |
| $K$ | Stiffness |
| $M$ | Mass of the device |
| $m_r(\omega_w)$ | Added mass |
| $\omega_w$ | Wave frequency |

## 2. MATHEMATICAL MODEL

### 2.1 Governing Equations

In this work, a cylindrical buoy, constrained to move in the vertical motion (heave motion) only, has been considered. Each term of the hydrodynamic model is described in table 1, with the dynamics governed by (1). $A(\omega_w)$, $B_{rad}(\omega_w)$, $K$, $M$ and $m_r(\omega_w)$ has been directly computed in the time domain using the seakeeping dedicated BEM code $ACHIL3D$ Clement (1997) The approach here focusses on simplifying the fully parameterized hydrodynamic model keeping the main characteristics of a realistic mathematical model. The governing equation is given by:

$$F_{ex}(t) = (M + m_r(\omega_w))\ddot{x}(t) + (B_{rad}(\omega_w) + B_{PTO}(t))\dot{x}(t) + Kx(t) \quad (1)$$

(1) can be conveniently expressed in state-space (companion) form, as:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{K}{M + m_r(\omega_w)} & -\dfrac{(B_{rad}(\omega_w) + B_{PTO}(t))}{M + m_r(\omega_w)} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{M + m_r(\omega_w)} \end{bmatrix} \quad (2)$$

(2) was directly coded using a zero-order hold (ZOH) discretization method to calculate the hydrodynamic model. During all this study, the excitation force will be assumed to be monochromatic.

$$F_{ex}(t) = A(\omega_w)\sin(\omega_w t) \tag{3}$$

where $A(\omega_w)$ is the amplitude of the excitation force at $\omega_w$.

### 2.2 Power and Energy

The Power Take-Off (PTO) device is represented by the damper. Subsequently, the power transferred to the damper $P_d$ for such a hydraulic system is given by:

$$P_d(t) = force \times velocity = B_{PTO}(t)\dot{x}(t) \times \dot{x}(t) = B_{PTO}(t)\dot{x}(t)^2 \tag{4}$$

The energy developed in the damper over a period $t_1$ is:

$$E_d(t_1) = \int_0^{t_1} P_d(t)dt = \int_0^{t_1} B_{PTO}(t)\dot{x}(t)^2 dt \tag{5}$$

The objective of our study is to maximize the energy developed in the damper. Theoretically, one can determine that energy is maximized when:

$$\omega_d = \sqrt{K/(M + m_r(\infty))} = \omega_w \tag{6}$$

where $m_r(\infty)$ is the added mass at the infinity of the device when the $\omega_w$ tends to the infinity, $\omega_d$ is the natural resonance frequency of the device and $\omega_w$ is the incident wave frequency. Under such a condition, the velocity profile of the device is in phase with the excitation force.

## 3. PARAMETRIZATION OF THE DAMPING FORCE

### 3.1 Sigmoid Parametrization

The main idea behind our study is borne by the parametrization of the damping profile $B_{PTO}$. Indeed, the choice of a general sigmoid function to evaluate the optimal loading regime is crucial. The form of the sigmoid function is detailed as followed:

$$B_{PTO}(t) = \frac{B_{max} - B_{min}}{1 + e^{(-\beta(t-t^*))}} + B_{min} \tag{7}$$

Such a function allows a certain range of possibilities for the damping profile. In particular, three well-distinguished parameterizations can be found with regards to (7): a latching-like, an invariant constant value and a declutching-like (as illustrated by fig. 1).

In order to achieve this degree of freedom within the parametrization, the four variables of (7) (see table 2) will be treated simultaneously. Fig. 1 shows that profiles of both phase control solutions are in stark contrast to each other. Latching initially employs a large damping value, effectively locking the device in position, whereas declutching initially employs a tiny damping value in order to achieve velocity build up. Since the wave energy absorbed is converted in the damping term (see (6)), the question arises as to which of these damping profiles is optimal in terms of energy conversion?

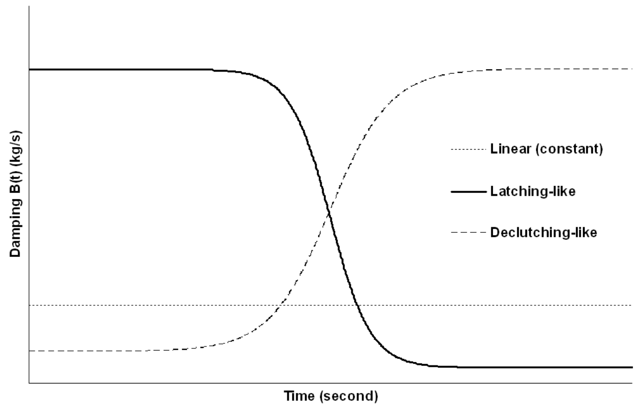


Fig. 1. Three possible damping profiles with the sigmoid

Table 2. Sigmoid parameters

| Parameter | Description |
|---|---|
| $B_{max}$ | upper boundary |
| $B_{min}$ | lower boundary |
| $\beta$ | slope |
| $t^*$ | time delay |

The sign of the parameter $\beta$ determines wether latching ($\beta$ negative) or declutching ($\beta$ positive) has to be applied. In other words, $\beta$ has to be considered as the decisive parameter for the purpose of this study.

### 3.2 Latching Control

Latching control consist of locking the motion of the body at the very moment when its velocity vanishes. Then, the body is released after the most favorable time to maintain the device motion in phase with the wave excitation force. An important number of studies has been done on latching control. In the early $80's$, Budal and Falnes (1975) introduced firstly this concept applied to a point absorber. It has been theoretically detailed by Greenhow and White (1997) as well as Eidsmoen (1995). Practical studies inspired from this work have been led by Korde (2002).

Probably the main advantage of latching remains on the no-need to deliver energy to the device while it is engaged. Practically, this can be achieved by means of a mechanical brake or open close valves on the hydraulic lines of the PTO system.

In our work, the sigmoid is implemented and synchronized on a period-by-period bias. Discrete control methods to configure latching are explored in the paper by Babarit *et al.* (2004) such as the ramps alternated maxima as well as the equal ending ramps strategies.

Here, latching is effected when $\beta$ is negative as illustrated in fig. 1. Moreover, the finite damping value reached after releasing the device, has to be chosen by considering energy absorption versus design limitations on the amplitude of the oscillation of the buoy. From Ringwood and Butler (2004), it can be proven that the choice of damping value has insignificant effect on the optimum latched time period. Thus, the single and most crucial control variable, for a point absorber system employing a latching strategy, becomes the latching duration, denoted $T_L$. Fig. 2 depicts the calculation of the latched time as a representation of the velocity and the position.

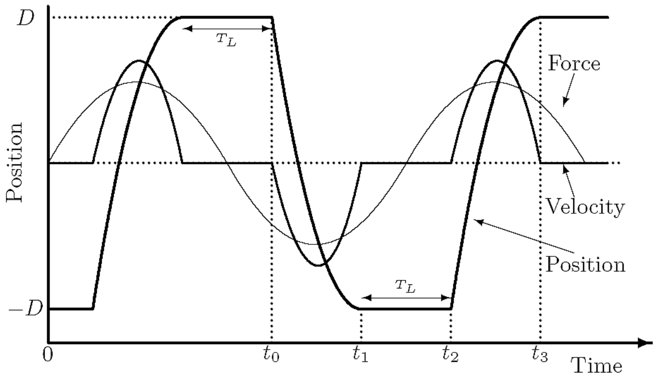


Fig. 2. Latching calculations

One can graphically evaluate the dynamic response period as (see figure 2):

$$t_1 - t_0 = t_3 - t_2 = T_\omega/2 - T_L \tag{8}$$

with $T_\omega$, the period of the incoming waves.

*3.3 Declutching Control*

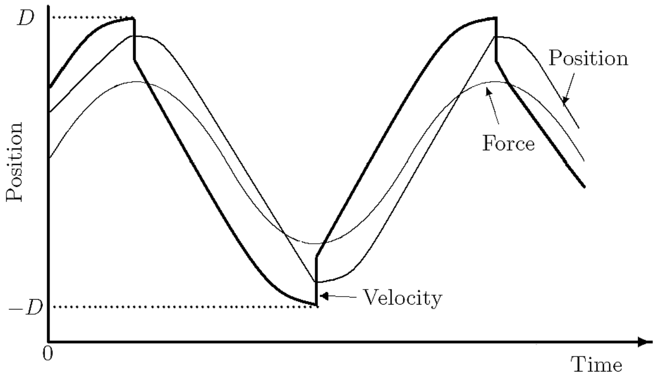


Fig. 3. Position and velocity controlled by declutching

Notionally, declutching can be seen as the reverse control strategy of latching (see fig. 2). Also called unlatching or freewheeling, as denoted by Wright *et al.* (2003), this strategy was originally introduced by Salter *et al.* (2002). Prior to Salter *et al.* (2002), declutching was mentioned by Justino and Falcao (2000) for an oscillating wave column device but not as a mean of controlling the PTO force. At some stage during a declutching procedure, the PTO force encountered a zero value.

To date, the article written by Babarit *et al.* (2009) explores precisely the issue of declutching. It is shown that declutching can assess the energy capture width of a point absorber as efficiently as a continuous controlled variation of the PTO force strategy. In their paper, Babarit *et al.* (2009) emphasize the fact that declutching requires only a by-pass valve in the circuit of the hydraulic cylinder to be implemented.

Unlike latching, declutching corresponds to a positive value of the slope of the damping profile. Once again, declutching strategy is passive. Therefore, it does not need reactive power which means no extra power contribution is required to control the PTO force. Unfortunately, Babarit *et al.* (2009) have proven that the issue of the future knowledge of the excitation force stays when irregular sea states are considered. Following the same pattern as fig. 2, fig. 3 shows the two main motion characteristics of the declutching.

## 4. OPTIMIZATION PROCEDURE

In this section, the tools used to compute the parametrization of the sigmoid function and run the optimization will be pointed out. Since our problem is non-convex and stochastic, a genetic algorithm (GA) and the Nelder-Mead algorithm have been chosen to find the global maximum of our objective function [5]. Finally, to improve the accuracy of the results obtained, the Levemberg-Marquardt has been used.

### *4.1 Genetic Algorithm*

The parameters of the sigmoid in (7) were adapted, using a genetic algorithm (Goldberg (1989)), in order to maximize the energy function over a wave period (5). During the optimization process, it is assumed that the shape of the damping profile remains independent of the model of our hydrodynamic system (2). Henceforth, the optimal damping profile expected will be accepted as the best one in order to harvest the maximum of energy. A genetic algorithm (GA), with elitism, was chosen since the performance surface to be searched is non-convex with respect to the sigmoid parameters.

The capabilities of such algorithms for optimizing control problems was examined by Gunn *et al.* (2009). Additionally, Gunn *et al.* (2009) explains GA suits particularly well problem which requires a parametrization within the objective function. Hereby, the main benefit of using GA rest on the wide range which can be attributed for each of the four variables used. Since the performance function to be searched is non-convex and the problem is stochastic, GA has been naturally employed. For the study, a binary type of coding was performed. Table 3 defines the main settings provided for this particular type of coding.

The range of each parameter are grouped together in the table 4. One can note that both latching and declutching cases can be achieved throughout such ranges, respecting what has been raised in section 3.1.

### 4.2 Nelder-Mead Algorithm

In order to check the results get from the GA simulations, one algorithm will be introduced.
Nelder-Mead algorithm is a well known method for optimizing a numerical problem. It is considered as part of the simplex algorithm type. Basically, the concept of a simplex relies on the extrapolation of the behavior of the objective function following the decision vector. Unlike modern optimization methods, the NelderMead heuristic can converge to a non-stationary point unless the problem satisfies stronger conditions than are necessary for modern methods.
For the purpose of our work, the simplex algorithm provides a second optimization method which is meant to ensure that directions taken by $B_{min}$, $B_{max}$, $t^*$ and $\beta$ via the GA are correct. Furthermore, the Nelder-Mead was run before all optimization for key initializations. The objective of such tests relies on the validation of the hydrodynamic model by checking few well-known results.

### 4.3 Levemberg-Marquardt Algorithm

Levemberg-Marquardt resembles the well-known Gauss-Newton algorithm. Actually, the minimization is based on a gradient descent method. Levemberg-Marquardt optimization is considered to be more robust than the Gauss-Newton as it finds a solution even if it starts very far off the global minimum. On the other hand, for well-behaved functions and reasonable initial parameters, the Levemberg-Marquardt tends to be a bit slower than the Gauss-Newton.
For the purpose of our study, the gradient algorithm Levemberg-Marquardt has refined the four values coming from the GA for each wave frequency experienced. In the mean time, the absorbed energy possibly to be captured was slightly improved.

## 5. RESULTS AND DISCUSSIONS

This section displays and comments the results brought by the optimization procedure. A set of 11 successive wave frequency has been experienced (from $\omega_w = 0.4$ rad/s to $\omega_w = 1.3$ rad/s plus the resonance case). Table 5 gathers the parameters optimized for each of them.
Around both sides of the resonance case, final values attained for the maximization of the energy clearly indicate the optimality of a specific strategy.
In terms of energy, fig. 4 provides the variation of the maximized amount of energy which can be harvested over one period with the wave frequency. One can appreciate

Table 3. Genetic algorithms settings

| Parameter | Binary |
|---|---|
| Population size | 100 |
| Individual size | 4 |
| Number of generations | 30 |
| Generation gap | 0.7 |
| Selection | Roulette Wheel |
| Recombination | Crossover shuffle |
| Crossover rate | 0.7 |
| Mutation | Simple mutation |
| Mutation rate | 0.035 |

Table 4. Range of parameter

| Range | $B_{min}$ | $B_{max}$ | $t^*$ | $\beta$ |
|---|---|---|---|---|
| Min | 0 | 0 | 0 | $-2000$ |
| Max | 50 | $1e20$ | $1Period$ | 2000 |

Table 5. Optimized parameters of the sigmoid

| $\omega_w$ | $B_{min}$ | $B_{max}$ | $t^*$ | $\beta$ | strategy |
|---|---|---|---|---|---|
| 0.4 | 21 | $3.7E+08$ | 4.2 | $-500$ | Latching |
| 0.5 | 38 | $4.3E+08$ | 2.6 | $-200$ | Latching |
| 0.6 | 33 | $8.2E+06$ | 1.7 | $-100$ | Latching |
| 0.7 | 6 | $5.5E+08$ | 1 | $-3000$ | Latching |
| 0.8 | 5 | $9.9E+05$ | 0.8 | $-3000$ | Latching |
| 0.865 | $2.6E+04$ | $2.6E+04$ | 0 | 0 | Linear |
| 0.9 | 35 | $9.2E+04$ | 1.9 | 3000 | Decluthcing |
| 1.0 | 17 | $6.8E+19$ | 3.4 | 100 | Decluthcing |
| 1.1 | 36 | $3.3E+06$ | 2.4 | 100 | Decluthcing |
| 1.2 | 36 | $5.7E+06$ | 2.2 | 100 | Decluthcing |
| 1.3 | 13 | $7.0E+16$ | 2.6 | 100 | Decluthcing |

the steadily decreasing curve followed by our target characteristic, the absorbed energy. One should also notice the benefit, in the energy absorption, brought by both declutching and latching strategies in comparison with the uncontrolled case.

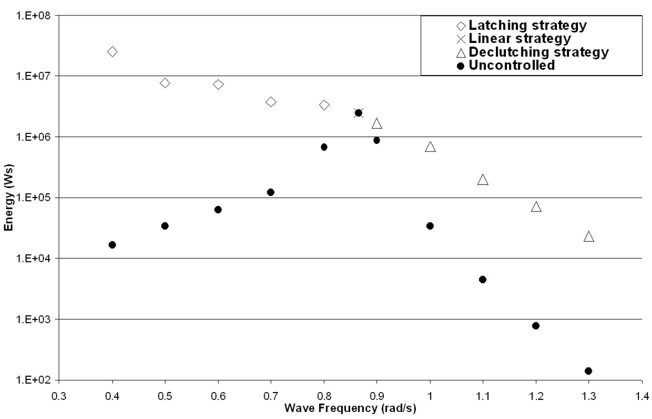


Fig. 4. Variation of the energy developed in the damper over one period with the wave frequency

### 5.1 $\omega_w < \omega_d$, Latching strategy

When the natural resonance frequency of the device is greater than the incoming waves frequency, the value of $\beta$ obtained is negative. The magnitude of the parameter $\beta$ is quite important. Hence, the transition between the two steady extreme states of the damping profile has to be almost instantaneous. In such a case, a latching strategy has to adopted. Fig. 5 plots the profile of the damping force which should be applied following the optimized parameters found when $\omega_w < \omega_d$.
Fig. 6 represents the motion of the device at the wave frequency $\omega_w = 0.5$. One can note that this graph is in consistence with fig. 2. One can also assess the important amplitude of the device elevation.

It is interesting to compare the optimized parameter $t^*$ with the theoretical optimal latching duration determined by extrapolation of (8). As illustrated in fig. 7, one can

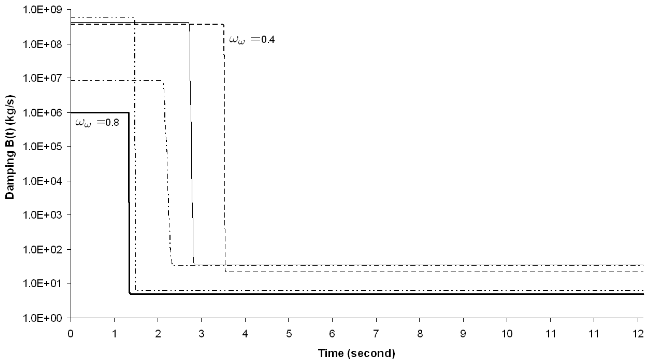


Fig. 5. Best damping profile strategy to apply when $\omega_w < \omega_d$, $\omega_w = 0.4; 0.5; 0.6; 0.7; 0.8$ (rad/s), Latching

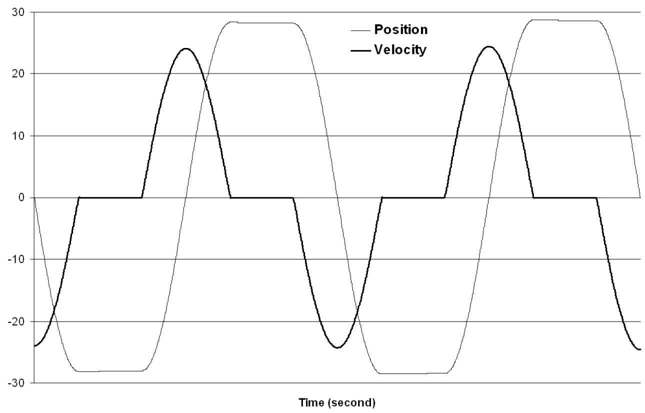


Fig. 6. Simulation of the motion of a heaving buoy wave energy converter with latching control

appreciate the very relevant adequacy of the value given though the optimization procedure to the theoretical latching duration expected.

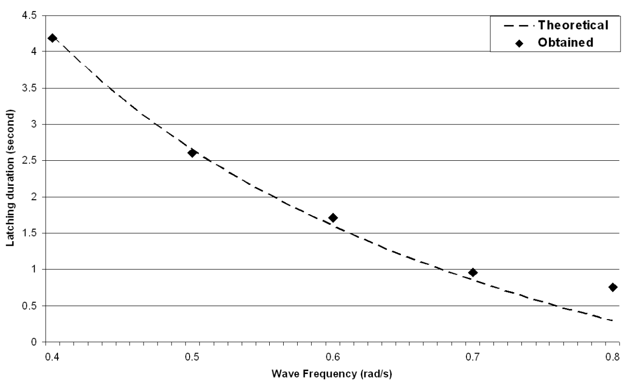


Fig. 7. Variation of the optimal latching time with the wave frequency

*5.2 $\omega_w = \omega_d$, Linear constant damping*

At resonance, $\omega_w = 0.865$ (rad/s), our optimization procedure leads to a linear damping profile. Besides, the constant damping value indicated for the PTO ($B_{PTO} = 26567$ (kg/s))is very close to the radiation damping at this very particular wave frequency ($B_{rad}(\omega_w = 0.865) = 24095$ (kg/s)).
To assess the accuracy of this result, fig. 8 confirms that the position and the excitation force are in phase. As a result, phase control is no longer required which support the linear damping profile pointed out by the optimization procedure. Furthermore, Falnes (2002) has proven that to magnify the capture of energy at resonance, the PTO has to fully compensate the radiation damping (consequence of the governing equation (1)).

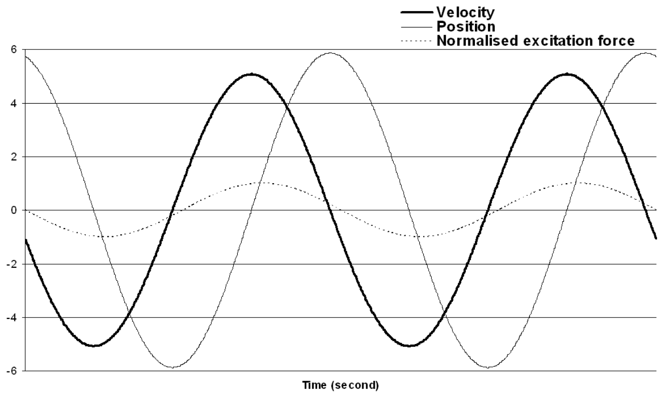


Fig. 8. Simulation of the motion of a heaving buoy wave energy converter at resonance

*5.3 $\omega_w > \omega_d$, Declutching strategy*

For a wave frequency greater than the natural resonant frequency of the device, our optimization procedure gives a declutching strategy. Unlike the case described in section 5.1, the best value of $\beta$ is negative. The typical damping profile for the wave frequency greater than $\omega_d$ is plotted in fig. 9 ($\omega_w = 1.2$ (rad/s)).

The motion of the device when $\omega_\omega = 1.2$ (rad/s) is given by fig. 10. One can note the smaller amplitude of the motion in comparison with both latching and linear cases observed in fig. 6 and 8. In addition, the sudden increase slope of the velocity notable in fig. 10 impose the body to maintain itself in phase with the excitation force. At the very moment the velocity suddenly changes, one can note that the body is thus force to oscillate faster.
Fig. 9, as well as fig. 5 includes only very sharp transition for the optimal damping profile. The absolute value of $\beta$ remains important which implies that the best strategy should implement an almost instantaneous transition between the two extreme damping states (initial and final damping values).

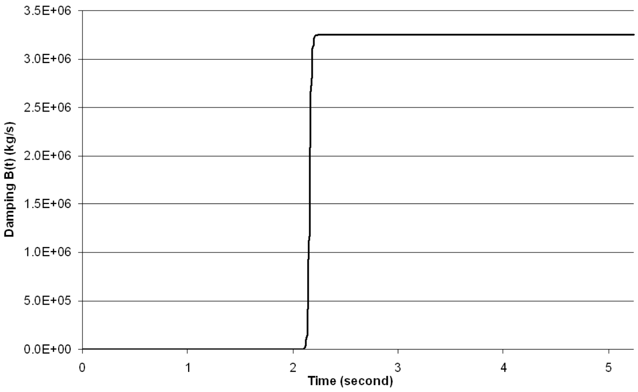


Fig. 9. Best damping profile strategy to apply for $\omega_w = 1.2(rad/s)$, Declutching

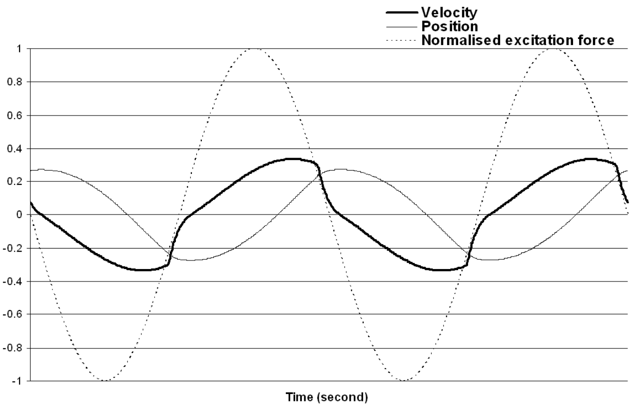


Fig. 10. Simulation of the motion of a heaving buoy wave energy converter with declutching control

## 6. CONCLUSION

In this paper, the damping profile for a heaving buoy over each wave period was parameterized by a general sigmoid function. By employing an optimization technique, we calculated the optimal damping profile for a range of incident wave frequencies, including the resonant frequency of the device. The optimal damping strategy moves from a latching strategy when $\omega_w < \omega_d$ to a declutching strategy when $\omega_w > \omega_d$. Interestingly, a hard switching function for $B_{PTO}(t)$ is preferred in *both* cases. This is consistent with what is traditionally known for latching, but also conforms the recommendations in Babarit *et al.* (2009) for economical declutching.

In order to focus on fundamental properties, this work has considered monochromatic waves; however, a greater range of (potential overlapping) possibilities exist for polychromatic seas and situations where latching and/or declutching is considered over multiple wave periods or pseudo-periods.

## ACKNOWLEDGEMENTS

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