



# Saturation non-linearities for Virtual Analog filters

Joseph Timoney  
Victor Lazzarini

Sound and Digital Music Technology Group, National University of Ireland, Maynooth, Ireland

## Summary

In the digital modeling of analog synthesizer filters using standard digital elements, a saturation element is usually included in the structure. This paper discusses some of the options available for soft clipping saturation elements. It then presents three different filter configurations that include saturation. A review of perceptual techniques for assessing distortion is made to select a suitable evaluation criterion: Timbral warmth being found as the most relevant. Experiments are carried out to apply this to the structures proposed. The conclusion suggests which structure might be most interesting musically.

PACS no. xx.xx.Nn, xx.xx.Nn

## 1. Introduction

Saturation or distortion effects are used in many electronic music applications. They are a common method of adding high-frequency components to an input audio source, thereby enriching the timbral quality of the sound. This is for instance, the typical case of the overdriven audio vacuum tube amplifiers used by guitarists and keyboard players [1]. The popularity of this sound spurred the development of separate overdrive and distortion effect units for musicians, particularly guitarists, as vacuum tube technology was superseded by transistor-based designs [2]. In essence these units are designed so that the amplitude of the input wave is shaped by the circuitry in such a way as to mimic certain saturation curves [3]. However, the phenomenon of the saturation effect is not confined to amplifiers and is also associated with the sound of the filter stages of analog subtractive synthesizers.

According to [4], the biggest contribution to the sound of a musical analog filter is its large signal distortion behavior. This [4] says is the reason for the sonic difference between Moog filter in comparison to others created using circuits implemented with transconductance amplifiers. For all analog filters, there is a saturation response characteristic associated with the transistor differential pair that results in a soft clipping of

the filter input. Other nonlinear circuit elements may also contribute to the overall sound, such as capacitors [5]. However, although there is anecdotal evidence in relation to synthesizers [6], there has been no study to prove this.

Two approaches are available to the designer of digital versions of analog musical filters: either (1) attempt to reproduce a particular analog filter design directly using a detailed circuit analysis or (2) to emulate the operation of a particular filter using standard digital elements that approximate the essence of the hardware design. While the first approach can work very well, it produces an algorithm that is computationally intensive and requires a significant oversampling factor to operate correctly [7], [8]. The second approach is less complex, and therefore cheaper, and much more flexible if parameter driven user tweaking is desirable. It was pointed out in [4] that the soft clipping phenomenon does not happen inherently with these digital filters and instead their saturation behavior leads to a much rougher, unpleasant sound.

To overcome this some form of additional saturation element is normally included in the filter structure. Sample choices are the soft-clipping saturation of [9], [10], a tanh nonlinearity, inverse square root, parabolic sigmoid, and the cubic sigmoid [11]. Care must be exercised though as the resulting increase in bandwidth caused by a non-linear element may generate frequency components that would in theory lie beyond the Nyquist rate but would

appear as possibly objectionable aliasing distortion in the output.

The aim of this work is to attempt to compile some ideas regarding the implementation of saturation effects, discuss their application in a digital filter structure, consider how to assess their output from a perceptual standpoint, and carry out some tests. These will be covered in sections 2, 3, 4 and 5 respectively.

## 2. Saturation effect and representation

Nonlinear saturation effects result in the distortion of the waveshape of an input signal. This is effectively a form of non-linear waveshaping, which is a widely studied method of sound synthesis [12]. The common denominator of all of these saturation effects is an increased spectral bandwidth, adding a number of extra harmonic components to each input signal partial. Musically, this has been explored to give more weight to tones that are originally thinner and less penetrating. This has been fundamental to the sound of popular music since the 1960s. Two outcomes are possible from a static nonlinearity:

Considering a single sinewave input, the output of the saturation effect will have new frequency components that are harmonically related to the input. This is Harmonic distortion. The location and magnitudes of these new components depend on the shape of the nonlinearity. If the saturation nonlinearity is an odd function, only odd order harmonics are produced, while if it is an even function, only even harmonics are produced. If the shape is not odd or even-symmetrical, components at all frequencies are created. An asymmetric nonlinearity can also result in a large DC component [13].

Intermodulation distortion results when two or more input spectral components interact with the nonlinearity. The output will contain distortion product components that exist at sum and differences of the original spectral components. The frequencies of these intermodulation components are not necessarily harmonically related to the input and thus could be perceived as unwanted noise. Strong intermodulation components are associated with sharp nonlinearities [14].

The symmetric or asymmetric saturation nonlinearity is commonly defined as a shaping function that has specific output properties that depend on the instantaneous value of the input amplitude. This is written as set of output

conditions invoked by particular input amplitudes. For example, a well-used expression is that for a soft-clipper where the input amplitude is  $x$  [9]

$$f(x) = \begin{cases} -2/3 & x \leq -1 \\ x - x^3/3 & -1 \leq x \leq 1 \\ 2/3 & x \geq 1 \end{cases} \quad (1)$$

We know, from the shape of this piecewise non-linear function, that its spectrum will produce odd harmonics of each component of the input. In particular, the central section polynomial tells us that, for signals in that range, the output will be bandlimited (if the input is bandlimited), producing an added third harmonic [15]. However, as this function is not smooth as a whole, for peak input amplitude values exceeding 1, we will have a non-bandlimited output.

In some cases, we have an analytic expression for the spectrum of the output, given a sinewave input. One example of such is an exponential saturation curve is given in [16]

$$g(x) = \pm A(1 - e^{-c|x|}) \quad (2)$$

where  $g(x) > 0, x > 0$ ;  $g(x) < 0, x < 0$ ;  $A$  is the maximum value of the function  $g(x)$ ; and  $(Ac)$  defines the slope of the curve at the origin. The first and third harmonic coefficients for (2) are obtained from

$$a_1 = 2A(I_1(cB) - L_1(cB)) \quad (4)$$

and

$$a_3 = a_1 - \frac{8A}{cB}(I_2(cB) - L_2(cB)) \quad (5)$$

where  $B$  is the amplitude of the input,  $I_k(\cdot)$  is the modified Bessel function of order  $k$ , and  $L_k(\cdot)$  is the modified Struve function of order  $k$ .

Another useful saturation curve with a well understood spectrum is the hyperbolic tangent or  $\tanh(\cdot)$  function whose spectrum is given in [12]. The arctangent function  $\text{atan}(\cdot)$  has also been proposed in [17], where a derivation of its spectrum is presented

Alternatively, the nonlinearity can be described as an order  $n$  polynomial that will generate  $n$  harmonics given a monocomponent input (as indicated above with regards to eq.1). This approach is very convenient as it is amenable to analysis using Chebyshev polynomials [18]. Two important works exist that use these to describe the relationship between the polynomial coefficients and the resulting spectrum, [15], [18], where the technique of waveshaping synthesis was

given a thorough theoretical treatment. In particular, a convenient matrix approach was given in [15] that was extended to account for phase in [19].

Here, we will use the exponential curve as the definition of the curve in (2) is flexible ranging from a very gentle slope to one that is more abrupt. This is illustrated by a plot of curves for a range of values of  $c$  shown in Figure 1. The value of  $A=B=1$ . They are only shown for positive values of  $x$  as the curve is a function around the origin.

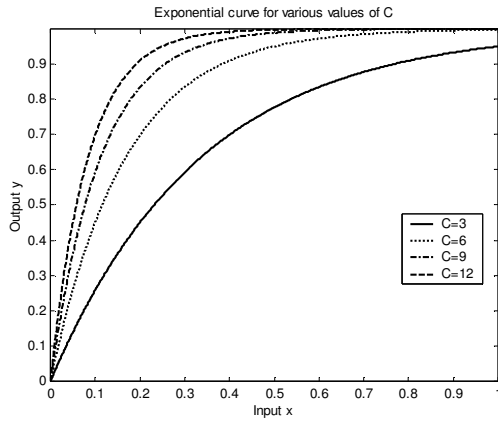


Figure 1 Shape of the exponential curve for four different values of parameter  $c$

A final advantage is because of the single parameter  $c$  it can be modulated easily allowing for a dynamic saturation behavior as exemplified in [20].

### 3. Saturation with Filter structure

Once selected, the saturation element needs to be included in the filter structure. The first model of an analog synthesizer filter using standard digital elements was the State variable filter of Chamberlin [21]. There is also a Sallen-key digital filter, a number of versions of the Moog filter [22], [23] and [24], and a model of the Korg MS-20 filter [25]. Only some of these explicitly contain a saturation element. There is no set position where the saturation element is placed: it can be before the filter section [10], [23] or in the feedback loop [25]. Others have mentioned the use of multiple saturation elements, one included with each 6dB/oct filter stage [26], [27]. In this work three configurations will be examined. The first is Configuration 1, as shown in Figure 2, where the saturation element precedes the filter, both connected in a feedback loop whose gain is  $\beta$ .

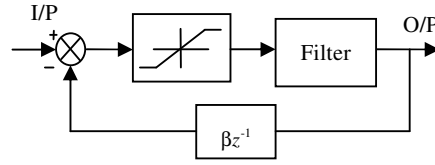


Figure 2 First Configuration of Saturation and Filter

The second is where the saturation element is placed after the filter, again in a feedback loop. This is referred to as Configuration 2 and is shown in Figure 3.

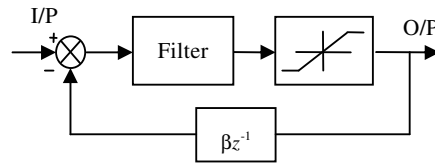


Figure 3 Second Configuration of Saturation and Filter

The last is Configuration 3 where the saturation element is moved to the feedback loop, just before the gain element  $\beta$ . Figure 4 shows this.

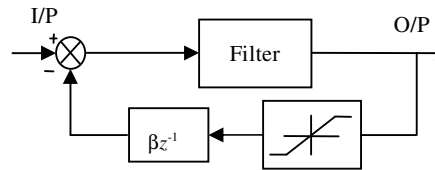


Figure 4 Third Configuration of Saturation and Filter

For ease of analysis the filter is chosen to be a single stage of the gain compensated Moog filter of [23]. This has a transfer function given by

$$H(z) = \frac{g(a + bz^{-1})}{1 - (1-g)z^{-1}} \quad (6)$$

where  $a=1/1.3$ ,  $b=0.3/1.3$  and for a normalized cut-off frequency  $\omega_c$ , the gain  $g$  is

$$g = 0.9892\omega_c - 0.4342\omega_c^2 + 0.1318\omega_c^3 - 0.0202\omega_c^4 \quad (7)$$

An essential feature of synthesizers is parameter modulation to give dynamic interest. In the three configurations given a number of options could be available for example:

- Modulation of the amplitude of the input

- Modulation of the curve parameter  $c$  for the saturation
- Modulation of the filter cut-off frequency  $f$

Alteration of the parameter  $c$  will increase/decrease the magnitude of additional harmonics created due to the saturating element [20]. Modulation of the filter cut-off can lead to extra components in the signal that may or may not be harmonic depending on the modulation frequency, when this is at audio ranges [28]. If these components are non-harmonic they will definitely lead to intermodulation distortion on passing through the saturating element. This may or may not be desirable. However, audio-rate modulation is not a commonly-used effect in musical applications.

#### 4. Perceptual evaluation of distortion

To assess or classify the timbral quality of the output of these configurations it is necessary to have some metric, preferably one that is objective. However, the perceptual effect of distortion, rather than saturation specifically, has been examined in a number of articles. Further, most applications considered have been inclined towards loudspeaker systems, but not exclusively so. An early paper used a dissonance metric derived from Consonance theory [29]. It found that a psychological assessment of dissonance did not correlate well with the standard objective measures of Harmonic distortion and Intermodulation distortion. However, it did match well with their Dissonance measure. Another study was carried out by [30] on the auditory perception of nonlinear distortion. This led to the following observations about the distortion nonlinearities themselves

- The masking effect of the human ear will tend to make higher order nonlinearities more audible than lower order ones.
- Nonlinear by-products that increase with level can be completely masked if the order of the nonlinearity is low.
- Nonlinearities that occur at low signal levels will be more audible than those that occur at higher signal levels.

Based on this they proposed the GedLee metric which was formed using a cosine weighted second derivative of the nonlinear function. In a second paper [31] noted that the ubiquitous Total Harmonic Distortion (THD) and Intermodulation Distortion (IMD) were poor predictors of subjective assessments of sound quality in

comparison to their GedLee metric which was shown to be a much better match.

Around the same time [32] also introduced a psychoacoustically-based measure Rnonlin that was computed using a Gammatone filterbank. They reported an excellent correlation with subjective ratings, but did emphasize that the results were specific to the nonlinearities examined and the subjects themselves. Another work was the evaluation of the timbral qualities of nonlinear guitar distortion effects that was presented in a number of works [33], [34]. They found that Zwicker's measure of spectral sharpness, essentially a weighted spectral centroid, correlated closely with subjective descriptions. [35] also mentioned that of the stimuli presented to subjects those with low spectral Kurtosis could be described as being more powerful.

Finally, a recent paper discussed the timbral features associated with the 'warmth' of a sound [36]. It denoted all the spectral energy from the signal's fundamental to 3.5 times above this as the warmth region. The signal's warmth then is computed by finding the ratio between the energy here and the energy in the remainder of the spectrum. Since analog musical processing systems have inherent nonlinearities their output is frequently described by aficionados to possess this warm quality in contrast to digital systems [37]. Thus, measure of warmth is most relevant to the application in hand because of its use as a common verbal descriptor for the result of signal saturation effects.

#### 5. Experimental assessment of output

To assess the warmth of the output for the three configurations given in the previous section a pair of unity amplitude bandlimited sawtooths were generated. The pitch of the first sawtooth was 441Hz and that of the second was 439Hz. They were combined as is typical for subtractive synthesis where oscillator waveforms are detuned slightly to produce a gentle chorusing effect. This had the added advantage here that inharmonic partials would be generated via intermodulation. The curve parameter of the exponential saturation of (2) was varied from  $c=1$  to  $c=9$  and the cutoff frequency of the filter ranged from 110Hz up to 1100Hz in steps of 20Hz. For each set of parameters the output was found and the timbral Warmth was computed. Figures 5, 6 and 7 plot this parameter for each configuration. The values for Warmth were converted to dB before plotting.

Higher values should imply a greater sensory perception of warmth in the sound.

For Configuration 1 the output is warmest for the lowest value of curve parameter and the largest filter cutoff. The maximum value reached is about 50dB. It is smallest when the cutoff is lowest and the curve parameter is highest. The response of the system is approximately linear to parameter changes.

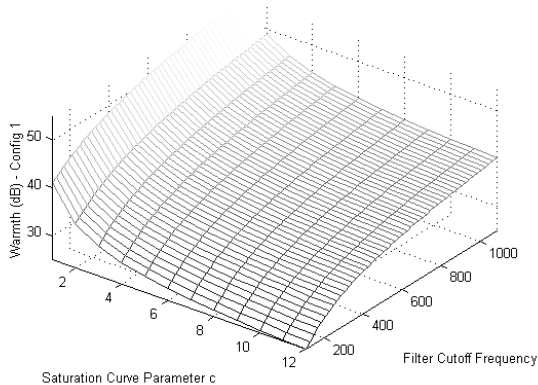


Figure 5 Warmth values computed for Configuration 1

In Figure 6, the warmest sound is for the highest cutoff and a midway curve parameter. This time the maximum is about 70dB. The response though is not linear with respect to the change parameters: there is not much change in the warmth at a high values of curve parameter no matter what the value of the cutoff is. However, for the lowest cut-off and lowest value of curve parameter the warmth is smallest.

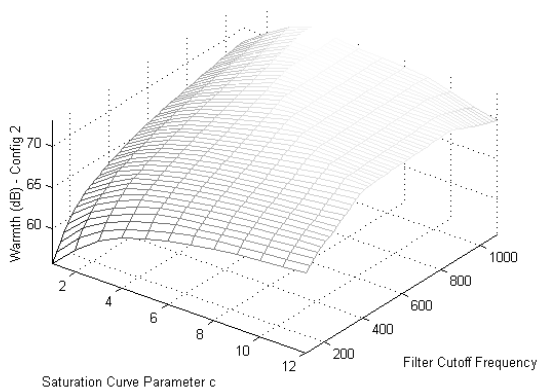


Figure 6 Warmth values computed for Configuration 2

For configuration 3, as seen in Figure 7, the maximum warmth is given for the lowest value of curve parameter and the widest cutoff. For greater values of curve parameter this falls off to give its lowest value for the highest curve parameter and the highest cutoff frequency. Again, for this

configuration the maximum value of warmth is about 70dB. The response is not linear showing a peakiness for a high curve parameter and low cutoff frequency.

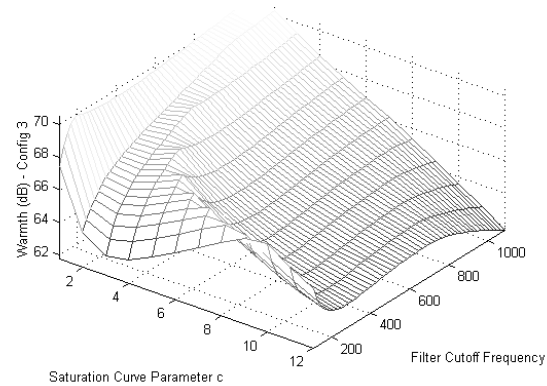


Figure 7 Warmth values computed for Configuration 3

Overall, the first configuration has the most predictable response with regard to the changing parameters. However, it does not exhibit the greatest measurement of warmth. The three systems appear to show different ranges of warmth value, being about 20dB for configuration 1 but approximately 10dB for Configurations 2 and 3, with the third configuration having the narrowest range.

## 6. Conclusions

This paper has discussed the use of nonlinear saturation effects in digital models for analog synthesizer filters. It mentioned some of the approaches to implementing saturation curves and chose an exponential model for the analysis. It introduced three closed loop Configurations that included a single stage digital filter and a saturation element. Considering was given to how to assess the impact of a saturation element on the timbral quality of the output and the quality of warmth was chosen. Experiments were carried out over a range of curve and filter cutoff parameters for the three configurations. The results showed the while the output of the first configuration did not have the greatest value for warmth in comparison to others it did have the most linear response with respect to the parameters. This suggests that it could be the most suitable one for implementation in digital filters structures as either user parameter tweaking or parameter modulation using a Low Frequency Oscillator (LFO) or envelope function would offer the smoothest perceptual effect.

## References

- [1] R.O. Hamm, 'Tubes vs. transistors – is there an audible difference,' *Jnl. Audio Eng. Soc.*, vol. 24, no. 4, pp. 267-273, Sept. 1972.
- [2] B. Santo, 'Volume Cranked Up in Amp Debate,' *Electric Engineering Times*, issue 817, Oct. 3, 1998. [www.trueaudio.com/at\\_eetjlm.htm](http://www.trueaudio.com/at_eetjlm.htm).
- [3] D. Yeh., J. Abel and J. O. Smith, 'Physically-informed models of distortion and overdrive guitar effect pedals,' *Proc DAFX-07*, Bordeaux, France, September 2007.
- [4] D. Rossum, 'Making digital filters sound analog', *Proc. Int. Comp. Music Conf.*, San Francisco, USA, 1992, pp. 30-33.
- [5] M. van der Veen, 'Non-linear distortions in capacitors', *AES convention 124*, Amsterdam, Netherlands, May 2008.
- [6] Sound differences: xox vs. 303, <http://www.adafruit.com/forums/viewtopic.php?f=12&t=10868&start=0>
- [7] M. Civolani and F. Fontana, 'A nonlinear digital model of the EMS VCS3 voltage controlled filter', *Proc. Dafx 2008*, Espoo, Finland, Sept. 2008.
- [8] T. Helie, 'Volterra Series and State Transformation for Real-Time Simulations of Audio Circuits Including Saturations: Application to the Moog Ladder Filter', *IEEE Trans. Speech and Audio Proc.*, Vol.18(4), May 2010, pp. 747-759.
- [9] C. Sullivan, 'Extending the Karplus-Strong Algorithm to Synthesize Electric Guitar Timbres with Distortion and Feedback', *Computer Music Journal*, 14(3), 1990, pp. 26-37.
- [10] B. Frei, 'Digital Sound Generation – Part 2 Filters', ICST, Zurich Uni. of the Arts, Switzerland, 2007. [http://www.icst.net/fileadmin/data/pdf/dsg/Digital\\_Sound\\_Generation\\_2.pdf](http://www.icst.net/fileadmin/data/pdf/dsg/Digital_Sound_Generation_2.pdf)
- [11] R. Borogrove, 'Fun with sigmoids', 2010, <http://www.tinygod.com/sigmoid/>
- [12] V. Lazzarini and J. Timoney, 'New Perspectives on Distortion Synthesis for Virtual Analog Oscillators', *Computer Music Journal*, 34 (1), pp. 28-40, March 2010.
- [13] S. Barbati and T. Serafini, 'A perceptual approach to clipping and saturation', SimulAnalog, 2010, <http://www.simulanalog.org/clip.pdf>
- [14] T. Rossing, *Springer handbook of acoustics*, Springer, USA, 2007.
- [15] D. Arfib, "Digital synthesis of complex spectra by means of multiplication of nonlinearly distorted sinewaves," *Jnl. Audio Eng. Soc.*, vol. 27, no. 10, pp. 757-768, October 1979.
- [16] R. J. Kavanagh, 'An approximation to the harmonic response of saturating devices', *Proc. IEE - Part C*, Vol. 107(11), March 1960, pp. 127-133.
- [17] A. dePaor, and J. Ringwood, 'A simple soft limiter describing function for biomedical applications', *IEEE Trans. on Biomedical Engineering*, Vol.53, No.7, July 2006, pp 1233-1240.
- [18] M. LeBrun, "Digital waveshaping synthesis," *Jnl. Audio Eng. Soc.*, vol. 27, no. 4, pp. 250-266, April 1979.
- [19] J. Timoney, V. Lazzarini, A. Gibney, and J. Pekonen, 'Digital Emulation of Distortion Effects by Wave and Phase Shaping Methods', *Proc. Dafx 2010*, Graz, Austria, Sept. 2010.
- [20] Y. Mitsuhashi, 'Waveshape parameter modulation in producing complex audio spectra', *Jnl. of the Audio Eng. Soc.*, Vol.28(12), DEc. 1980, pp. 879-896.
- [21] H. Chamberlin, *Musical applications of microprocessors*, Hayden book company, USA, 1985.
- [22] T. Stilson, and J. Smith, 'Analyzing the Moog VCF with Considerations for Digital Implementation', *Proc. Int. Comp. Music Conf.*, San Fransisco, USA, 1996, pp. 398-401.
- [23] V. Välimäki and A. Huovilainen, 'Oscillator and filter algorithms for virtual analog synthesis' *Computer Music Journal*, vol. 30, no. 2, pp. 19-31, 2006.
- [24] A. Huovilainen, 'Nonlinear Digital Implementation of the Moog Ladder Filter', *Proc. Dafx 2004*, Naples, Italy, pp. 61-64.
- [25] A. Huovilanen, *Design of a scalable polyphony-midi synthesizer for a low cost DSP*, Ph.D. thesis, Aalto University, Espoo, Finland, May 2010.
- [26] T. Stilson, *Efficiently-Variable Non-Oversampled Algorithms in Virtual-Analog Music Synthesis - A Root-Locus Perspective*, Ph.D. thesis, Stanford University, Stanford, CA, USA, June 2006. <http://ccrma.stanford.edu/~stilti/papers/Welcome.html>.
- [27] D. Berners, 'The inner workings of the moog multimode filter', Universal Audio, May 26, 2010. <http://www.uaudio.com/blog/moog-multimode-filter-design/>
- [28] M. Cherniakov, *An Introduction to Parametric Digital Filters and Oscillators*, Wiley, 2003.
- [29] H. Yahiro, A. Kameoka, M. Kuriyagawa, 'Psychological Evaluation of Nonlinear Distortion' *AES Convention 37*, Oct. 1969.
- [30] E. Geddes, and L. Lee, 'Auditory perception of nonlinear distortion - Theory', *AES Convention 115*, New York, USA, Oct. 2003
- [31] E. Geddes, and L. Lee, 'Auditory perception of nonlinear distortion', *AES Convention 115*, New York, USA, Oct. 2003
- [32] C. Tan, B. Moore, N. Zacharov and V. Mattila, 'Predicting the Perceived Quality of Nonlinearly Distorted Music and Speech Signals', *Jnl. of the Audio Eng. Soc.*, Vol.52(7/8), July 2004, pp. 699-711.
- [33] A. Marui and W. Martens, 'Perceptual and Semantic Scaling for User-Centered Control Over Distortion-Based Guitar Effects', *AES Convention 110*, Amsterdam, The Netherlands. May 2001.
- [34] A. Marui and W. Martens, 'Timbre of nonlinear distortion effects: Perceptual attributes beyond sharpness?', *Conference on Interdisciplinary Musicology (CIM05)*, Montreal, Canada. Mar. 10-12, 2005.
- [35] W. Martens and A. Marui, 'Psychophysical calibration of sharpness for multiparameter distortion effects processing', *AES convention 114*, Amsterdam, Netherlands, Mar. 2003.
- [36] T. Brookes and D. Williams, 'Perceptually motivated audio morphing: warmth', *AES convention 128*, London, UK, May 2010.
- [37] H. Robjohns, 'Analogue warmth: the sound of tubes, tape and transformers', *Sound on Sound*, Feb. 2010.